Robust Model Reference Adaptive Control Design for 1DOF Flexible Arm with Payload Variations based on Fractional Order MIT Law

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Abstract: - Flexible manipulators, due to their inherent structural flexibility, present significant control challenges, especially when manipulating non-fixed payloads. This paper introduces a fractional-order model reference adaptive control (FOMRAC) design to deal with the problem of controlling a single-link flexible manipulator which is subject to joint friction and variation in the load. This control system allows removing the effects of nonlinear friction by implementing an adaptive regulator for the motor position that compensates the coupling torque and Coulomb friction. The tip position control is carried out by decoupling the system dynamics using a positive feedback closed loop around the tip position and then applying a FOMRAC adaptive regulator for the decoupled system when the tip load is varying. Simulations of ideal and disturbed control scenarios are given to validate the efficiency of the proposed fractional-order adaptive control algorithms applied to a single-link flexible manipulator and demonstrate its superior performance when managing variations in payload and disturbance rejection comparatively with the standard integer MRAC scheme.

Key-Words: - Fractional order control, adaptive control, FOMRAC, flexible robot manipulator, robustness, parallel feed-forward.

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1 Introduction

Today, the development of industrial fabric cannot be imagined without the increased use of manipulator robots. In assembly, handling, welding, or painting lines, very simple single-degree-of-freedom robotic systems can be very useful, even essential. Most existing robotic manipulators are designed and constructed to maximize the rigidity, in order to minimize end effectors vibration and achieve good positioning accuracy.

This is largely justified by the fact that all mechanical devices are subject to deformations under load, hence the inherent requirement in the design to stiffen the mechanical structure at the cost of degraded performance (bulky design, limitation of the robot operating speed, increase in the size of the actuators, high energy consumption, increase in the overall cost, etc.). In addition, the payload/weight ratio of the robot in such situations is low.

To overcome these effects, flexible robots have attracted increasing interest in industrial and aerospace fields over the last two decades, [1], [2].

Flexible robotic manipulators offer many advantages over rigid robots: they require less material (usually they are lighter, more maneuverable and transportable), consuming less power (with smaller actuators), longer reach (more access and space due to thinner construction), less risky operation (avoiding damage due to compliance and low inertia), more economic and providing a higher payload/robot weight ratio, [3], [4], [5].

But, flexibility is an undesirable characteristic of robotic manipulators because it leads to challenging control problems such as static deflections and vibrations.

If one does not want to sacrifice the advantages associated with lightness, one must develop precise models and efficient controllers.

A first experiment on the control of a single-link flexible robot moving in a plane was carried out in

[6]. The subject has seen enormous developments since then, with improved controls applied to rigid robotic bodies. The proportional derivative/proportional integral (PD/PID) controller is distinguished by its simplicity and efficiency [7], [8], the linear quadratic (LQR) controller [9], IA based PID control [10], hybrid PID-LQR control [11].

A significant problem arises when the peak payload can change. The controllers, generally set to a nominal payload, then perform poorly with other payload values. Many adaptive control strategies have been synthesized to solve this problem: in [12] proposed a robust tip trajectory tracking of a very lightweight single-link flexible arm in presence of large payload changes. [13] developed a position control of very lightweight single-link flexible arms with large payload variations by means of a disturbance. A robust adaptive control strategy was proposed for robotic manipulators with output constraints in [14]. More recently, [15] proposed a fixed-time passivity-based model-free adaptive control for robotic manipulators; [16] developed an adaptive neural network tracking control method whereas [17] investigated the adaptive control of robot manipulators with closed architecture. An interesting adaptive control scheme was also introduced in [18].

The application of such adaptive control methods to the 1 dof or single-link flexible manipulators has been realized in many works, [19], [20], [21].

Robot arms are widely used in industrial plants, as they provide more power, precision and rapidity to the manufacturing systems. Their control problem is among the most attractive and challenging research thematic, [22], [23].

Recently many automation researchers have focused their efforts on the introduction of fractional-order systems and operators in adaptive control schemes, and especially in model reference adaptive control (FOMRAC), [24], [25]. The main reason for this development is the ability of fractional-order operators to enhance the controlled system performance and robustness, [26].

Nowadays, there is a growing trend for utilization of FOMRAC in the solution of science and engineering problems. Many applications concern various aspects of physical sciences fields, such as mechanics, electricity, chemistry, biology, economics, modeling, time and frequency domain, system identification and notably control theory, mechatronics and robotics, with encouraging results and advantageous performance, [27], [28], [29].

There have also been some works introducing the FOMRAC approach in industrial robots control systems, particularly on robotic manipulators, [30], [31], [32].

They were able to demonstrate superior behavior of robotic systems when using fractional-order modelization and/ fractional order control, [33], [34]. Many categories of robotic systems were experimented like: Complex Soft Robot Motions [35], dual-arm robots [36], flexible link manipulator robot [37], articulated robotic arm [38], soft robotic arm [39], single link flexible robot with disturbances and payload changes [40], flexible-joint robots [41], mobile robot control [42], and so on.

The motivation of this research is to design a robust controller capable of reaching the precise final position of the flexible arm tip at one degree of freedom and at the same time reducing the resulting vibrations and thus compensating for payload variations. The adopted approach is essentially based on fractional order reference model adaptive control (FOMRAC), to improve the robustness of the closed-loop control system a parallel feed-forward path has been.

The remaining of this manuscript is organized as follows:
Section 2 presents the considered single-link flexible robot modeling. Basic elements of Fractional Calculus are then presented in section 3. In Section 4 the configuration of the fractional order MRAC controller is introduced whereas the proposed controller design is detailed in Section 5. Simulation results illustrating the efficiency and robustness of the FOMRAC controller are given in Section 6. Finally, Section concluding remarks are given in Section 7.

2 Modeling of Single-Link Flexible Manipulator

The proposed single-link flexible robot is illustrated in Figure 1, consists of two separate subsystems: the DC motor and the beam or link. The interaction between these parts can be ignored, as it will be compensated by the controller.

The flexible arm, which is cantilevered and carries a concentrated mass (represents the payload and/or gripper) at its tip, mounted directly on the vertical shaft of a DC motor, [43], [44].

First, the motor rotates the arm in a horizontal plane so that gravity can be ignored. Second, the arm is nearly massless and the mass of the payload is several times greater than that of the link.

Simultaneously, the mass of the payload is concentrated enough that its inertia can be ignored (no torque is produced at the tip).

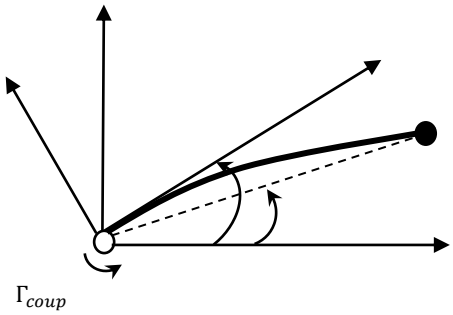

Fig. 1: Diagram of a single-link flexible arm

Based on the above considerations and according to [45], the mechanical system behaves practically like an ideal, single- degree-of freedom, undamped spring-mass system, can be expressed as follows:

$$\ddot{\beta} = \omega_n^2(\beta_m - \beta_t) \quad (1)$$

with ω_n representingthe natural resonance frequency of the beam, calculated as $\sqrt{\frac{c}{ml^2}}$, where m is the mass at the end (payload), l and c are the length and the stiffness of the beam respectively, β_m is the angle of the motor, and β_t is the angle of the tip.

The dynamic model of the DC motor can be obtained using Newton's second law:

$$K\, i(t) = I\ddot{\beta}_m + \mu\dot{\beta}_m + C_t(t) + C_f(t) \quad (2)$$

where K is a motor constant, $i(t)$ is the current of the direct drive motor in amperes, I is the motor inertia, μ is the viscous friction coefficient, $C_f(t)$is the Coulomb's friction torque and $C_t(t)$ is the coupling torque between the motor and the link, [46].

3 Basics of Fractional Calculus

The domain of Fractional Calculus (FC) initially based on the definition of integration and differentiation of arbitrary orders takes history with famous researchers like Cauchy, Riemann-Liouville and Leitnikov in the 19th century. Its use has affected both mechanics since at least the 1930s and electrochemistry since the 1960s. In the automatic control domain, the first works appeared in the Soviet Union in 1962 is [47], and later, a great number of scientists have been involved in fractional order systems research projects with application in various systems, [48].

Definitions of these concepts were diverse in mathematical literature. The most popular ones are those of Riemann-Liouville, Grünwald-Letnikov and Caputo, [49].

3.1 Definition of Fractional Integration

Riemann-Liouville defined the fractional-order integral as:

$$D_a^{-\gamma} f(t) = \frac{1}{\Gamma(\gamma)} \int_a^t (t-\zeta)^{\gamma-1} f(\zeta) d(\zeta) \quad (3)$$

where the Gamma function given by:

$$\Gamma(x) = \int_0^\infty y^{x-1} e^{-y}\, dy \quad (4)$$

with $(a, t) \in \Re^2$ with $a < t$ and $(0 < \gamma < 1, \gamma \in \Re)$.

This aspect of fractional systems has also been addressed extensively by the mathematical and engineering research community. Many discretization algorithms have been proposed to address this need with varying degrees of complexity and accuracy, [25], [50].

3.2 Numerical Approximation of the Fractional Order Integral

An obvious formulation of numerical approximation for Riemann-Liouville integral is used in this work based on the rectangular approximation method, [25].

By putting, $t = k\Delta$ where t is the actual time, k an integer and $\Delta$the sampling period, we obtain:

$$I^{\gamma}\big(f(k\Delta)\big) = \frac{\Delta^{\gamma}}{\Gamma(\gamma)} \sum_{\tau=0}^{k-1} (k-\tau)^{\gamma-1} f(\tau\Delta) \quad (5)$$

Another widely used numerical approximation is based on the Grünwald-Leitnikov definition, [49]. For a causal function $f(t)$, with the time operator $t = kh$, the fractional order derivative is computed by [51]:

$$D_t^{\alpha} f(t) = h^{-\alpha} \sum_{r=0}^{\frac{t}{h}} (-1)^r \binom{\alpha}{r} f(kh - rh) \quad (6)$$

The fractional order integral is given by:

$$I_t^{\alpha} f(t) = D_t^{-\alpha} f(kh) \approx h^{\alpha} \sum_{r=0}^{\frac{t}{h}} (-1)^r \binom{-\alpha}{r} f(khrh) \quad (7)$$

where $\binom{-\alpha}{r}$ is calculated from equation:

$$\binom{\alpha}{0} = 1, \quad \binom{\alpha}{r} = \frac{\alpha(\alpha-1)\cdots(\alpha-r+1)}{r!} \quad (8)$$

with $r \in N$ and h denotes the sampling period.

3.3 Approximation of Fractional-Order Transfer Function

In order to implement our adaptive regulators we need to approximate the fractional-order components by rational transfer functions. For this objective, we use the so-called singularity function method proposed in [51] which provides a simple algorithm for the approximation of transfer function with a single pole or two poles.

In this work we are concerned with a second-order plant model, and for the case of fractional first order-like system of the form:

$$G(s) = \left(\frac{1}{1+\frac{s}{p_T}}\right)^{\delta} \tag{9}$$

The method developed in [51] is based on the approximation of the Fractional Laplace Operator s^{δ} through a rational function of the form:

$$G(s) = k\prod_{k=1}^{N}\frac{s+z_k}{s+p_k} \tag{10}$$

The zeros and poles of the rational transfer function can be obtained by following these steps. The location ratios $a$and $b$are defined as:

$$a = 10^{\left(\frac{\varepsilon_p}{10(1-\delta\delta)}\right)} = \frac{z_0}{p_0} = \cdots = \frac{z_{N-1}}{p_{N-1}} \tag{11}$$

$$b = 10^{\left(\frac{\varepsilon_p}{10\delta}\right)} = \frac{p_1}{z_0} = \cdots = \frac{p_N}{z_{N-1}} \tag{12}$$

The total number of singularities, as determined by the frequency band of the system, is N + 1:

$$\mathrm{N} = \frac{\ln\left[\frac{\omega_{\max}}{\mathrm{p_0}}\right]}{\ln(\mathrm{ab})} + 1 \tag{13}$$

where $\omega_{max}$is the maximum frequency of the system and $p_0$is calculated as:

$$p_0 = p_T\sqrt{b} \tag{14}$$

The poles and zeros of the rational transfer function can be computed as follows:

$$P_j = (ab)^j p_0 \quad \mathrm{j} = 1,2,3,\ldots \mathrm{N} \tag{15}$$

$$Z_i = (ab)^i a p_0 \qquad i = 2,3,\ldots N-1 \tag{16}$$

4 Fractional-order Model Reference Adaptive Control

FOMRAC is an extension of MRAC control scheme to the fractional order systems. Since the first propositions in [24] and [25], many authors have proposed FOMRAC schemes based on the Lyapunov stability theorem or the gradient approach.

One can cite the works [52] in discrete time, [53] for a FOMRAC algorithm where the orders are switched among a fractional value in the interval (0, 1) and 1 at certain time instants, [54] for a scheme with an adaptive $PI^{\lambda}D^{\mu}$ controller, and many others... In MRAC approach, the desired performance is expressed in terms of a reference model (a model that describes the desired input-output properties of the closed-loop system) and the parameters of the controller are adjusted based on the error between the reference model output and the system output, [55].

The model reference adaptive control system can be represented by the scheme of Figure 2.

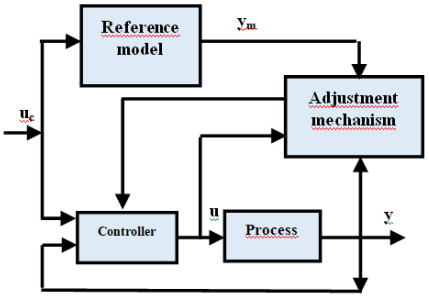

Fig. 2: Direct model reference adaptive control scheme

4.1 Direct MRAC with MIT rule

The plant model is given as:

$$y = G.u = k_p\frac{N(s)}{D(s)}u \tag{17}$$

where y is the system output and u is the control input.

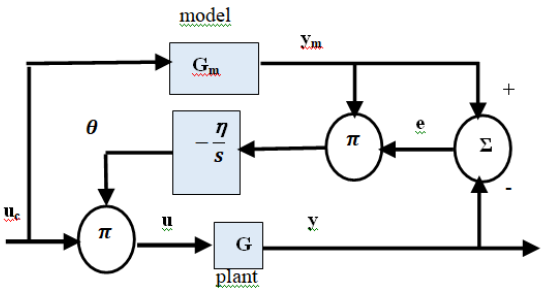

Fig. 3: Adaptation algorithm

Consider a closed loop system (Figure 3) where the controller has an adjustable parameter vector θ. We choose a reference model"

$$y_m(t) = G_m.u_c = k_m\frac{N_m(s)}{D_m(s)}u_c \tag{18}$$

which output is y_m specifies the desired closed loop response and u_c the reference signal. Let e be the error between the closed loop system output y and the model one y_m, we adjust the parameters such that the cost function:

$$J(\theta) = \frac{1}{2}e^2 = \frac{1}{2}(y - y_m)^2 \quad (19)$$

is minimized. Thus, the parameters are changed in the direction of negative gradient of J, so:

$$\frac{d\theta}{dt} = -\eta\frac{\partial J}{\partial \theta} = -\eta e\frac{\partial e}{\partial \theta} \quad (20)$$

where θ is the parameter vector and J the quadratic error criterion. We get [56],

$$\theta = -\frac{\eta}{s}y_m e \quad (21)$$

The control law is calculated using this relation,

$$u = \phi^T.\theta \quad (22)$$

where ϕ is the regression vector containing the measured input (control) u and output y signals and the input reference signal u_c.

5 Controller Design

The control system consists of two nested loops: an inner loop for the motor position and an outer loop for the tip position. We first close the inner loop for the motor position after compensating for the friction and coupling between the tip and the motor position, and then closing the outer loop for the tip position, [56]. The control signal generated by the outer loop is the motor position reference for the inner loop. The general control scheme is shown in Figure 4.

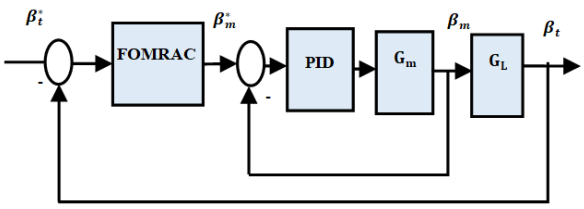

Fig. 4: General Control Scheme

5.1 Motor Position Control Loop (Inner Loop Controller)

We want to achieve two goals with this loop.

- To remove modeling errors and any nonlinearities introduced by Coulomb friction.
- To make the response of the motor position much faster than the response of the tip position control loop (outer loop in Figure 4), so that its transfer function can be approximated to one.

To simplify the design of the inner loop, the system described in equation (1) can be linearized by compensating for Coulomb friction and coupling torque as illustrated in Figure 5.

Since the beam is nearly massless, the coupling torque can be approximated by

$$C_t(t) = c\big(\beta_m(t) - \beta_t(t)\big) \quad (23)$$

and the coulomb friction is:

$$C_f(t) = f.sign\big(\dot{\beta}_m\big) \quad (24)$$

where f is the Coulomb friction coefficient.

These terms, can be removed from the differential equation by adding them to the control current of the motor.

$$i_c(t) = \frac{1}{K}\big[c(\beta_m - \beta_t) + f.sign\big(\dot{\beta}_m\big)\big] \quad (25)$$

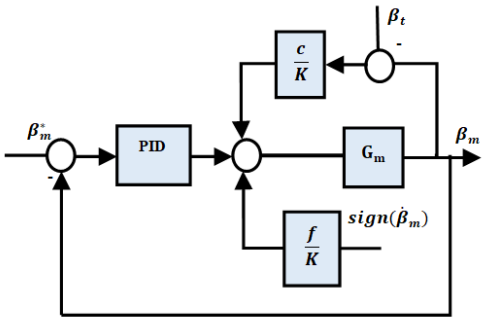

Fig. 5: Inner Loop Control Scheme

After compensating for the friction and coupling torque, the transfer function between the angle of the motor and the current is given by

$$G_m(s) = \frac{\beta_m(s)}{i(s)} = \frac{K/I}{s(s+\mu/I)} \quad (26)$$

The fulfillment of the objectives allows us to substitute the inner loop by an equivalent block whose transfer function is approximately equal to one; i.e., the error in motor position is small and is quickly removed. The controller PID is responsible for eliminating modeling errors and any nonlinear dynamics caused by Coulomb friction. Its goal is to ensure that the motor position has a faster dynamic than the tip position, so that its transfer function can be approximated to one.

The step response of the motor position in Figure 6 shows that the settling time is 0.1 seconds and no overshoot is observed, indicating that the motor position control system provides both fast and stable behavior in response to the input step command.

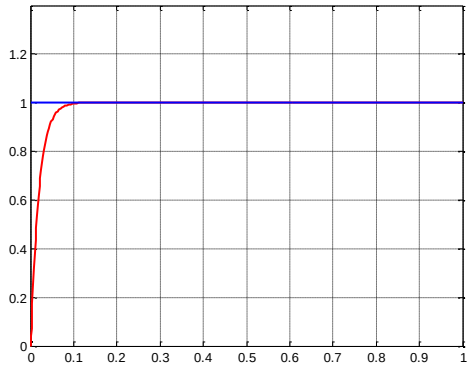

Fig. 6: Inner Loop Step Response.

5.2 Tip Position Control Loop (Outer Loop Controller)

A block diagram of the tip position control system may be drawn as shown in Figure 4. The tip position control system contains inside its loop the motor position control system. The transfer function of the motor control system was assumed to be equal to 1 for the sake of simplicity in the design of the tip controller.

As previously underlined, the decoupling process is crucial because it helps to simplify the system dynamics to that of a double integrator. This is done simply by closing a positive feedback unity gain loop around the tip position.

$$G(s) = \frac{G_L(s)}{1 - G_L(s)} = \frac{\omega_n^2}{s^2} \qquad (27)$$

The response is shown in Figure 7 and Figure 8.

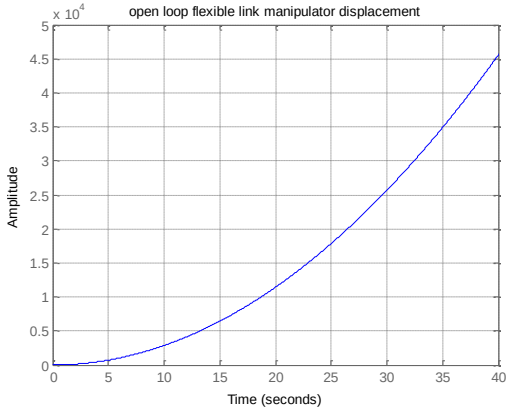

Fig. 7: Open-loop FL arm displacement

The outer loop gives a fast response of the tip position by adding feed-forward term and an output filter significantly reduces oscillations in the overall system response and minimizes the risk of instability. We chose to utilize a Bode diagram to select a feed-forward block and a filter that smoothes the system's response and improves the phase margin of the closed-loop system (Figure 9 and Figure 10).

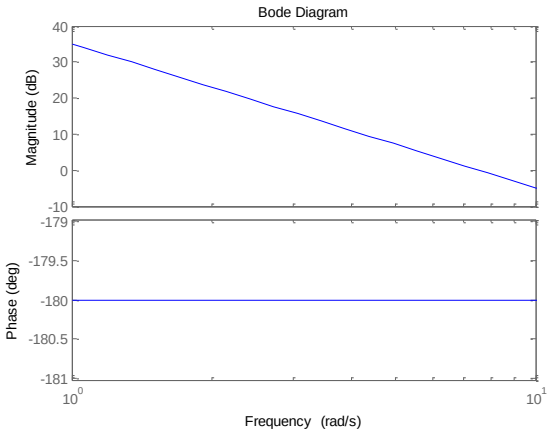

Fig. 8: Open-loop bode diagram for FL arm

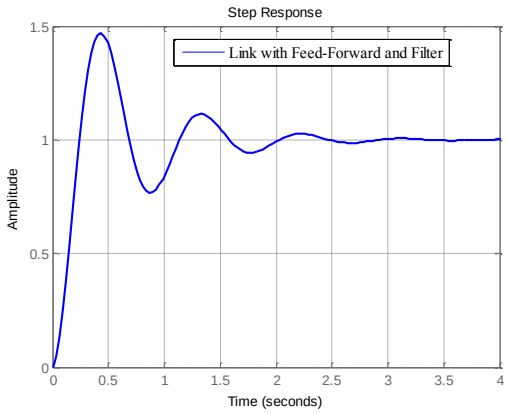

Fig. 9: Open-loop displacement for FL arm with FF & filter

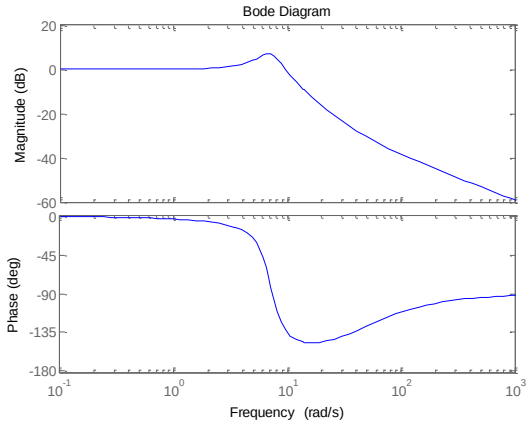

Fig. 10: Open-loop bode diagram for FL arm with FF & filter

The function of the outer loop controller is to suppress the tip vibration and accurately track the desired trajectory. In addition, it is robust enough to changes in the tip payload.

In this section, an outer loop control strategy based on a robust FOMRAC is developed to accomplish the above two tasks and overcome the defects of existing PID-type control methods.

The final control scheme for the outer loop is shown in Figure 11.

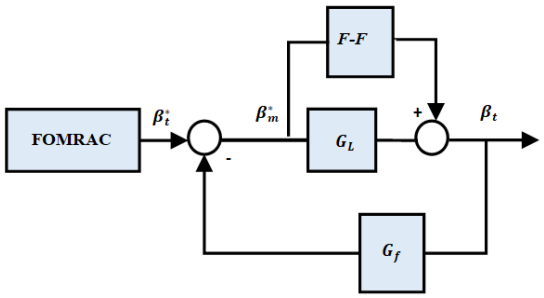

Fig. 11: Outer Loop Control Scheme

6 Simulation Results and Discussion

In this section, we will demonstrate the effectiveness of the developed method by showing some numerical simulations. The numerical simulations are realized by MATLAB.

The nominal parameters of the flexible manipulator and the motor are shown in Table 1. The parameters are all taken from [13].

Table 1. The parameters of the single-link flexible manipulator and DC motor

DC Motor Parameters		
Motor inertia	I	6.87x10^{-5} (kg m^2)
Viscous friction	μ	1.041x10^{-3} (Nm s)
Electromechanical constant	K	0.21((N m)/V)
Motor Coulomb friction torque	f	119.7x10^{-3} (N m)
Flexible Manipulator rameters		
Nominal mass	m_0	0.05 (kg)
Length	l	0.7 (m)
Stiffness	c	1.395 (N m)
Nominal natural frequency	ω_n	7.546 (rad/s)

To define the desired characteristics of the tip position dynamics and response, we first choose an integer-order reference model of the form:

$$G_{em}(s) = \frac{1}{0.04s+1} \tag{28}$$

In our case, we opted for a fractional-order reference model mainly to exploit the robustness of fractional computation in improving the performance of the control system, in particular to minimize the tracking error, and to increase the robustness against variations in plant parameters.

$$G_{fm}(s) = \frac{1}{(0.04s+1)^{\delta}} \tag{29}$$

In order to evaluate the performance of the different control schemes results, we consider the following index functions on the simulation window:

The quadratic error criterion *Je*:

$$J_e = \sqrt[2]{\sum e^2} \tag{30}$$

The quadratic control input (energy) criterion *Ju*:

$$J_u = \sum u^2 \tag{31}$$

The combined quadratic error and energy criterion *Jeu*:

$$J_{eu} = \sqrt{\lambda \sum e^2 + \upsilon \sum u^2} \tag{32}$$

where λ and υ are two positive real numbers to be chosen.

In Table 2, we present the comparative results obtained for the criteria J_e, J_u and J_{eu} with $\lambda = 0.6$ and $\upsilon = 0.4$ when applying different values of δ from 0.1 to 0.9 with adequate values of the adaptation gain. It can be noted from the values presented in Table 2 that the best value resulted for δ is 0.9, for which the quadratic error criterion and the combined quadratic criterion are equal to $J_e =$ $\mathbf{140.3412}$, $J_{eu} =$ $\mathbf{270.2767}$ respectively.

Table 2. Adaptive control comparative performance evaluation vs integration order

Control scheme	J_e	J_{eu}	T_s	Peak overshoot
MRAC	*149.4739*	273.7596	0.773	26.5263
FOMRAC				
$\boldsymbol{\delta = 0.1}$	145.2324	271.824	0.5735	28.5098
0.2	144.8283	271.6944	0.5735	28.5098
0.3	144.2532	271.5103	0.5735	28.5098
0.4	143.536	271.2823	0.5735	28.5098
0.5	142.7684	271.039	0.5735	28.5099
0.6	141.9832	270.7913	0.5735	28.5099
0.7	141.2375	270.557	0.5735	28.5099
0.8	140.6223	270.3645	0.5735	28.51
0.9	***140.3412***	***270.2767***	0.5735	***28.51***

We get the approximate model using the singularity function method introduced in Section 3, with $\delta = 0.9$ and a tolerance of 1.5 dB

$$G_{fm}(s) \approx \frac{\begin{array}{c}2.456\ 10^{58}\ s^7+2.31810^{71}\ s^6+4.614\ 10^{82}\ s^5 \\ +1.978\ 10^{92}\ s^4+1.826\ 10^{100}\ s^3+3.634\ 10^{106}\ s^2 \\ +1.55710^{111}\ s+1.407\ 10^{114}\end{array}}{\begin{array}{c}5.72910^{55}\ s^8+7.937\ 10^{68}\ s^7+2.319\ 10^{80}\ s^6+1.459\ 10^{90}\ s^5 \\ +1.97810^{82}\ s^4+5.776\ 10^{104}\ s^3+3.634\ 10^{109}\ s^2 \\ +4.924\ 10^{112}\ s+1.407\ 10^{114}\end{array}} \tag{33}$$

This fractional order model is discretized with a sampling period $\Delta = 0.05$ s. The sampling period Δ has been chosen in accordance to the Shannon

theorem, so that all the useful information is saved in the discretized signals.

6.1 Simulation Results with $\mathbf{m = m_0}$

In the first, we start with simulations of the ideal case; we apply the MRAC and FOMRAC control laws to the flexible robot system.

Figure 12 shows the responses obtained for the case of integer order model and fractional order model whereas the tracking error is depicted in Figure 13 for $\beta_t^* = 25^o$. From the numerical simulations, we can notice that the performance level is better with the FOMRAC in terms of response time and tracking error.

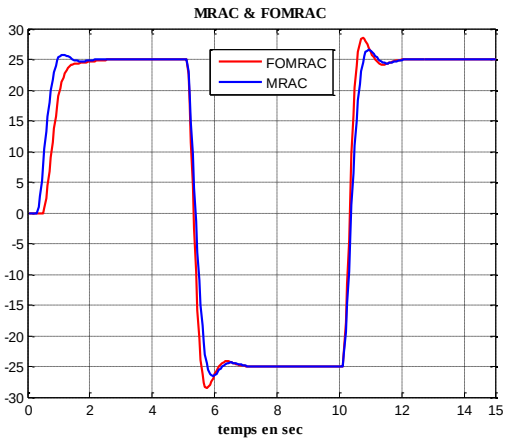

Fig. 12: Tip position response for MRAC and FOMRAC with $\beta_t^* = 25^o$

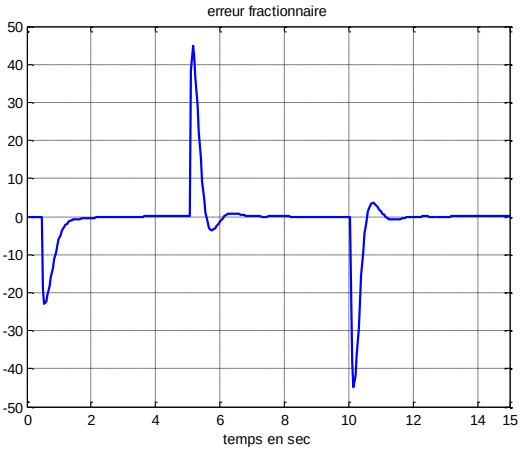

Fig. 13: Tracking error for $\beta_t^* = 25^o$

6.2 Robustness Test

Since the controller was designed for an ideal case and a payload of 50g ($\omega_n = 6.61$rad/s).

This section mainly tests the robustness of the proposed controller by considering the possible effects, if the system contains unmodeled dynamic behavior caused by tip mass variation or by time-varying disturbances such as wind force, base movement during outdoor operation, sensor noise, which may lead to unwanted vibrations and inaccuracies in the end-effector trajectory, ultimately affecting the overall performance and accuracy of the manipulator.

6.2.1 Payload Variations

Firstly, we changed the value of the natural frequency ω_n to simulate the system response for two different values 10.67 rad/s and 6.38 rad/s for masses of 25g and 70g, respectively.

The simulation results are shown in Figure 14 and Figure 15. The adaptive control scheme, applied in both fractional and integer order cases, effectively allows the manipulator to adapt to the uncertainties arising from payload variations. Furthermore, from the simulation results, we can see that FOMRAC is the most robust, both in terms of stability and tracking performance, because its tracking accuracy is the highest and the tracking trajectory is almost unchanged with the tip mass varying.

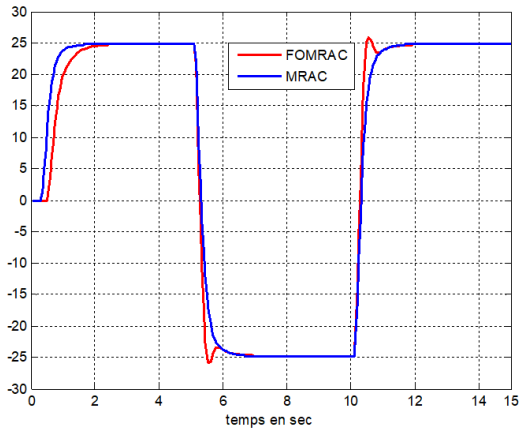

Fig. 14: Tip position response for a payload of 25 g and $\beta_t^* = 25^o$

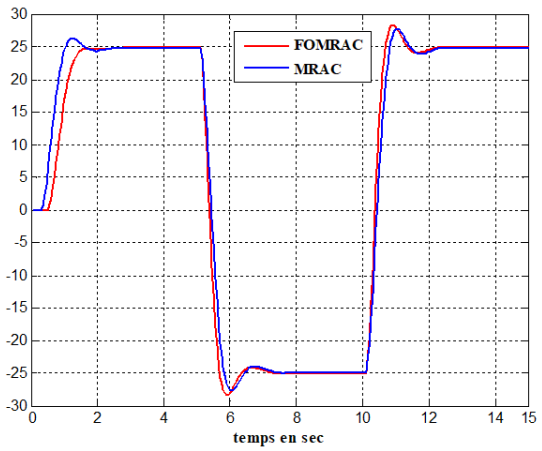

Fig. 15: Tip position response for a payload of 70 g and $\beta_t^* = 25^o$

6.2.2 Disturbance and Noise Rejection

In addition, additive output random noises of 1% reference signal amplitude were added to the system, and the simulation results are shown in Figure 16. It illustrates also the robustness of this adaptive control scheme against additive output noises, [57].

Under the same disturbance, Table 3 clearly shows the improvement obtained with the proposed fractional order adaptive control scheme.

Table 3. Comparative performance evaluation of the different adaptive control schemes

Control scheme	J_e	J_u	J_{eu}	Peak overshoot
MRAC ideal	*149.4739*	1.5385 10^5	273.7596	26.5263
MRAC +payload 25g	131.9042	1.5901 10^5	272.1079	0
MRAC +payload 70g	161.6182	1.3966 10^5	267.4617	27.6762
MRAC + noises	148.6415	1.5404 10^5	273.6268	27.1441
FOMRAC Ideal	140.3412	1.5308 10^5	270.2767	28.51
FOMRAC +payload25g	124.0903	1.5814 10^5	269.2472	25.9207
FOMRAC +payload 70g	152.548	1.3984 10^5	264.3846	28.3753
FOMRAC + noises	139.5421	1.5291 10^5	269.9051	28.391

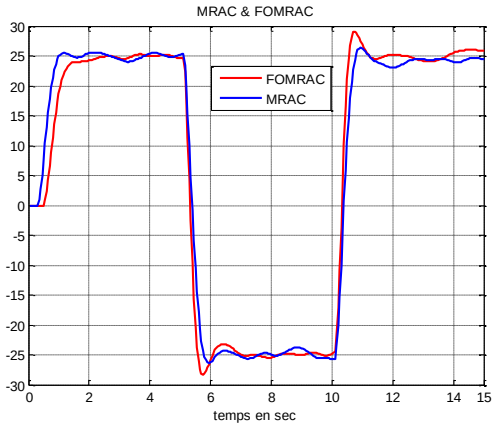

Fig. 16: Tip position response in the presence of output noise, $\beta_t^* = 25^o$

7 Conclusion

In this paper, we introduce a robust control scheme for a lightweight flexible single link manipulator with friction in the joint and a mass attached to the tip.

This scheme is composed of two nested control loops plus an adaptation loop. The inner loop is designed to compensate for friction and to make the motor response fast. The outer loop decouples the dynamics of the system (reduces its transfer function to a double integrator), and for improved stability and robustness, a parallel feed-forward path and a feedback filter are included to reduce oscillations. The fractional-order model reference adaptive control (Tip position control loop) is designed in this paper.

According to the simulation results, the proposed method is excellent in terms of tracking accuracy, robustness to payload variations and disturbance rejection compared with the classical integer order control system.

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The authors equally contributed in the present research, at all stages from the formulation of the problem to the final findings and solution.

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Conflict of Interest

The authors have no conflicts of interest to declare.