Performance Analysis of Ball and Beam System using Modern Control Techniques

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Abstract: - The ball-and-beam system is an attractive laboratory experimental tool due to its inherent nonlinearity and open-loop instability. However, designing an effective ball-and-beam system controller is a challenge for researchers and engineers. In this paper, a mathematical model of the ball-and-beam system has been derived and the state space system model has been obtained. This system is an open loop unstable system because the system output (ball position) increases without a limit for a fixed input. To ensure that the ball will not fall down and only move or roll on the beam, a controller should be designed to control the system and make it stable. In this paper, modern control techniques such as full-state feedback controller and full-state feedback with integral action have been employed to study the system's response and achieve desired outcomes. In addition, a modern optimization control strategy known as linear quadratic regulator (LQR) has been employed too. MATLAB software has been used to design the controllers, analyze the system's behavior, and evaluate the effectiveness of the controllers in achieving stability. The step response of the system states using the suggested controller has been investigated and compared, in order to determine which control scheme provided the best performance .Finally, the results show that the integral full-state feedback controller is the most effective compared to other control methods, as it gives the best performance.

Key-Words: - Ball beam system, mathematical model, modern controller design, state feedback, integral state feedback, LQR.

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1 Introduction

Most systems are nonlinear in nature, which plays an important role in the field of control engineering. One of these systems is the ball and beam balancer system. This system is available in most control engineering department laboratories to teach the students the principle of control engineering and apply different controller techniques for it. It is a highly nonlinear benchmark control problem in the field of control engineering. This is similar to practical control problems like balance control, position control, and tracking control problems, [1]. The concept of the ball and beam system has broad applicability across various domains, including stabilizing aircraft during landing, controlling liquid-carrying tankers on roads, aircraft yaw roll control, and industrial robot applications for handling goods. Controlling an under-actuated system, where there are fewer control inputs than state variables to be controlled, is a significant research area.

The ball and beam system can be classified into two configurations. The first configuration is the ball and beam balancer, depicted in Figure 1, where the beam is supported in the middle and rotates around its central axis.

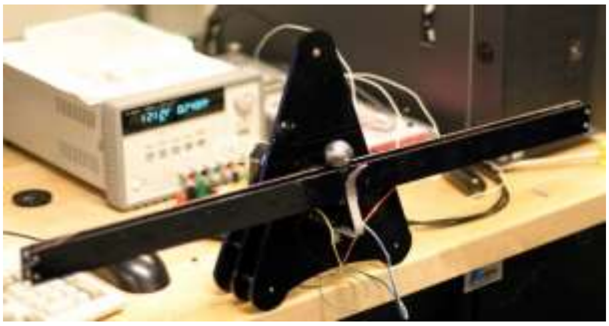

Fig. 1: Ball and beam balancer system, [2]

The second configuration of the ball and beam system is the ball and beam module. In this setup, two lever arms on both sides support the beam. One of the level arms serves as the pivot, while the other is coupled to the motor's output gear. This configuration offers the advantage of being able to use a relatively small motor, thanks to the presence of a gearbox. However, it introduces challenges in deriving a mathematical model due to the additional mechanical components that need to be considered, [3].

The problem with the ball and beam balancer system is, that when the system is powered ON, the beam will tilt to a certain angle. The steel ball will move with an acceleration that is proportional to the angle of the beam. If the beam tilts without a limit of angle, the ball will fall down from the beam because the acceleration of the ball increases. The control objective is to keep the ball in a desired position on the beam while controlling the ball position by manipulating the angle of the beam. Many students and researchers in many organizations previously have controlled the ball and beam balancer system, [4], [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19], [20], [21]. The contribution of this research is to investigate the effect of full-state feedback, integral full-state feedback, and linear quadratic regulator techniques on the performance of the system states.

2 Mathematical Model of Ball and Beam System

The complete description of the dynamics of the ball rolling on the beam is quite complicated and for control system design, a simplified derivation is used to give a model that is good for controller design. This is the basic model of the ball and beam system. As seen from Figure 2, three main components have moments and forces acting on them, the motor, the beam, and the ball. Starting with the ball along the beam, it experiences a force due to the rolling constraint along the beam and a downward component due to gravity that depends on the angle of the beam ∅.

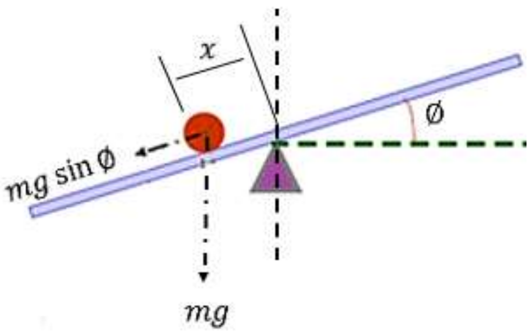

Fig. 2: Ball and beam balancer motion

The sum of the force:

$$\mathrm{mg}\sin\emptyset - F_r = \mathrm{m}\ddot{x} \qquad (1)$$

where

m = mass of the ball

g = gravity

F_r= rolling constraint force on the ball

x = position of the ball along the beam

By geometry, position can be defined as:

$$x = \alpha \cdot \acute{a}$$

α = angular displacement of the ball

$\acute{a}$=distance between the axis of the ball and the point of contact of the ball with the beam

The torque balance of the ball τ_b is given by equation;

$$\sum \tau_b = F_r\acute{a} = J_b\ddot{\alpha} \qquad (2)$$

and

$$J_b = \frac{2}{5}\mathrm{m}a^2$$

where

J_b = moment of inertia of the ball

a = radius of the ball

After all equations have been simplified and combined, two equations can be obtained which are;

$$\left[1 + \frac{2}{5}\left(\frac{a}{\acute{a}}\right)^2\right]\ddot{x} = \mathrm{g}\sin\emptyset \qquad (3)$$

Some assumptions need to be made and this system is expected to operate at or around 0° beam angle that makes equation (3) can be linearized using small angle approximations by:

$$\left[1 + \frac{2}{5}\left(\frac{a}{\acute{a}}\right)^2\right]\ddot{x} = \mathrm{g}\emptyset$$

$$\left(\frac{a}{\acute{a}}\right) \cong 1$$

$$\left[1 + \frac{2}{5}\right]\ddot{x} = \mathrm{g}\emptyset \qquad (4)$$

Next, the moment and force balances can be determined for the beam and motor. The beam bears the load of the ball as well as the input torque of the motor. The torque balance is given by:

$$\tau_{in} = J_{bm}\ddot{\emptyset} = k_t I_{in} \qquad (5)$$

where bm denotes the beam and motor and τ_{in} represents the torque generated by motor, k_t is motor torque constant and I_{in} is current supplied to motor.

Let assume the system state variable to be:

$$\mathrm{x}_1(\text{ball position}) = \mathrm{x}$$

$$\mathrm{x}_2(\text{ball velocity}) = \dot{\mathrm{x}}$$

$$\mathrm{x}_3(\text{beam angle}) = \emptyset$$

$$\mathrm{x}_4(\text{beam angular velocity}) = \dot{\emptyset}$$

Therefore, the overall system in state-space form can be rewritten as follow:

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{6}$$

$$y(t) = C\,x(t) \tag{7}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & H & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ K \end{bmatrix},$$

$$C = [1 \quad 0 \quad 0 \quad 0]$$

The ball and beam system parameters are taken from [22].

$$H = \frac{5g}{7} = 7$$

$$K = \frac{k_t}{J_{bm}} = 85$$

3 Modern Controller Design

This section discusses the design of state feedback controllers and optimal control system (LQR) to control the ball position of the system. The states of the system can be transferred to another desired state over a finite time by using input only if the system is controllable. So, the controllability test is checked using MATLAB software for this system before proceeding with the following design controller.

3.1 Full State Feedback Controller Design

A full state feedback controller is a modern control system that utilizes measurements of all state variables of a system to compute the control signal. It aims to regulate the system's behavior by directly influencing its states through feedback.

The state variable design process assumes that all states are available for feedback, that have access to the complete state x(t) for all t, the system should be controllable [23], then the control gains are computed by placing the desired closed loop poles at desired locations in the left half plane, [24]. Using the measured states and the calculated control gains, the control signal is obtained that represents the input to the ball and beam system. The full-state feedback block diagram is illustrated in Figure 3.

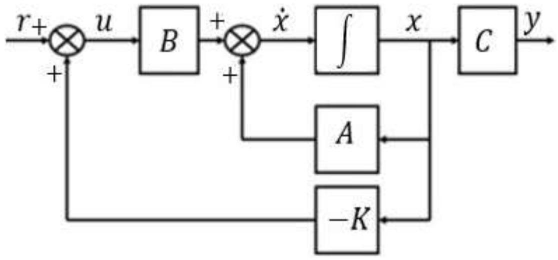

Fig. 3: Full state feedback block diagram

The control signal of the full-state feedback is:

$$u(t) = -Kx(t) + r(t) \tag{8}$$

where K is $1 \times n$ state feedback gains matrix.

The desired root location is chosen so that, the system response has a settling time of 2 seconds and less than 5% overshoot which satisfy the roots location $-2 \pm 2i$ and the other two roots are located to the left of the real part of the desired dominant roots. Therefore, the desired root location locations are chosen to be at:

$$s_{1,2,3,4} = -2 + 2i, -2 - 2i, -7, -10$$

Then, the feedback gains matrix is determined by equating the desired characteristic equation by the closed loop characteristic equation:

desired chara. Equ. $= |sI - (A - BK)|$

Therefore, the matrix gain:

$$K = [0.9412 \quad 0.6992 \quad 1.7176 \quad 0.2471]$$

3.2 Full State Feedback with Integral Controller Design

When incorporating integral control action into the state feedback controller, it becomes an integral state feedback controller. The designed controller increases the system type, improves the system's steady-state performance by eliminating steady-state error, and achieves the desired transient response. The basic principle design is to insert an integrator in the feedforward path between the error compensator and the plant. The block diagram of the integral state feedback controller is shown in Figure 4.

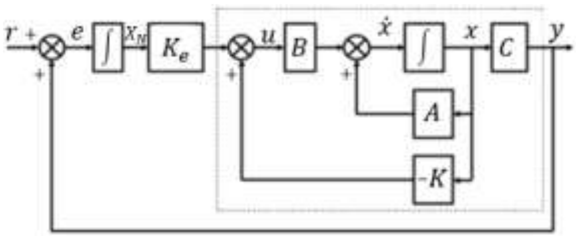

Fig. 4: Full state feedback with integral action block diagram

The integral state feedback control signal is:

$$u(t) = -Kx + K_e x_N \quad (9)$$

where K is $1 \times n$ state feedback gains matrix and K_e is the integral gain

Substitutes equ. (9) into equ. (6) to obtain the integral state feedback closed loop model as:

$$\begin{bmatrix} \dot{x} \\ \dot{x}_N \end{bmatrix} = \begin{bmatrix} (A - BK) & Bk_e \\ -C & 0 \end{bmatrix} \begin{bmatrix} x \\ x_N \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r \quad (10)$$

The desired roots locations for this controller are chosen to be at:

$$s_{1,2,3,4,5} = -2 + 2i, -2 - 2i, -7, -10, -20$$

Then, the feedback gains matrix and the integral gain are:

$$K = [14.9244 \quad 5.6067 \quad 6.6588 \quad 0.4824]$$

$$K_e = [-18.8235]$$

3.3 LQR Controller Design

The linear quadratic regulator is an optimal control technique used in modern control systems. It is a theory of optimal control concerned with operating a dynamic system at minimum cost. The purpose of design is to realize a system with practical components that will provide the desired performance. The LQR controller takes the state equation of the system as feedback and generates a feedback error signal, as shown in Figure 5.

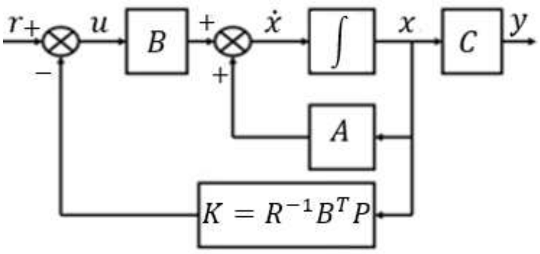

Fig. 5: LQR controller block diagram

It minimizes a quadratic cost function by adjusting control inputs based on the system's state. The cost function in LQR is defined as the quadratic function of the state and control inputs, that allows to specification the relative importance of different system states and control efforts, [25].

The design of the system is based on minimizing performance index, [26], [27], [28]. Therefore, systems that are adjusted to provide a minimum performance index are called optimal control systems. The quadratic performance index of a control system, in terms of a state variable system, is expressed as:

$$J = \int_0^{t_1} \left(x^T Q x + u^T R u\right) dt \quad (11)$$

where x is the state vector, u is the control vector, Q is the state weighting matrix, and R is the control weighting matrix.

The optimal control law and optimal gain matrix are represented as:

$$u_{opt}(t) = -Kx(t) \quad (12)$$

$$K = R^{-1} B^T P$$

One approach to finding a controller that minimizes the LQR cost function is based on the positive-definite solution of the following algebraic Riccati equation (ARE).

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \quad (13)$$

P is the unique positive definite solution matrix to the ARE. In this research, Q is chosen to be $C' \times C$ and $R = 1$. Hence, the feedback gains matrix using LQR is:

$$K = [1 \quad 1.5461 \quad 4.8669 \quad 1.0557]$$

4 Results and Discussions

The objective of these design techniques is to set the ball position at $0.1m$ from the beam center with an almost good transient response and zero steady-state error. The state responses of the system using a full-state feedback controller are shown in Figure 6.

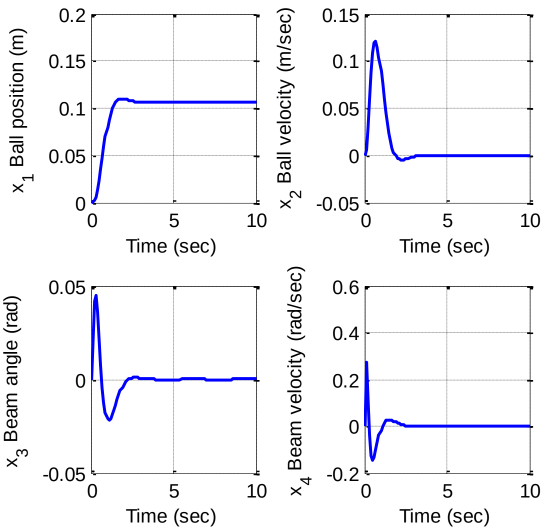

Fig. 6: System states response using full state feedback

It is clear that the control approach is effective in achieving the desired system performance in terms of transient response. In contrast, the system using this controller has a steady state error of about 0.0062 *m*. This ensures the effectiveness of a full-state feedback controller in achieving the desired performance and stability. Therefore, to cancel the steady-state error in this system, it is suggested to use integral state feedback.

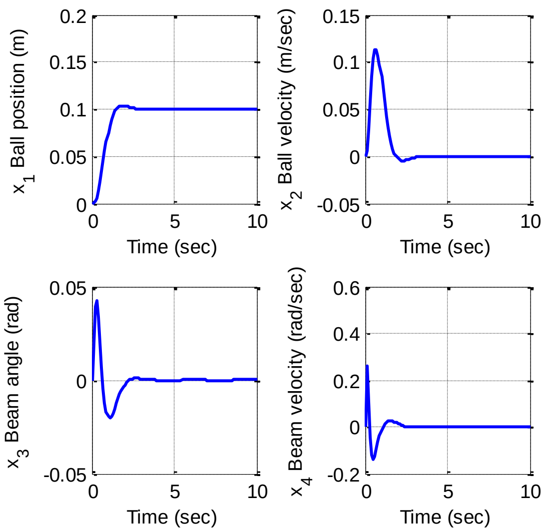

Fig. 7: System states response using full state feedback with integral

The integral state feedback controller result is shown in Figure 7. Since the state x_N is added to the full state feedback controller which represents the integral action. Its effect can be seen clearly in eliminating the existing error in the system. The ball is at a desired position, which means that, the controller achieved its goal.

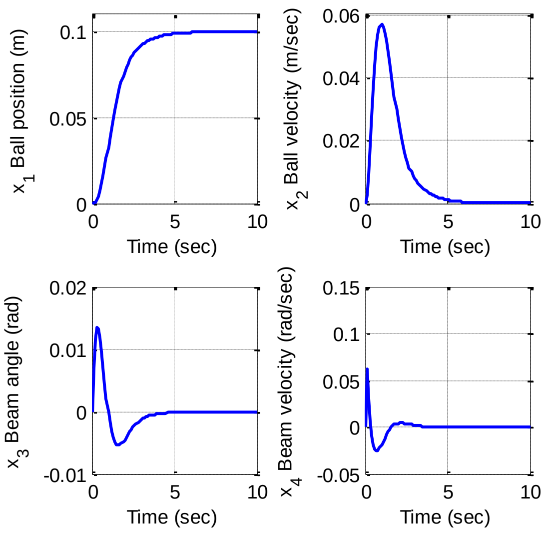

Fig. 8: System states response using LQR

Using these controllers to achieve the desired performance. The designer should have to take care of the control signal because too high controller gains may affect the other system states. Moreover, to balance between the control signal and the desired response specifications, the optimal controller such as the LQR controller is used in this research. Figure 8 shows the state responses using the LQR controller.

It is clear that the ball position reached its desired position with no overshoot and zero steady-state error. In addition, using the LQR controller, the beam angle has settled with angles approximately between (0.012 rad and -0.005 rad), which is smaller than other used controllers.

Table 1 summarizes the performance comparison of the output response of ball position. From this table, the difference in characteristics between the responses can be clearly seen. Among the three controllers, full-state feedback (FSFB) and integral full-state feedback (IFSFB) have the fastest response with a rise time of 0.86 seconds, whereas the LQR controller has the slowest response with 2.22 seconds. LQR has no overshoot compared to FSFB and IFSFB. However, it takes 4.43 seconds to reach the steady state value, which is more than other controllers are. The last characteristic is the steady-state error. Zero steady-state error is obtained using IFSFB with the shortest time.

Table 1. Performance comparison of output response of ball position

Controller / Characteristics	FSFB	IFSFB	LQR
Rise time (sec.)	0.86	0.86	2.22
Settling time(sec)	2.35	2.35	4.43
Overshoot (%)	3.65	3.65	0
S.S error	0.0062	0	0

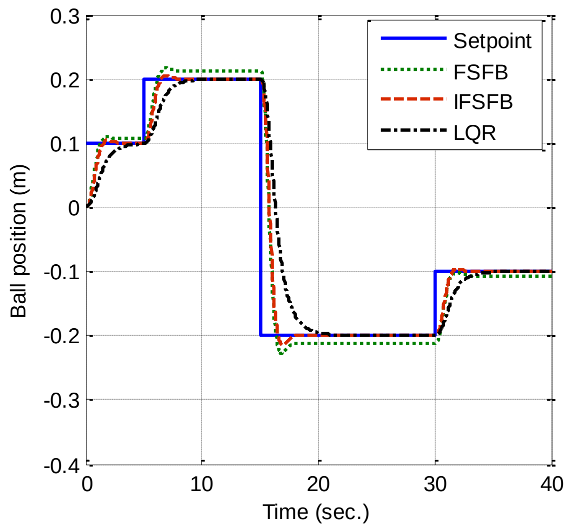

Fig. 9: Ball position response for stetpoint step changes

To examine the controlled system robustness, a step change test and disturbance rejection test are applied. The setpoint tracking is performed and the setpoint-tracking signal contains changing the set point during the operation as shown in Figure 9.

The ball position using the designed controllers has tracked the change in the setpoint even to the left or right of the beam center.

The disturbance or noise always is an undesirable input due to its negative affect on the system, [29]. In this research, the disturbance is considered as an external force acting on the ball with 0.05 Newton step at a time of 6 seconds. The output response under the effect of this disturbance is shown in Figure 10.

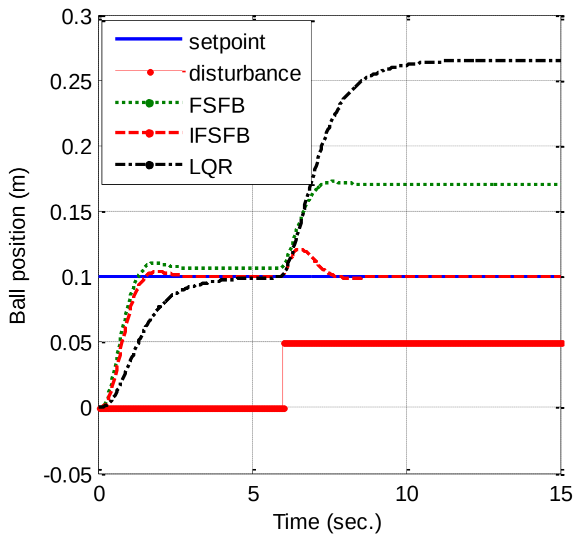

Fig. 10: Ball position response under the effect of disturbance input

The result showed that the disturbance affected the system at a time of 6 seconds. The FSFB and LQR cannot retrain the ball back to its desired setpoint and they have an error of 0.055m and 0.154m respectively. In contrast, the effect of disturbance is rejected in a few seconds by using the IFSFB controller.

5 Conclusion

In this paper, three modern controllers are designed and implemented to control the ball and beam balancer system. First, a mathematical model of the ball and beam system is derived. Before the controller design process, one important property of the system is verified, the controllability test. It is confirmed that the system can be controlled effectively.

Based on the results in the previous section, it is obvious that all the controllers exhibited satisfactory performance in terms of transient response and steady-state error. The full-state feedback controller demonstrated a rapid response characterized by fast rise and settling time. The benefit of adding the integral to the state feedback is seen in the result, which eliminates the steady-state error. The response using the LQR controller has no overshoot and zero steady-state error. On the other hand, for this system, the LQR controller exhibited a slower response compared to the other controllers. However, the designed controllers performed well in a tracking step change for the setpoint. The integral full-state feedback controller, is the only controller that has rejected the disturbance effect. Therefore, it can be concluded that, the IFSFB controller is the best choice for this system due to its robustness.

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Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

- Mustafa Saad and Khaled Mustafa carried out the simulation and the optimization.
- Mustafa Saad and Noor Hdood have organized and executed the controller design and simulation of sections 3 and 4.
- Mustafa Saad, Khaled Mustafa, and Amna Alsharef were responsible for the text writing.

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Conflict of Interest

The authors have no conflicts of interest to declare.