Fractional-Order Uncertain Chaotic Cryptovirology in Blockchain System Stabilization using a Robust Adaptive Sliding Mode Controller

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Abstract: - One of the most critical security problems today concerns our data on the internet. Cybercrimes, perpetrated through malicious programs, have become commonplace, posing risks to personal and public information during data exchanges or in data storage. In recent studies, the worm propagation dynamics in cryptovirology in blockchain systems could be successfully modeled by means of three-dimensional fractional-order chaotic systems based on the SEI nonlinear epidemic representation. This paper introduces an adaptive sliding mode control design for the robust stabilization of fractional-order cryptovirology in blockchain systems in presence of disturbance. The problem of stabilizing chaos in cryptovirology systems is solved analytically following the methodology of Lyapunov's theorem. We were able to show that the application of the proposed adaptive SMC control approach allows the system states to converge towards stable values. Furthermore, this is largely validated by numerical simulations on Matlab/Simulink.

Key-Words: - Cryptovirology in blockchain, fractional-order control, fractional order chaotic system, sliding mode control, robust adaptive control, stability analysis.

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1 Introduction

From the beginning of the twentieth century to the present day, technology tools have undergone immense development for several purposes like artificial intelligence, cyber-networks, industry 4.0 and many others, [1].

In particular, the internet is the most powerful tool ever developed by humankind, with an immediate and indelible effect on all aspects of life, work, and leisure. This has made it highly vulnerable to attacks and the cybercrime that has developed alongside it to exploit this opportunity, modifying information and creating a significant risk to sensitive data, [2]. In this context, the most feared risk for engineers is worm propagation or viral malware that can harm weaknesses in used software. This powerful malicious software is synthesized in cryptovirology by means of cryptography technique, [3], [4].

In [5], the authors modeled the spread of computer viruses by drawing analogous biological disease from a green epidemiological model. In [6], they investigated the infection dynamics on scale-free networks, showing that it is possible to stop the virus propagation in an efficient defense algorithm, [7]. The study in [8], produced an interesting study on modelization and analysis of worm development. In [9], blockchain systems were addressed and a queuing model designed for its understanding. Based on the hypothesis that computer and biological virus spread are similar, many authors have proposed models that consider SEI (susceptible-exposed-infected) configuration, [10].

Malware modeling thus plays a crucial role in limiting viruses to acceptable proportions. Furthermore, in order to limit the spread of the virus in the internet network, it is necessary to identify the chaotic behavior of the system and stabilize it, [11], [12]. The control of chaotic systems has developed considerably in recent years due to its numerous applications in various fields of engineering.

Several approaches to chaos control have been proposed in the literature, such as adaptive control [13], sliding mode control [14], fuzzy control [15], model predictive control [16], etc.

Applied mathematics includes a field that has become very popular in physics and engineering: fractional calculus. While its origins date back more than three centuries, its success is only recent, due to its ability to model many physical phenomena and its highly advantageous properties, especially in complex systems, such as memory and robustness. Its principle is to extend the order of differentiation to the set of real numbers and even complex numbers, [17], [18].

This renewed interest in chaotic fractional-order systems is justified by their diverse applications in engineering and nonlinear sciences affecting telecommunications, electronic circuits, chemical reactors, mechanical, biological and medical systems, [19]. These highly nonlinear behaviors can be very troublesome when they induce irregularities in the observed dynamics. Eliminating these unwanted oscillations becomes a requirement in order to use these systems effectively. Recently, a variety of control techniques for fractional chaos have emerged, such as fractional-order $PI^{\lambda}D^{\mu}$ Control [20], robust stabilization design using the matrix's singular value decomposition (SVD) method [21], active control method [22], simple linear feedback control [23], impulsive control [24].

Among these control strategies, fractional order sliding mode control (FOSMC) is one of the most popular solutions to deal with this particular class of nonlinear systems because of its robustness and easy implementation, [25], [26]. The effectiveness and robustness of FOSMC control strategies for stabilizing fractional-order chaotic systems is demonstrated over a growing number of different algorithms in recent literature, [27], [28].

However, SMC controllers present the serious disadvantage of chattering and high frequency oscillations. Fortunately, adaptive control gives an effective solution to limit these undesired phenomena, as it is able to treat badly modelized or time-varying models, [29], [30]. The result is that adaptive sliding mode control combines the advantages of the robustness offered by SMC and the flexibility offered by adaptive control, [31]. The specialized literature has been enriched with many papers on fractional order adaptive SMC which proved to be a better solution for synchronizing or stabilizing fractional-order uncertain chaotic systems, [32], [33].

In this work, a new adaptive sliding mode control scheme based on Lyapunov's stability theorem is proposed for the control chaos in the fractional-order chaotic cryptovirology in blockchain system. The aim is to force the system states to asymptotic stability. The considered class of systems is the three dimensional fractional-order systems controlled by an adaptive law following the Lyapunov theory. This control scheme is a general framework that can deal with all three dimensional chaotic systems of fractional-order using a time-varying parameter k which converges to a final value $k^{\star}$. We consider that no information is a priori known about k estimate.

The remainder of this study is organized as follows. In Section 2, the fractional-order chaotic systems are introduced and definitions of basic fractional calculus are presented. Fractional chaotic crypto-virology in blockchain systems is presented in Section 3, whereas the proposed control scheme based on fractional-order sliding mode control and the gain adaptation law design is detailed in Section 4. In order to validate the effectiveness of this scheme, a numerical example on Matlab/Simulink is performed in Section 5. Finally, some concluding remarks are given in Section 6.

2 Fractional-order Systems

The fractional-order derivative and integral are extensions of standard differential operators to real orders instead of integer ones only. This gives a new concept of non-integer operator which is defined from a general fundamental operator ${}_aD_t^q$. Its mathematical definition is as follows, [34], [35]:

$$
{}_aD_t^q = \begin{cases} \frac{d^q}{dt^q} & q > 0 \\ 1 & q = 0 \\ \int_a^t (d\tau)^{-q} & q < 0 \end{cases} \tag{1}
$$

where a is the initial value, and q is the fractional-order.

If *f(t)* is a continuous time function, for the order
n where $n \in [0, 1]$. The most popular definitions for the fractional-order derivatives of *f(t)* are [36], [37]:

- The Grünwald-Letnikov's derivative definition:

$$ {}_aD_t^{\mu} f(t) = \lim_{h \to 0} h^{-\mu} \sum_{r=0}^{\frac{t-a}{h}} (-1)^r \left(\begin{array}{c} \mu \\ r \end{array} \right) f(t - rh) \tag{2} $$

The binomial coefficients (r > 0) are computed by:

$$ \left(\begin{array}{c} \mu \\ 0 \end{array} \right) = 1, \ \left(\begin{array}{c} \mu \\ r \end{array} \right) = \frac{\mu(\mu-1)\ldots(\mu-r+1)}{r!} \tag{3} $$

Here *h* is the sampling period.

-The Caputo's derivative definition of the form:

$$ {}_aD_t^m f(t) = \frac{1}{\Gamma(n-m)} \frac{d^{\eta}}{dt^{\eta}} \left[\int_a^t \frac{f^{(n)}(\tau)}{(t-\tau)^{m-n+1}} d\tau \right] \tag{4} $$

-The Riemann-Liouville definition expresses as:

$$ {}_aD_t^m f(t) = \frac{1}{\Gamma(\eta-m)} \frac{d^{\eta}}{dt^{\eta}} \left[\int_a^t \frac{f(\tau)}{(t-\tau)^{m-\eta+1}} d\tau \right] \tag{5} $$

where:

$$ \Gamma(m) = \int_t^a y^{m-1} e^{-y} dy \tag{6} $$

Γ(.) is the Euler's Gamma function.

These three approximations of the fractional order derivative are very complementary and allow the easy handling of different situations ranging from analytical calculation to numerical implementations.

3 Fractional-order Cryptovirology in Blockchain system

The Lorenz chaotic system model was developed in 1963, opening the way to a multitude of chaotic model creations that could represent physical behavior and whose purpose was to study its behavior and characteristics. Many chaotic systems, such as Chen system , Liu system , Lü system , and Qi system have been proposed inspired by the Lorenz system, [38], [39], [40].

The fractional-order Cryptovirology in blockchain model is derived from the VVP (Virus, Vulnerable and Protected class) model [41] as follows:

$$ \begin{cases} D_t^q x(t) = -d_1\, x(t) - \alpha_1\, y(t) \\ D_t^q y(t) = -d_2\, y(t) + \alpha_2\, x(t) - \beta_2\, x(t)z(t) \\ D_t^q z(t) = -d_3\, z(t) + \beta_1\, x(t)y(t) \end{cases} \tag{7} $$

x, *y* and *z* represent the three population levels of the malware virus, vulnerable systems and protected systems class, and the parameters are:

- d_1: is the coefficient of decay of virus;
- d_2: is the coefficient of decay of vulnerable systems;
- d_3: is the coefficient of decay of protected systems;
- α_2: is the coefficient of susceptibility of vulnerable system;
- α_1: is the coefficient of availability of vulnerable system;
- β_1: is the coefficient of encounter between virus and protected systems;
- β_2: is the coefficient of interaction between virus and vulnerable system.

$d_1, d_2, \alpha_1, \alpha_2, \beta_1, \beta_2$, being real parameters.

The system (7) exhibits chaos for the initial conditions I.C. (3, 2.5, 15.2) and the fractional-order $q = 0.99$ (see for instance [4]) as confirmed by Figure 1 and Figure 2.

Figure 3 illustrates the phase portraits for different fractional-orders state.

4 Adaptive Sliding Mode Control Design

Let us consider the fractional chaotic cryptovirology in blockchain system (7) without external disturbance. Our aim is to stabilize this system using an adaptive sliding mode controller designed in this Section.

The controlled system contains the input *u(t)* in the second state equation in order to device chaos giving,

$$ \begin{cases} D_t^q x(t) = -d_1\, x(t) - \alpha_1\, y(t) \\ D_t^q y(t) = -d_2\, y(t) + \alpha_2\, x(t) - \beta_2\, x(t)z(t) + u(t) \\ D_t^q z(t) = -d_3\, z(t) + \beta_1\, x(t)y(t) \end{cases} \tag{8} $$

4.1 Control Design

With the aim of designing a fractional-order sliding mode control law adapted to our system, the approach is as follows: First, we will impose the desired dynamics through an appropriate choice of the fractional-order sliding surface, then it will be necessary to derive a switching law that ensures the controlled system is always in sliding mode. It is then that an adaptation law will be established to allow the automatic adjustment of control gains and the convergence of states in a finite time.

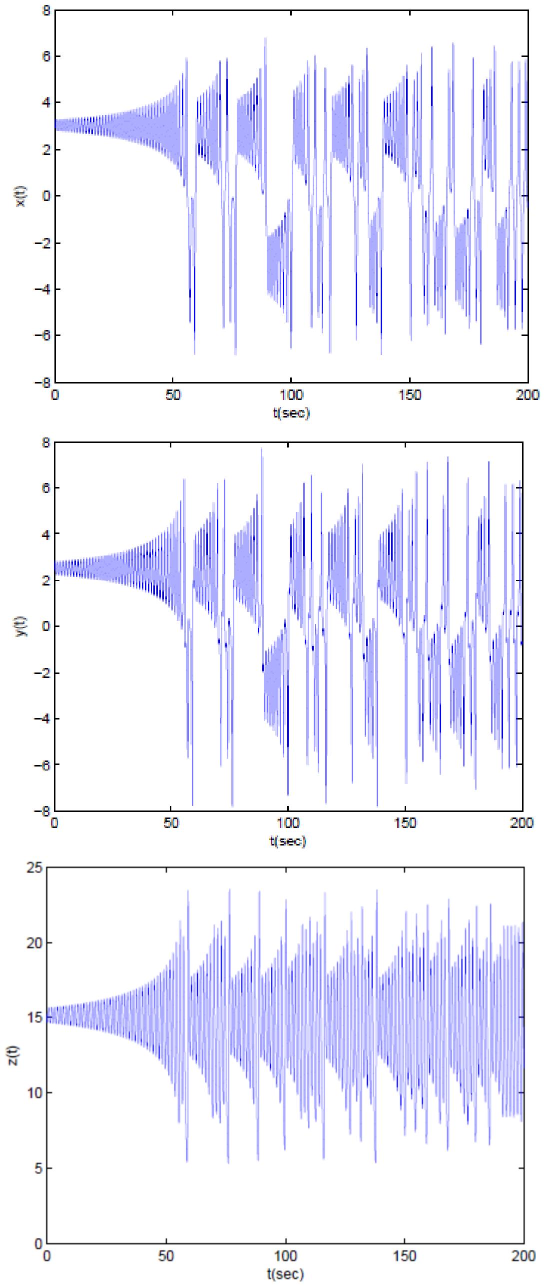

Fig. 1: State trajectories of fractional-order cryptovirology in blockchain system
Source: created by the authors

The objective of this paper is to design a suitable adaptive sliding mode controller such that the state variables are asymptotically stable.

A condition for the validity of this design method is that the sliding surface S and its derivative $\dot{S}$ must verify,

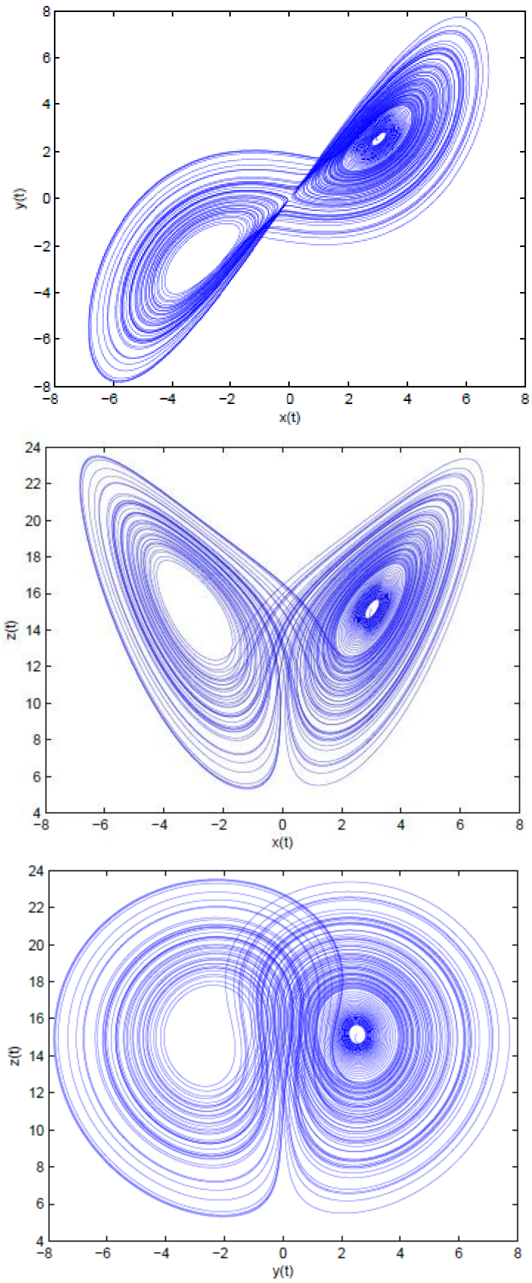

Fig. 2: Phase portraits of the system (7)
Source: created by the authors

$$\forall t \in \Re^+, \quad \begin{cases} S(t) = 0 \\ \dot{S}(t) = 0 \end{cases}$$

A good option for the sliding surface, is to take:

$$S(t) = D_t^{q-1} y(t) + \zeta(t) \tag{9}$$

where

$$\zeta(t) = \int_0^t (\alpha_1 x(t) + \beta_1 z(t) x(t) + d_2 y(t))\, dt \tag{10}$$

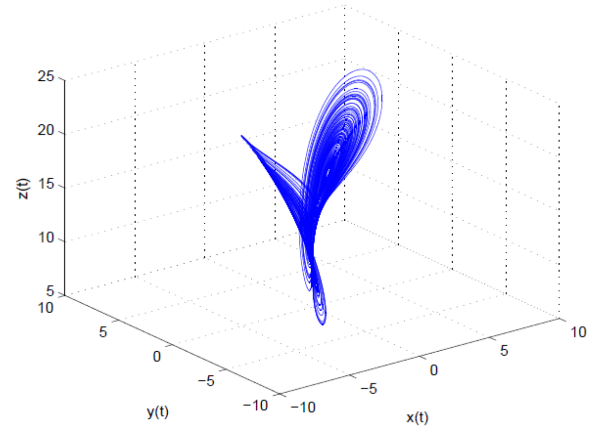

Fig. 3: Chaotic attractors of the fractional-order Cryptovirology in Blockchain system
Source: created by the authors

The SMC law is obtained by taking the derivative of (9) as follows:

$$\begin{aligned}\dot{S}(t) &= D_t^q y(t) + (\alpha_1 x(t) + \beta_1 z(t)x(t) + d_2 y(t)) \\ &= (\alpha_1 + \alpha_2)\, x(t) + (\beta_1 - \beta_2)\, x(t)z(t) + u(t) = 0\end{aligned} \tag{11}$$

Thus, the equivalent control law is given by:

$$u_e(t) = -(\alpha_1 + \alpha_2)\, x(t) - (\beta_1 - \beta_2)\, x(t)z(t) \tag{12}$$

A suitable choice for the discontinuous reaching law ensuring permanent sliding is as follows,

$$u_r(t) = -k_r\, sgn(S(t)) \tag{13}$$

where

$$sgn(S(t)) = \begin{cases} -1 \;\; if \;\; S(t) < 0 \\ \;\;0 \;\; if \;\; S(t) = 0 \\ +1 \;\; if \;\; S(t) > 0 \end{cases} \tag{14}$$

and the total control law can be defined as:

$$u(t) = u_e(t) + u_r(t) \tag{15}$$

And the positive gain k_r is adapted according to the following adaptation law,

$$\dot{k}_r = -\delta\, |S(t)| \tag{16}$$

4.2 Stability Analysis

The main result of this adaptive SMC control design is expressed by the following theorem,

Theorem 1. *Consider the fractional-order Cryptovirolgy in blokchain system (8), if it is controlled using the adaptive SMC control (12), (13), (15) and the gain adaptation law (16), then it is globally asymptotically stable.*

Proof. In order to study the closed-loop stability of the system, the following Lyapunov candidate function is considered:

$$V = \frac{1}{2} S^2 + \frac{1}{2\delta} \tilde{k}_r^T \tilde{k}_r \tag{17}$$

where δ is a positive constant and $\tilde{k}_r = k_r - k^\star$, is the gain optimal value of the sliding mode control.

Taking the derivative of the function V along the augment system trajectories, we get:

$$\dot{V} = S\dot{S} + \frac{1}{\delta} \dot{k}_r \tilde{k}_r \tag{18}$$

thus,

$$\begin{aligned}\dot{V} &= S(t) \tfrac{d}{dt} \left\{ D_t^{q-1} y(t) + \int_0^t (\alpha_1 x(t) + \beta_1 z(t)x(t) + d_2 y(t)) \right. \\ &= S(t) \left\{ D_t^q y(t) + \alpha_1 x(t) + \beta_1 z(t)x(t) + d_2 y(t) \right\} + \tfrac{1}{\delta} \dot{k}_r \tilde{k}_r\end{aligned} \tag{19}$$

where,

$$D_t^q y(t) = -d_2\, y(t) + \alpha_2\, x(t) - \beta_2\, x(t)z(t) + u(t) \tag{20}$$

From equations (19), (12), (13) and (16) we get,

$$\begin{aligned}\dot{V} &= S(t) \left\{ -k_r\, sgn(S(t)) \right\} + \tfrac{1}{\delta} \dot{k}_r \tilde{k}_r \\ &= -k_r\, |S(t)| + \tfrac{1}{\delta} \dot{k}_r k_r - \tfrac{1}{\delta} \dot{k}_r k^\star \\ &= -k_r \left| S(t) - \tfrac{1}{\delta} \dot{k}_r \right| - \tfrac{1}{\delta} \dot{k}_r k^\star \\ &= -\tfrac{1}{\delta} \dot{k}_r k^\star\end{aligned} \tag{21}$$

Substituting the gain k_r adaptation law (16) in (21) we have:

$$\begin{aligned}\dot{V} &= S(t) \left\{ -k_r\, sgn(S(t)) \right\} \\ &= -k_r\, |S(t)| < 0\end{aligned} \tag{22}$$

□

It has thus been proven that the proposed control law meets the Lyapunov stability condition. Thus, the fractional-order Cryptovirolgy in blokchain system (8) augmented by the feedback controller (15) is globally asymptotically stable.

4.3 Robustness Analysis

In this work step, it is assumed that system (8) suffers from model uncertainties and is subject to external perturbations, as represented by:

$$\begin{cases} D_t^q x(t) = -d_1\, x(t) - \alpha_1\, y(t) + \Delta(t) \\ D_t^q y(t) = -d_2\, y(t) + \alpha_2\, x(t) - \beta_2\, x(t)z(t) + \Delta(t) + u(t) \\ D_t^q z(t) = -d_3\, z(t) + \beta_1\, x(t)y(t) + \Delta(t) \end{cases} \tag{23}$$

Assumption 1. *Δ(t)* is assumed to be bounded, the upper bound is some $\rho \in R^+$ i.e. $\forall t \in R^+, \Delta(t) \leq \rho$.

Then, we have the following result,

Theorem 2. Consider the fractional-order *Cryptovirolgy in blokchain system (23) subject to bounded uncertainties and an external disturbance Δ(t) that verifies Assumption 1, if it is controlled using the adaptive SMC control (12), (13), (15) and the gain adaptation law (16),*
then it is is globally asymptotically stable.

Proof. Consider again the Lyapunov candidate function given by (17),

$$V = \frac{1}{2}S^2 + \frac{1}{2\delta}\tilde{k}_r^T\tilde{k}_r$$

one has

$$\dot{V} = S\dot{S} + \frac{1}{\delta}\dot{k}_r\tilde{k}_r \tag{24}$$

By the same procedure given by equation (19), (20), (12) and (15), we obtain the time derivative,

$$\dot{V} = -\frac{1}{\delta}\dot{k}_r k^\star \tag{25}$$

In this case, the second state is given by,

$$D_t^q y(t) = -d_2\, y(t) + \alpha_2\, x(t) - \beta_2\, x(t)z(t) + \Delta(t) + u(t) \tag{26}$$

thus,

$$\begin{aligned}\dot{V} &= S(t)\left\{\Delta(t) - k_r\ sgn(S(t))\right\} + \tfrac{1}{\delta}\dot{k}_r\tilde{k}_r \\ &= S(t)\Delta(t) - k_r\ |S(t)| + \tfrac{1}{\delta}\dot{k}_r k_r - \tfrac{1}{\delta}\dot{k}_r k^\star \\ &= S(t)\Delta(t) - k_r\ \left|S(t) - \tfrac{1}{\delta}\dot{k}_r\right| - \tfrac{1}{\delta}\dot{k}_r k^\star \\ &= S(t)\Delta(t) - \tfrac{1}{\delta}\dot{k}_r k^\star \\ &= S(t)\Delta(t) - |S(t)|\, k^\star \\ &\leq -|S(t)|\, [k^\star - \rho[\end{aligned} \tag{27}$$

The closed loop system under the adaptive sliding mode controller (15) and the adaptive law (16) is globally asymptotically stable when the optimal gain
$k^\star \geq \rho$, in this case $\dot{V} \leq 0$.

□

5 Numerical Simulation Example

In this example, we consider the system given by (7) [4]. Chaos is observed in the system for the initial condition (3, 2.5, 15.2) and for fractional order q = 0.99.

Figure 3 illustrates the chaotic trajectories of the states x, y, z of the fractional chaotic cryptovirology in blockchain system.

Let us apply the proposed adaptive SMC control to this system with a sliding surface given by (9) and (10) in presence of the control laws (12), (13) and (15), and external disturbances given by:

$$\Delta(t) = 0.5\ \sin(\pi t)\cos(\pi x)\sin(2\pi y) \tag{28}$$

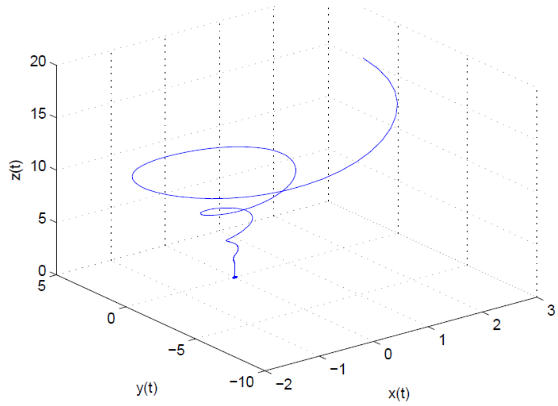

Fig. 4: Phase space-trajectory.
Source: created by authors

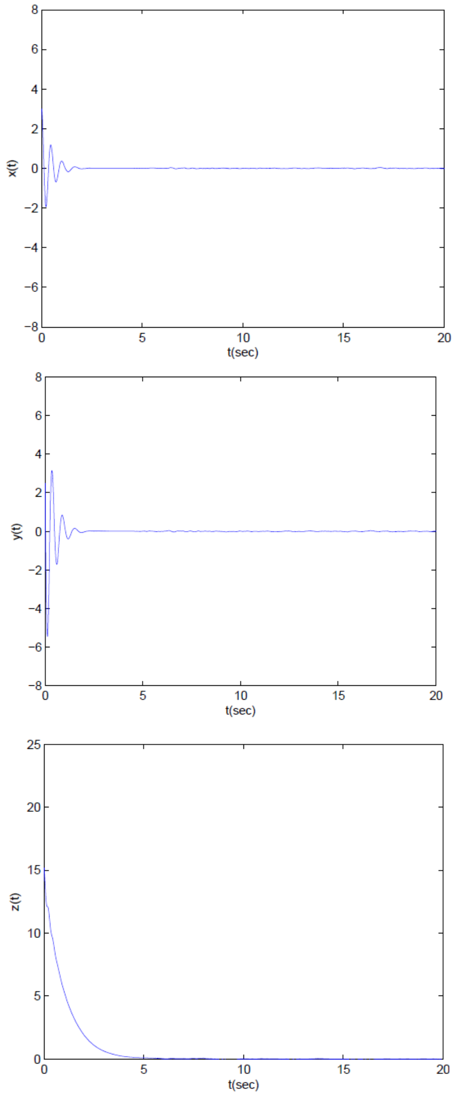

Fig. 5: Time responses of the controlled system (7) states in the presence of external disturbance.
Source: created by the authors

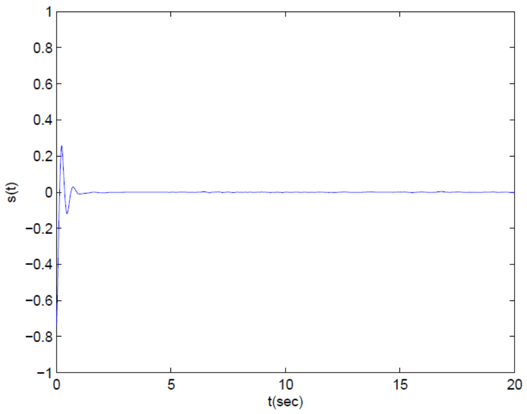

Fig. 6: Sliding surface *S(t)* vs time
Source: created by the authors

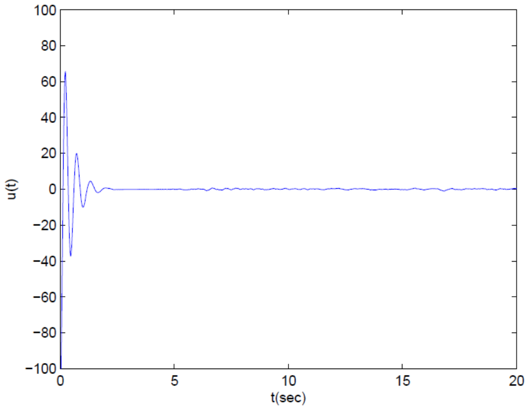

Fig. 7: Control signal *u(t)*
Source: created by the authors

The phase space trajectory of the controlled system is presented in Figure 4 whereas the states time responses are given in Figure 5. They show that the fractional-order chaotic cryptovirology in blockchain system states converge rapidly to the stable origin.

The sliding surface of the stabilized system (7) is illustrated in Figure 6. Figure 7 shows a slightly oscillating control signal *u(t)* vs time.

One can notice that the stabilization performance of this system is good despite the non-negligible effect of disturbance and uncertainties, which proves the anti-interference ability of the proposed adaptive SMC controller.

All the signals remain bounded and converge rapidly to the final values. This confirms that FO adaptive mechanisms are more effective in attenuating the disturbance effect, achieving faster responses with smaller deviation amplitudes.

The performance indexes like accuracy and precision are improved under fractional-order SMC control.

6 Conclusion

In this control design contribution, an adaptive sliding mode control scheme is introduced for the stabilization of the fractional-order chaotic cryptovirology in blockchain system at an equilibrium point. The synthesis of an efficient and appropriate adaptive SMC law is achieved and robust stability analysis is concluded based on the Lyapunov stability method.
An illustrative numerical simulation example of the fractional-order chaotic cryptovirology in blockchain system controlled by the proposed adaptive SMC law is provided to illustrate its effectiveness even in presence of uncertainties and external additive disturbance.

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