Robustness of Moving Average-Exponential Weighted Moving Average Control Chart with the Light-Tailed Distribution

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Abstract: - Control charts are the most significant statistical process control tools for ensuring industrial process reliability and efficiency. This research uses moving average – exponentially weighted moving average (MA-EWMA) control charts to investigate the dispersion characteristics. The control chart performance compares the arithmetic mean of run length (AMRL) and standard deviation of run length (SDRL) profiles of MA-EWMA and synthetic charts. Student's t-distributions are compared for dispersion processes. Finally, we present a case to show how essential control charts are in practice.

Key-Words: - Process dispersion, Monte Carlo simulation, Variability chart, Average run length, Standard deviation of run length, Monitoring, Sensitivity.

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1 Introduction

Statistical Process Control (SPC) utilizes statistical analysis to monitor and improve the quality of processes across various industries, extending its reach beyond traditional manufacturing applications. Control charts, introduced by [1], serve as the cornerstone of SPC methodology. These charts effectively identify abnormal variations within a process, ensuring consistent performance and adherence to quality specifications. Initially developed for the manufacturing sector, control charts have seen widespread adoption in diverse fields, including nuclear engineering, healthcare, and education, [2], [3], [4], [5].

Prioritizing the analysis of dispersion parameters before location parameters is crucial for establishing a robust understanding of process variability. Higher dispersion indicates a broader process output. At the same time, low dispersion suggests that the output is closely clustered around a central trend. To characterize process features, failing to assess and control dispersion before estimating position parameters might drastically impair their interpretability due to the potentially deceptive effects of excessive variance. As a result, a thorough examination of dispersion gives critical insights into process variability and guides the selection of the best position measurement for robust characterization.

Although much control chart research focuses on the fundamentals of normality, it is also helpful for verification processes governed by standard distributions, such as the Student's t-distribution and the mixture-specific distribution, which are common in industrial applications. Furthermore, control chart features can be carefully chosen for processes with non-normal distributions to achieve a given shift magnitude. Conversely, control chart designs are suited to processes with unknown distributions and predefined target shift sizes. It is a daunting challenge.

Shewhart chart is excellent at spotting substantial process changes despite their reliance on recent observations. It is, however, less responsive to changes. Control charts with memory, such as cumulative sum (CUSUM) and exponentially weighted moving average (EWMA) control charts [6], [7], can be used to address this problem. These graphics make use of both historical and current data. Enhances ability to notice small process changes. Although, [8] suggests moving average (MA) charts as an alternative, their performance may not always match that of CUSUM or EWMA charts in all circumstances.

The never-ending quest for better process parameter changes detection has driven the recent development of advanced control charts. Researchers used known approaches, such as EWMA and MA charts, to present an innovative control chart. [9] show a modified EWMA (mEWMA) chart. It is specifically developed to increase average verification while [10] used a different strategy. [11], who pioneered the MA-EWMA chart for processes with exponential distributions, presented EWMA-MA charts, which integrate strategic features from both control charts. Their findings convincingly indicate the improved effectiveness of MA-EWMA charts in identifying parameter changes in a wide range of processes. It includes symmetric and asymmetric distributions for all change sizes.

The use of control charts is divided into two situations: Phase I and Phase II where Phase I retrospectively focuses on thoroughly understanding the process and assessing its stability. This distance ensures the process functions within the inherent variability at the desired goal level. In addition, Phase I includes estimating essential process parameters and determining control limits. Following this, Phase II, the prospective phase, leverages the control chart to monitor processes in real-time. Its primary objective is the detection of incipient process shifts, facilitating the timely implementation of corrective actions. Phase II assesses control chart performance, particularly its efficacy in identifying process changes. In this paper, In Phase II, we focus on effective control charts for process dispersion parameters to solve problems with position parameters. For EWMA charts, [12]. Leveraging the established framework of EWMA charts, [13] introduced a ground-breaking approach to process variability monitoring. Their methodology centers on log-transformed sample variance, explicitly targeting the detection of nascent increases in variability that can critically impact product quality. This innovative approach outperforms traditional range or s^2 chart by enabling the swift identification of even minute standard deviation increases within a normally distributed process. [14], go into much detail about tracking distributions via normalized transformation. Their study looked at using EWMA control charts created utilizing the sample variance transformed logarithmically. They introduced a new control chart known as the NEWMA chart. The strategy includes a selective deletion of negative observations. As a result, it can improve the efficiency of detecting fragmentation changes, particularly for little differences.

[15], extended the study of process variability by employing one-sided and two-sided EWMA charts. Their simulations confirmed the accuracy of the preceding chart in detecting upward drift. The chart below outperforms current approaches for identifying shifts. [16], examine the choice of control charts for variability. Eight configurations were rigorously evaluated using standard deviation estimators for normal and non-normal distributions. They include calculation variables for control limits, greatly aiding the operator in chart selection. [17], compared the average performance (AMRL) of two new memory charts (Float T-S^2 and U-S^2) to CUSUM and EWMA charts. Their findings suggest fragmentation changes are detected more accurately, particularly for specific change sizes. [18], evaluated the efficacy of moving average standard deviation (MA-S) control charts in detecting process variability changes. Their study compared the performance of the MA-S chart to the standard S control chart, which used a moving average of sample standard deviations to measure process variance.

This article offers a combination control chart used to monitor process fragmentation. One distinguishing characteristic is using EWMA and MA statistics to estimate dispersion depending on change magnitude. Identifying this constant will mainly cause the control chart to differ. Monte Carlo simulations are crucial. It gives essential measures such as AMRL and SDRL. These indicators enable us to evaluate the chart's performance in various conditions. Comprehensively, this guarantees that adequate performance is evaluated under various scenarios.

As a result, this work presents an effective MA-EWMA control chart for monitoring process dispersion parameters. The design structure and performance were investigated, particularly in distinct primary conditions where the process dispersion factors differed. The motivation and inspiration for this investigation came from, [19]. Before moving on to the MA EWMA chart's basic structure, we describe the robust estimators in the next section. This paper is structured as follows: Sec. 2 comprehensively develops the control chart for standard deviation, detailing its construction; Sec. 3 meticulously evaluates its performance, scrutinizing effectiveness; Sec. 4 encompasses the simulation study; Sec. 5 includes comparative analysis and Sec. 6 discusses the illustrating example; and finally, Sec. 7 encapsulates the study's findings, making a definitive statement about the research.

2 Control Charts

This section outlines the theoretical basis for control charts. It begins by outlining the assumptions regarding the underlying data distribution, specifically the Student's t distribution, followed by a detailed explanation of control chart properties.

2.1 Distribution

This study investigates the performance measurement of control charts utilizing the arithmetic mean of run length and standard deviation of the run length with the t-value distribution shown below.

However, t_ν has heavier tails, and the parameter ν controls the probability mass of the tails. For $\nu = 1$, the Student's t distribution t_ν is transformed into the standard Cauchy distribution, which has very fat tails, whereas for $\nu \to \infty$, it is transformed into the standard average distribution N(0,1), which has thin tails. The probability density function for the Student's t-distribution is:

$$f(t) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\pi\nu}\,\Gamma\left(\frac{\nu}{2}\right)}\left(1+\frac{t^2}{\nu}\right)^{-(\nu+1)/2^2} \quad ; \ -\infty \le t \le \infty. \qquad (1)$$

The parameter ν is the number of degrees of freedom. The expectation of t distribution is 0, and the variance of the distribution is $\nu / \nu - 2$.

2.2 Control Chart

The author's research focuses on the performance of the moving average control chart - exponential moving average (MA-EWMA$_S$) with standard deviation. Then, compare the detection efficiency. The Student's t-distribution is an example of a symmetrical distribution. Control chart performance is determined by the arithmetic mean of run length when the manufacturing process is outside of the arithmetic mean of run length control (AMRL$_1$) and the run length's standard deviation. The theories and related research are discussed below.

2.2.1 S Chart

The standard deviation control chart (S-chart) is the most basic chart for detecting variations in a process's standard deviation. The standard deviation control limit is computed using the probability limit, or 3σ in the approach, which is $\mu_S \pm 3\sigma_S$, where μ_S and σ_S are the process's mean and standard deviation, [20].

Thus, the chart's upper and lower control limits are given as (2). When $\hat{\sigma}$ is an unknown parameter, it can be calculated by $\bar{S} / c_4$.

$$UCL / LCL = c_4\hat{\sigma} \pm B_1\sqrt{1-c_4^2}\,\hat{\sigma} \qquad (2)$$

where B_1 is the coefficient of control limit of the S chart. The process is unstable if a sample point plot is outside the control limit. c_4 is the factor for calculating the control limit of the control chart. $\bar{S}$ is the mean of the standard deviation of the process.

2.2.2 MA-S Chart

To detect process fluctuations, [18] investigated standard moving average (MA) control charts. MA charts may immediately identify departures from the control for changes. Minor and Major Process Variability MA charts have two possible statistical values:

$$MA-S_i = \begin{cases} \dfrac{S_i + S_{i-1} + S_{i-2} + \ldots + S_1}{i} & ; i < l \\ \dfrac{S_i + S_{i-1} + \ldots + S_{i-\omega+1}}{l} & ; i \ge l. \end{cases} \qquad (3)$$

When l is the width of the MA-S chart. The expectation of the MA-S statistic is denoted as

$$E(MA-S_i) = E\left(\frac{1}{i}\sum_{j=1}^{i} S_j\right) = \frac{1}{i}\sum_{j=1}^{i} E\left(S_j\right) = c_4\hat{\sigma} \qquad (4)$$

and the variance of MA statistic can be divided into two cases as follows:

when $i < l$,

$$Var(MA-S_i) = Var\left(\frac{1}{i}\sum_{j=1}^{i} S_j\right) = \frac{1}{i^2}\sum_{j=1}^{i} Var\left(S_j\right)$$

and when $i \ge l$,

$$Var(MA-S_i) = Var\left(\frac{1}{l}\sum_{j=i-l+1}^{i} R_j\right) = \frac{\sigma^2(1-c_4^2)}{l}.$$

Therefore, the variance of MA-S can be rewritten as

$$Var(MA-S_i)=\begin{cases}\dfrac{(1-c_4^2)\hat{\sigma}^2}{i}, & i<l\\ \dfrac{(1-c_4^2)\hat{\sigma}^2}{l}, & i\geq l.\end{cases} \quad (5)$$

Therefore, the upper and lower control limits are given as follows:

$$UCL/LCL=\begin{cases}c_4\hat{\sigma}+B_2\sqrt{\dfrac{(1-c_4^2)\hat{\sigma}^2}{i}}, & i<l\\ c_4\hat{\sigma}-B_2\sqrt{\dfrac{(1-c_4^2)\hat{\sigma}^2}{l}}, & i\geq l.\end{cases} \quad (6)$$

Where B_2 is the coefficient of control limit of the MA chart.

2.2.3 EWMA-S chart

[6], introduced EWMA charts, which [13] investigated further. EWMA charts are a great alternative to Shewhart charts for detecting minor changes in process parameters and monitoring fluctuations. The procedure is based on statistics, [20].

$$EWMA-S_i=\lambda\bar{S}_i+(1-\lambda)EWMA_{i-1},\ i=1,2,... \quad (7)$$

Where λ is the weighting parameter of the data in the past having the value from 0 to 1, and $\bar{S}_i$ is the average standard deviation at the time *i*. The mean and variance of EWMA-S are:

$$E(EWMA-S_i)=c_4\hat{\sigma}$$

and

$$Var(EWMA-S_i)=(1-c_4^2)\hat{\sigma}^2(\lambda/2-\lambda). \quad (8)$$

Therefore, the control limit of the EWMA-S chart is as follows:

$$UCL/LCL=c_4\hat{\sigma}\pm B_3\sqrt{(1-c_4^2)\hat{\sigma}^2(\lambda/2-\lambda)}. \quad (9)$$

Where B_3 is the coefficient of control limit of the EWMA-S chart.

2.2.4 MA-EWMA$_S$ Chart

The MA-EWMA$_S$ chart combines the MA-S and EWMA-S charts, [11]. Let Z_i is statistical data for the EWMA-S chart, which is input to the MA-S chart. Thus, the statistics of the MA-EWMA$_S$ chart are as follows:

$$MA-EWMA_{S_i}=\begin{cases}\dfrac{Z_i+Z_{i-1}+Z_{i-2}+...+Z_1}{i}; & i<l\\ \dfrac{Z_i+Z_{i-1}+...+Z_{i-\omega+1}}{l}; & i\geq l.\end{cases} \quad (10)$$

When *l* is the width of the MA-EWMA$_S$ chart. The mean and variance of statistics MA-EWMA$_S$ are

$$E(MA-EWMA_{S_i})=c_4\hat{\sigma}$$

and

$$Var(MA-EWMA_{S_i})=\begin{cases}\dfrac{(1-c_4^2)\hat{\sigma}^2}{i}\left(\dfrac{\lambda}{\lambda-2}\right), & i<l\\ \dfrac{(1-c_4^2)\hat{\sigma}^2}{l}\left(\dfrac{\lambda}{\lambda-2}\right), & i\geq l.\end{cases} \quad (11)$$

Therefore, the control limits of the MA-EWMA$_S$ chart are as follows

$$UCL/LCL=\begin{cases}c_4\hat{\sigma}+B_4\sqrt{\dfrac{(1-c_4^2)\hat{\sigma}^2}{i}\left(\dfrac{\lambda}{\lambda-2}\right)}, & i<l\\ c_4\hat{\sigma}-B_4\sqrt{\dfrac{(1-c_4^2)\hat{\sigma}^2}{l}\left(\dfrac{\lambda}{\lambda-2}\right)}, & i\geq l.\end{cases} \quad (12)$$

Where B_4 is the coefficient of control limit of the MA-EWMA$_S$ chart.

2.3 The Performance of Control Chart

Commonly, the efficiency of control charts is measured from the mean of run length or arithmetic mean of run length (AMRL). The AMRL is the estimated number of observations from a process under control before the control chart incorrectly flags out of control. It is divided into two phases: Phase I is in control (represented by AMRL$_0$), and Phase II is out of control (represented by AMRL$_1$). The AMRL can be defined as follows:

$$AMRL=\sum_{i=1}^{T}RL_i\Big/T. \quad (13)$$

In this case, the sample being examined before the process surpasses the control limits for the first time is indicated by RL_i. *T*, set to 200,000, is the number of experiment repetitions in the simulation during round *i*.

The standard deviation of the run length (SDRL) can be computed as follows:

$$SDRL=\sqrt{E(RL_i)^2-AMRL^2} \quad (14)$$

3 Analyses of Results

This study investigates the efficiency of control charts in detecting changes in process standard deviation. The analysis employed a student distribution with 15, 30, and 50 degrees of freedom and sample sizes of 5 and 10. Monte Carlo simulations allow for the calculation of numerical results. The in-control arithmetic mean of run length ($AMRL_0$) is set at 370. The $AMRL_1$ measure is used to determine the effectiveness of the control chart. The figure displays the lowest $AMRL_1$ readings, regarded as the most effective at detecting changes. The findings of the study are separated into two parts:

3.1 Performance of MA-EWMA Chart

The run length evaluation method for a mixed moving average – exponentially weighted moving average control chart (MA-EWMA) for process dispersion. The weighting factors (λ) of the MA-EWMA control charts were 0.50, and the width of the moving average (*l*) was 2, 3, 5, 10, and 15. The number of repetitions of the MA-EWMA chart is 5,000 iterations. The changing sizes of the process (δ) were 1.10, 1.20, 1.30, 1.40, 1.50, 1.75, 2.00, 2.50, and 3.00. The estimated arithmetic mean of run lengths (AMRLs) for each case can be explained as follows:

Table 1 (Appendix) presents the arithmetic mean of run length (AMRL) of the MA-EWMA chart for t-distributed data with a parameter value of 15 and a subgroup size of 5. The AMRLs of the MA-EWMA chart with smoothing parameters (*l)* of 2, 3, 5, 10, and 15 are compared. The results show that when the process shift increases, the MA-EWMA chart with a smoothing parameter (*l)* of 2 is the most effective in detecting changes in the standard deviation, as it has the lowest $AMRL_1$ value.

Table 2 (Appendix) displays the AMRL values of the MA-EWMA control chart based on data distributed as the t-distribution, utilizing parameter settings of 15 and a subgroup size of 10. This research examines the AMRL performance of the MA-EWMA control chart with smoothing parameter (*l*) settings of 2, 3, 5, 10, and 15. The results imply that the MA-EWMA control chart with smoothing grows with the extent of the process change. Because of the lowest $AMRL_1$ value, the parameter (*l*) set to 2 outperforms others in detecting standard deviation differences.

Table 3 and Table 4 (Appendix) show the arithmetic mean of run length (AMRL) of the MA-EWMA charts when the data is assumed to follow a t-distribution with parameters of 30 and subgroup sizes of 5 and 10, respectively. Furthermore,

Table 5 and Table 6 (Appendix) display the AMRL values of the MA-EWMA charts under the *t* distribution assumption, with the parameter set to 50 and the subgroup sizes set to 5 and 10, respectively. The results reveal that when process variability grows, parameter (*l)* value 2 for MA-EWMA charts performs the best in identifying changes in standard deviation. This result mirrors the outcomes observed when the t-distribution parameters were set at 15.

3.2 AMRL and SDRL Performance of the Control Chart

To comparison, the MA-EWMA control chart is compared with the S chart, moving average-standard deviation (MA), exponentially weighted moving average-standard deviation (EWMA), measured by the out-of-control arithmetic mean of run length ($AMRL_1$), and standard deviation of run length (SDRL). The parameter of in-control for the MA-EWMA chart is given $AMRL_0$ = 370, $\lambda = 0.5$, and $l = 2$. The parameter changes at δ = 1.10, 1.20, 1.30, 1.40, 1.50, 1.75, 2.00, 2.50, and 3.00. The results of comparing the efficiency of control charts for measuring dispersion can be explained as follows:

Table 7 (Appendix) specifies data assumed to follow a t-distribution with parameters set at 15 and subgroup sizes of 5. The numerical data analysis reveals that when there is a change in the process, the measurement of dispersion increases. The MA chart is the most efficient control chart for detecting these changes, exhibiting the lowest $AMRL_1$ value.

Subsequently, Table 8 (Appendix) outlines data presumed to conform to a student t-distribution with parameters set at 15 and subgroup sizes of 10. Analysis of the numerical data indicates that the dispersion measurement falls within the range of 1.00 to 2.00 in the event of a process change. S charts seem to be the best control charts for spotting these changes. MA charts also outperform in identifying changes, as evidenced by the lowest $AMRL_1$ value.

Table 9 (Appendix) displays the results from numerical investigations expected to follow a t-distribution with parameters set to 30 and a subgroup size of 5, the dispersion metric increases when the method changes. MA charts appear to be the best control charts for detecting such changes. It outperforms other charts, each with the lowest $AMRL_1$ value.

Table 10 (Appendix) demonstrates that with data from the t-distribution and parameters set to 30 with a subgroup size of 10, numerical analysis suggests that S-charts are the most successful at

Additionally, when the dispersion measurement reaches 3.00 or higher due to a process change, the MA chart outperforms all presented charts, exhibiting the lowest $AMRL_1$ value in change detection.

Table 11 and Table 12 (Appendix) compare control charts' arithmetic mean of run length outcomes under a t-distribution with parameters set at 50 and subgroup sizes of 5 and 10, respectively. The findings suggest that the most effective chart consistently aligns with the scenario where the data follows a student-t distribution with parameters set at 30, regardless of the level of change.

3.3 A Real Application

This section illustrates the practical application of an accelerometer dataset within the control chart examined in this research. Accelerometers, versatile devices with applications spanning manufacturing vibration measurement, car accident detection, pollution monitoring, scientific research, medicine, and more, are the focus. For this study, we have utilized a smartphone accelerometer dataset for monitoring objectives, specifically implementing control charts for accelerometer data. Following the methodology outlined by [21], we have divided the data into 10 subgroups, each comprising 40 entries. The appropriate distribution for this data is a student- t distribution with an average of 0.843 and a standard deviation of 0.257. The performance measurement results in detecting data changes through graphical representations can be explained.

Figure 1 demonstrates that the S-chart statistic falls between the upper and lower boundaries, which leads to the conclusion that S chart cannot detect changes in data. MA chart, like S chart, cannot detect changes in data since the statistics do not surpass the top and lower bounds, as seen in Figure 2.

Figure 3 indicates that the EWMA chart cannot detect changes in the data. Finally, the analysis detects variations in the standard deviation. Figure 4 illustrates how the MA-EWMA chart successfully identified nine process improvements when comparing the performance of the charts above. Finally, the MA-EWMA chart outperforms other methods for detecting standard deviations.

4 Conclusion and Further Research

This research aims to construct the new mixed MA-EWMA for monitoring the standard deviation and detecting changes within a range. Dispersion measurement ranges from 1.00 to 2.50.

examines the AMRL and SDRL of control charts for a Student t-distribution method. The preliminary study gives operators insight into control charts to aid in process monitoring and prevent delays in identifying process changes. The mixed MA-EWMA chart is robust to the light-tail process as student-t distribution to detect a process dispersion based on standard deviation for large magnitudes of shift sizes.

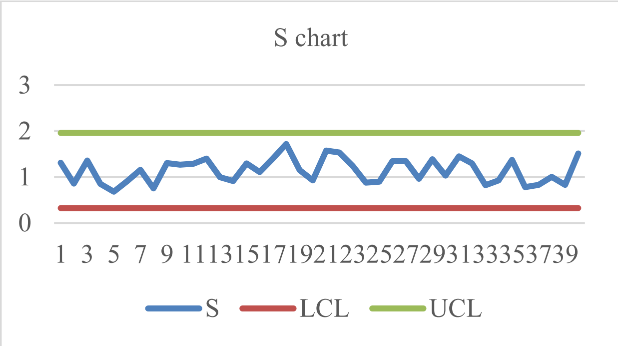

Fig. 1: Performance of S chart

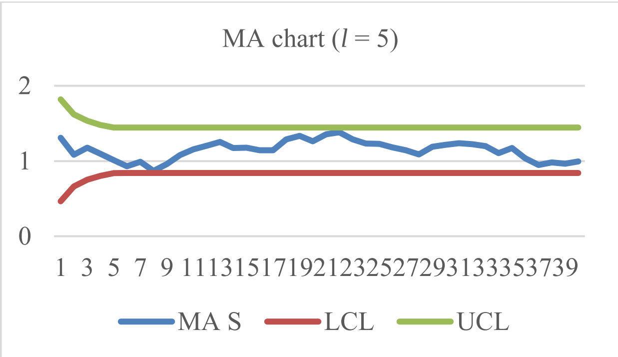

Fig. 2: Performance of MA chart

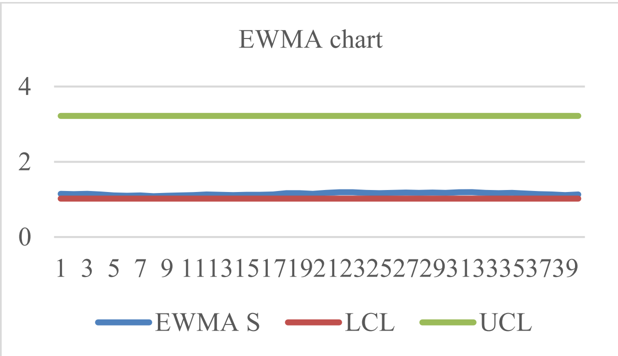

Fig. 3: Performance of EWMA chart

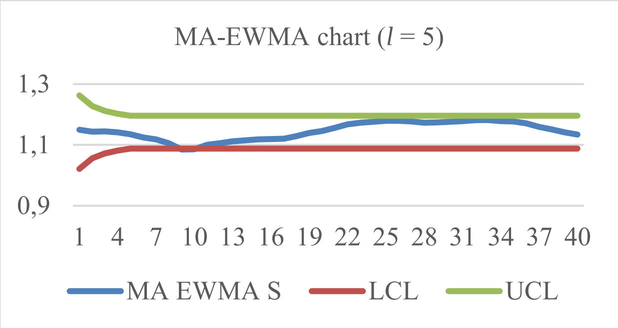

Fig. 4: Performance of MA-EWMA chart

In addition, the results obtained by simulation studies reveal that the mixed MA-EWMA based on standard deviation outperformed as well as for MA chart for moderate to large shift sizes when the parameter of student-t distribution is increased. In future research, the verification can be extended to heavy-tail processes and compared with other control charts. Other performance metrics include the median run length (MRL) and the percentile of the run length distribution. In most real-world settings, comparing these charts helps determine process parameters such as target mean and standard deviation from Phase I datasets. Because so little information is available, the process parameters must be guessed.

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Declaration of Generative AI and AI-assisted Technologies in the Writing Process

During the preparation of this work the authors used Google Gemini in order to study the source and importance of research. After using this tool/service, the authors reviewed and edited the content as needed and take full responsibility for the content of the publication.

References:

[1] W. A. Shewhart, "Statistical control," in *Economic Control of Quality Manufactured Product*," NY: D. Van Nostrand Company, (1931).
[2] S. A. Abbasi, "On the performance of EWMA chart in the presence of two-component measurement error," *Quality Engineering*, 22, 199–213, (2010), doi: 10.1080/08982111003785649
[3] S. L. Hwang, J. T. Lin, G. F. Liang, Y. J. Yau, T. C. Yenn, and C. C. Hsu, "Application control chart concepts of designing a pre-alarm system in the nuclear power plant control room," *Nuclear Engineering and Design*, 238, 3522–3527,)2008(, doi: 10.1016/j.nucengdes.2008.07.011
[4] Z. Wang and R. Liang, "Discuss on applying SPC to quality management in university education," in *Proceedings of the 9th International Conference for Young Computer Scientists, ICYCS*, Hunan, China, 2372–2375, (2008), doi: 10.1109/ICYCS.2008.301.
[5] W. H. Woodall, "The use of control charts in healthcare and public-health surveillance," *Journal of Quality Technology*, 38, 89–104,)2006(, doi: 10.1080/00224065.2006.11918593.
[6] S. W. Roberts, "Control chart tests based on geometric moving average," *Technometrics*, 1, 239-250, (1959), doi: 10.2307/1266443.
[7] E. S. Page, "Continuous inspection scheme," *Biometrika*, 41(1-2), 100-115, (1954), doi: 10.2307/2333009.
[8] B. C. Khoo, "A moving average control chart for monitoring the fraction non-conforming," *Quality and Reliability Engineering International*, 20, 617-635, (2004), doi: 10.1002/qre.576.
[9] A. K. Patel, and J. Divecha, "Modified exponentially weighted moving average (EWMA) control chart for an analytical process data," *Journal of Chemical Engineering and Material Science*, 2(1), 12–20, (2011), doi: 10.5897/JCEMS.9000014.
[10] N. Khan, M. Aslam, and C. H. Jun, "A EWMA control chart for exponential distributed quality based on moving average statistics," *Quality and Reliability Engineering International*, 32) 3(, 1179–1190, (2016), doi: 10.1002/qre.1825.
[11] S. Sukparungsee, Y. Areepong, and R. Taboran, "Exponentially weighted moving

average-moving average charts for monitoring the process mean," *PloS ONE*, 15) 2: e0228208, (2020), doi: 10.1371/journal.pone.0228208.

[12] I. M. Zwetsloot, M. Schoonhoven, and R. J. M. M. Does, "A robust Phase I exponentially weighted moving average chart for dispersion." *Qual Reliab. Eng. Int.*, 31(6), 989-999, (2015), doi: 10.1002/qre.1655.

[13] S. V. Crowder, and M. D. Hamilton, "An EWMA for monitoring a process standard deviation," *Journal of Quality Technology*, 24(1), 12-21. (1992), doi: 10.1080/00224065.1992.11979369.

[14] L. Shu, and W. Jiang, "A new EWMA chart for monitoring process dispersion," *Journal of Quality Technology*, 40(3), 319-331, (2008), doi: 10.1080/00224065.2008.11917737.

[15] L. Huwang, C. J. Huang and Y. H. Wang, "New EWMA control chart for monitoring process dispersion," *Computational Statistics and Data Analysis*, 54(10), 2328-2342, (2010), doi: 10.1016/j.csda.2010.03.011.

[16] S. A. Abbasi and A. Miller, "On proper choice of variability control chart for normal and non-normal processes," *Quality and Reliability Engineering International*, 28(3), 279-296, (2012), doi: 10.1002/qre.1244.

[17] N. Abbas, M. Riaz, and R. J. Does, "Memory-type control charts for monitoring the process dispersion," *Quality and Reliability Engineering International*, 30(5), 623-632, (2014), doi: 10.1002/qre.1514.

[18] O. A. Adeoti, and J. O. Olaomi, "A moving average S control chart for monitoring process variability," *Quality Engineering*, 28(2), 212-219, (2016), doi: 10.1080/08982112.2015.1104542.

[19] S. Phantu, Y. Areepong, and S. Sukparungsee, "Analysis process dispersion variation tracked using a mixed MA-EWMA control chart." *WSEA Transactions on Systems and Control*, 19, 217-226, (2024), https://doi.org/10.37394/23203.2024.19.23.

[20] D. C. Montgomery, "Statistical quality control: a modern introduction," New York: John Wiley & Sons, (2009).

[21] M. Riaz, M. Abid, H.Z. Nazir, and S.A. Abbasi. "An enhanced nonparametric EWMA sign control chart using sequential mechanism. " *PLoS One*, 14(11), e0225330. doi: 10.1371/journal.pone.0225330.

Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

- P. S.: writing an original draft, software, data analysis, data curation, proof, and validation. Y. A.: investigation, methodology, validation, reviewing
- S. S.: conceptualization, investigation, funding acquisition, project administration, reviewing, and editing.

Sources of Funding for Research Presented in a Scientific Article or Scientific Article Itself

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Conflict of Interest

The authors have no conflict of interest to declare.

APPENDIX

Table 1. $AMRL_1$ of MA-EWMAS charts when the data are from *t(15)* with $AMRL_0$ =370, n=5 and λ=0.5

Shift size	*l=2*	*l=3*	*l=5*	*l=10*	*l=15*
	B_4 = 3.124	B_4 = 3.113	B_4 = 3.136	B_4 = 3.125	B_4 = 3.143
1.00	370.667	370.449	370.447	370.170	370.892
1.10	*352.726*	355.624	358.461	362.486	364.252
1.20	*327.148*	330.853	331.843	333.644	335.490
1.30	*291.570*	292.482	295.173	298.456	310.189
1.40	*267.901*	267.056	269.434	275.162	286.953
1.50	*244.116*	258.511	267.131	270.667	271.148
1.75	*189.053*	202.134	253.835	264.160	266.545
2.00	*144.279*	153.502	196.957	211.760	238.471
2.50	*83.056*	86.468	106.831	158.806	204.199
3.00	*49.392*	49.994	56.377	72.721	83.335

Note: Italics number represents the lowest $AMRL_1$.

Table 2. $AMRL_1$ of MA-$EWMA_S$ charts when the data are from *t(15)* with $AMRL_0$ =370, n=10 and λ=0.5

Shift size	*l=2*	*l=3*	*l=5*	*l=10*	*l=15*
	B_4 = 3.107	B_4 = 3.115	B_4 = 3.119	B_4 = 3.121	B_4 = 3.125
1.00	370.797	370.209	370.210	370.414	370.174
1.10	*350.237*	354.958	357.066	365.182	367.205
1.20	*325.249*	329.425	331.107	338.925	345.716
1.30	*284.324*	289.341	290.143	294.312	297.856
1.40	*254.822*	258.821	260.347	262.470	275.314
1.50	*248.723*	255.001	261.133	268.213	272.558
1.75	*186.683*	231.115	237.073	245.840	248.512
2.00	*134.565*	166.357	173.987	181.801	192.608
2.50	*68.557*	78.794	103.557	159.087	172.579
3.00	*35.540*	37.997	43.778	53.497	59.378

Note: Italics number represents the lowest $AMRL_1$.

Table 3. $AMRL_1$ of MA-$EWMA_S$ charts when the data are from *t(30)* with $AMRL_0$ =370, n=5 and λ=0.5

Shift size	*l=2*	*l=3*	*l=5*	*l=10*	*l=15*
	B_4 = 3.109	B_4 = 3.112	B_4 = 3.120	B_4 = 3.124	B_4 = 3.125
1.00	370.018	370.049	370.016	370.299	370.836
1.10	*360.449*	361.204	365.780	367.564	368.056
1.20	*355.092*	356.101	357.315	359.930	360.417
1.30	*347.868*	349.724	350.443	352.946	356.410
1.40	*323.542*	326.829	330.216	343.928	347.281
1.50	*309.954*	314.645	319.955	339.068	340.928
1.75	*275.962*	282.378	294.507	322.848	331.998
2.00	*241.405*	248.910	270.843	317.289	322.096
2.50	*176.970*	181.798	201.243	242.719	274.396
3.00	*126.273*	127.269	138.049	161.846	177.374

Note: Italics number represents the lowest $AMRL_1$.

Table 4. $AMRL_1$ of MA-EWMA$_S$ charts when the data are from *t(30)* with $AMRL_0$ =370, n=10 and λ=0.5

Shift size	*l=2*	*l=3*	*l=5*	*l=10*	*l=15*
	B_4 = 3.105	B_4 = 3.114	B_4 = 3.118	B_4 = 3.124	B_4 = 3.126
1.00	370.015	370.641	370.534	370.521	370.296
1.10	*351.967*	353.187	356.822	360.478	361.854
1.20	*344.630*	346.859	349.558	352.064	358.416
1.30	*325.418*	330.714	333.561	336.205	342.758
1.40	*312.546*	316.852	320.141	324.189	337.286
1.50	*309.064*	315.986	317.717	319.902	327.354
1.75	*271.972*	293.404	308.508	312.539	318.041
2.00	*232.434*	255.135	293.359	304.849	308.863
2.50	*160.521*	174.281	202.015	252.182	287.580
3.00	*104.469*	106.640	119.966	136.785	144.904

Note: Italics number represents the lowest $AMRL_1$.

Table 5. $AMRL_1$ of MA-EWMA$_S$ charts when the data are from *t(50)* with $AMRL_0$ =370, n=5 and λ=0.5

Shift size	*l=2*	*l=3*	*l=5*	*l=10*	*l=15*
	B_4 = 3.109	B_4 = 3.116	B_4 = 3.119	B_4 = 3.125	B_4 = 3.126
1.00	370.524	370.568	370.268	370.173	370.096
1.10	*349.786*	352.748	357.809	361.270	364.593
1.20	*347.152*	349.617	352.641	355.276	358.947
1.30	*343.268*	345.501	348.286	350.421	352.119
1.40	*339.107*	341.226	343.028	347.564	349.871
1.50	*336.352*	338.328	341.881	340.819	345.469
1.75	*315.385*	318.340	327.536	336.919	338.771
2.00	*293.496*	296.135	307.660	321.987	327.538
2.50	*245.854*	247.676	260.307	288.648	306.944
3.00	*198.407*	199.040	209.393	232.726	246.549

Note: Italics number represents the lowest $AMRL_1$.

Table 6. $AMRL_1$ of MA-EWMA$_S$ charts when the data are from *t(50)* with $AMRL_0$ =370, n=10 and λ=0.5

Shift size	*l*=2	*l*=3	*l*=5	*l* =10	*l* =15
	B_4 = 3.108	B_4 = 3.118	B_4 = 3.121	B_4 = 3.125	B_4 = 3.128
1.00	370.637	370.020	370.342	370.084	370.026
1.10	*352.411*	355.967	357.187	359.721	360.187
1.20	*349.724*	351.092	355.131	357.413	358.943
1.30	*343.528*	345.201	348.664	350.217	352.323
1.40	*339.642*	340.829	342.511	345.976	348.665
1.50	*335.655*	337.017	339.812	341.139	342.956
1.75	*312.999*	320.704	330.587	334.313	338.704
2.00	*287.991*	297.356	316.148	329.476	332.221
2.50	*233.555*	243.308	262.604	294.001	315.031
3.00	*179.770*	186.303	198.041	214.766	222.886

Note: Italics number represents the lowest $AMRL_1$.

Table 7. Comparison $AMRL_1$ of S, MA, EWMA, and MA-$EWMA_S$ charts from *t(15)* with *l*=2, λ=0.5, and n=5

Shift size	S chart		MA chart		EWMA chart		MA-$EWMA_S$ chart	
	B_1 = 3.156		B_2 = 3.075		B_3 = 3.154		B_4 = 3.124	
	AMRL	SDRL	AMRL	SDRL	AMRL	SDRL	AMRL	SDRL
1.00	370.114	0.971	370.096	0.973	370.352	0.978	370.667	0.974
1.10	265.464	0.927	*261.287*	*0.915*	274.689	0.967	352.726	0.961
1.20	260.794	0.894	*258.164*	*0.894*	263.478	0.935	327.148	0.902
1.30	259.799	0.851	*253.461*	*0.824*	256.974	0.912	291.570	0.854
1.40	254.637	0.790	*249.347*	*0.781*	252.962	0.880	267.901	0.785
1.50	250.747	0.743	*243.793*	*0.728*	248.385	0.862	244.116	0.731
1.75	199.445	0.616	*188.855*	*0.587*	192.659	0.698	189.053	0.588
2.00	156.547	0.483	*144.024*	*0.457*	145.375	0.536	144.279	0.458
2.50	96.280	0.308	*82.938*	*0.264*	82.759	0.300	83.056	0.265
3.00	60.980	0.194	*49.332*	*0.158*	48.727	0.171	*49.392*	*0.158*

Note: Italics number represents the lowest $AMRL_1$.

Table 8. Comparison $AMRL_1$ of S, MA, EWMA, and MA-$EWMA_S$ charts from *t(15)* with *l*=2, λ=0.5 and n=10

Shift size	S chart		MA chart		EWMA chart		MA-$EWMA_S$ chart	
	B_1 = 3.158		B_2 = 3.082		B_3 = 3.167		B_4 = 3.107	
	AMRL	SDRL	AMRL	SDRL	AMRL	SDRL	AMRL	SDRL
1.00	370.835	0.972	370.745	0.972	370.545	0.980	370.797	0.972
1.10	*295.146*	*0.944*	304.107	0.946	293.107	0.953	350.237	0.954
1.20	*270.764*	*0.914*	290.174	0.915	273.165	0.924	325.249	0.897
1.30	*264.822*	*0.832*	272.316	0.889	269.208	0.873	284.324	0.834
1.40	*230.486*	*0.715*	256.864	0.824	264.815	0.852	254.822	0.815
1.50	*227.612*	*0.687*	248.707	0.741	259.988	0.697	248.723	0.741
1.75	*170.475*	*0.533*	186.659	0.582	254.287	0.590	186.683	0.582
2.00	*126.960*	*0.402*	134.526	0.426	195.113	0.513	134.565	0.426
2.50	70.625	*0.214*	*68.551*	0.219	97.726	0.450	68.557	0.319
3.00	40.496	*0.113*	*35.535*	0.134	49.165	0.209	*35.540*	*0.134*

Note: Italics number represents the lowest $AMRL_1$.

Table 9. Comparison $AMRL_1$ of S, MA, EWMA, and MA-$EWMA_S$ charts from *t(30)* with *l* =2, λ=0.5 and n=5

Shift size	S chart		MA chart		EWMA chart		MA-$EWMA_S$ chart	
	B_1 = 3.164		B_2 = 3.068		B_3 = 3.159		B_4 = 3.109	
	AMRL	SDRL	AMRL	SDRL	AMRL	SDRL	AMRL	SDRL
1.00	370.097	0.969	370.065	0.969	370.054	0.984	370.018	0.969
1.10	336.842	0.945	*331.052*	*0.946*	338.461	0.979	360.449	0.953
1.20	328.603	0.937	*325.118*	*0.922*	330.820	0.970	355.092	0.931
1.30	322.849	0.916	*320.176*	*0.906*	325.649	0.962	347.868	0.914
1.40	317.297	0.891	*315.279*	0.882	318.502	0.956	323.542	0.875
1.50	313.791	0.874	*309.982*	*0.868*	313.581	0.945	309.954	0.868
1.75	281.899	0.812	*275.979*	*0.786*	279.658	0.938	275.962	0.800
2.00	249.459	0.740	*241.420*	*0.720*	244.241	0.847	241.405	0.723
2.50	189.362	0.584	*176.990*	*0.552*	177.440	0.645	176.970	0.552
3.00	139.837	0.443	*126.286*	*0.400*	123.566	0.453	*126.273*	*0.400*

Note: Italics number represents the lowest $AMRL_1$.

Table 10. Comparison $AMRL_1$ of S, MA, EWMA, and $MA\text{-}EWMA_S$ charts from *t(30)* with *l*=2, λ=0.5 and n=10

Shift size	S chart		MA chart		EWMA chart		$MA\text{-}EWMA_S$ chart	
	B_1 = 3.168		B_2 = 3.083		B_3 = 3.176		B_4 = 3.105	
	AMRL	SDRL	AMRL	SDRL	AMRL	SDRL	AMRL	SDRL
1.00	370.016	0.970	370.028	0.987	370.071	0.989	370.015	0.972
1.10	*346.441*	*0.954*	349.537	0.965	376.759	0.967	351.967	0.955
1.20	*334.749*	*0.919*	338.601	0.922	372.303	0.952	344.630	0.921
1.30	*323.467*	*0.907*	327.642	0.901	367.546	0.937	325.418	0.904
1.40	*310.149*	*0.884*	317.058	0.885	361.037	0.918	312.546	0.886
1.50	*300.023*	*0.851*	309.066	0.870	352.483	0.890	309.064	0.870
1.75	*261.972*	*0.770*	271.973	0.794	335.078	0.864	271.972	0.794
2.00	*223.923*	*0.678*	232.434	0.701	311.042	0.827	232.434	0.701
2.50	*156.675*	*0.491*	160.528	0.503	243.168	0.792	160.521	0.503
3.00	107.090	*0.339*	*104.469*	*0.330*	163.594	0.771	*104.469*	*0.330*

Note: Italics number represents the lowest $AMRL_1$.

Table 11. Comparison $AMRL_1$ of S, MA, EWMA, and $MA\text{-}EWMA_S$ charts from *t(50)* with *l*=2, λ=0.5 and n=5

Shift size	S chart		MA chart		EWMA chart		$MA\text{-}EWMA_S$ chart	
	B_1 = 3.172		B_2 = 3.079		B_3 = 3.165		B_4 = 3.109	
	AMRL	SDRL	AMRL	SDRL	AMRL	SDRL	AMRL	SDRL
1.00	370.013	0.971	370.561	0.975	370.563	0.985	370.524	0.971
1.10	358.647	0.986	*348.307*	*0.969*	355.941	0.975	349.786	0.965
1.20	350.228	0.974	*345.922*	*0.945*	351.036	0.952	347.152	0.947
1.30	345.976	0.965	*341.546*	*0.938*	343.550	0.949	343.268	0.938
1.40	342.208	0.934	*338.250*	*0.922*	340.312	0.934	339.107	0.924
1.50	338.162	0.920	*336.364*	*0.917*	339.047	0.921	*336.352*	*0.918*
1.75	319.640	0.886	*315.425*	*0.875*	319.235	0.899	*315.385*	*0.880*
2.00	298.749	0.847	*293.510*	*0.838*	297.674	0.871	*293.496*	*0.839*
2.50	254.419	0.751	*245.873*	*0.731*	248.432	0.852	*245.854*	*0.733*
3.00	209.985	0.639	*198.485*	*0.609*	200.026	0.713	*198.407*	*0.609*

Note: Italics number represents the lowest $AMRL_1$.

Table 12. Comparison $AMRL_1$ of S, MA, EWMA, and $MA\text{-}EWMA_S$ charts from *t(50)* with *l*=2, λ=0.5 and n=10

Shift size	S chart		MA chart		EWMA chart		$MA\text{-}EWMA_S$ chart	
	B_1 = 3.175		B_2 = 3.087		B_3 = 3.182		B_4 = 3.108	
	AMRL	SDRL	AMRL	SDRL	AMRL	SDRL	AMRL	SDRL
1.00	370.833	0.972	370.229	0.971	370.524	0.992	370.637	0.972
1.10	*345.411*	*0.961*	347.023	0.965	349.602	0.984	352.411	0.968
1.20	*341.869*	*0.955*	343.556	0.957	344.856	0.962	349.724	0.960
1.30	*337.528*	*0.930*	340.528	0.934	340.929	0.950	343.528	0.954
1.40	*333.142*	*0.917*	338.641	0.920	339.528	0.934	339.642	0.936
1.50	*331.990*	*0.909*	335.231	0.914	336.430	0.911	*335.655*	*0.915*
1.75	*308.280*	*0.868*	312.643	0.874	313.138	0.893	*312.999*	*0.875*
2.00	*282.932*	*0.816*	287.609	0.825	288.414	0.871	*287.991*	*0.826*
2.50	*230.997*	*0.696*	233.311	0.701	234.775	0.862	*233.555*	*0.701*
3.00	180.136	0.561	*179.585*	*0.557*	180.647	0.850	*179.770*	*0.557*

Note: Italics number represents the lowest $AMRL_1$.