Modeling and Control of a Coaxial Parallel Spherical Mechanism

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Abstract: - This paper presents a method for modeling and controlling a coaxial parallel spherical mechanism. This mechanism consists of elements connected to allow them to move relative to each other in a parallel and coaxial manner, forming a spherical structure capable of rotating and moving in multiple directions. The operation of the mechanism is controlled by a high-order sliding controller to improve control performance, reduce chattering phenomena, and mitigate disturbances during operation. The system's stability using the suggested control approach is demonstrated and analyzed in detail through experimental validation on a real-time controller, TMS320F28379D.

Key-Words: - Coaxial parallel spherical mechanism, high-order sliding control, system modeling, chattering phenomenon, real-time controller, balance control.

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1 Introduction

Parallel mechanisms consist of dynamically linked kinematic chains forming a closed loop, where the platform is controlled for motion and connected to the base through two or more links, [1]. Due to their parallel structure, they often exhibit higher load-bearing capacity, speed, robustness, and accuracy compared to open-loop mechanisms with similar functions, thus finding applications in systems such as object folding robots [2], orienting [3] and positioning devices, and tracking equipment [4], [5], medical [6]. However, they also have some drawbacks, such as limited workspace and complexity of motion, which make control and simulation of the mechanism difficult, especially in determining the workspace.

The control of parallel spherical robots is a challenging task that has garnered significant attention from researchers due to its multivariable nature, strong coupling, and nonlinearity [7]. To date, various modeling and control methods have been suggested to enhance the stability and dynamic performance of these robots, such as equation-based modeling methods using Newton's law or other modeling approaches [8], along with control methods including PID control and optimal control [9], [10]. Sliding mode control is a suitable choice for systems that require fast response and are subject to unpredictable disturbances. It has many advantages, as it is robust and stable under changes in system parameters, with low sensitivity [11], [12]. Non-singular terminal sliding mode (NTSM) is being researched as a new method to improve the quality of sliding mode control, helping to reduce chattering and enhance control performance [13]. Therefore, in this paper, NTSM controllers for flow and current control of parallel spherical mechanisms are proposed. The advantages of the proposed method are specifically demonstrated through experimental results.

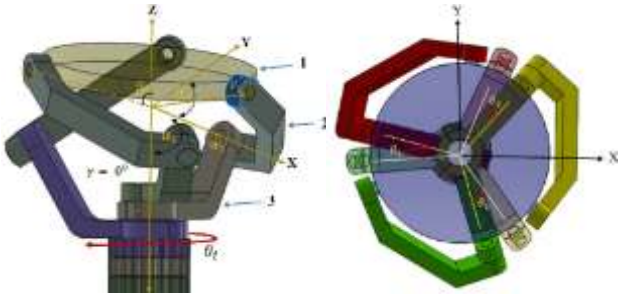
The structure of the article consists of four parts. Part 1 is the introduction; system modeling and controller design are presented in Part 2. Part 3 covers the control results of the robot based on the designed controller, and Part 4 presents the conclusion and directions for future research.

2 Mathematical model of the Coaxial Parallel Spherical Mechanism

2.1 Modeling of Coaxial Parallel Spherical Mechanism

The analysis of the motion of parallel spherical mechanisms originates from the general motion of parallel robot mechanisms published in studies [14], [15]. The upper base of the mechanism is called the movable plane (or working plane), which is assembled to the fixed base via three equal links numbered i = 1, 2, 3 respectively in the counterclockwise direction. Each link has two joints with curved shapes: the lower and upper joints, with

angles α_1 and α_2 representing the curvature of these two joints. The axes of the joints of the fixed base, intermediate links, and movable base intersect at the rotational center and are determined by the unit vectors ui, vi, and wi directed from the rotational center towards the corresponding joints.



(a) (b)

Fig. 1: (a) Coordinate system in the robot, (b) Positive directions of the control joints

The independent system is defined as shown in Figure 1(a) and must follow the right-hand rule. The z-axis is perpendicular to the base and the center axis of the base's coaxial rotation axis. The vertical x-axis is determined by the direction of the z-axis and the vertical plane when the controller is in an angular position, while the y-axis is determined from the remaining two axes. The beta angle is defined in relation to the z-axis. The configuration of the parallel robot mechanism is selected so that the three lower links are separated by an angle of 120^0. In this case, the working plane lies horizontally, perpendicular to, and on the positive side of the z-axis. The joint position inputs consist of the vector $\theta = [\theta_1; \theta_2; \theta_3]^T$ calculated from the planes defined by the z-axis and the unit vectors Wi (i=1, 2, 3). The joint position vectors are configured so that $\theta = [\theta_1; \theta_2; \theta_3]^T$ presents the initial arrangement.

The coordinate system of the SPM has its rotational center at the origin, with the positive directions as show in Figure 1(b). The Z-axis is always perpendicular to the working surface and extends upward. The y-axis is created by the z-axis and the x-axis. The unit vector u_i is defined as:

$$u_i = [0\ 0\ -1]^T \quad (1)$$

The direction and orientation of the unit vectors Wi (where i = 1, 2, 3) for the axes of rotation joints are determined by the input angle θi (i = 1, 2, 3):

$$W_i = Ri(\theta i)\ Zy \quad (2)$$

where $zy = [0\ 1\ 0]^T$. From forward kinematic, we can obtain rotation matrix Ri(θi), thus, Eq. (2) can be expressed specifically as:

$$W_i = \begin{bmatrix} (\cos\eta_i .\sin\theta_i - \sin\eta_i .\cos\gamma .\cos\theta_i) .\sin\alpha_1 \\ (\sin\eta_i .\sin\theta_i + \cos\eta_i .\cos\gamma .\cos\theta_i). \sin\alpha_1 \\ -\cos\gamma .\cos\alpha_1 \end{bmatrix} \quad (3)$$

The vectors v_i are in the same direction as the upper rotation axis of the links, depending on the orientation of the upper platform, and they are represented by a matrix Q (where Q is a rotation matrix). Thus. we have:

$$v_i = Q.v_i^* \quad (4)$$

where v_i^* are the vectors corresponding to the axes of the upper joints, given as:

$$v_i^* = [-\sin\eta_i .\sin\beta \quad \cos\eta_i .\sin\beta \quad \cos\beta]^T \quad (5)$$

Modelling the entire spherical parallel manipulator involves the following relationship:

$$w_i.v_i = \cos\alpha_2 \quad \text{with } (i = 1, 2, 3) \quad (6)$$

From equations (3), (4) and (6), we obtain:

$$A_i.t_i^2 + 2.B_i.t_i + C_i = 0 \quad (i = 1, 2, 3) \quad (7)$$

where $t_i = \tan(\theta_i/2)$. The kinematic parameters and characteristics of the mobile platform are determined by Ai, Bi and Ci.

$$\begin{aligned} A_i &= v_{xi}(\cos\beta_1 .\cos\eta_i .\sin\alpha_1 + \cos\alpha_1 .\cos\eta_i .\sin\beta_1) \\ &\quad + v_{yi}(\cos\beta_1 .\sin\alpha_1 .\sin\eta_i - \cos\alpha_1 .\sin\beta_1 .\sin\eta_i) \\ &\quad - v_{zi}(\cos\alpha_1 .\cos\beta_1 + \sin\alpha_1 .\sin\beta_1) - \cos\alpha_2, \\ B_i &= -(v_{xi} .\sin\alpha_1 .\sin\eta_i + v_{yi} .\cos\eta_i .\sin\alpha_1), \\ C_i &= v_{xi}(-\cos\beta_1 .\cos\eta_i .\sin\alpha_1 + \cos\alpha_1 .\cos\eta_i .\sin\beta_1) \\ &\quad - v_{yi}(\cos\beta_1 .\sin\alpha_1 .\sin\eta_i + \cos\alpha_1 .\sin\beta_1 .\sin\eta_i) \\ &\quad - v_{zi}(\cos\alpha_1 .\cos\beta_1 + \sin\alpha_1 .\sin\beta_1) - \cos\alpha_2. \end{aligned}$$

In this context, v_{xi}, v_{yi}, v_{zi} refer to the components of the vector v_i. The parameters α_1, α_2 and β_1 are determined by the geometry of the manipulator under investigation. The angles η_i (for i = 1, 2, 3) denote the angles between the projections of the axes of the driven revolutions onto the base plane of the stationary pyramid and a defined reference in that plane. Utilizing symmetry, we can represent the relationships as follows:

$$\eta_1 = 0, \eta_2 = \frac{2\pi}{3}, \eta_3 = \frac{4\pi}{3}$$

In this equation, the specific solution of the forward kinematics equation has not yet been determined. To ascertain the direction and angle of rotation for control facilities, we must first establish a system of equations:

$$v_i.v_j = \cos\alpha_3 \quad i, j = 1,2,3,\ i \neq j \text{ and } \|v_j\| = 1 \quad (8)$$

The angle between the above two joints is expressed by α3.

2.2 NTSM Design

The modeling of the Spherical Parallel Manipulator is represented by the following equation:

$$\begin{aligned}\dot{x}_1 &= ב_1(x_i, y_i, z_i),\\ \dot{x}_2 &= ב_2(x_i, y_i, z_i)+g.(x_i, y_i, z_i)+B(x_i, y_i, z_i).u\end{aligned} \quad (9)$$

where $ב_1$ and $ב_2$ are vector functions determined by the system, and g represent the unknown parameters and distribution, such that $\|g.(x_i, y_i, z_i)\| \le l_g$, where $l_g \ge 0$, B is a system matrix, and the control vector is defined as $u = (u_1, u_2, u_3)^T \in R^n$. It is further assumed that $(x_i, y_i, z_i) = (0, 0, 0)$ is valid if and only if $(x_1, \dot{x}_i) = (0, 0)$. The NTSM for this manipulator can be designed as follows, [13].

$$S = x_1 + א.\dot{x}_1^{p/q} \quad (10)$$

where: $א = \mathrm{diag}(\tau_1, \tau_2, \tau_3)$, $(\tau_i > 0)$ for i = 1, 2, 3 and $\dot{x}_1^{p/q}$ is represented as:

$$\dot{x}_1^{p/q} = (\dot{x}_1^{p1/q1}, \dot{y}_1^{p2/q2}, \dot{z}_1^{p3/q3})$$

The control signal is designed as shown in (11). Then, the dynamic systems of the spherical parallel manipulator (9) will converge to the equilibrium point in finite time:

$$u = -\frac{\begin{pmatrix} (\lg\left\|\frac{\partial ב_1}{\partial x_2}\right\|+\eta)\frac{S}{\|S\|} \\ +\frac{\partial ב_1}{\partial x_1}ב_1(x_i,y_i,z_i)+\frac{\partial ב_1}{\partial x_2}ב_2(x_i,y_i,z_i) \\ +\varepsilon^{-1}א^{-1}\mathrm{diag}(\dot{x}_1^{2-\frac{p1}{q1}}, \dot{y}_1^{2-\frac{p2}{q2}}, \dot{z}_1^{2-\frac{p3}{q3}}) \end{pmatrix}}{\left(\frac{\partial ב_1}{\partial x_2}B(x_i,y_i,z_i)\right)} \quad (11)$$

where $\varepsilon = \mathrm{diag}\left({}^{p1}/_{q1}, {}^{p2}/_{q2}, {}^{p3}/_{q3}\right)$, p_i and q_i are positive odd integers, with $q_i < p_i < 2q_i$ for i = 1, 2, 3.

Proof. Consider the Lyapunov function

$$V = 0.5\, S^T S$$

Differentiating V with respect to time, and combine (10), (11) into it yields:

$$\begin{aligned}\dot{V} &= S^T\dot{S} = S^T\left(\dot{x}_1 + \varepsilon.א.\mathrm{diag}(\dot{x}_1^{(\frac{p}{q}-1)}).\dot{x}_2\right)\\ &= S^T\varepsilon.א.\mathrm{diag}(\dot{x}_1^{(\frac{p}{q}-1)})\left(\varepsilon^{-1}.א^{-1}.\mathrm{diag}(\dot{x}_1^{(2-\frac{p}{q})}) + \dot{x}_2\right)\\ &= \varepsilon.א.\mathrm{diag}(\dot{x}_1^{(\frac{p}{q}-1)})\left(S^T.g.(x_i, y_i, z_i) - \left(\lg\left\|\frac{\partial ב_1}{\partial x_2}\right\| + \eta\right)\frac{S^TS}{\|S\|}\right)\end{aligned}$$

With condition: $\|g.(x_i, y_i, z_i)\| \le l_g$, and $l_g \ge 0$. That is:

$$\dot{V} \le -\eta\|S\| < 0 \quad \text{for } \|S\| \ne 0$$

3 Experiment Result

Fig. 2: Experimental model

The experimental model consists of three components: a computer, a control board, and a parallel real-world robot model, as described in Figure 2. To test the system response, the control unit sends signals to change the joint angles while transmitting signals from the accelerometer sensors attached to the base plate to the controller. The NTSM controller was designed with parameters set to $p_i = 5$, $q_i = 3$ and $\tau_i = 0.0001$.

Figure 3 illustrates the roll angle response of the manipulator in comparison to the roll angle signal directly obtained from the accelerometer sensor. We observe that the robot's motion closely follows the real signal, with the largest errors occurring only during moments of rapid change in angular velocity. The rotational motion in the three roll-pitch-yaw angles of the robot, when receiving signals from the accelerometer sensor, is depicted in Figure 4.

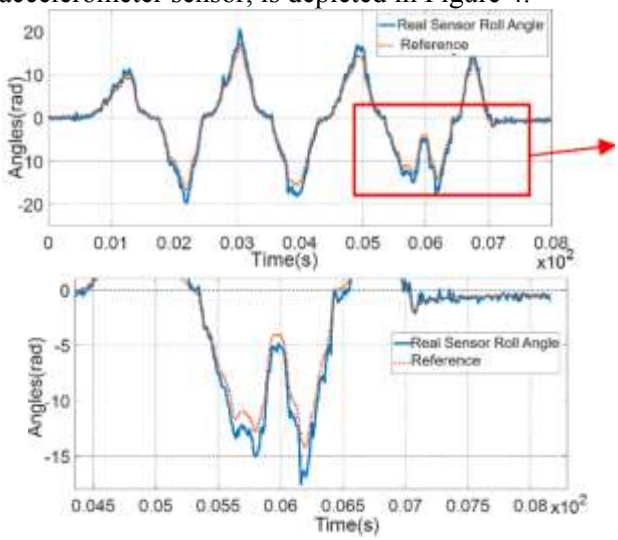


Fig. 3: Accelerometer roll angle and roll angle of the manipulator signal

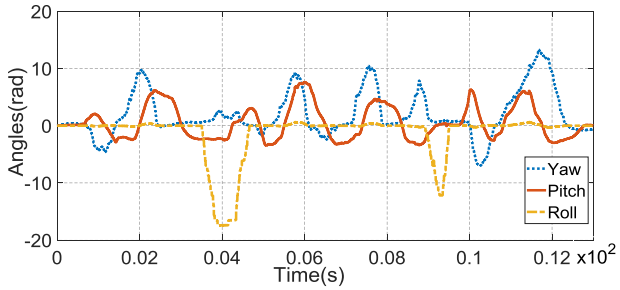


Fig. 4: Roll, Pitch, Yaw Angles Control

4 Conclusion

In this study, the research team successfully developed a model and control system for a parallel spherical robot structure using high-order sliding mode control theory. This method employs a nonlinear sliding surface to manage the robot's motion, resulting in vibration-free control that approaches zero with faster response times and higher accuracy. The research findings were verified through experiments on the real-time control board TMS320F28379D, ensuring high reliability. In the future, research will focus on designing larger detailed structures to create seating for electric wheelchairs in scenarios where the vehicle moves up and down stairs. These designs could also serve as fixtures for CNC machines, and as seating for virtual reality simulation systems for automobiles, aircraft, and ships. Applications will integrate artificial intelligence algorithms to predict singular points and plan complex trajectories for robotic operations.

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Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

- Nguyen Tan No has researched, designed, and built the control algorithm of Section 2.
- Duong Tan Dat has implemented the Algorithm and carried out the simulation and the optimization of Section 3.
- Le Hong Ky has organized and executed the experiments and is responsible for the Statistics.

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Conflict of Interest

The authors have no conflicts of interest to declare.