Solving Fully Fuzzy Linear Programming Problems using The Fuzzy Chaos Approach

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Abstract: - In this paper, an application of fuzzy chaos to a fuzzy linear programming problem with fuzzy goals, fuzzy constraints, and fuzzy variables is proposed. The application of the fuzzy chaos approach to a fuzzy linear programming problem is demonstrated in a concrete case. The model is illustrated with a numerical example of the maximization of the profit of a manufacturing company. The optimal solution to the investigated problem is discussed. The performed investigations show that centroids of the fuzzy numbers, satisfying the constraints can exceed the optimal crisp values and, consequently, the optimal value of the crisp objective function. On the other hand, the fuzzy representation of the objective function also contributes to the centroid of the optimal value of the objective function which can slightly exceed the optimal crisp value. It should be noted that when optimizing certain real processes, the representation of both the constraints and the objective function might be fuzzy. Therefore, it is purposive to solve the problem using the fuzzy approach without its conversion to the crisp linear programming problem, which is more effective. The discussion of the numerical example results shows that it is purposeful to solve the optimization problem using the fuzzy approach without its conversion to the crisp linear programming problem, which is more effective.

Key-Words:- linear programming; fuzzy chaos; fuzzy simulation; fully fuzzy linear programming; fuzzy constraints; fuzzy numbers.

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1 Introduction

The first technique to solve the fuzzy linear programming problem was introduced in [1]. To solve linear programming problems with fuzzy goals three approaches are described in [2]. Linear programming problem with fuzzy variables is considered in [3], [4] and [5]. Linear programming with fuzzy constraints is considered in [4], [6] and [7]. It should be noted that a majority of these efforts transform fuzzy linear programming problems into clearly defined, traditional linear programming problems.

It is reasonable to optimize current procedures employing the objective function and the constraints of fuzzy representation. By doing this, the fuzzy problem of linear programming is solved and not converted to a classical problem form of linear programming. This approach improves the efficiency a lot. In [8], a method is proposed to find the most suitable solution to fully fuzzy linear programming (FFLP) problems with all parameters and variables represented by *LR* flat fuzzy numbers. The sorting relationship in the objective function is the same as the constraint inequalities. Based on the definition of the order relationship, FFLP can be converted to equivalent to multi-objective linear programming (MOLP), which is an easy-to-solve crisp programming. Taking into consideration the MOLP problem, the classical fuzzy programming method was modified to obtain the optimal solution.

A general model for resolving the system of *n* fuzzy linear equations with *n* fuzzy variables and the FFLP problem is described in [9]. This study also presents a requirement for the existence of a positive fuzzy solution to the fully fuzzy linear system (FFLS). It is clear from a detailed comparison of the suggested approach with the ones that already exist that the suggested method is more appropriate and general.

A non-negative solution of a fully fuzzy generalized system of linear equations is given in [10]. In this regard, two new methods based on single and double parametric forms of fuzzy numbers are presented. The double parametric approach was found to be easy and straightforward. This method keeps the order of the original system without any changes. Triangular and trapezoid fuzzy numbers were considered for the analysis.

Portfolio management to maximize return while minimizing unsystematic risk is considered in [11]. Fundamental definitions of this problem are given in fuzzy logic theory and fuzzy logic approach. In the fuzzy logic model price/earnings ratio and accumulation/distribution index are added according to the model developed by Werner.

A technique for utilizing *LR* fuzzy numbers to tackle the fully fuzzy programming problem has been presented in [12]. Fuzzy calculus is employed to convert the FFLP problem into an MOLP problem, which is subsequently dealt with using the lexicography approach along with precise linear programming.

In [13], a voltage division circuit problem is modeled to determine the values of the centered resistors, so that the resistance voltage divider impedance will be minimal. This problem is equivalent to maximizing the resistance-related admittance, which is defined as the part of the electrical current and voltage measured at Siemens. For the components of linear programming, three cases were analyzed in this study: real numbers, fuzzy numbers of type-1, and fuzzy sets of type-2.

The work [14] focuses on solving a fully fuzzy linear programming problems that involve triangular fuzzy integers as parameters. This is achieved by using a new lexicographic ordering based on triangular fuzzy numbers and transforming the FFLP into a multi-objective linear programming problem with three objective functions, building upon an existing solution approach. When imprecise data are represented as non-negative triangular fuzzy numbers, this research provides a new technique for reducing computing complexity in certain existing FFLP problems.

The study [15] examines a new technique that employs triangular fuzzy numbers to resolve FFLP problems involving fuzzy decision parameters and variables. To obtain the optimal completely fuzzy solution for real-world problems, the authors suggest using a revised triangular fuzzy number method based on alpha-cut theory. This technique considers the issue to be a fully fuzzy one. The objective function and decision variables are then optimized using the triangular fuzzy number's recently proposed definition.

In this paper, an application of fuzzy chaos to fully fuzzified linear programming with fuzzy goals, fuzzy constraints, and fuzzy variables is presented. It should be mentioned that fuzzy chaos is receiving a great deal of attention in works [16], [17], [18], but with a small amount of applications.

In this study, a practical instance is utilized to exemplify the fuzzy chaotic approach to fuzzy linear programming issues. The consequences of this method display considerable efficiency when applied to linear programming problems.

2 Optimization Problem Statement

A fuzzy linear programming issue with fuzzy variables, fuzzy constraints, and fuzzy goals will be discussed:

$$\max \tilde{Z} = \tilde{c}\tilde{x} \tag{1}$$

$$A\tilde{x} \le \tilde{b} \tag{2}$$

$$\tilde{x} \ge 0 \tag{3}$$

where $\tilde{c} = (\tilde{c}_1, \tilde{c}_2, ..., \tilde{c}_n)$, $\tilde{x} = (\tilde{x}_1, \tilde{x}_2, ..., \tilde{x}_n)$, $\tilde{b} = (\tilde{b}_1, \tilde{b}_2, ..., \tilde{b}_m)$ are vectors of fuzzy numbers (for example, triangle, trapezoidal) and A=(a_{ij}) is an $m \times n$ matrix of fuzzy numbers.

Unlike the crisp linear programming problem, there is no easy way to solve the problems (1) - (3). There is no algorithm available to determine the precise value of the optimal $\tilde{x}_i, i = \overline{1, n}$, even if there exists an "optimal" value of $\tilde{x}_i, i = \overline{1, n}$. This section examines a approximate solution to the (1)–(3) problem using chaotic generation of fuzzy numbers $\tilde{x}_i, i = \overline{1, n}$.

With preserving generality, for simplicity, we represent $\tilde{x}_i, i = \overline{1, n}$ as the fuzzy triangular numbers. Let's note that a triangular fuzzy number $\tilde{N}$ can be described by three numbers a<b<c, where 1) $\mu_{\tilde{N}}(x) = 0$ for x≤a or x≥c; 2) $\mu_{\tilde{N}}(b) = 1$; 3) the graph of $\mu_{\tilde{N}}(x)$ is a straight-line segment from (a,0) to (b,1) on [a,b] and a straight-line segment from (b,1) to (c,0) to on [b,c]. We can write $\tilde{N}$ =(a,b,c) for triangular fuzzy numbers.

3 Fuzzy Chaos Approach to Fuzzy Linear Programming Problem

In [19], which covers methods for creating chaotic fuzzy sets, fuzzy chaos is applied to optimization, fuzzy programming, and fuzzy maximum flow. This study constructs a series of fuzzy integers using the following approach: Chaotic numbers are produced by applying the subsequent chaotic function:

$$f(x)=\lambda x(M-x)/M \tag{4}$$

where *f* maps [0,M] into [0,M].

Assume that we want to generate a chaotic sequence of triangular fuzzy number set $\tilde{x}_n = (a_n, b_n, c_n)$ in [0,M]. If $\tilde{x}_{n+1} = (a_{n+1}, b_{n+1}, c_{n+1})$, then choosing $\lambda_i, i = \overline{1,3}$ such that *f* is chaotic and considering that $f_i(x) = \lambda_i x(M - x)/M, i = \overline{1,3}$, the values of $a_{n+1}, b_{n+1}, c_{n+1}$ can be determined from the following relationships, [4]:

$$\begin{aligned} a_{n+1} &= \min\{f_1(a_n), f_2(b_n), f_3(c_n)\} \\ c_{n+1} &= \max\{f_1(a_n), f_2(b_n), f_3(c_n)\} \end{aligned} \tag{5}$$

b_{n+1} is the middle value of $f_1(a_n), f_2(b_n), f_3(c_n)$.

For every current chaotically generated set of fuzzy numbers, $\tilde{x}_i, i = \overline{1, n}$ (n is the number of fuzzy numbers) we check if the fuzzy constraints of this linear programming problem are satisfied. This checking for every fuzzy constraint is performed. As a result of performing particular operations over fuzzy variables in the left part of the fuzzy constraint, we get the fuzzy number $\tilde{N}$. Note that all possible operations over fuzzy numbers are described in detail in [20]. The right part of the fuzzy constraint is represented as the fuzzy number $\tilde{M}$.

Checking of the set of fuzzy numbers $\tilde{x}_i, i = \overline{1, n}$ if they satisfy the fuzzy constraint is the same as checking if the inequality takes place. Given two fuzzy numbers $\tilde{M}, \tilde{N}$ and a parameter x constrained by $\tilde{M}$, the possibility and necessity measures that $\tilde{M}$ is greater than or equal to $\tilde{N}$ are defined as follows, [21]:

$$PG(\tilde{M},\tilde{N}) = \sup_x \min(\mu_{\tilde{M}}(x), \mu_{[\tilde{N},+\infty)}(x)) \tag{6}$$

$$NG(\tilde{M},\tilde{N}) = \inf_x \max(1-\mu_{\tilde{M}}(x), \mu_{[\tilde{N},+\infty)}(x)) \tag{7}$$

Definition [21]. The set of numbers possibly greater than or equal to $\tilde{A}$ is denoted as [$\tilde{A}$,+∞) with membership function:

$$\mu_{[\tilde{A},+\infty)}(y) \equiv \sup_{x \le y} \mu_{\tilde{A}}(x) \tag{8}$$

(6) implies the grade of the possibility of the proposition " $\tilde{M}$ as it is greater than or equal to $\tilde{N}$ ". It estimates the maximum chance that an event " $\tilde{M} \ge \tilde{N}$ " will occur. Eq. (7) implies the grade of the necessity of the proposition " $\tilde{M}$ is greater than or equal to $\tilde{N}$ ". It provides an index that estimates the minimum chance of an event " $\tilde{M} \ge \tilde{N}$ " occurs.

Next, the approach used to compare fuzzy temporal parameters is presented. Given two fuzzy temporal parameters $\tilde{M}$ and $\tilde{N}$, the degree that $\tilde{M}$ is greater than or equal to $\tilde{N}$ is defined as the weighted sum of PG($\tilde{M}, \tilde{N}$) and NG($\tilde{M}, \tilde{N}$), [21]:

$$g(\tilde{M} \ge \tilde{N}) = \beta \times PG(\tilde{M},\tilde{N}) + (1-\beta) \times NG(\tilde{M},\tilde{N}) \tag{9}$$

where β is the optimism-pessimism index, $0 \le \beta \le 1$.

A researcher's mindset determines parameter β. If β is found to be larger than 0.5, it indicates that he has an optimistic mindset. Conversely, if β is smaller than 0.5, it indicates that he has a pessimistic mindset. After $g(\tilde{M} \ge \tilde{N})$ is determined, the relationship between $\tilde{M}$ and $\tilde{N}$ can be identified by the following decision rules, [21]:

$$\begin{aligned} &\text{if } g(\tilde{M} \ge \tilde{N}) > g(\tilde{N} \ge \tilde{M}) \text{ then } \tilde{M} \ge \tilde{N} \\ &\text{else if } g(\tilde{M} \ge \tilde{N}) < g(\tilde{N} \ge \tilde{M}) \text{ then } \tilde{N} \ge \tilde{M} \end{aligned} \tag{10}$$

4 Algorithm for the Problem Solution

The following describes the algorithm used to solve problems (1)–(3) using the chaotic technique.

1. Generate chaotically an initial set of fuzzy numbers $\tilde{x}_i, i = \overline{1,n}$. Test $\tilde{x}_i, i = \overline{1,n}$ to see if they are triangular fuzzy numbers (or trapezoidal, if we describe $x_i, i = \overline{1,n}$ by trapezoidal fuzzy numbers).
2. Verify that the constraints (2) and (3) are satisfied by the test set of generated fuzzy numbers. If yes, proceed to step 3; if not, create a new set of fuzzy numbers $\tilde{x}_i, i = \overline{1, n}$ in a chaotic manner.
3. Determine $\tilde{Z}$ and save the objective function's current value.
4. Generate chaotically a new set of fuzzy numbers $\tilde{x}_i, i = \overline{1, n}$, test this set of fuzzy numbers in accordance with step 2.
5. Compute the current value of $\tilde{Z}^c$ and compare to the previous value of $\tilde{Z}$. If $\tilde{Z}^c$ is better than the previous $\tilde{Z}$, then replace the previous "best" value of $\tilde{Z}$ by $\tilde{Z}^c$.

6. Repeat the process until the objective function does not improve any further.

5 Case

Let's examine a brief example to gain a better understanding of when and how fuzzy logic is employed in linear programming.

For the upcoming year, the Manufacturing Company needs to decide on the best combination of its commercial products, B and C. The B and C product lines are produced by the company. The average profit is \$400 for each B and \$800 for each C. However, fabrication and assembly are constricted by available resources, with a monthly capacity of 5,000 hours (three hours for each B and five hours for each C). What is the optimal number of units of each product to manufacture each month to maximize profit, given a monthly assembly capacity maximum of 3,000 hours, with one hour required to assemble each B unit and four hours required to assemble each C unit?

Solution. The objective function is to maximize

$$Z=400x_1 + 800x_2, \tag{11}$$

where Z = the monthly profit from B and C; x_1 = the number of B produced each month; x_2 = the number of C produced each month.

The constraints for the solution of the linear programming problem are presented in Table 1.

Table 1. The constraints for the linear programming problem solution

B(x_1)	C(x_2)		
Required Time/Unit	Required Time/Unit	Available Time/Month	
3	5	5,000	Fab
1	4	3,000	Assy

The company manager formulates the production scheduling problem as follows:

$$3x_1 + 5x_2 \le 5{,}000 \quad \text{Fab} \tag{12}$$

$$x_1 + 4x_2 \le 3{,}000 \quad \text{Assy} \tag{13}$$

$$x_1, x_2 \ge 0 \quad \text{Nonnegativity}$$

However, as the company manager has the power to request overtime, the total number of labor hours available may not be entirely accurate. Hence, there can be some flexibility in the restrictions. Therefore, the resource definition is where the fuzziness becomes apparent.

It may not be apparent that B and C generate profits of 400 and 800 units per unit, respectively, due to the market's constant instability. Therefore, these figures can only be interpreted as maximum potential values. Using fuzzy integers to represent objective coefficients makes sense in light of this. The value of labour hours required to produce B and C can only be estimated to be around three and five units, respectively, due to human effort's inconsistency. As such, using fuzzy integers to represent these coefficients is appropriate.

Let's consider the following example of the fuzzy linear programming.

$$\tilde{Z} = (300,400,550)\tilde{x}_1 + (700,800,950)\tilde{x}_2 \tag{14}$$

$$(2,3,5)\tilde{x}_1 + (4,5,7)\tilde{x}_2 \le (3000,5000,7500) \tag{15}$$

$$(0,1,3)\tilde{x}_1 + (3,4,6)\tilde{x}_2 \le (1000,3000,5500) \tag{16}$$

$$\tilde{x}_1, \tilde{x}_2 \ge 0 \tag{17}$$

The initial set of fuzzy numbers $\tilde{x}_{01}$, $\tilde{x}_{02}$ are chosen randomly. Let them be as $\tilde{x}_{01}$=(550, 600, 700), $\tilde{x}_{02}$=(150,200,400). Note that each of the three numbers inside the parentheses must take values from the interval [0, M]. Here we take M=1000.

Before we calculate the value of the objective function for this set of fuzzy values, let's check if they satisfy the constraints (15) and (16) because $\tilde{x}_{01}$, $\tilde{x}_{02}$ ≥0. First, we apply them to (15)

(2,3,5)(550,600,700)+(4,5,7)·(150,200,400)≤ (3000,5000,7500)

By using the operations of summation and multiplication described in detail in [15], we have the fuzzy number $\tilde{N}_1$ at the right hand of this inequality

$\tilde{N}_1$ =(2,3,5)·(550,600,700)+(4,5,7)·(150,200,400)
≈(2·550, 3·600, 5·700)+(4·150, 5·200,7·400)
=(1100, 1800, 3500)+(600, 1000, 2800)=
(1100+600,1800+1000, 3500+2800)
=(1700,2800,6300)

The right part of this inequality can be represented as $\tilde{M}_1$ =(3000,5000,7500). Thus, checking if the fuzzy numbers satisfy the constraint (15) is the same process as checking if the inequality $\tilde{N}_1 \le \tilde{M}_1$ is true.

Let's first calculate $PG(\tilde{M}_1, \tilde{N}_1)$. Based on formula (6) as a result of operations done in the computer simulation of this problem, we have

$PG(\widetilde{M}_1, \widetilde{N}_1) =$ 1.000. Then, based on formula (7), we calculate $NG(\widetilde{M}_1, \widetilde{N}_1)$ =1.000.

Using the formula (9), we determine the weighted sum of $PG(\widetilde{M}_1, \widetilde{N}_1)$ and $NG(\widetilde{M}_1, \widetilde{N}_1)$
$g(\widetilde{M}_1 \geq \widetilde{N}_1)$ =β× $PG(\widetilde{M}_1, \widetilde{N}_1)$ +(1-β)×
$NG(\widetilde{M}_1, \widetilde{N}_1)$ =.1.000.

In this problem, we assign β as the value 0.6.

Similarly, we determine $PG(\widetilde{N}_1, \widetilde{M}_1)$ =0.6000, $NG(\widetilde{N}_1, \widetilde{M}_1)$ =0.0203 and $g(\widetilde{N}_1 \geq \widetilde{M}_1)$ =0.3681.

And finally, using inequalities (10) we prove that $\widetilde{N}_1 \leq \widetilde{M}_1$ because
$g(\widetilde{M}_1 \geq \widetilde{N}_1) > g(\widetilde{N}_1 \geq \widetilde{M}_1)$
0.6000>0.0203.

Then, we apply the values of $\widetilde{x}_{01}$, $\widetilde{x}_{02}$ into (16)
(0,1,3)·(550,600,700)+(3,4,6)·(150,200,400)≤(1000, 3000,5500)

In the left part of this inequality, we get the number $\widetilde{N}_2$ ≈(0·550, 1·600, 3·700)+(3·150, 4·200, 6·400)=(0+450, 600+800, 2100+2400)=(450, 1400, 4500).

The right part is the number $\widetilde{M}_2$ =(1000,3000, 5500).

Similarly, the satisfaction of the inequality $\widetilde{N}_2 \leq \widetilde{M}_2$ can be checked. We determine:
$PG(\widetilde{M}_2, \widetilde{N}_2)$ =1.000, $NG(\widetilde{M}_2, \widetilde{N}_2)$ =0.8644,
$PG(\widetilde{N}_2, \widetilde{M}_2)$ =0.6863, $NG(\widetilde{N}_2, \widetilde{M}_2)$ =0.1356,
$g(\widetilde{M}_2 \geq \widetilde{N}_2)$ =0.9458, $g(\widetilde{N}_2 \geq \widetilde{M}_2)$ =0.4660.

And lastly, using the inequalities (10) we prove that $\widetilde{N}_2 \leq \widetilde{M}_2$. Because
$g(\widetilde{M}_2 \geq \widetilde{N}_2) > g(\widetilde{N}_2 \geq \widetilde{M}_2)$
0.9458>0.4660.

These numbers satisfy both constraints, they can be applied to formula (14) for calculating the values of the objective function $\widetilde{Z}$. By using the operations of summation and multiplication of fuzzy numbers, described in detail in [1], for this set of fuzzy numbers $\widetilde{x}_{01}$=(550, 600, 700), $\widetilde{x}_{02}$ =(150,200,400), we get the following fuzzy value for the objective function:

$$\tilde{Z} = (300,400,550)\cdot(550,600,700) +$$
$$(700,800,950)\cdot(150,200,400) \approx$$
$$(300\cdot 550, 400\cdot 600, 550\cdot 700) +$$
$$(700\cdot 150, 800\cdot 200, 950\cdot 400) =$$
$$(165000,\ 240000,\ 385000) + (105000,\ 160000,\ 380000)$$
$$= (270000,\ 400000, 765000)$$

Using the chaotic function (4), we generate a sequence of fuzzy numbers. As three numbers characterize every fuzzy number, we use the three chaotic functions created based on function (4) for the chaotic generation:
$f_1(a_{n+1})=\lambda_1 a_n(M-a_n)/M$,
$f_2(b_{n+1})=\lambda_1 b_n(M-b_n)/M$,
$f_3(c_{n+1})=\lambda_1 c_n(M-c_n)/M$.

As it was noted above, in this problem, we take M=1000. [0, M] is the interval from which the three above functions get their values $\lambda_1, \lambda_2, \lambda_3$ must be chosen so that all these functions remain chaotic. In this example, $\lambda_1, \lambda_2, \lambda_3$ are assigned to 3.9, 3.95, 3.99, respectively.

Using the relation (5), we get the following set of fuzzy numbers $\widetilde{x}_{1i} = (a_{1i}, b_{1i,}, c_{1i})$, $i = \overline{1,2}$. Thus, we have:
for $\widetilde{x}_{11} = (a_{11}, b_{11}, c_{11})$
$f_1(a_{01})$=3.9·550·(1000-550)/1000=965.250
$f_2(b_{01})$=3.95·600·(1000-600)/1000=948.000
$f_3(c_{01})$=3.99*700·(1000-700)/1000=837.900
a_{11}=min{$f_1(a_{01})$, $f_2(b_{01})$, $f_3(c_{01})$}=min{965.250, 948.000, 837.900}=837.900
c_{11}=max{$f_1(a_{01})$, $f_2(b_{01})$, $f_3(c_{01})$}=max{965.250, 948.000, 837.900}=965.250
b_{11}=middle value of $f_1(a_{01})$, $f_2(b_{01})$, $f_3(c_{01})$=middle value of 965.250, 948.000, 837.900=948.000
for $\widetilde{x}_{12} = (a_{12}, b_{12}, c_{12})$
$f_1(a_{02})$= 3.9·150·(1000-150)/1000=497.250
$f_2(b_{02})$=3.95·200·(1000-200)/1000=632.000
$f_3(c_{02})$=3.99*400·(1000-400)/1000=957.600
a_{12}=min{ $f_1(a_{02})$, $f_2(b_{02})$, $f_3(c_{02})$}=min{497.250, 632.000, 957.900}=497.250
c_{12}=max{$f_1(a_{02})$, $f_2(b_{02})$, $f_3(c_{02})$}=max{497.250, 632.000, 957.900}=957.600
b_{12}=middle value of $f_1(a_{02})$, $f_2(b_{02})$, $f_3(c_{02})$=middle value of 497.250, 632.000, 957.600=632.000

Thus,

$\tilde{x}_{11}=(837.900, 948.000, 965.250)$,

$\tilde{x}_{12}=(497.250, 632.000, 957.600)$.

Applying these values to (15) we have,

(2,3,5) ·(837.900, 948.000, 965.250)+(4,5,7) ·(497.250, 632.000, 957.600)

and we get,

$\tilde{N}_1 \approx(2\cdot 837.900,\ 3\cdot 948.000,\ 5\cdot 965.250)$
$+(4\cdot 497.250,\ 5\cdot 632.000,\ 7\cdot 957.600)=$
(1675.8, 2844, 4826.25)+(1989, 3160, 6703.2)
=(1675.8+1989, 2844+3160, 4826.25+6703.2)
=(3664.8, 6004, 11529.45).

Thus, checking if the fuzzy numbers satisfy the constraint (5) is the same as checking if the inequality $\tilde{N}_1 \le \tilde{M}_1$ holds true.

To this end we first calculate $PG(\tilde{M}_1,\tilde{N}_1)$. On the basis of formula (6) as a result of the operations performed during the computer simulation when solving this problem we get $PG(\tilde{M}_1,\tilde{N}_1)=0.7925$. Then, based on formula (7) we calculate $NG(\tilde{M}_1,\tilde{N}_1)$=0.3077.

Using the formula (9), we determine the weighted sum of $PG(\tilde{M}_1,\tilde{N}_1)$ and $NG(\tilde{M}_1,\tilde{N}_1)$

$g(\tilde{M}_1 \ge \tilde{N}_1)=\beta\times PG(\tilde{M}_1,\tilde{N}_1)+(1-\beta)\times NG(\tilde{M}_1,\tilde{N}_1)=0.5986$.

Similarly, we determine $PG(\tilde{N}_1,\tilde{M}_1)$=1.000, $NG(\tilde{N}_1,\tilde{M}_1)$=0.6923 and $g(\tilde{N}_1 \ge \tilde{M}_1)$=0.8769. And lastly, using the inequalities (10), we found out that $\tilde{N}_1 \le \tilde{M}_1$ is false as well as:

$g(\tilde{M}_1 \ge \tilde{N}_1) < g(\tilde{N}_1 \ge \tilde{M}_1)$

0.5986<0.8769, and thus $\tilde{N}_1 \ge \tilde{M}_1$.

Then, using the given sequence of fuzzy numbers $\tilde{x}_{11},\tilde{x}_{12}$, we chaotically generate a set of fuzzy numbers. We have $\tilde{x}_{21}$ =(133.834, 194.719, 529.712) and $\tilde{x}_{22}$=(162.003 918.675 974.971). Consequently, performing the above-mentioned operations, we found out that this sequence of numbers does not satisfy the given constraints. Hence, we do not use these values to calculate the objective function but use them for chaotically generating the next set of fuzzy numbers. Then, we get:

$\tilde{x}_{31}$=(452.099 619.374 993.978) $\tilde{x}_{32}$=(97.368 295.109 529.456).

We can see that this set of numbers satisfies the constraints (15) and (16). Hence, we calculate the value of the objective function:

$\tilde{Z}^c$=(203787.1875 483836.7434 1049671.0262).

Now we have to compare the two fuzzy numbers:

$\tilde{Z}$=(270000, 400000, 765000) and $\tilde{Z}^c$= =(203787.1875, 483836.7434, 1049671.0262).

Finally, we calculate:

$PG(\tilde{Z} \ge \tilde{Z}^c)$=0.8700, $NG(\tilde{Z} \ge \tilde{Z}^c)$=0.4785,
$PG(\tilde{Z}^c \ge \tilde{Z})$=1.000, $NG(\tilde{Z}^c \ge \tilde{Z})$=0.5215,
$g(\tilde{Z} \ge \tilde{Z}^c)$=0.7134, $g(\tilde{Z}^c \ge \tilde{Z})$=0.8086.

Because

$$g(\tilde{Z} \ge \tilde{Z}^c) < g(\tilde{Z}^c \ge \tilde{Z})$$
$$0.7134<0.8086.$$

It is concluded that the current value of the objective function is better than the previous, and the former should be replaced by the latter:

$\tilde{Z}=(203787.1875, 483836.7434\quad 1049671.0262)$.

Using the current set of fuzzy numbers $\tilde{x}_{n1}$, $\tilde{x}_{n2}$, we can chaotically create the next (n+1)th set of fuzzy numbers $\tilde{x}_{n+1,1}$ $\tilde{x}_{n+1,2}$ by repeating all of the above operations again.

All the above processes are repeated until there are no improvements in the objective function. After that, the process of chaotic generation of random numbers can be stopped. The optimal value of the fuzzy objective function, achieved by our experiment at 1237 generation, was:

$$\tilde{Z}_{opt}=\left(461591.5530,\ 731812.9373,\ 1199640.1725\right).$$

The centroid of this number is equal to 797381.4512. The fuzzy optimal numbers leading to this optimal value are:

$\tilde{x}_{1opt}$=(743.2823, 867.5932, 995.0630),

$\tilde{x}_{2opt}$=(340.8670, 480.9696, 686.6900).

The centroids of these numbers are equal to 868.6230 and 502.7090, respectively. In the crisp linear programming problem:

$Z=400x_1+800x_2$

$3x_1+5x_2 \le 5000$

$x_1 + 4x_2 \leq 3000$
$x_1, x_2 \geq 0$

The optimal value of the objective function Z=743144 is reached at x_1=715 and x_2=571.43. The difference in the optimal values of the objective function and the values of x_1, x_2 can be a result of several reasons: the right part of the constraints in the crisp linear programming problem consists crisp numbers 5000 and 3000. Besides, those positive values x_1, x_2 that satisfy the constraints (12) and (13), in the left part, make the values not exceed 5000 and 3000, respectively.

In the considered fuzzy linear programming problem, the right part of the constraints consists of fuzzy numbers (3000, 5000, 7500) and (1000, 3000, 5500), the right spreads of which are 7500 and 5500, respectively. The centroids of these fuzzy numbers are 5165.8476 and 3166.1697, respectively. Thus, the non-negative fuzzy numbers $\tilde{x}_1, \tilde{x}_2$ in the left part, which satisfy the constraints (15) and (16), make the numbers and the centroids exceed 5000 and 3000, respectively. The centroids of the fuzzy numbers $\tilde{x}_1, \tilde{x}_2$, satisfying these constraints can exceed the optimal crisp values x_1, x_2, and, consequently, the optimal value of the crisp objective function.

On the other hand, the fuzzy representation of the objective function also contributes to the centroid of the optimal value of the objective function which can slightly exceed the optimal crisp value. It should be noted that when optimizing certain real processes, the representation of both the constraints and the objective function might be fuzzy. Therefore, it is purposive to solve the problem using the fuzzy approach without its conversion to the crisp linear programming problem, which is more effective.

6 Discussion and Conclusion

In this paper, application of the fuzzy chaos to fuzzy simulation is considered. A fully fuzzified linear programming problem with a fuzzy goal, fuzzy constraints, and fuzzy variables was solved using fuzzy simulation. In a particular case, the fuzzy chaos technique is applied to a fuzzy linear programming problem. The model is illustrated with a numerical example of maximization of the profit of the manufacturing company. The obtained optimal solution from the given approach to the problems of linear programming shows a high efficiency.

The discussion of the numerical example results shows that it is purposeful to solve the optimization problem using the fuzzy approach without its conversion to the crisp linear programming problem, which is more effective.

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References:

[1] Bellman, R.E., Zadeh, L.A. Decision-making in a fuzzy environment, *J. Management Science*, Vol. 17, No.4, 1970, pp. B141-B164. https://doi.org/10.1287/mnsc.17.4.B141.

[2] Delgado, M., Verdegay, J.L., Vila, M.A. Relating different approaches to solve linear programmimg problems with imprecise costs. *Fuzzy Sets and Systems*, Vol. 37, 1990, pp 33-42. http://dx.doi.org/10.1016/0165-0114(90)90061-A.

[3] Ebrahimnejad, A., Cao, B-Y., Nasseri, S.H. *Fuzzy Linear Programming Solution Techniques and Applications*. Springer Nature Switzerland AG, 2020. ISBN-10: 3030183947.

[4] Wang, L-X. *A Course in Fuzzy Systems and Control*. Prentice Hall PTR, 1997. ISBN-13: 978-0135408827.

[5] Werner, B. An interactive fuzzy programming system, *Fuzzy Sets and Systems*, Vol. 23, 1987, pp.131-147. https://doi.org/10.1016/0165-0114(87)90105-9.

[6] Lai, Y.J., Hwang, C.L. *Fuzzy Mathematical Programming*. Springer-Verlag, NY, 1992. doi10.1007/978-3-642-48753-8.

[7] Chandra, S., Aggarwal, A. On solving fuzzy linear programming problems: a revisit to Zimmermann's approach, *Journal of Intelligent & Fuzzy Systems*, Vol. 27, No. 5, 2014, pp. 2603-2610. http://dx.doi.org/10.3233/IFS-141233.

[8] Yang, X-P., Zhou, X-G., Cao, B-Y., Nasseri, S.H. A MOLP method for solving fully fuzzy linear programming with *LR* fuzzy parameters, *Mathematical Problems in Engineering*, Vol. 2014, Article ID 782376. http://dx.doi.org/10.1155/2014/782376.

[9] Otadi, M. Solving fully fuzzy linear programming. *Int. J. Industrial Mathematics*,

Vol. 6, No. 1, 2014, Article ID IJIM-00303. ISSN: 2008-5621.

[10] Behera, D., Chakraverty, S. New approach to solve fully fuzzy system of linear equations using single and double parametric form of fuzzy numbers, *Sadhana,* Vol. 40, Part 1, 2015, pp.35–49, [Online]. https://www.ias.ac.in/article/fulltext/sadh/040/01/0035-0049 (Accessed Date: November 20, 2024).

[11] Konak, F., Bağcı, B. Fuzzy linear programming on portfolio optimization: empirical evidence from FTSE 100 index, *Global Journal of Management and Business Research: C Finance*, Vol. 16, No. 2, 2016, pp. 57-61. Online ISSN: 2249-4588 & Print ISSN: 0975-5853.

[12] Hosseinzadeh, A., Edalatpanah, S.A. A new approach for solving fully fuzzy linear programming by using the lexicography method, *Advances in Fuzzy Systems*, Vol. 2016, No. 1, 2016, Article ID 1538496, pp. 1-6. http://dx.doi.org/10.1155/2016/1538496.

[13] Bertone, A.M.A., Jafelice, R.S.M., Camara, M.A. Fuzzy linear programming: optimization of an electric circuit model, *Tema*, Vol. 18, No. 3, 2017, pp. 419-434. http://dx.doi.org/10.5540/tema.2017.018.03.0419.

[14] Ebrahimnejad, A. An effective computational attempt for solving fully fuzzy linear programming using MOLP problem, *Journal of Industrial and Production Engineering*,Vol. 36, Iss. 2, 2019, pp. 59-69. https://doi.org/10.1080/21681015.2019.1585391.

[15] Ghoushchi, S.J., Osgooei, E., Haseli, G., Tomaskova, H. A novel approach to solve fully fuzzy linear programming problems with modified triangular fuzzy numbers, *Mathematics*, Vol. 9, No. 22, 2021, 2937, pp. 1-13. https://doi.org/10.3390/math9222937.

[16] Diamond, P. Chaos and fuzzy representation of dynamical systems, *2nd Internat. Conf. on Fuzzy Logic and Neural Networks*, Iizuka, Japan, 1992, pp.51-58, [Online]. https://cir.nii.ac.jp/crid/1570009749681724032 (Accessed Date: November 20, 2024).

[17] Kloeden, P.E. Chaotic iterations of fuzzy sets, *Fuzzy Sets and Systems*, Vol. 42, 1991, pp. 37-42. https://doi.org/10.1016/0165-0114(91)90087-7.

[18] Teodorescu, H-N. Chaos in fuzzy systems and sygnals, *2nd Internat. Conf. on Fuzzy Logic and Neural Networks*, Iizuka, Japan, 1992, pp.21-50, [Online]. https://cir.nii.ac.jp/crid/1572261549495410944 (Accessed Date: November 20, 2024).

[19] Buckley, J.J., Hayashi, Y. Applications of fuzzy chaos to fuzzy simulation, *Fuzzy Sets and Systems,* Vol. 99, 1998, pp. 151-157. https://doi.org/10.1016/S0165-0114(97)00023-7.

[20] R.A. Aliev R.R. Aliev, Soft Computing and its Application. World Scientific, New Jersey, London, Singapure, Hong Kong, 2001. ISBN: 9813105879, 9789813105874.

[21] Wang, J. A Fuzzy project scheduling approach to minimize schedule for product development, *Fuzzy Sets and Systems*, Vol. 127, 2002, pp. 99-116. http://dx.doi.org/10.1016/S0165-0114(01)00146-4.

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Conflict of Interest

The authors have no conflicts of interest to declare.