New Stability Criteria for Neutral Cohen–Grossberg Neural Networks with Discrete Delays

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Abstract: - This research study examines global asymptotic stability of Cohen–Grossberg neural networks that involve both discrete time delay terms in neuron states and neutral delay terms in the time derivatives of neurons' states. In this context, a suitable Lyapunov functional, constructed as a linear combination of three complementary main Lyapunov functional candidates, is employed to present new alternative sufficient criteria guaranteeing global asymptotic stability of neutral-type neural system possessing discrete delay components. The obtained criteria are established through explicit algebraic inequalities that utilize key matrix properties and structural characteristics of the system functions. The proposed results are basically stated in terms of parameters associated with the considered neutral neural system. These conditions are completely independent of the delay terms and can directly be tested by checking a set of simple algebraic inequalities. In order to illustrate some efficiency aspects of the derived stability results, a numerical example is analyzed.

Key-Words: - Stability Analysis, Neutral Systems, Lyapunov Functionals, Discrete Delay Terms, Cohen-Grossberg Neural Networks, Matrix Theory.

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1 Introduction

Recently, different classes of neural networks have received an immense attention as many neural network models have commendably been utilized to find the solutions to various engineering problems related to fixed point computations, signal and image processing, optimization, fault diagnosis, associative memories and pattern recognition, [1], [2], [3]. In the applications, where dynamical neural networks are employed, the information, presented to the system for processing, is generally in the various forms of stable states of neurons. In this context, studying the convergence properties of equilibrium points associated with neural networks proved to be a prominent concept. In electronically implementing neural networks for real world related engineering problems, time delays are unavoidable resulting from finite varying speed of electronics system components and signal remittance times among the neurons. These time delay components usually make some undesired negative impacts on targeted dynamical performances of neural networks and they can result in various unsolicited nonlinear phenomena associated with instability, bifurcation and oscillation. In consequence, for the sake of observing the precise influence of time delay components on dynamics of neural systems, when conducting stability analysis regarding these systems, involving time delay terms in mathematical statements of neural systems is of great interest.

In the past literature, stability issues associated with neural systems containing time delay terms have been commonly investigated and numerous sets of useful stability criteria have been given, [4], [5], [6], [7], [8], [9], [10], [11]. It should be mentioned here that, beside dynamical neural networks whose states involve time delay terms, there are dynamical neural systems which are expressed by neutral differential equations where time derivatives of states further retain delay terms. A neural network admitting delay components in both states and their time derivatives is commonly named as neutral-type neural network. Since standard dynamical neural networks may not perfectly identify the complete dynamical properties of a neural reaction process for the reason that the neurons in the real world possess complex dynamic behaviors. Additionally, it may usually be needed to appraise the information connected to time derivatives of past states to conduct a satisfactory analysis related to exact dynamics of such delayed neural network models. Neutral-type neural networks proved to be alternative systems to represent complete dynamics of delayed systems and have appropriately been applied to miscellaneous engineering applications including distributed networks retaining lossless-transmission line models, robotics in contact with rigid environment and heat exchanger models, [12], [13], [14]. Hence, stability analysis of neutral systems possessing time and neutral delay arguments has obviously emerged to be a major research area.

We note here that, in the past, many researcher articles have remarkably studied stability issues of different neutral neural network models and submitted numerous sets of stability conditions, [15], [16], [17], [18], [19], [20], [21], [22], [23], [24], [25]. In the current paper, by using a linear combination of a set of quadratic-type Lyapunov functionals, we will present novel stability results regarding neutral Cohen–Grossberg neural networks admitting discrete delay parameters. The presented conditions are stated in terms of system components and are of independent of delay terms. These conditions can also be tested by exploiting some simple algebraic inequalities.

2 Neutral Neural Systems

This study will essentially conduct a stability analysis related to neutral Cohen-Grossberg neural networks whose dynamical models are generally defined by some nonlinear differential equations stated in the form given below:

$$\begin{aligned}\dot{y}_i(t) =& d_i(y_i(t))\Big(- c_i(y_i(t)) + \sum_{j=1}^{n} a_{ij} g_j(y_j(t)) \\ &+ \sum_{j=1}^{n} b_{ij} g_j(y_j(t-\tau_j)) + u_i\Big) \\ &+ \sum_{j=1}^{n} e_{ij}\dot{y}_j(t-\zeta_j), \quad i = 1,\dots,n. \qquad (1)\end{aligned}$$

where n indicates number of neurons that system (1) possesses, $y_i(t)$ indicates a state related to ith neuron. The constant terms a_{ij} and b_{ij} indicate network interconnection elements, $d_i(y_i(t))$ indicate some amplification functions, $c_i(y_i(t))$ indicate the behaved functions, $g_i(y_i(t))$ denote activation functions, τ_i are constant time delays, ζ_i are constant neutral delays, constant e_{ij} are weight coefficients related to time derivatives of states which involve neutral delays and u_i are the constant inputs, $i, j = 1,\dots,n$. In neutral system (1), let $\tau_M = max\{\tau_i\}$ and $\zeta_M = max\{\zeta_i\}$ with $\nu = max\{\tau_M, \zeta_M\}$. (1) possesses the initial states: $y_i(t) = \Psi_i(t)$ together with $\dot{y}_i(t) = \theta_i(t) \in C([-\nu,0],R)$ having $C([-\nu,0],R)$ as the sets of real initial conditions functions ranging from $[-\nu,0]$ to R.

The system elements $d_i(y_i(t))$, $c_i(y_i(t))$ and $g_i(y_i(t))$ included in neural system (1) are supposed to attain some main assumptions given below:

$\mathcal{A}_1$: The amplification functions $d_i(y)$ satisfy

$$0 < \psi_i \le d_i(y) \le \varsigma_i, \ \forall y \in R, \ \forall i$$

with ψ_i and ς_i are being positive numbers.

$\mathcal{A}_2$: The behaved functions $c_i(y)$ satisfy

$$\varrho_i \le \frac{c_i(y) - c_i(x)}{y - x} \le \pi_i, \ \forall x, y \in R, x \neq y, \ \forall i$$

with ϱ_i and π_i are being positive numbers.

$\mathcal{A}_3$: The activation functions $g_i(y)$ satisfy

$$|g_i(y) - g_i(x)| \le \ell_i |y - x|, \ \ \forall x, y \in R, \ \forall i$$

where ℓ_i are positive constants.

3 Stability Analysis

This section is aiming at proposing sufficient criteria regarding global asymptotic stability for system described with (1) by establishing a main Lyapunov functional, which is a linear combination of a set of positive definite quadratic-type Lyapunov functionals. To this end, neural system (1) is transformed into an equivalent neural network model. Suppose that y_i^* represent equilibrium point of ith state of system stated in (1). Describe the shifting operation $x(t) = y(t) - y^*$. By this transformation, our original system given in (1) is transformed into the system which is expressed by the equations:

$$\begin{aligned}\dot{x}_i(t) =& \alpha_i(x_i(t))\Big(- \beta_i(x_i(t)) + \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) \\ &+ \sum_{j=1}^{n} b_{ij} f_j(x_j(t-\tau_j))\Big) \\ &+ \sum_{j=1}^{n} e_{ij}\dot{x}_j(t-\zeta_j), \qquad \forall i \qquad (2)\end{aligned}$$

where nonlinear system functions are now stated as $\alpha_i(x_i(t)) = d_i(x_i(t)+y_i^*), \beta_i(x_i(t)) = c_i(x_i(t)+y_i^*) - c_i(y_i^*)$ and $f_i(x_i(t)) = f_i(x_i(t)+y_i^*) - f_i(y_i^*), \forall i$. Based on $\mathcal{A}_1 - \mathcal{A}_3$, these functions satisfy the conditions:

C_1: $\psi_i \leq \alpha_i(x_i(t)) \leq \varsigma_i$, $\forall i$,

C_2: $\varrho_i x_i^2(t) \leq x_i(t)\beta_i(x_i(t)) \leq \pi_i x_i^2(t)$, $\forall i$,

C_3: $|f_i(x_i(t))| \leq \ell_i |x_i(t)|$, $\forall i$.

The vector-matrix representation of (2) is

$$\dot{x}(t) = \Gamma(x(t))\Big(-\beta(x(t)) + Af(x(t)) + Bf(x(t,\tau))\Big) + E\dot{x}(t,\zeta) \quad (3)$$

In (3), A, B and E represent constant system matrices, whose entries are respectively a_{ij}, b_{ij} and $e_{ij}, \forall i, j$. $x(t) = (x_1(t), x_2(t), ..., x_n(t))^T$ indicates transformed system vector related to new state variables, $x(t,\tau)$, $f(x(t))$, $f(x(t,\tau))$, $\dot{x}(t,\zeta)$ and $\beta(x(t))$ are representing the transformed system vectors, whose entries are respectively $x_i(t-\tau_i)$, $f_i(x_i(t))$, $f_i(x_i(t-\tau_i))$, $\dot{x}_i(t-\zeta_i)$ and $\beta_i(x_i(t))$. $\Gamma(x(t))$ is observed to be a diagonal matrix possessing the components $\alpha_i(x_i(t)), \forall i$.

Note that neural systems (1) and (2) have the equivalent stability properties. Therefore, we will simply conduct a stability analysis for origin of neural system given in (2) and present the following major theorem of this paper:

Theorem 1. Let system (2) satisfy the conditions $C_1 - C_3$. Then, neural system (2) is globally asymptotically stable if the following two sets of criteria hold:

$$\epsilon_i = h_1(\psi_i \varrho_i^2 - \kappa_i - \mu_i) + h_2(\psi_i^2 \varrho_i^2 - \hat{\kappa}_i - \hat{\mu}_i) + h_3(\varrho_i^2 - \tilde{\kappa}_i - \tilde{\mu}_i) > 0, \ \forall i.$$

and

$$\varepsilon_i = h_1(\frac{1}{\varsigma_i} - \iota_i) + h_2(1 - \hat{\iota}_i) + h_3(\frac{1}{\varsigma_i^2} - \tilde{\iota}_i) > 0, \ \forall i.$$

where h_1, h_2 and h_3 are some nonnegative constants with $h_1 + h_2 + h_3 = 1$,

$$\kappa_i = (1+\delta_1+\hat{\delta}_1)\sum_{j=1}^{n} \varsigma_j \Big|\sum_{k=1}^{n} a_{ki}a_{kj}\Big|\ell_i^2,$$

$$\hat{\kappa}_i = (1+\delta_2+\hat{\delta}_2)\sum_{j=1}^{n} \varsigma_j^2 \Big|\sum_{k=1}^{n} a_{ki}a_{kj}\Big|\ell_i^2$$

$$\tilde{\kappa}_i = (1+\delta_3+\hat{\delta}_3)\sum_{j=1}^{n} \Big|\sum_{k=1}^{n} a_{ki}a_{kj}\Big|\ell_i^2,$$

$$\mu_i = (1+\frac{1}{\delta_1}+\tilde{\delta}_1)\sum_{j=1}^{n} \varsigma_j \Big|\sum_{k=1}^{n} b_{ki}b_{kj}\Big|\ell_i^2$$

$$\hat{\mu}_i = (1+\frac{1}{\delta_2}+\tilde{\delta}_2)\sum_{j=1}^{n} \varsigma_j^2 \Big|\sum_{k=1}^{n} b_{ki}b_{kj}\Big|\ell_i^2,$$

$$\tilde{\mu}_i = (1+\frac{1}{\delta_3}+\tilde{\delta}_3)\sum_{j=1}^{n} \Big|\sum_{k=1}^{n} b_{ki}b_{kj}\Big|\ell_i^2$$

$$\iota_i = (1+\frac{1}{\hat{\delta}_1}+\frac{1}{\tilde{\delta}_1})\sum_{j=1}^{n} \frac{1}{\psi_j} \Big|\sum_{k=1}^{n} e_{ki}e_{kj}\Big|,$$

$$\hat{\iota}_i = (1+\frac{1}{\hat{\delta}_2}+\frac{1}{\tilde{\delta}_2})\sum_{j=1}^{n} \Big|\sum_{k=1}^{n} e_{ki}e_{kj}\Big|,$$

$$\tilde{\iota}_i = (1+\frac{1}{\hat{\delta}_3}+\frac{1}{\tilde{\delta}_3})\sum_{j=1}^{n} \frac{1}{\psi_j^2} \Big|\sum_{k=1}^{n} e_{ki}e_{kj}\Big|, \quad \forall i,$$

where the real constant elements $\delta_i, \hat{\delta}_i, \tilde{\delta}_i, (i = 1, 2, 3)$ are determined to be positive.

Proof. Describe two Lyapunov functionals:

$$W(t) = 2\sum_{i=1}^{n}\int_0^{x_i(t)} \Big(h_1 + h_2\alpha_i(s) + \frac{h_3}{\alpha_i(s)}\Big)\beta_i(s)ds \quad (4)$$

and

$$Q(t) = \sum_{i=1}^{n}\Big((h_1\mu_i + h_2\hat{\mu}_i + h_3\tilde{\mu}_i)\int_{t-\tau_i}^{t} x_i^2(\hat{s})d\hat{s} + (h_1\iota_i + h_2\hat{\iota}_i + h_3\tilde{\iota}_i)\int_{t-\zeta_i}^{t} \dot{x}_i^2(\tilde{s})d\tilde{s}\Big) \quad (5)$$

Construct the Lyapunov functional for system given with (2):

$$V(t) = W(t) + Q(t) \quad (6)$$

When inspecting time derivative of $W(t)$, described by (4), it can be determined that it has the form

$$\begin{aligned}\dot{W}(t) =& 2\sum_{i=1}^{n}\beta_i(x_i(t))\Big(h_1 + h_2\alpha_i(x_i(t)) + \frac{h_3}{\alpha_i(x_i(t))}\Big)\dot{x}_i(t) \\ =& 2\beta^T(x(t))\Big(h_1 + h_2\Gamma(x(t)) + h_3\Gamma^{-1}(x(t))\Big)\dot{x}(t) \quad (7)\end{aligned}$$

Now, when adding and then subtracting the term

$$\Big(h_1\dot{x}^T(t)\Gamma^{-1}(x(t))+h_2\dot{x}^T(t)+h_3\dot{x}^T(t)\Gamma^{-2}(x(t))\Big)\dot{x}(t)$$

in (7), we simply determine the equation:

$$\begin{aligned}\dot{W}(t) =&\Big(2\beta^T(x(t))(h_1+h_2\Gamma(x(t))\\&+h_3\Gamma^{-1}(x(t)))+h_1\dot{x}^T(t)\Gamma^{-1}(x(t))\\&+h_2\dot{x}^T(t)+h_3\dot{x}^T(t)\Gamma^{-2}(x(t))\\&-h_1\dot{x}^T(t)\Gamma^{-1}(x(t))-h_2\dot{x}^T(t)\\&-h_3\dot{x}^T(t)\Gamma^{-2}(x(t))\Big)\dot{x}(t)\\=&h_1\Big(2\beta(x(t))+\Gamma^{-1}(x(t))\dot{x}(t)\Big)^T\dot{x}(t)\\&+h_2\Big(2\Gamma(x(t))\beta(x(t))+\dot{x}(t)\Big)^T\dot{x}(t)\\&+h_3\Big(2\Gamma^{-1}(x(t))\beta(x(t))+\Gamma^{-2}(x(t))\\&\times\dot{x}(t)\Big)^T\dot{x}(t)-h_1\dot{x}^T(t)\Gamma^{-1}(x(t))\dot{x}(t)\\&-h_2\dot{x}^T(t)\dot{x}(t)-h_3\dot{x}^T(t)\Gamma^{-2}(x(t))\dot{x}(t)\end{aligned}\tag{8}$$

On the basis of (3), (8) yields

$$\begin{aligned}\dot{W}(t) =&h_1\Big(\beta(x(t))+Af(x(t))+Bf(x(t-\tau))\\&+\Gamma^{-1}(x(t))E\dot{x}(t-\zeta)\Big)^T\times\Gamma(x(t))\\&\times\Big(-\beta(x(t))+Af(x(t))+Bf(x(t-\tau))\\&+\Gamma^{-1}(x(t))E\dot{x}(t-\zeta)\Big)+h_2\Big(\beta(x(t))\\&+Af(x(t))+Bf(x(t-\tau))+\Gamma^{-1}(x(t))\\&\times E\dot{x}(t-\zeta)\Big)^T\times\Gamma^2(x(t))\times\Big(-\beta(x(t))\\&+Af(x(t))+Bf(x(t-\tau))+\Gamma^{-1}(x(t))\\&\times E\dot{x}(t-\zeta)\Big)+h_3\Big(\beta(x(t))+Af(x(t))\\&+Bf(x(t-\tau))+\Gamma^{-1}(x(t))E\dot{x}(t-\zeta)\Big)^T\\&\times\Big(-\beta(x(t))+Af(x(t))+Bf(x(t-\tau))\\&+\Gamma^{-1}(x(t))E\dot{x}(t-\zeta)\Big)-h_1\dot{x}^T(t)\\&\times\Gamma^{-1}(x(t))\dot{x}(t)-h_2\dot{x}^T(t)\dot{x}(t)\\&-h_3\dot{x}^T(t)\Gamma^{-2}(x(t))\dot{x}(t)\end{aligned}\tag{9}$$

The following three inequalities are now noted:

$$\begin{aligned}&\Big(\beta(x(t))+Af(x(t))+Bf(x(t-\tau))\\&+\Gamma^{-1}(x(t))E\dot{x}(t-\zeta)\Big)^T\times\Gamma(x(t))\\&\times\Big(-\beta(x(t))+Af(x(t))+Bf(x(t-\tau))\\&+\Gamma^{-1}(x(t))E\dot{x}(t-\zeta)\Big)\\\le&-\beta^T(x(t))\Gamma(x(t))\beta(x(t))+(1+\delta_1+\hat{\delta}_1)\\&\times f^T(x(t))A^T\Gamma(x(t))Af(x(t))\\&+(1+\frac{1}{\delta_1}+\tilde{\delta}_1)f^T(x(t-\tau))B^T\Gamma(x(t))\\&\times Bf(x(t-\tau))+(1+\frac{1}{\hat{\delta}_1}+\frac{1}{\tilde{\delta}_1})\\&\times\dot{x}^T(t-\zeta)E^T\Gamma^{-1}(x(t))E\dot{x}(t-\zeta),\end{aligned}\tag{10}$$

$$\begin{aligned}&\Big(\beta(x(t))+Af(x(t))+Bf(x(t-\tau))\\&+\Gamma^{-1}(x(t))E\dot{x}(t-\zeta)\Big)^T\times\Gamma^2(x(t))\\&\times\Big(-\beta(x(t))+Af(x(t))+Bf(x(t-\tau))\\&+\Gamma^{-1}(x(t))E\dot{x}(t-\zeta)\Big)\\\le&-\beta^T(x(t))\Gamma^2(x(t))\beta(x(t))+(1+\delta_2+\hat{\delta}_2)\\&\times f^T(x(t))A^T\Gamma^2(x(t))Af(x(t))\\&+(1+\frac{1}{\delta_2}+\tilde{\delta}_2)f^T(x(t-\tau))B^T\Gamma^2(x(t))\\&\times Bf(x(t-\tau))+(1+\frac{1}{\hat{\delta}_2}+\frac{1}{\tilde{\delta}_2})\\&\times\dot{x}^T(t-\zeta)E^TE\dot{x}(t-\zeta)\end{aligned}\tag{11}$$

and

$$\begin{aligned}&\Big(\beta(x(t))+Af(x(t))+Bf(x(t-\tau))\\&+\Gamma^{-1}(x(t))E\dot{x}(t-\zeta)\Big)^T\times\Big(-\beta(x(t))\\&+Af(x(t))+Bf(x(t-\tau))\\&+\Gamma^{-1}(x(t))E\dot{x}(t-\zeta)\Big)\\\le&-\beta^T(x(t))\beta(x(t))+(1+\delta_3+\hat{\delta}_3)\\&\times f^T(x(t))A^TAf(x(t))+(1+\frac{1}{\delta_3}+\tilde{\delta}_3)\\&\times f^T(x(t-\tau))B^TBf(x(t-\tau))\\&+(1+\frac{1}{\hat{\delta}_3}+\frac{1}{\tilde{\delta}_3})\dot{x}^T(t-\zeta)\\&\times E^T\Gamma^{-2}(x(t))E\dot{x}(t-\zeta)\end{aligned}\tag{12}$$

Using (10)-(12) in (9) results in

$$\begin{aligned}\dot{W}(t) \leq & h_1\Big(-\beta^T(x(t))\Gamma(x(t))\beta(x(t))\\ &+(1+\delta_1+\hat{\delta}_1)f^T(x(t))A^T\Gamma(x(t))\\ &\times Af(x(t))+(1+\frac{1}{\delta_1}+\tilde{\delta}_1)\\ &\times f^T(x(t-\tau))B^T\Gamma(x(t))Bf(x(t-\tau))\\ &+(1+\frac{1}{\hat{\delta}_1}+\frac{1}{\tilde{\delta}_1})\dot{x}^T(t-\zeta)E^T\Gamma^{-1}(x(t))\\ &\times E\dot{x}(t-\zeta)-\dot{x}^T(t)\Gamma^{-1}(x(t))\dot{x}(t)\Big)\\ &+h_2\Big(-\beta^T(x(t))\Gamma^2(x(t))\beta(x(t))\\ &+(1+\delta_2+\hat{\delta}_2)f^T(x(t))A^T\Gamma^2(x(t))\\ &\times Af(x(t))+(1+\frac{1}{\delta_2}+\tilde{\delta}_2)f^T(x(t-\tau))\\ &\times B^T\Gamma^2(x(t))Bf(x(t-\tau))\\ &+(1+\frac{1}{\hat{\delta}_2}+\frac{1}{\tilde{\delta}_2})\dot{x}^T(t-\zeta)E^TE\dot{x}(t-\zeta)\\ &-\dot{x}^T(t)\dot{x}(t)\Big)+h_3\Big(-\beta^T(x(t))\beta(x(t))\\ &+(1+\delta_3+\hat{\delta}_3)f^T(x(t))A^TAf(x(t))\\ &+(1+\frac{1}{\delta_3}+\tilde{\delta}_3)f^T(x(t-\tau))B^T\\ &\times Bf(x(t-\tau))+(1+\frac{1}{\hat{\delta}_3}+\frac{1}{\tilde{\delta}_3})\\ &\times\dot{x}^T(t-\zeta)E^T\Gamma^{-2}(x(t))E\dot{x}(t-\zeta)\\ &-\dot{x}^T(t)\Gamma^{-2}(x(t))\dot{x}(t)\Big) \qquad (13)\end{aligned}$$

Under the assumption C_1 and C_3, the following four key inequalities can be stated:

$$\begin{aligned}&f^T(x(t))A^T\Gamma(x(t))Af(x(t))\\ &\leq\sum_{i=1}^{n}\sum_{j=1}^{n}\varsigma_j\Big|\sum_{k=1}^{n}a_{ki}a_{kj}\Big|\ell_i^2x_i^2(t) \qquad (14)\end{aligned}$$

$$\begin{aligned}&f^T(x(t-\tau))B^T\Gamma(x(t))Bf(x(t-\tau))\\ &\leq\sum_{i=1}^{n}\sum_{j=1}^{n}\varsigma_j\Big|\sum_{k=1}^{n}b_{ki}b_{kj}\Big|\ell_i^2x_i^2(t-\tau_i), \qquad (15)\end{aligned}$$

$$\begin{aligned}&f^T(x(t))A^T\Gamma^2(x(t))Af(x(t))\\ &\leq\sum_{i=1}^{n}\sum_{j=1}^{n}\varsigma_j^2\Big|\sum_{k=1}^{n}a_{ki}a_{kj}\Big|\ell_i^2x_i^2(t) \qquad (16)\end{aligned}$$

and

$$\begin{aligned}&f^T(x(t-\tau))B^T\Gamma^2(x(t))Bf(x(t-\tau))\\ &\leq\sum_{i=1}^{n}\sum_{j=1}^{n}\varsigma_j^2\Big|\sum_{k=1}^{n}b_{ki}b_{kj}\Big|\ell_i^2x_i^2(t-\tau_i), \qquad (17)\end{aligned}$$

In the light of C_1, we can state

$$\begin{aligned}&\dot{x}^T(t-\zeta)E^T\Gamma^{-1}(x(t))E\dot{x}(t-\zeta)\\ &\leq\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{\psi_j}\Big|\sum_{k=1}^{n}e_{ki}e_{kj}\Big|\dot{x}_i^2(t-\zeta_i) \qquad (18)\end{aligned}$$

$$\begin{aligned}&\dot{x}^T(t-\zeta)E^T\Gamma^{-2}(x(t))E\dot{x}(t-\zeta)\\ &\leq\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{\psi_j^2}\Big|\sum_{k=1}^{n}e_{ki}e_{kj}\Big|\dot{x}_i^2(t-\zeta_i) \qquad (19)\end{aligned}$$

$$-\dot{x}^T(t)\Gamma^{-1}(x(t))\dot{x}(t)\leq-\sum_{i=1}^{n}\frac{1}{\varsigma_i}\dot{x}_i^2(t) \qquad (20)$$

and

$$-\dot{x}^T(t)\Gamma^{-2}(x(t))\dot{x}(t)\leq-\sum_{i=1}^{n}\frac{1}{\varsigma_i^2}\dot{x}_i^2(t) \qquad (21)$$

Under the assumption C_3, we can also obtain that

$$\begin{aligned}&f^T(x(t))A^TAf(x(t))\\ &\leq\sum_{i=1}^{n}\sum_{j=1}^{n}\Big|\sum_{k=1}^{n}a_{ki}a_{kj}\Big|\ell_i^2x_i^2(t), \qquad (22)\end{aligned}$$

and

$$\begin{aligned}&f^T(x(t-\tau))B^TBf(x(t-\tau))\\ &\leq\sum_{i=1}^{n}\sum_{j=1}^{n}\Big|\sum_{k=1}^{n}b_{ki}b_{kj}\Big|\ell_i^2x_i^2(t-\tau_i), \qquad (23)\end{aligned}$$

We also note that

$$\begin{aligned}&\dot{x}^T(t-\zeta)E^TE\dot{x}(t-\zeta)\\ &\leq\sum_{i=1}^{n}\sum_{j=1}^{n}\Big|\sum_{k=1}^{n}e_{ki}e_{kj}\Big|\dot{x}_i^2(t-\zeta_i), \qquad (24)\end{aligned}$$

In the light of assumptions defined by C_1 and C_2, the following two basic inequalities can be stated

$$\begin{aligned}&-\beta^T(x(t))\Gamma(x(t))\beta(x(t))\\ &=-\sum_{i=1}^{n}\alpha_i(x_i(t))\beta_i^2(x_i(t))\leq-\sum_{i=1}^{n}\psi_i\varrho_i^2x_i^2(t) \quad (25)\end{aligned}$$

and

$$-\beta^T(x(t))\Gamma^2(x(t))\beta(x(t)) = -\sum_{i=1}^{n}\alpha_i^2(x_i(t))\beta_i^2(x_i(t)) \le -\sum_{i=1}^{n}\psi_i^2\varrho_i^2x_i^2(t) \quad (26)$$

Using (14)-(26) in (13) yields

$$\begin{aligned}\dot{W}(t) \le\ & h_1\sum_{i=1}^{n}\Bigg(-\psi_i\varrho_i^2x_i^2(t)\\
&+\sum_{j=1}^{n}\Bigg((1+\delta_1+\hat{\delta}_1)\varsigma_j\Big|\sum_{k=1}^{n}a_{ki}a_{kj}\Big|\ell_i^2x_i^2(t)\\
&+(1+\frac{1}{\delta_1}+\tilde{\delta}_1)\varsigma_j\Big|\sum_{k=1}^{n}b_{ki}b_{kj}\Big|\ell_i^2x_i^2(t-\tau_i)\\
&+(1+\frac{1}{\hat{\delta}_1}+\frac{1}{\tilde{\delta}_1})\frac{1}{\psi_j}\Big|\sum_{k=1}^{n}e_{ki}e_{kj}\Big|\dot{x}_i^2(t-\zeta_i)\\
&-\frac{1}{\varsigma_i}\dot{x}_i^2(t)\Bigg)\Bigg)+h_2\sum_{i=1}^{n}\Bigg(-\psi_i^2\varrho_i^2x_i^2(t)\\
&+\sum_{j=1}^{n}\Bigg((1+\delta_2+\hat{\delta}_2)\varsigma_j^2\Big|\sum_{k=1}^{n}a_{ki}a_{kj}\Big|\ell_i^2x_i^2(t)\\
&+(1+\frac{1}{\delta_2}+\tilde{\delta}_2)\varsigma_j^2\Big|\sum_{k=1}^{n}b_{ki}b_{kj}\Big|\ell_i^2x_i^2(t-\tau_i)\\
&+(1+\frac{1}{\hat{\delta}_2}+\frac{1}{\tilde{\delta}_2})\Big|\sum_{k=1}^{n}e_{ki}e_{kj}\Big|\dot{x}_i^2(t-\zeta_i)\\
&-\dot{x}_i^2(t)\Bigg)\Bigg)+h_3\sum_{i=1}^{n}\Bigg(-\varrho_i^2x_i^2(t)\\
&+\sum_{j=1}^{n}\Bigg((1+\delta_3+\hat{\delta}_3)\Big|\sum_{k=1}^{n}a_{ki}a_{kj}\Big|\ell_i^2x_i^2(t)\\
&+(1+\frac{1}{\delta_3}+\tilde{\delta}_3)\Big|\sum_{k=1}^{n}b_{ki}b_{kj}\Big|\ell_i^2x_i^2(t-\tau_i)\\
&+(1+\frac{1}{\hat{\delta}_3}+\frac{1}{\tilde{\delta}_3})\frac{1}{\psi_j^2}\Big|\sum_{k=1}^{n}e_{ki}e_{kj}\Big|\dot{x}_i^2(t-\zeta_i)\\
&-\frac{1}{\varsigma_i^2}\dot{x}_i^2(t)\Bigg)\Bigg)\\
=&\sum_{i=1}^{n}\big(-h_1(\psi_i\varrho_i^2-\kappa_i)-h_2(\psi_i^2\varrho_i^2-\hat{\kappa}_i)\\
&-h_3(\varrho_i^2-\tilde{\kappa}_i)\big)x_i^2(t)+\sum_{i=1}^{n}(h_1\mu_i+h_2\hat{\mu}_i\\
&+h_3\tilde{\mu}_i)x_i^2(t-\tau_i)+\sum_{i=1}^{n}(h_1\iota_i+h_2\hat{\iota}_i+h_3\tilde{\iota}_i)\\
&\times\dot{x}_i^2(t-\zeta_i)-\sum_{i=1}^{n}(h_1\frac{1}{\varsigma_i}+h_2+h_3\frac{1}{\varsigma_i^2})\dot{x}_i^2(t) \quad (27)\end{aligned}$$

Calculating time derivative of $Q(t)$, described by (5), leads to

$$\begin{aligned}\dot{Q}(t) =& \sum_{i=1}^{n}(h_1\mu_i+h_2\hat{\mu}_i+h_3\tilde{\mu}_i)x_i^2(t)\\
&-\sum_{i=1}^{n}(h_1\mu_i+h_2\hat{\mu}_i+h_3\tilde{\mu}_i)x_i^2(t-\tau_i)\\
&+\sum_{i=1}^{n}(h_1\iota_i+h_2\hat{\iota}_i+h_3\tilde{\iota}_i)\dot{x}_i^2(t)\\
&-\sum_{i=1}^{n}(h_1\iota_i+h_2\hat{\iota}_i+h_3\tilde{\iota}_i)\dot{x}_i^2(t-\zeta_i) \quad (28)\end{aligned}$$

Combining (28) with (27) yields

$$\begin{aligned}\dot{V}(t) \le& -\sum_{i=1}^{n}\Big(h_1(\psi_i\varrho_i^2-\kappa_i-\mu_i)+h_2(\psi_i^2\varrho_i^2\\
&-\hat{\kappa}_i-\hat{\mu}_i)+h_3(\varrho_i^2-\tilde{\kappa}_i-\tilde{\mu}_i)\Big)x_i^2(t)\\
&-\sum_{i=1}^{n}\Big(h_1(\frac{1}{\varsigma_i}-\iota_i)+h_2(1-\hat{\iota}_i)\\
&+h_3(\frac{1}{\varsigma_i^2}-\tilde{\iota}_i)\Big)\dot{x}_i^2(t)\\
=& -\sum_{i=1}^{n}(\varepsilon_ix_i^2(t)+\epsilon_i\dot{x}_i^2(t))\\
\le& -\varepsilon_m\|x(t)\|_2^2-\epsilon_m\|\dot{x}(t)\|_2^2 \quad (29)\end{aligned}$$

with $\varepsilon_m = \min\{\varepsilon_i\}$ and $\epsilon_m = \min\{\epsilon_i\}$. We need to examine time derivative of employed main functional $V(t)$ in different distinct cases for establishing the desired stability result. Firstly, in (29), consider that $x(t) \ne 0$ or $\dot{x}(t) \ne 0$. Obviously, such a case directly leads us to make the conclusion that $\dot{V}(t)$ is clearly negative definite. Secondly, consider another case in which $x(t) = \dot{x}(t) = 0$. This new distinct case directly yields to the conditions that $\dot{W}(t) = 0$ together with

$$\begin{aligned}\dot{Q}(t) =& -\sum_{i=1}^{n}(h_1\mu_i+h_2\hat{\mu}_i+h_3\tilde{\mu}_i)x_i^2(t-\tau_i)\\
&-\sum_{i=1}^{n}(h_1\iota_i+h_2\hat{\iota}_i+h_3\tilde{\iota}_i)\dot{x}_i^2(t-\zeta_i) \quad (30)\end{aligned}$$

Finally, in (30), if $x_i(t-\tau_i) \ne 0$ or $\dot{x}_i(t-\zeta_i) \ne 0$, for at least one index i, then (30) will assure that employed main functional $V(t)$, given by (6), will be absolutely negative definite, which immediately indicates that $\dot{V}(t) = 0$ only if $x_i(t) = x_i(t-\tau_i) = \dot{x}_i(t) = \dot{x}_i(t-\zeta_i) = 0, \forall i$, holds. Based on these results and Lyapunov stability theorems in the literature, it is obvious to precisely indicate that origin of transformed system given with (2) is justified to be asymptotically stable. Besides, we may clearly observe that employed main functional $V(t)$ is definitely radially unbounded since the required criterion $W(t) \longrightarrow \infty$ as $\|x(t)\| \longrightarrow \infty$ is satisfied, which simply ensures that system given with (2) is also globally asymptotically stable. Thus, we have directly proved the criteria proposed in the main theorem of our paper.

4 A Numerical Example and Simulation Results

The section aims at studying a numerical example to illustrate some efficiency aspects of stability results derived by Theorem 1.

Example: Let the system matrices and nonlinear system functions of neural network (1) be as follows:

$$A = \frac{1}{20}\begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & -1 & 1 & 1 \\ -1 & 1 & -1 & 1 \\ 1 & -1 & -1 & 1 \end{bmatrix},$$

$$B = \frac{1}{16}\begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & -1 & 1 & 1 \\ -1 & 1 & -1 & 1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

and

$$E = \frac{1}{8}\begin{bmatrix} 1 & 1 & 1 & -1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & -1 & -1 & -1 \end{bmatrix}.$$

The behaved functions are selected as $c_i(y_i(t)) = 0.5y_i(t)$, the activation functions are chosen as $g_i(y_i(t)) = 0.05\tanh(y_i(t))$, and the amplification functions are defined by $d_i(y_i(t)) = 1.2 + 0.3\tanh^2(y_i(t))$, $i = 1,2,3,4$.

For this example, we choose $h_1 = 0.2$, $h_2 = 0.3$, $h_3 = 0.5$, and select $\delta_1 = \hat{\delta}_3 = 0.5$, $\delta_2 = \tilde{\delta}_2 = 0.3$, $\delta_3 = 0.2$, $\hat{\delta}_1 = 0.4$, $\hat{\delta}_2 = \tilde{\delta}_3 = 0.1$, $\tilde{\delta}_1 = 0.6$. For the system functions, we can determine that $\psi_1 = \psi_2 = \psi_3 = \psi_4 = \varsigma_1 = \varsigma_2 = \varsigma_3 = \varsigma_4 = \ell_1 = \ell_2 = \ell_3 = \ell_4 = 1$ and $\varrho_1 = \varrho_2 = \varrho_3 = \varrho_4 = 0.6$. Then, the conditions of Theorem 1 are obtained as $\epsilon_i = 0.262875 > 0$ and $\varepsilon_i = 0.260417 > 0$, $i = 1,2,3,4$.. Thus, Theorem 1 ensures stability of neutral neural system (1) possessing the components of this example.

Now, choose the time delay and neutral delay parameters as follows: $\tau_1 = 15.2$, $\tau_2 = 7.3$, $\tau_3 = 11.2$, $\tau_4 = 8.5$, $\zeta_1 = 2.1$, $\zeta_2 = 3.0$, $\zeta_3 = 0.8$, $\zeta_4 = 1.5$. In this case, $\tau_M = \max\{\tau_1, \tau_2, \tau_3, \tau_4\} = 15.2$ and $\zeta_M = \max\{\zeta_1, \zeta_2, \zeta_3, \zeta_4\} = 3.0$ and hence $\nu = \max\{\tau_M, \zeta_M\} = 15.2$. In the case of choosing the initial conditions as $y(t) = (0.3, 0.2, -0.2, -0.3)^T$, $t \in [-15.2, 0]$, the time response of the neural system of this example is depicted in Figure 1.

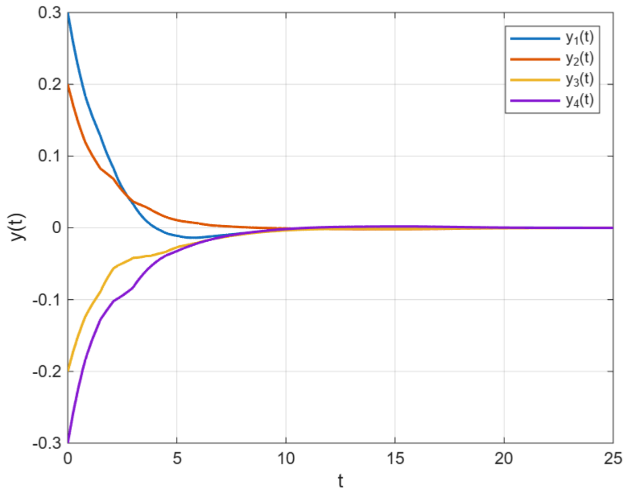

Fig. 1: Time response of the states of system (1). Source: created by the authors.

In the case of the same delay parameters, when choosing new initial conditions $y(t) = (0.4, 0.1, -0.4, -0.2)^T$, $t \in [-15.2, 0]$, the time response of system (1) is obtained to be in the form shown in Figure 2.

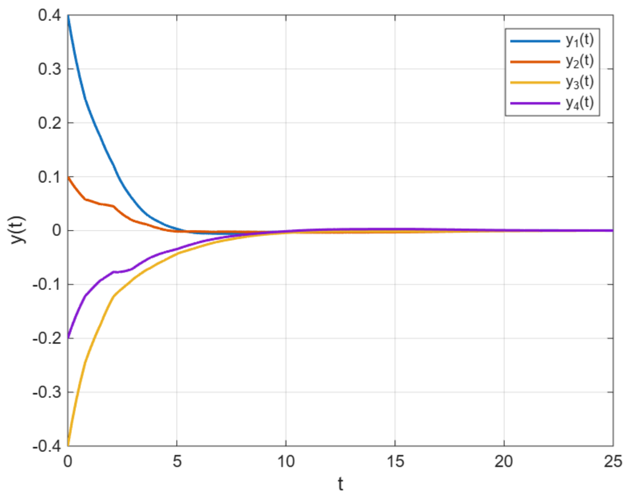

Fig. 2: Time response of the states of system (1). Source: created by the authors.

To further examine the impact of different delay terms on convergent dynamics of system (1), we conduct the simulations for two other cases. Choose the delay parameters are as $\tau_1 = 2.1$, $\tau_2 = 1.2$, $\tau_3 = 0.8$, $\tau_4 = 1.5$, $\zeta_1 = 5.2$, $\zeta_2 = 1.8$, $\zeta_3 = 2.6$, $\zeta_4 = 4.5$. In this case, $\tau_M = \max\{\tau_1, \tau_2, \tau_3, \tau_4\} = 2.1$ and $\zeta_M = \max\{\zeta_1, \zeta_2, \zeta_3, \zeta_4\} = 5.2$ and hence $\nu = \max\{\tau_M, \zeta_M\} = 5.2$. In this case, for the initial conditions $y(t) = (0.3, 0.2, -0.2, -0.3)^T$, $t \in [-5.2, 0]$, the time response of the neural system of this example is depicted in Figure 3.

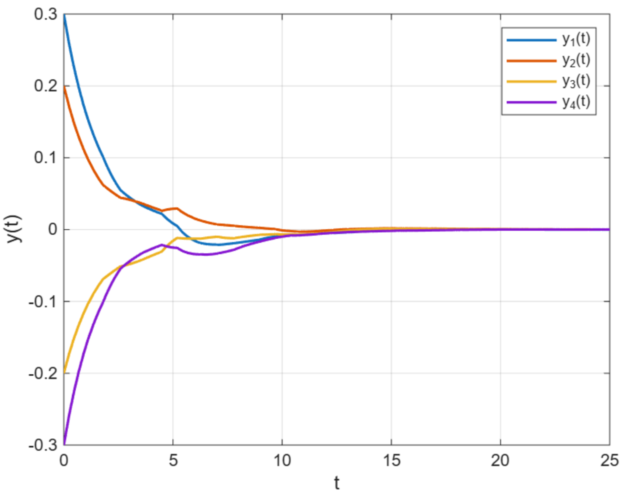

Fig. 3: Time response of the states of system (1). Source: created by the authors.

For the same delay parameters, when choosing the initial conditions $y(t) = (0.4, 0.1, -0.4, -0.2)^T$, $t \in [-5.2, 0]$, the time response of system (1) is obtained to be in the form given by Figure 4.

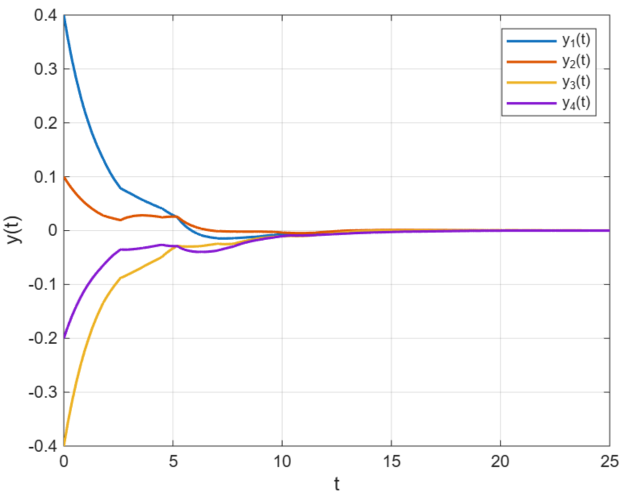

Fig. 4: Time response of the states of system (1). Source: created by the authors.

5 Conclusion

This research study has basically examined stability of a Cohen–Grossberg neural network model, whose mathematical representation mainly possesses many discrete delay terms in both neuron states and in their time derivatives. In this context, a suitable Lyapunov functional, constructed as a linear combination of three complementary main Lyapunov functional candidates, is employed to present new alternative and useful sufficient criteria guaranteeing global asymptotic stability for a neutral neural system possessing many discrete delay components. The obtained criteria are established through explicit algebraic inequalities that utilize key matrix properties and structural characteristics of the system functions. The proposed results are stated with respect to parameters associated with the considered neutral neural system, and they are of independent of the delay terms. These conditions can also be tested by exploiting some simple algebraic inequalities. The mathematical and analytical methods and techniques utilized in this article can be further exploited for investigation of stability conditions for various models of linear and nonlinear neutral systems. A numerical example, together with the simulation results, has also been studied to illustrate some efficiency aspects of the derived stability conditions.

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Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

The authors equally contributed in the present research, at all stages from the formulation of the problem to the final findings and solution.

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Conflicts of Interest

The authors have no conflicts of interest to declare that are relevant to the content of this article.