Determining the Degree of Preference Liking and Effectiveness of DMUs over a Long Period of Time by Means of a New Approach based on the Cross Efficiency Value Chain in Harmony with Fuzzy Arithmetic

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Abstract: - In this paper, an alternative approach is presented for the evaluation of the likeability preference and effectiveness of DMUs, based on the DEA and fuzzy DEA models. In the magnitudes of variable values according to input-output levels, over the time period, some have not completely clear (fuzzy) values obtained from perceptions and surveys. For a more realistic assessment of effectiveness and determination of the degree of preference liking, to avoid accidental fluctuation values, and to get as close as possible to the trend of the process's progress, dynamic analysis of smoothing of the time series is applied to the input-output value levels. This is done according to a k-order moving average, determining the new levels of the input-output values. The approach is applied in two phases. In the first phase, the efficiency value chain matrix is determined, applying conventional DEA models with constant and variable returns to scale, evaluation of super efficiencies, fuzzy efficiency, and cross-efficiency. The data and the comparison of the models are analyzed, focusing in particular on the cross-efficiency value chain. In the second phase, fuzzy triangular numbers are composed of of the chain of cross-efficiency values for each DMU. Then based on fuzzy arithmetic as well as the concept given by the geometric probability model is determined and the transition matrix of the degree of preference liking, the evaluation of the ranking is obtained according to the degree of preference liking of each DMU in relation to other DMUs. In the paper, the contributions of the approach to the evaluation of the effectiveness and the degree of preference liking with the relevant conclusions are highlighted.

Key-Words: - cross-efficiency, degree of preference liking, ranking, dynamic analysis, DEA models, fuzzy triangular numbers, super efficiencies.

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1 Introduction

The approach presented is based on real-life data, which aims to explore the distinctive features of each DMU assessed as best practices as well as to identify the impact of factors on the effectiveness and the degree of preference likeability related to the standard of living on the basis of the prefecture and region. In Albania, according to regions and prefectures, there are tangible differences in internal migration as well as in the natural rate of population growth. For the study analysis, the method of data envelopment analysis (DEA) is used as a very applied and powerful method in the study and evaluation of the effectiveness, ranking, and evaluation of the influence of the factors on the efficiency value. In real life, the values of the variable quantities are not all completely determined, where DEA also shows "weakness" if the inputs and outputs with which the DMUs operate do not have completely clear quantitative values or the data are vague as they can be those given in the field of perceptions, surveys, etc. Therefore, in addition to the basic DEA models with constant returns to scale and variable returns to scale, [1], [2], many models of its extension are also applied, as well as Fuzzy DEA models. The study analysis is over an extended period of time (2016-2022), so the data can be presented formatted as a time series, where seasonal variations, random variations (or accidental variations influenced by the conditions in which the observations are carried out) are encountered, etc. In order to soften the variations in the values of variable quantities and to get as close as possible to the trend of the period, decomposing time series and smoothing transformations on time series is done, as is the series of moving averages of order k, [3]. In the multiplier model with linear programming in the

DEA method, the DMUs choose their most favorable weights, but the weights can have zero values or very small values where it can be said that specific inputs or outputs are ignored or misinterpreted, and the performance of DMUs decreasing and the power of distinguished between them. To increase the distinguishing power in the DEA rankings, many approaches have been applied, such as the super-efficiency approach developed for the first time by [4], as well as the connection of the DEA performance with other approaches with canonical correlation analysis such as [5]. The approach called cross efficiency was presented for the first time by [6] for evaluating the performance of DMUs, but it can also be said as a likability evaluation for management strategies. The evaluation of efficiency of each DMU is evaluated with its own weights, but also with the weights of other units, which is called cross efficiency, where each DMU is compared with every other unit in the set of DMUs, then it is evaluated average efficiency values for each DMUs. Applications of Cross-efficiency can be found in many papers such as [7], [8], [9], where in [9] the cross-efficiency evaluation method is used for 102 DMUs (for the years 2012 and 2017), which use 4 inputs and 4 outputs, where two of the outputs have a qualitative nature. Fuzzy DEA is developed based on the theory of fuzzy sets. [10] is the first to present the Fuzzy set, also [11], in addition to the generalization of the conventional Fuzzy set, connected it with the so-called membership function, giving the concept of linguistic variable. The authors [12] and [13] give the classification of approaches applied in Fuzzy DEA, classifying them into 6 types. [14] provides an approach to Fuzzy DEA in a form characterized by numbers reflected through perception, also proposing an extension of the Fuzzy DEA model in the relationship between DEA and linear regression. The ranking is related to the comparison of Fuzzy numbers. [15] makes a comparison of Fuzzy numbers based on the concept of probability, giving examples compared with other approaches. [16] develops the approach of programming possibilities with a certain level of possibility based on three components, so that Fuzzy numbers are realistic to represent approximations and use the concept of possibility by comparing fuzzy numbers. [17] developed the approach in the case of Fuzzy linear programming and linear programming with multiple objectives, giving a modified model for each case. Based on different applications for the ranking of fuzzy numbers, in the coefficient of variation of the distance of the central point and the initial point [18] proposes a modification of the approach based on the distance called sign distance. [19] proposes a new ranking function for the ranking of the real number and the fuzzy number with an acceptance rate and then extends it to the ranking of two fuzzy numbers. The ranking of fuzzy numbers is interpreted as an instrument in many application models. To evaluate the measurement of efficiency using the concept of the set of fuzzy numbers in the context of DEA, [20] brings fuzzy mathematical programming, to contribute to an optimal solution in the evaluation of efficiency, fuzzy regression to illustrate and types of different options that are available. Efficiency evaluation and ranking of DMUs with Fuzzy data, where the CCR fuzzy model is transformed into a crisp linear programming problem applying α-cut approach illustrated and with numerical examples is given in [21]. [22] presents fuzzy DEA models based on fuzzy arithmetic formulated as a linear programming where the fuzzy efficiency of decision-making units can be evaluated and an analytical approach of fuzzy ranking developed according to fuzzy rank efficiencies for performance evaluation. [23] proposed finding a common set of weights in fuzzy DEA by evaluating the upper bounds of the weights in the solution of the problem presented in linear programming., demonstrate the flexibility of the procedure illustrated and with examples. In [24] a fuzzy expected value approach is proposed for DEA analysis, in which we first obtain the weights of the values for the inputs and outputs. These weights are used to measure the optimistic and pessimistic efficiency of DMUs. Then the geometric mean is evaluated. Fuzzy models are built based on fuzzy arithmetic and α-level sets, determining the ranking approach for fuzzy efficiencies. [25] provides a model of fuzzy DEA dynamics in a study to compare discriminating power and perceived improvement with the aim of improving the performance of DMUs operating with 56 railways in computational time and discriminating power. [26] presents the model in the fuzzy context to evaluate efficiency and productivity in an uncertain environment with different α levels, where decision-makers can evaluate economic and environmental factors in the selection of sustainable suppliers with a probability distribution. [27] presents a new approach for priorities in the process of fuzzy analytical hierarchy, where the fuzzy nature of the data is maintained in all the steps of the approach, further determines the level of consistency, gives the pairwise comparison matrix with appropriate index with the aim of selecting a better ventilation system. Considering the input and output data that may be inaccurate in [28] a possible approach to solving DEA models with fuzzy data is proposed, where the fuzzy data are constructed as trapezoidal fuzzy numbers to evaluate the technical efficiency. To study the impact of undesirable factors in a banking system where 12 DMUs operate [29] analyses their effectiveness by proposing the integration of the cross-efficiency model from DEA and the α-cut model of fuzzy DEA. In the approach presented in [30], the approach for an extension of DEA is presented, where more components are operated with variables that take the form of fuzzy numbers for the evaluation of the fuzzy technical efficiency and the ranking of DMUs, giving linear programming in numerical applications. The application of Fuzzy DEA in a two-phase process with multiple objectives investigating the effects on the efficiency value with the application of multiple linear regression is seen in [31]. [32] presents an approach based on the lexicographic language in a linear program with many objectives for efficiency evaluation in fuzzy efficiency models. This paper presents an alternative approach for evaluating the performance of DMUs along a time course according to a given period with the evaluation of the degree of preference based on Fuzzy arithmetic. The evaluation of the efficiency value chain is given by the application of DEA models, CCR efficiency, super efficiency, Cross-efficiency value chain, and Fuzzy efficiency where the respective rankings are determined. The data of variable quantities are processed in what is called statistics smoothing of the time series. This is done as the data may not be clearly defined (obtained from perceptions or from surveys). The evaluation of the efficiency according to the different models is done to provide the most detailed performance evaluation, comparing the models and the advantage of the application of Cross efficiency in the composition of fuzzy numbers and the application in the evaluation of the degree of preference in relation to the others models. For the evaluation of the degree of preference based on Fuzzy arithmetic, from the chain of cross-efficiency values evaluated in the relevant time course, the composition of fuzzy numbers is done first. In harmony with fuzzy arithmetic, for two Fuzzy triangular numbers $(\widetilde{\omega},\tilde{\eta})$, where $\widetilde{\omega}-\tilde{\eta}$ is again a triangular fuzzy number, which can be projected geometrically as a location, and then the geometric model of the probability of the event is applied, enabling presentation of the matrix of degree of preference. This alternative approach can be applied in certain fields in the evaluation of the degree of preference of DMUs that operate along a given time course.

2 Methodology

The study of this work in the basic conception is based on first evaluating the data in their dynamic analysis, displayed during a period of time (which can be obtained from perceptions and surveys). By evaluating the efficiency of DMUs according to the values obtained from data processing according to a module k (or order k), $M_{(k)}$, determined according to the moving average (dynamic time series analysis). The data is applied to DEA and Fuzzy DEA models for evaluating effectiveness, ranking DMUs, and relevant analysis by comparing the models applied, and determining the advantages of each model. Over an extended period of time, data can be viewed as a time series for any variable. If the time series is given followed by consecutive values $X_1, X_2, \ldots, X_N$, we determine the "series" of values according to the $M_{(k)}$ module of the moving averages. The series of values according to the model $M_{(k)}$ is given by the countable sums, [33], [34]:

$$M_{(k)}: \quad \frac{X_1+X_2+\cdots+X_k}{k}, \quad \frac{X_2+X_3+\cdots+X_{k+1}}{k}, \quad \ldots, \frac{X_{(N-k+1)}+X_{(N-k+2)}+\cdots+X_N}{k}.$$

These values, defined as input-output levels, are applied for a more realistic assessment and examination of the periodic trend of instantaneous changes and the evaluation of the effectiveness of each DMU according to the relevant DEA models. By successively following the chain of efficiency values with the data according to the $M_{(k)}$ module, the matrix of the chain with efficiency values is also built.

Efficiency values are calculated by applying the DEA model input oriented according to constant and variable returns to scale (CRS and VRS), [35]:

$$h^* = \min\ h \tag{1}$$

$$\text{s.t:}\quad hx_{io} - \sum_{j=1}^{n}\lambda_j x_{ij} \geq 0 \qquad i=1,2,\ldots,m,$$

$$\sum_{j=1}^{n}\lambda_j y_{rj} \geq y_{ro} \qquad r=1,2,\ldots,s,$$

$$\lambda_j \geq 0 \qquad j=1,2,\ldots,n$$

$$\theta^* = \min\ \theta \tag{2}$$

$$\text{s.t}\quad \sum_{j=1}^{n}\lambda_j x_{ij} \leq \theta x_{io} \qquad i=1,2,\ldots,m,$$

$$\sum_{j=1}^{n}\lambda_j y_{rj} \geq y_{ro} \qquad r=1,2,\ldots,s,$$

$$\lambda_j \geq 0 \qquad j=1,2,\ldots,n$$

The matrix is constructed with the chain of efficiency values as below, also giving the indicator of the change in the efficiency value (Ind. EC).

$$M_{(k)}^{Efficiency} = \begin{matrix} & E_{fM1} & E_{fM2} & \cdots & E_{fMk} \\ D1 \\ D2 \\ \cdots \\ Dn \end{matrix} \begin{bmatrix} E_{f1M1} & E_{f1M2} & \cdots & E_{f1Mk} \\ E_{f2M1} & E_{f2M2} & \cdots & E_{f2Mk} \\ \cdots & \cdots & \cdots & \cdots \\ E_{fnM1} & E_{fnM2} & \cdots & E_{fnMk} \end{bmatrix} \begin{bmatrix} I_{1EC} \\ I_{2EC} \\ \cdots \\ I_{nEC} \end{bmatrix},$$

where, Ind. EC = $\frac{\sum_{k=2}^{k} \frac{Ef\,(k)}{Ef(k-1)}}{k-1}$

To increase the distinguishing power in the ranking of DMUs (where several DMUs with the efficiency $Ef_{CRS}=1$), the super-efficiency evaluation model is applied as follows [4], [35]:

$$\min \theta^{super}$$
$$\text{st: } \sum_{\substack{j=1 \\ j\neq 0}}^{n} \lambda_j x_{ij} \le \theta^{super} x_{i0}, \qquad i=1,2,\ldots,m; \tag{3}$$
$$\sum_{\substack{j=1 \\ j\neq 0}}^{n} \lambda_j y_{rj} \ge y_{r0}, \qquad r=1,2..,s;$$
$$\lambda_j \ge 0, \qquad j \ne 0$$

Based on the conclusions of the efficiency values (CRS and VRS), the scale efficiency is also determined, which also enables the classification of inefficiencies, [36]. For each DMU, the impact of variable factors on efficiency values is also determined using the formula:

W(I_i) = $\frac{Eff(I_i)}{\sum_{i=1}^{m} Eff(I_i)} \cdot 100\%$ ($Eff(I_i)$ is the efficiency value according to the I-th input.)

Following the study, to determine the degree of preference likeability based on efficiency values, triangular fuzzy numbers are composed (from matrices of efficiency values of one model).

- **Fuzzy background:**

The definition of fuzzy set theory was first proposed by [10]. Based on this concept, the field of applications has been expanded in the presentation of solutions to problems that use it in research operations and other areas from the practical life of the real world, where the data are not completely clearly defined. Fuzzy sets are related to a characteristic function called the membership function. In accordance with the definitions given by [37], [38] are given:

Definition 1: A Fuzzy set $\tilde{A}$ given over a collection of objects X is defined by the set of ordered pairs $\tilde{A}$ = {(x, $\mu_{\tilde{A}}(x)$: xϵ X}, where $\mu_{\tilde{A}}(x)$ is the membership function, of the membership degree value of each element x and that $\mu_{\tilde{A}}(x)$: X→ [0,1].

Definition 2: The set of elements of x (collection) such that the value of the membership function is equal to 1, $\mu_{\tilde{A}}(x)$=1, represents Core ($\tilde{A}$), core of Fuzzy set.

Mathematically, this can be given by the equation core($\tilde{A}$)= {x: $\mu_{\tilde{A}}(x)$=1}.

Definition 3: (α-cut) (Figure 1)

Let it be the geometric projection of a trapezoidal number Fuzzy $\tilde{A}$= ($\gamma_1, \gamma_2, \gamma_3, \gamma_4$), where $\gamma_1, \gamma_2, \gamma_3, \gamma_4$ are all real numbers ($\gamma_1 < \gamma_2 < \gamma_3 < \gamma_4$).

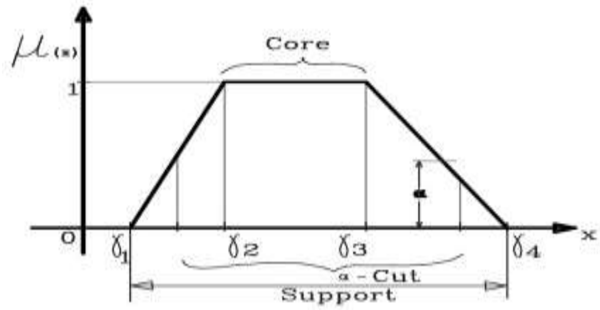

Fig. 1: core, α –cut, and support of fuzzy set, [38]

The α-cut set of a Fuzzy number $\tilde{A}$ (trapezoidal) is the set given and denoted $\tilde{A}_\alpha$={x ϵ R: $\mu_{\tilde{A}}(x) \ge \alpha$}. Let us express the membership function,

$$\mu_{\tilde{A}}(x) = \begin{cases} \mu_{\tilde{A}}^{L}(x): \frac{X-\gamma_1}{\gamma_2-\gamma_1}, & \gamma_1 \le x \le \gamma_2 \\ 1 \ : & \gamma_2 \le x \le \gamma_3 \\ \mu_{\tilde{A}}^{R}(x): \frac{\gamma_4-X}{\gamma_4-\gamma_3}, & \gamma_3 \le x \le \gamma_5 \\ 0 & Otherwise \end{cases}$$

where $\mu_{\tilde{A}}^{L}(x)$ and $\mu_{\tilde{A}}^{R}(x)$ are membership functions of the left side and the right side. If according to (α-cut) we have $\mu_{\tilde{A}}(x) = \alpha$ then we say that:

$$x = \begin{cases} \alpha\gamma_2 + (1-\alpha)\gamma_1, & if\ \gamma_1 \le x \le \gamma_2 \\ \alpha\gamma_3 + (1-\alpha)\gamma_4, & if\ \gamma_3 \le x \le \gamma_4 \end{cases}$$

for x ϵ [γ_2, γ_3], $\mu_{\tilde{A}}(x)$= 1.

In Figure 1, where the geometric projection of the Fuzzy number is given, core ($\tilde{A}$), α −cut, and support of $\tilde{A}$ are presented respectively. Support of Fuzzy set is presented when α-cut we have α=0, 0-cut. For a Fuzzy triangular number $\tilde{A}$ = ($\gamma_1, \gamma_2, \gamma_3$) we have the following :

$$\mu_{\tilde{A}}(x) = \begin{cases} \mu_{\tilde{A}}^{L}(x): \frac{x-\gamma_1}{\gamma_2-\gamma_1}, & \gamma_1 \le x \le \gamma_2 \\ \mu_{\tilde{A}}^{R}(x): \frac{\gamma_3-x}{\gamma_3-\gamma_2}, & \gamma_2 \le x \le \gamma_3 \\ 0, & otherwise \end{cases}$$

- Let two Fuzzy numbers be given: $\tilde{A}$= $(\gamma_1, \gamma_2, \gamma_3)$ and $\tilde{B}$ = $(\delta_1, \delta_2, \delta_3)$ and k a scalar [38], [37], [39]:

a) Addition of two fuzzy numbers: $\tilde{\xi}= \tilde{A}+\tilde{B}$ = $(\gamma_1+\delta_1, \gamma_2+\delta_2, \gamma_3+\delta_3)$.

b) Subtraction of two fuzzy numbers: $\tilde{\xi}= \tilde{A}-\tilde{B}$ = $(\gamma_1-\delta_3, \gamma_2-\delta_2, \gamma_3-\delta_1)$.

c) Multiplication of a fuzzy number by a scalar number: $\tilde{\xi}= k\tilde{A} = \begin{cases}(k\gamma_1, k\gamma_2, k\gamma_3) \ if \ \ k \ge 0 \\ (k\gamma_3, k\gamma_2, k\gamma_1) \ if \ \ k < 0\end{cases}$

• Positive Fuzzy Triangular Numbers and Negative Fuzzy Triangular Numbers

A triangular fuzzy number given $\tilde{A}$ = $(\gamma_1, \gamma_2, \gamma_3)$ is classified as positive if $\gamma_i > 0$ (i=1,2,3) and classified as negative if $\gamma_i < 0$ (i=1,2,3).

They are classified as partial negative if $\gamma_1 < 0 \ dhe \ \gamma_3 > 0$.

If we have n DMUs, where the levels of inputs and outputs can also be characterized by fuzzy triangular numbers ($\tilde{x}_{ij}$= (x_{ij}^{L}, x_{ij}^{M}, x_{ij}^{U}) and $\tilde{y}_{rj}$= (y_{rj}^{L}, y_{rj}^{M}, y_{rj}^{U}).The values of fuzzy efficiencies $(\theta^{L}, \theta^{M}, \theta^{U})$ [22] are calculated from the models (4), (5) and (6).

$$\begin{aligned} &\max \ \theta_p^{L} = \textstyle\sum_{r=1}^{s} \omega_r y_{rp}^{L} \\ &\text{st: } \textstyle\sum_{i=1}^{m} \upsilon_i x_{ip}^{U} = 1 \qquad (4) \\ &\textstyle\sum_{r=1}^{s} \omega_r y_{rj}^{U} - \sum_{i=1}^{m} \upsilon_i x_{ij}^{L} \le 0 \\ &\quad \omega_r \ \ge \varepsilon, \forall \text{ r}, \upsilon_i \ \ge \varepsilon, \forall \text{ i.} \end{aligned}$$

$$\begin{aligned} &\max \ \theta_p^{M} = \textstyle\sum_{r=1}^{s} \omega_r y_{rp}^{M} \\ &\text{st: } \textstyle\sum_{i=1}^{m} \upsilon_i x_{ip}^{M} = 1 \qquad (5) \\ &\textstyle\sum_{r=1}^{s} \omega_r y_{rj}^{U} - \sum_{i=1}^{m} \upsilon_i x_{ij}^{L} \le 0 \\ &\quad \omega_r \ \ge \varepsilon, \forall \text{ r}, \upsilon_i \ \ge \varepsilon, \forall \text{ i.} \end{aligned}$$

$$\begin{aligned} &\max \ \theta_p^{U} = \textstyle\sum_{r=1}^{s} \omega_r y_{rp}^{U} \\ &\text{st: } \textstyle\sum_{i=1}^{m} \upsilon_i x_{ip}^{L} = 1 \qquad (6) \\ &\textstyle\sum_{r=1}^{s} \omega_r y_{rj}^{U} - \sum_{i=1}^{m} \upsilon_i x_{ij}^{L} \le 0 \\ &\quad \omega_r \ \ge \varepsilon, \forall \text{ r}, \upsilon_i \ \ge \varepsilon, \forall \text{ i.} \end{aligned}$$

To evaluate the ranking of DMUs, the geometric mean is evaluated $\theta_p^{GM}= \sqrt[3]{\theta_p^{L} \cdot \theta_p^{M} \cdot \theta_p^{U}}$ [40].

• Cross-efficiency evaluation

In addition to the evaluation of efficiencies according to the above models, the cross efficiency evaluation model is also applied, which has a better distinguishing power for ranking DMUs, [35].

In conventional DEA models, DMUs have the nature of self-evaluation in the selection of weights for each input and output, where we can have several efficient evaluated DMUs, where Eff^{CRS}=1. The Cross efficiency model is the approach presented by [6] and [41]. The efficiency values of each DMUs with the cross efficiency model are evaluated not only with their own weights, but also with the input -output weights of other units. The average representing the Cross efficiency result for a DMUj (j=1, 2,...,n) is given: $\overline{Ef_j}= \frac{1}{n}\sum_{k=1}^{n} Ef_{kj}$, where this average is the average of the values according to the column presented in the Cross efficiency matrix, [41]. From the cross efficiency value chain, the corresponding efficiency value chain matrix is formatted, where the harmonic cross efficiency for the given period and the corresponding rankings are also evaluated. From the cross efficiency value chain matrix for each DMU, the triangular fuzzy numbers $\tilde{A}$, are composed, where $(\gamma_1 < \gamma_2 < \gamma_3)$, where this matrix fully enables this composition, which is advantageous over other models.

After determining the triangular fuzzy numbers from the cross efficiency values, based on fuzzy arithmetic and the geometric model of the concept of event probability, the approach for determining the degree of preference likeability is then applied.

In the following figures, the cases of geometric projections of triangular fuzzy numbers are given when their comparison is required. If two triangular fuzzy numbers $\tilde{\omega}$= $(\gamma_1, \gamma_2, \gamma_3)$ and $\tilde{\eta}$ = $(\delta_1, \delta_2, \delta_3)$ according to fuzzy arithmetic and fuzzy triangular number $\tilde{A}$ =$(\tilde{\omega}-\tilde{\eta})$ = $(\gamma_1-\delta_3, \gamma_2-\delta_2, \gamma_3-\delta_1)$ where for each of them their geometric projections can be given in a coordinate plane. For each case, it can be judged according to the concept of the geometric model of the probability of event A, the probability of this event is estimated. *Case 1:* In the Figure 2, the geometric projection of the two fuzzy numbers $\tilde{\omega}$= $(\gamma_1, \gamma_2, \gamma_3)$ = (5, 7, 9.5) and $\tilde{\eta}$ = $(\delta_1, \delta_2, \delta_3)$ = (1, 2, 3.5) is given. The fuzzy triangular number $\tilde{A}$ =$(\tilde{\omega}-\tilde{\eta})$ = (1.5, 5, 8.5), where it seems that the fuzzy triangular number $\tilde{A}$ is classified as positive, since each of its components is greater than zero, so it can be said that in this case P(A) = ($\widetilde{\omega} > \widetilde{\eta}$) =1. From the projections, it is noted that the condition ($\gamma_1 \geq \delta_3$) must be satisfied.

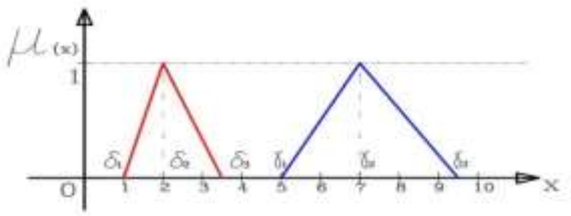

Fig. 2: Geometric projections of fuzzy numbers, $\widetilde{\omega}$= $(\gamma_1, \gamma_2, \gamma_3)$ = (5, 7, 9.5) and $\widetilde{\eta}$ = $(\delta_1, \delta_2, \delta_3)$ = (1, 2, 3.5)

Case 2: (Figure 3) If two triangular fuzzy numbers $\widetilde{\omega}$ = (2, 4, 5) and $\widetilde{\eta}$ = (5.5, 6, 7) are given, as well as $\tilde{A}$ =($\widetilde{\omega} - \widetilde{\eta}$) = (-5, -2, -0.5), where it can be said that the triangular fuzzy number $\tilde{A}$ =($\widetilde{\omega} - \widetilde{\eta}$) is classified as negative, so in this case it is said that P(A) = ($\widetilde{\omega} > \widetilde{\eta}$) = 0. From the projections it is noted that the condition $\gamma_3 \leq \delta_1$ must be fulfilled.

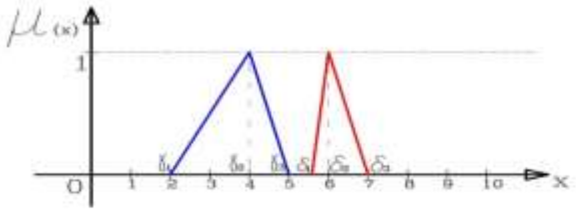

Fig. 3: Geometric projections of fuzzy numbers, $\widetilde{\omega}$ = (2, 4, 5) and $\widetilde{\eta}$ = (5.5, 6, 7)

Case 3: In the Figure 4 and Figure 5 if fuzzy triangular numbers $\widetilde{\omega}$ = (1, 4, 6), $\widetilde{\eta}$ = (2, 5, 7) and $\tilde{A}$ = ($\widetilde{\omega} - \widetilde{\eta}$) = (-6, -1, 4) are given. This number $\tilde{A}$ is partial negative triangular fuzzy number, so based on the concept of the geometric model of the probability of the event P(A) = P($\widetilde{\omega} > \widetilde{\eta}$) = $\frac{S(p)}{S(\bar{p})+S(p)}$, where S(p) is the area of the event A and $S(\bar{p}) + S(p)$ = S(Ω) is the area containing all the elementary events of a zone Z. $\Omega = A \cup \bar{A}$ dhe $A \cap \bar{A} = \emptyset$. In the case of the Figure 5.

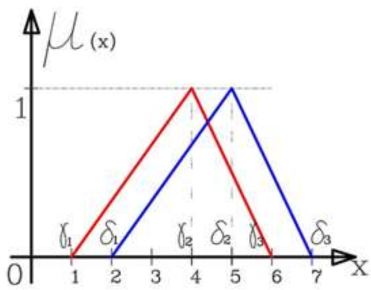

Fig. 4: Geometric projections of fuzzy numbers, $\widetilde{\omega}$ = (1, 4, 6), $\widetilde{\eta}$ = (2, 5, 7)

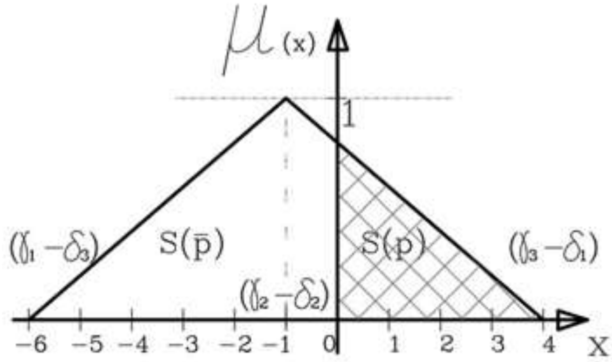

Fig. 5: Geometric projections of numbers, $\tilde{A} = (\widetilde{\omega}$-$\widetilde{\eta})$ = (-6, -1, 4)

S(p) = $\int_0^{\gamma_3-\delta_1} \frac{[(\gamma_3-\delta_1)-x]}{[(\gamma_3-\delta_1)-(\gamma_2-\delta_2)]}\,dx = \frac{1}{2}\frac{(\gamma_3-\delta_1)^2}{(\gamma_3-\delta_1+\delta_2-\gamma_2)}$.

S(Ω) = S($\bar{p}$) + $S(p)$ = $\frac{1}{2}$ [$(\gamma_3 - \delta_1) - (\gamma_1 - \delta_3)$],

so P($\widetilde{\omega} > \widetilde{\eta}$)= $\frac{(\gamma_3-\delta_1)^2}{(\gamma_3-\delta_1+\delta_2-\gamma_2)(\gamma_3-\gamma_1+\delta_3-\delta_1)}$,

from the presented case it is said that the condition $(\gamma_3 > \delta_1) \cap (\gamma_2 \leq \delta_2)$ is fulfilled. *Case 4:* (Figure 6 and Figure 7) Where $\widetilde{\omega}$ = (1.5, 4.5, 5.5), $\widetilde{\eta}$ = (0.5, 2.5, 3.5) and $\tilde{A}$= (-2, 2, 5).

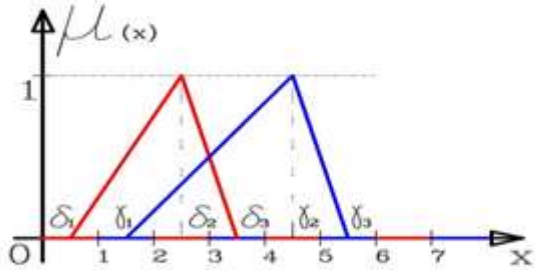

Fig. 6: Geometric projections of fuzzy numbers, $\widetilde{\omega}$ = (1.5, 4.5, 5.5), number $\tilde{A} = (\widetilde{\omega}$-$\widetilde{\eta})$ = (-2, 2, 5)

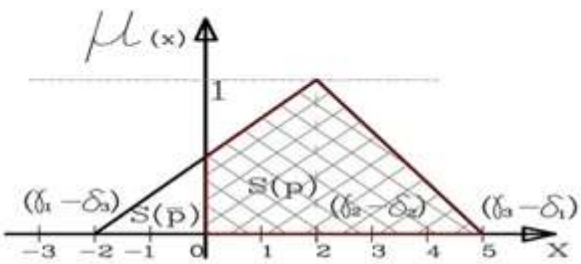

Fig. 7: Geometric projections of fuzzy $\widetilde{\eta}$ = (0.5, 2.5, 3.5)

In this case, it can be written P($\widetilde{\omega} > \widetilde{\eta}$) = 1- $\frac{S(\bar{p})}{S(\bar{p})+S(p)}$. It is calculated

$S(\bar{p})$ = $\frac{1}{2}\frac{(\delta_3-\gamma_1)^2}{(\delta_3-\gamma_1+\gamma_2-\delta_2)}$.

P($\widetilde{\omega} > \widetilde{\eta}$)= 1- $\frac{(\delta_3-\gamma_1)^2}{(\delta_3-\gamma_1+\gamma_2-\delta_2)(\gamma_3-\gamma_1+\delta_3-\delta_1)}$.

From the presented case it is said that the condition $(\gamma_2 > \delta_2) \cap (\gamma_1 < \delta_3)$ is fulfilled.

Based on the calculation of the probabilities according to the cases presented above, the matrix of the degree of liking and preference is formed as follows

$$M_p = \begin{matrix} & D1 & D2 & \dots & Dn \\ D1 & - & p_{12} & \cdots & p_{1n} \\ D2 & p_{21} & - & \cdots & p_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ Dn & p_{n1} & p_{n2} & \dots & - \end{matrix}.$$

Statistical tests are also applied in the paper.

3 Numerical Application

The aim of the numerical application is to evaluate the effectiveness and determine the degree of likability preference in the performance presented by 15 DMUs (the 15 DMUs are the administrative divisions in the prefectures and regions in Albania). The study covers the 7-year period (2016-2022). The prefectures and regions between them show differences in their multifaceted development of production and standard of living. DMUs operate with three inputs and two outputs. The data are taken from the Statistical Yearbooks (INSTAT) (2017-2023) and the Labour Market (2016-2023) [42], [43]. Both outputs carry values as variables not fully determined (surveys): 1. Distribution of households according to the area used for housing (over 90 square meters, in percentage), 2. Ownership of long-term devices with multiple functions in households according to prefectures (in percentage). The three inputs are related to employment in the labour market according to the three main sectors of the economy in percentage (agriculture, industry and construction, services) for each prefecture and region. The study follows the following course:

- smoothing of the values that present as a time series each input and output according to the movement of the average of the order k (treated in the methodology)

Table 1 (Appendix) shows the values of the overall average data for each variable size.

The application of models is done according to the following steps:

Step 1. Application of linear programming models ((1), (2), (3)) for evaluation of CRS, VRS, super - efficiencies for efficient units evaluated by the CRS efficiency model. This evaluation is done according to each grouping with Eff-M_i(3) (i=1,2,3,4,5). Classifications of the source of inefficiencies are given by comparing scale efficiency with VRS efficiency.

Step 2: Evaluation of Cross efficiency according to each grouping Eff-M_i(3) (i=1,2,3,4,5), calculation and evaluation of fuzzy technical efficiency.

The impact of variable factors on the cross-eff value is calculated, at the beginning of the period (Eff - M_1(3)) and at the end of the period (Eff - M_5(3)), (multiple linear regression) where efficiency values are considered as dependent variable and the respective output values as independent variables. The summary results are given in Table 4 (Appendix).

With significance level α = 0.05, $F_{0.05,2,12}$ = 3.89 where from the Table 3 (Appendix), F $>$ $F_{0.05,2,12}$ for both cases. Then the hypothesis H_0: $\beta_1 = \beta_2$ = 0 is rejected. Table 4 (Appendix) shows that at the beginning of the period the first output has the greatest impact ($\frac{\beta_1}{\beta_2} = \frac{0.4856}{0.1968} \approx 2.47$) while at the end of the period the second output has the greatest impact (($\frac{\beta_1}{\beta_2} = \frac{0.2454}{0.3338} \approx 0.74$). The results obtained from the second step are given in Table 5 (Appendix).

From the summary rankings obtained from the three models, the overall ranking is estimated. Table 6 (Appendix) presents the overall ranking estimate as well as the triangular fuzzy numbers composed of the cross efficiency value matrix.

Step 3: Determining the degree of preference liking based on fuzzy arithmetic and the concept of the geometric probability model (based on the methodology). The composition of fuzzy numbers is given according to the matrix of cross-efficiency values given in Appendix in Table 5 and (Appendix) 6. The matrix of cross efficiency values is used, according to the fuzzy efficiency values ($\theta^L, \theta^M, \theta^U$) there are DMUs that have the fuzzy efficiency values $\theta^L = \theta^M = \theta^U$=1 as D9 and D4 ($\theta^M = \theta^U$=1).

From Table 7 (Appendix), it is noted the comparative preference between the decision-making units.

$$D_5 \overset{74.87\%}{\succ} D_3 \overset{40.99\%}{\succ} D_8 \overset{86.73\%}{\succ} D_7 \overset{56.31\%}{\succ}$$

$$\succ D_1 \overset{52.93\%}{\succ} D_9 \overset{49.59\%}{\succ} D_{13} \overset{64.90\%}{\succ} D_{12} \overset{63.90\%}{\succ}$$

$$\succ D_4 \overset{51.67\%}{\succ} \succ D_6 \overset{54.77\%}{\succ} D_{15} \overset{51.39\%}{\succ} D_{10} \overset{83.16\%}{\succ}$$

$$\succ D_2 \overset{17.54\%}{\succ} D_{14} \overset{89.85\%}{\succ} D_{11}$$

where $D_5 \overset{74.87\%}{\succ} D_3$ shows that D_5 has a better performance than D_3 to the extent to 74.87%, the same can be said for the others.

In the summary of the results obtained according to the rankings, it is noted that the Spearman coefficient (ρ_{Sp}) shows these values:

1) The rank correlation between: the cross efficiency (harmonic) ranking and the ranking of the degree of preference, ρ_{Sp1} = 0.98.
2) The rank correlation between: CRS efficiency ranking (overall efficiency) and preference degree ranking, ρ_{Sp2}= 0.95
3) The rank correlation between: CRS (overall efficiency) efficiency ranking and Cross (harmonic) efficiency ranking, ρ_{Sp3}= 0.90
4) The rank correlation between: ranking of the fuzzy geometric mean and overall efficiency, ρ_{Sp4}= 0.87
5) The rank correlation between: overall ranking (Table 6, Appendix) and degree of liking preference ranking (Table 7, Appendix), ρ_{Sp5}= 0.98

For each of the above cases, the hypothesis test is taken: H_0: ρ_{sp}= 0 dhe H_1: $\rho_{sp} \neq 0$. For n=15 and a significant level of α = 0.005, the critical value is 0.654. This shows that H_0 is rejected.

Even applying the statistic_t, where t = $\frac{r}{S_r}$, ku S_r = $\sqrt{\frac{1-r^2}{n-2}}$ and r = ρ_{sp}. For the level of α = 0.05, $|t| \geq t_{\frac{\alpha}{2},n-2}$ = $t_{0.025,13}$ = 2.160. For the above five points, the calculated t-values are t_1=17.756; t_2 = 10.97; t_3 = 7.44; t_4 = 6.3025 and t_5 = 17.756, so H_0 is rejected.

The values of the Spearman rank correlation coefficient and hypothesis testing indicate strong correlations between the rankings.

In conclusion of the results obtained, it can be said that certain DEA models can be used to assess the effectiveness of DMUs with appropriate objectives. From the above results, the following DMUs are found to be best practices: D_5, D_3, D_8, D_9, D_7, D_1. The general feature for them is the impact that employment represents in the service sector and in the industry and construction, being higher than other DMUs (Table 2, Table 3 and Table 4 in Appendix). In the classification of inefficiencies (Table 2, Appendix), managerial inefficiency is most evident. While the weakest practices are the DMUs that have the highest percentage of employment in agriculture.

While assessing the degree of preference liking, the most advantageous approach is the application of cross efficiency in harmony with Fuzzy arithmetic and the geometric model of probability. From the results obtained with the data in the above tables, the performance of each DMU is determined according to the objectives, the degree of preference, and the analysis judgments.

4 Conclusion

In this work, the evaluation of the performance of DMUs was dealt with by determining the degree of liking preference and effectiveness (efficiency value) of each DMU according to the input-output levels with the processed data as well as determining the respective rankings. The levels of quantities of input-output values where DMUs operate, where some of them do have not completely clear (fuzzy) values, according to the principle of dynamic analysis of a time series, smoothing of the time series is applied by determining the input-output levels in processed values, which are applied to the respective models. Conventional DEA models, cross-efficiency model, super efficiency models, Fuzzy efficiency evaluation models are applied in this paper. The alternative approach was applied according to a two-phase process, where the degree of liking preference was determined based on the cross-efficiency value chain in harmony with fuzzy arithmetic. We emphasize that in harmony with the fuzzy arithmetic, the matrix of the cross efficiency value chain was used, because with the fuzzy efficiency values ($\theta^L, \theta^M, \theta^U$) it may happen that we can have DMU where these values are equal or two of them are equal. Thus, two of the DMUs (specifically D4 and D9) do not enable the composition of the fuzzy number (in the order $\gamma_1 < \gamma_2 < \gamma_3$), so the model of the application of the value chain of cross efficiency enables the composition of triangular fuzzy numbers, so it can be said that it is more advantageous to apply it in harmony with fuzzy arithmetic. In summary, the contribution of the application of this approach can be said to be in fulfilling these objectives:

1. We better evaluate the meaningfulness of uncertain (fuzzy) data for their application in DEA and DEA Fuzzy models. Using several models provides a more realistic and informative assessment of effectiveness.

2. An alternative method is reflected as an approach of a two-phase process. The first phase determines efficiency value matrices according to models that assess the efficiency of DMUs. In the second phase, fuzzy numbers are composed of the cross-efficiency value matrix, and based on fuzzy arithmetic and the concept of the geometric probability model, the degree of preference likeability and preference ranking are determined.
   The first phase determines the matrix of the value of the cross efficiency along an extended time course and the second phase, based on fuzzy arithmetic and the concept of the geometric model of probability determines the degree of likability preference and preference ranking.
3. The evaluation of the application of this approach shows the advantages of the application in the evaluation of the effectiveness and the determination of the degree of likability preference over a long period of time in relation to the conventional DEA models, which also present limitations. The review and analysis of data according to each model is also selected based on the relevant objectives that can be set.

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APPENDIX

Table 1. Average data values (for 15 DMUs) over time

I/O	Average data for 15 DMUs (%)							The data by smoothing of time series (%)				
	t_1	t_2	t_3	t_4	t_5	t_6	t_7	$M_1(3)$	$M_2(3)$	$M_3(3)$	$M_4(3)$	$M_5(3)$
I_1	46.3	43.5	43.5	43.4	43.9	42.0	41.2	44.4	43.5	43.6	43.1	42.4
I_2	18.4	18.5	18.2	18.6	18.8	20.2	19.6	18.4	18.4	18.5	19.2	19.5
I_3	35.3	38.1	38.3	38.2	37.3	37.8	39.1	37.2	38.2	37.9	37.8	38.1
O_1	38.7	39.7	38.1	36.6	38.6	37.8	42.3	38.8	38.1	37.8	37.7	39.6
O_2	30.3	42.5	50.9	60.4	64.8	68.6	75.7	41.2	51.3	58.7	64.6	69.7

Table 2. Values of efficiency, super -efficiency, rankings and classification of sources of inefficiencies according to each grouping

DMU	Eff - $M_1(3)$			Eff -$M_2(3)$			Eff -$M_3(3)$			Eff -$M_4(3)$			Eff -$M_5(3)$			Ind. CE	Overall Rank
	Eff.	C-I_{eff}.	Rank	Eff.	C-I_{eff}.	Rank	Eff.	C-I_{eff}.	Rank	Eff.	C-I_{eff}.	Rank	Eff.	C-I_{eff}.	Rank		
D1	0.906	b	9	0.853	a	10	0.902	a	9	0.954	a	8	0.967	b	9	1.017	8
D2	0.912	a	8	0.751	a	15	0.854	b	13	0.849	b	14	0.821	b	15	0.981	13
D3	**1.122**	eff	3	**1.029**	eff	3	**1.003**	eff	5	**1.001**	eff	5	**1.135**	eff	3	1.000	3
D4	0.982	b	6	**1.015**	eff	4	**1.152**	eff	3	**1.247**	eff	2	**1.351**	eff	1	1.004	6
D5	**1.178**	eff	2	**1.237**	eff	2	**1.257**	eff	2	**1.172**	eff	3	**1.053**	eff	4	1.000	2
D6	0.846	a	10	0.860	a	9	0.869	a	11	0.891	a	12	0.962	b	10	1.033	10
D7	**1.001**	eff	4	**1.000**	eff	5	**1.000**	eff	6	**1.000**	eff	6	**1.001**	eff	5	1.000	4
D8	**1.797**	eff	1	**1.654**	eff	1	**1.358**	eff	1	**1.386**	eff	1	**1.223**	eff	2	1.000	1
D9	**1.000**	eff	5	**1.000**	eff	6	**1.000**	eff	7	**1.000**	eff	7	**1.000**	eff	6	1.000	5
D10	0.758	a	14	0.810	a	12	0.889	a	10	0.910	a	11	0.883	a	11	1.040	11
D11	0.758	b	13	0.767	b	14	0.790	b	15	0.786	b	15	0.875	b	12	1.037	15
D12	0.827	b	11	0.877	a	8	0.930	a	8	0.941	b	9	0.986	b	7	1.045	9
D13	0.947	b	7	0.997	b	7	**1.080**	eff	4	**1.032**	eff	4	0.978	b	8	1.008	7
D14	0.657	a	15	0.795	a	13	0.863	a	12	0.889	a	13	0.861	a	14	1.074	14
D15	0.791	a	12	0.843	b	11	0.840	b	14	0.915	a	10	0.873	a	13	1.026	12

Note: Eff- values of efficiency and super efficiency; C-I_{eff}- Classification of inefficiencies [38] (a-management inefficiency, $Ef_{CRS} < 1$, Ef_{VRS} <1 and SE < Ef_{VRS}, SE-scale efficiency; b-scale inefficiency, $Ef_{CRS} < 1$, Ef_{VRS} =1 dhe SE < Ef_{VRS}; c-managerial inefficiency and scale inefficiency, both together, $Ef_{CRS} < 1$, Ef_{VRS} <1 dhe SE> Ef_{VRS}; eff.- Efficient DMU); Ind. EC = $\frac{\sum_{k=2}^{k}\frac{Ef(k)}{Ef(k-1)}}{k-1}$ (k indicates the number of grouping (in the case of the study k= 5) according to the module of moving averages of order k);

Table 3. The impact of each input on the efficiency value (Ef_{CRS})

W(I_i)	M_1 (3)	$M_2(3)$	$M_3(3)$	$M_4(3)$	$M_5(3)$	Average	Parameter estimation $= \frac{(\%) impact\ on\ efficiency}{(\%)\ of\ employment}$
W(I_1)	12.08 %	12.50%	14.59%	14.26%	14.87%	13.66%	$\frac{13.6\%}{43.4\%}$ = 0.3147
W(I_2)	39.83%	37.07%	35.87%	36.67%	33.29%	36.55%	$\frac{36.55\%}{18.8\%}$ = 1.9442
W(I_3)	48.09%	50.43%	49.54%	49.07%	51.84%	49.79%	$\frac{49.79\%}{37.84\%}$ = 1.3158

Table 4. Statistical results

Applied model	Multiple R	R square	F	Significance F	β_1	β_2
$M_1(3)$	0.97	0.94	102.48	2.86367E-08	0.4856	0.1968
$M_5(3)$	0.94	0.89	49.78	1.54903E-06	0.2454	0.3338

Table 5. Cross efficiency values, rankings according to each grouping, geometric mean efficiency ($\theta^{L,}\theta^{M,}\theta^{U}$) fuzzy, ranking

DMU	Cross efficiency												Fuzzy DEA	
	Eff - $M_1(3)$		Eff - $M_2(3)$		Eff - $M_3(3)$		Eff - $M_4(3)$		Eff - $M_5(3)$		Cross H-Eff	Rank		
	Cross Eff.	Rank	Cross Eff.	Rank	Cross Eff.	Rank	Cross Eff.	Rank	Cross Eff.	Rank			GM = $\sqrt[3]{\theta^{L}\cdot\theta^{M}\cdot\theta^{U}}$	Rank of GM
D1	0.814	7	**0.807**	8	0.858	6	**0.863**	4	**0.855**	3	0.839	5	0.639	8
D2	0.769	8	**0.720**	13	**0.770**	14	0.720	14	**0.672**	15	0.728	13	0.568	13
D3	**0.902**	3	**0.873**	4	0.893	4	**0.964**	1	0.956	1	0.916	2	0.879	2
D4	0.677	13	**0.675**	15	**0.812**	11	**0.851**	6	0.826	7	0.760	12	0.689	6
D5	**0.901**	4	0.963	2	**0.968**	1	**0.956**	2	0.911	2	0.939	1	0.739	5
D6	0.756	9	**0.750**	12	**0.814**	10	**0.785**	12	0.811	8	0.782	9	0.597	10
D7	**0.902**	2	0.872	5	**0.851**	7	**0.796**	9	0.810	9	0.844	4	0.805	3
D8	**0.946**	1	**0.996**	1	0.957	2	**0.821**	7	0.842	4	0.907	3	0.766	4
D9	**0.885**	5	**0.829**	6	0.819	9	**0.816**	8	0.836	5	0.837	6	1.000	1
D10	**0.693**	12	0.792	9	**0.838**	8	**0.787**	11	0.751	11	0.769	11	0.587	11
D11	0.661	14	**0.753**	11	0.752	15	**0.644**	15	**0.681**	14	0.695	15	0.532	15
D12	**0.716**	10	**0.826**	7	**0.880**	5	0.790	10	0.832	6	0.805	8	0.647	7
D13	0.814	6	**0.884**	3	**0.915**	3	0.890	3	**0.706**	12	0.834	7	0.583	12
D14	**0.598**	15	0.688	14	**0.776**	13	0.766	13	**0.706**	13	0.701	14	0.538	14
D15	**0.700**	11	**0.778**	10	0.800	12	**0.856**	5	0.761	10	0.776	10	0.628	9

Note: Cross H-Eff- cross harmonic efficiency

Table 6. Overall ranking, composition of Fuzzy numbers from the cross efficiency value matrix

DMU	Ranking			Composition of fuzzy triangular numbers (Cross Eff.)
	Weak R_i	GMR_i=$\sqrt[3]{R_i^{CRS}\cdot R_i^{Cross}\cdot R_i^{Fuzzy}}$	Overall R_i	
D1	8	6.83990	6	(0.807, 0.855, 0.863)
D2	13	12.99999	13	(0.672, 0.720, 0.770)
D3	3	2.28943	2	(0.873, 0.902, 0.964)
D4	12	7.55953	7	(0.675, 0.812, 0.851)
D5	5	2.15443	1	(0.901, 0.956, 0.968)
D6	10	9.65489	10	(0.750, 0.785, 0.814)
D7	4	3.63424	5	(0.796, 0.851, 0.902)
D8	4	2.28943	3	(0.821, 0.946, 0.996)
D9	6	3.04461	4	(0.816, 0.829, 0.885)
D10	11	11	12	(0.693, 0.787, 0.838)
D11	15	15	15	(0.644, 0.681, 0.753)
D12	9	7.95811	8	(0.716, 0.826, 0.880)
D13	12	8.37772	9	(0.706, 0.884, 0.915)
D14	14	14	14	(0.598, 0.706, 0.776)
D15	12	10.25986	11	(0.700, 0.778, 0.856)

Note: Weak R_i- The weakest ranking value; GMR_i – geometric mean of rankings according to the three models applied; Overall R_i – positioning in the overall ranking on which the effectiveness of the DMUs is judged during the period.

Table 7. The matrix of the degree of liking preference from Cross efficiencies and rankings according to the degree of liking preference

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12	D13	D14	D15	Rank
D1	....	1.00000	0.00000	0.90349	0.00000	0.99576	0.43641	0.05863	0.52931	0.95309	1.00000	0.76358	0.50239	1.00000	0.91174	5
D2	0.00000	...	0.00000	0.17571	0.00000	0.02902	0.00000	0.00000	0.00000	0.16843	0.73860	0.06836	0.05861	0.66910	0.14987	13
D3	1.00000	1.00000	...	1.00000	0.21513	1.00000	0.94668	0.40988	0.98877	1.00000	1.00000	0.99781	0.90268	1.00000	1.00000	2
D4	0.09651	0.82429	0.00000	...	0.00000	0.51666	0.11583	0.01619	0.09644	0.56104	0.89845	0.36096	0.25362	0.86040	0.54185	9
D5	1.00000	1.00000	0.78487	1.00000	...	1.00000	0.99996	0.64191	1.00000	1.00000	1.00000	1.00000	0.99197	1.00000	1.00000	1
D6	0.00424	0.97098	0.00000	0.48334	0.00000	...	0.02250	0.00000	0.00000	0.57056	0.99952	0.30192	0.20680	0.97244	0.54773	10
D7	0.56359	1.00000	0.05332	0.88417	0.00004	0.97750	...	0.13275	0.58944	0.93371	1.00000	0.76057	0.53448	1.00000	0.89683	4
D8	0.94137	1.00000	0.59012	0.98381	0.35809	1.00000	0.86725	...	0.90592	0.99487	1.00000	0.94258	0.85323	1.00000	0.98191	3
D9	0.47069	1.00000	0.01123	0.90356	0.00000	1.00000	0.41056	0.09408	...	0.96615	1.00000	0.74076	0.49592	1.00000	0.92373	6
D10	0.04691	0.83157	0.00000	0.43896	0.00000	0.42944	0.06629	0.00513	0.03385	...	0.91598	0.29834	0.21554	0.87004	0.48606	12
D11	0.00000	0.26140	0.00000	0.10155	0.00000	0.00048	0.00000	0.00000	0.00000	0.08402	...	0.02662	0.02774	0.46402	0.06971	15
D12	0.23642	0.93164	0.00219	0.63904	0.00000	0.69808	0.23943	0.05742	0.25924	0.70166	0.97338	...	0.35098	0.94169	0.67467	8
D13	0.49761	0.94139	0.09732	0.74638	0.00803	0.79320	0.46552	0.14677	0.50408	0.78446	0.97226	0.64902	...	0.94825	0.75857	7
D14	0.00000	0.33090	0.00000	0.13960	0.00000	0.02756	0.00000	0.00000	0.00000	0.12996	0.53598	0.05831	0.05175	...	0.11695	14
D15	0.08826	0.85013	0.00000	0.45815	0.00000	0.45227	0.10317	0.01809	0.07627	0.51394	0.93029	0.32533	0.24143	0.88305	...	11