# Identification of Linear Systems Having Time Delay Connected in Series

CHAIMAE ABDELAALI[1], ALI BOUKLATA[1], MOHAMED BENYASSI[2], ADIL BROURI[1]
[1]AEEE Department, ENSAM,
Moulay Ismail University,
MOROCCO

[2]Electrical Engineering Department, ESTM,
Moulay Ismail University,
MOROCCO

*Abstract:* - Nonlinear system identification has been a hot research field over the past two decades. A substantial portion of the research work has been carried out based on block-structured models. Time delay is a problem occurring in most industrial applications. The time delay can destabilize the system. Then, the latter should be determined to control the system. This work aims to present an approach allowing the identification of a linear system having a time delay connected in series. In this study, an identification method is proposed to determine the system parameters. This method is based on sine inputs / or periodic stepwise input.

*Key-Words:* - Systems identification, time delay, series connections of linear and time delay, stability, time delay estimation, Least Square Method (LSM).

Received: July 9, 2023. Revised: June 6, 2024. Accepted: July 7, 2024. Published: August 13, 2024.

## 1 Introduction

Nonlinear system identification has been a hot research field over the past two decades, [1], [2], [3], [4], [5], [6]. The research in this way is still ongoing, [7], [8], [9], [10]. Several available papers have been focused on the identification of nonlinear systems structured by the series connection of linear and nonlinear blocks, [11], [12], [13], as well as the parallel connection of linear and nonlinear blocks [14]. The nonlinear system identification is often addressed in the case of Wiener and Hammerstein models, [1], [15], [16], [17], [18]. The identification techniques have been used in several application domains, [19], [20], [21].

Several techniques and solutions have been used to identify the nonlinear system parameters, e.g., stochastic methods [22], deterministic recursive techniques [23], and frequency methods [24], [25].

The research on nonlinear systems focuses not only on identifying their nonlinearity but also on control, to mitigate the negative effects of non-linearity on an affected system's performance, e.g., adaptive control using the backstepping method has been proposed, [26], [27], [28], fuzzy fixed-time control [29], passive robust control [30].

In this work, the focus is on system identification rather than compensating for the effects of nonlinearity. Knowing these nonlinearities makes other operations, including control, easier. In this way, the most studied solutions are proposed in the case of a series connection of linear and nonlinear blocks, i.e., the case of Hammerstein, Wiener, Wiener-Hammerstein, or Hammerstein-Wiener models. To increase the complexity of nonlinear system models and to make the model more general, the parallel connections of linear and nonlinear subsystems can be proposed, [31], [32], [33], [34].

Presently, the problem of identification of linear systems connected in series with a time delay is addressed. The proposed approach can be applied to a linear system. Furthermore, this system can describe many industrial systems. The rest of the paper is organized as follows. Section 2 is devoted to the presentation of the identification problem. In this section, a mathematical description of a studied nonlinear system is also presented. The identification method of the system (linear with time delay) is developed in Section 3. Then, examples of simulations are proposed in Section 4.

## 2 Problem Statement

Most of the studied identification problems have been focused on linear or nonlinear blocks, e.g., the Hammerstein system (composed of nonlinear element followed by a linear subsystem) and the Wiener system (having a linear subsystem followed by a nonlinear block). Presently, the identification problem is focused on a linear system followed by a time delay system. The linear system is described by a transfer function $G(s)$ and the time delay is of value $D$. When the latter is null (i.e., $D = 0$), a constant linear system is obtained.

Accordingly, the studied system can be mathematically described as follows:

$$y(t) = g(t - D) * u(t - D) \quad (1)$$

where '$*$' denotes the convolution product, $D$ is the time delay, $u(t)$ is the control signal, $y(t)$ denotes the output signal, and $g(t)$ is impulse response of the linear system (i.e., $g(t)$ is the inverse Laplace transform).

In discrete form, the transfer function $G(s)$ can be written as the ratio between two polynomials $A(q)$ and $B(q)$:

$$A(q) = 1 + \sum_{k=1}^{n_a} a_k \, q^{-k} \quad (2)$$

$$B(q) = \sum_{k=1}^{n_b} b_k \, q^{-k} \quad (3)$$

where $q$ is the offset operator, i.e., $q^{-k}u(t) = u(t - k)$. In this case, the transfer function $G(q)$ can be expressed as:

$$G(q) = \frac{B(q)}{A(q)} = \frac{\sum_{k=1}^{n_b} b_k \, q^{-k}}{1 + \sum_{k=1}^{n_a} a_k \, q^{-k}} \quad (4)$$

Unlike several previous works (e.g., [1], [12], [15], [32]), this transfer function is not necessarily of nonzero static gain. Let $v(t)$ denotes the signal before time delay (the output of linear system). Then, one has immediately:

$$v(t) = g(t) * u(t) \quad (5)$$

which can be expressed using (4):

$$\frac{v(t)}{u(t)} = \frac{\sum_{k=1}^{n_b} b_k \, q^{-k}}{1 + \sum_{k=1}^{n_a} a_k \, q^{-k}} \quad (6)$$

The latter leads to the following recursive equation:

$$v(t) = \sum_{k=1}^{n_b} b_k \, u(t - k) - \sum_{k=1}^{n_a} a_k \, v(t - k) \quad (7)$$

The output $y(t)$ can thus be written as:

$$y(t) = v(t - D) \quad (8)$$

Let us suppose that the time delay $D$ is a multiple of the sampling time. This means that $v(t - D) \equiv q^{-D} v(t)$. By combining this remark with (7), one has:

$$y(t) = v(t - D) = \sum_{k=1}^{n_b} b_k \, u(t - D - k) - \sum_{k=1}^{n_a} a_k \, v(t - D - k) \quad (9)$$

It is readily seen that the output $y(t)$ can be rewritten as the recursive expression:

$$y(t) = \varphi^T(t)\theta \quad (10)$$

where the data vector $\varphi(t)$ is defined as:

$$\varphi(t) = [u(t - D - 1), \dots, u(t - D - n_b), -v(t - D - 1), \dots, -v(t - D - n_a)]^T, \quad (11)$$

and the parameter vector $\theta$ is defined as follows:

$$\theta = \left[b_1, \dots, b_{n_b}, a_1, \dots, a_{n_a}\right]^T \quad (12)$$

Replacing $v(t - D)$ with $y(t)$ in (9), one immediately gets:

$$y(t) = \sum_{k=1}^{n_b} b_k \, u(t - D - k) - \sum_{k=1}^{n_a} a_k \, y(t - k), \quad (13)$$

which can be also written as the recursive expression:

$$y(t) = \phi^T(t)\theta \quad (14)$$

where the parameter vector $\theta$ is given in (12) and the data vector $\phi(t)$ is given as:

$$\phi(t) = [u(t - D - 1), \dots, u(t - D - n_b), -y(t - 1), \dots, -y(t - n_a)]^T, \quad (15)$$

In the case where the time delay is known, the data vector $\phi(t)$ given in (15) become known for any time $t \geq max(D + n_b, n_a)$. Indeed, the expression of $\phi(t)$ in (15) contains only the past values of input $u(t - i)$ and output $y(t - j)$, for $i = D + 1 \dots D + n_b$ and $j = 1 \dots n_a$, which can be fully determined for any time $t \geq max(D + n_b, n_a)$. Furthermore, the expression of $y(t)$ is affine according to the parameters, $b_1 \dots b_{n_b}$ and $a_1 \dots a_{n_a}$. Then, the parameter vector $\theta$ can be easily identified using, e.g., the least square method (LSM). Let $\hat{\theta}(t)$ denoting the parameter vector estimate. Accordingly, the LSM algorithm is described by the following equation system (16)-(17):

$$\dot{\hat{\theta}}(t) = -P(t)\phi(t)y(t) \quad (16)$$

for any arbitrary initial estimate value $\hat{\theta}(0)$ and the gain matrix $P(t)$ is defined as:

$$\dot{P}(t) = P(t) - P(t)\phi(t)\phi^T(t)P(t) \quad (17)$$

for any arbitrary initial value $R(0) = R^T(0) > 0$. The problem that arises at this stage is related to the fact that the time delay $D$ is not known. The data vector $\phi(t)$ in (15) is not known. If an upper bound $D_{max}$ of time delay $D$, the least square method (16)-(17) remains applicable for $t \geq max(D_{max} + n_b, n_a)$.

Presently, any knowledge of time delay is required $D$. The estimator algorithm (16)-(17) cannot be used directly. In order to overcome this problem, an estimate method of time delay $D$ is proposed. In this respect, the system is excited using a null control input, or any other constant control, for $t \geq max(D + n_b, n_a)$. Note that the time delay $D$ is not known at this stage. To ensure that $t \geq max(D + n_b, n_a)$, the null control is applied until the system output becomes null (or constant). Then, another value (different from zero) of control is applied ,and observing the time value when the output begins to change. This time value corresponds to $max(D + n_b, n_a)$. Once an upper bound of time delay $D$ delay is estimated, the estimator algorithm (16)-(17) can be used.

Other details of this study can be given in simulation section.

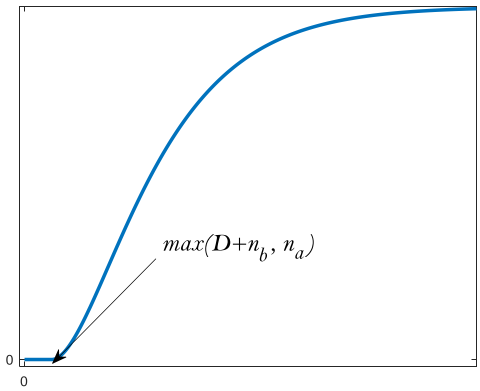


Fig. 1: Example of obtained results in the estimate of $max(D + n_b, n_a)$

## 3 Simulation

Presently, the aim is to identify the linear system parameters and the time delay $D$. The linear part of system considered in simulation is given as follows: The latter can be characterized by their transfer functions $G_i(s)$ and $G_o(s)$, respectively. Then, the latter have as parameters the module of gains $|G_i(j\omega)|$ and $|G_o(j\omega)|$, respectively, and the phases $\varphi_i(\omega)$ and $\varphi_o(\omega)$, respectively. Firstly, the considered system (Figure 1) is excited by the following signal:

$$G(q) = \frac{B(q)}{A(q)} = \frac{q^2+q}{q^2-0.7q+0.01} = \frac{q^{-2}+q^{-1}}{0.01q^{-2}-0.7q^{-1}+1} \quad (18)$$

where:

$$b_1 = b_2 = 1;\ a_1 = -0.7;\ a_2 = 0.01; \quad (19)$$

The time delay value is $D = 1$. In this work, the time delay is not supposed to be known. To estimate $D$, the method described in section 2 will be used. Then, the system is excited firstly by a control of zero value until the output returns to zero. The system is excited with a control different from zero. Example of obtained results for $v(t) = 1$ is shown in Figure 2.

To ensure that $t \geq max(D + n_b, n_a)$, a control of zero value is applied until the system output becomes null (or constant). Then, another value of the control $v(t)$ is applied and observing the time value when the output begins to change. This time value corresponds to $max(D + n_b, n_a)$. The result shown by Figure 2 allows us to estimate the time delay value $D$. Specifically, the latter is $D \approx 1s$.

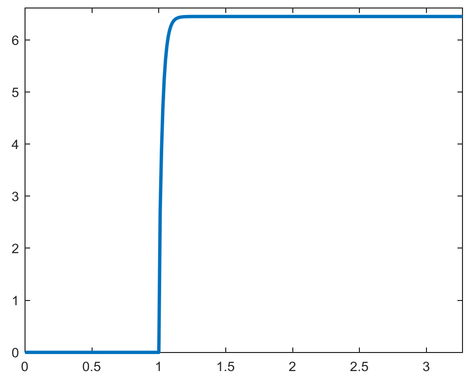


Fig. 2: System output for $v(t) = 1$

Once an upper bound of time delay $D$ delay is estimated, the estimator algorithm (16)-(17) can be used. In this respect, the studied system is excited using a signal with multiple frequencies. The used control is shown in Figure 3.

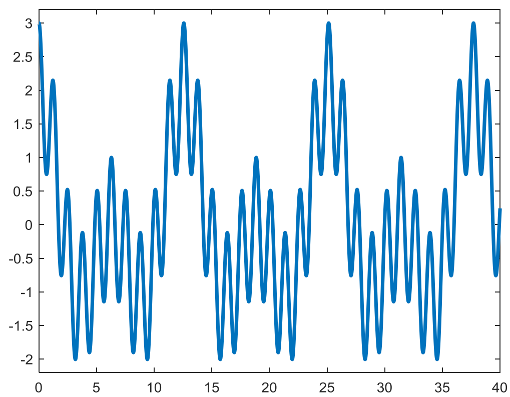


Fig. 3: The used control $v(t)$ in the estimate of linear parameters

Then, it follows from (14)-(15) that the output $y(t)$ can be expressed as the following recursive form:

$$y(t) = \phi^T(t)\theta \qquad (20)$$

where the true parameter vector $\theta$ is given as:

$$\theta = [b_1, b_2, a_1, a_2]^T = [1, 1, -0.7, 0.01]^T \qquad (21)$$

and the data vector $\phi(t)$ is given as:

$$\phi(t) = [u(t-D-1), u(t-D-2), -y(t-1), -y(t-2)]^T, \qquad (22)$$

Using the estimator algorithm (16)-(17), one obtains the estimates of $a_1$, $a_2$, $b_1$, and $b_2$ shown in Figure 4, Figure 5, Figure 6 and Figure 7, respectively.

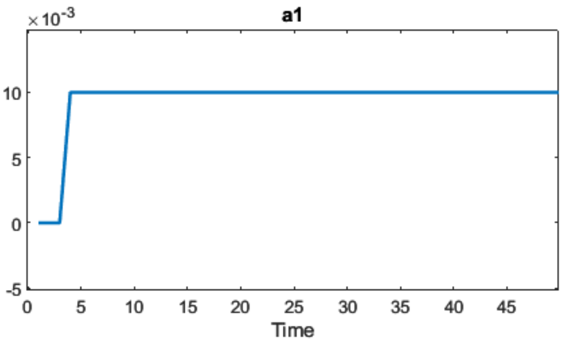


Fig. 4: The estimate of parameter $a_1$

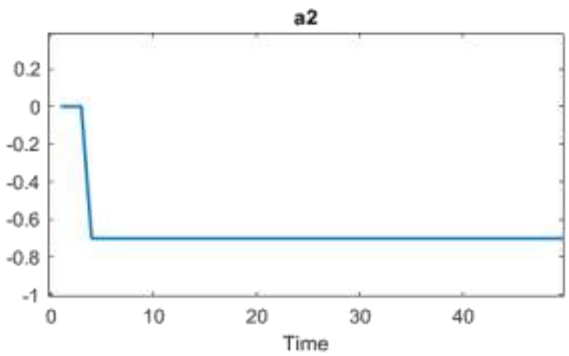


Fig. 5: The estimate of parameter $a_2$

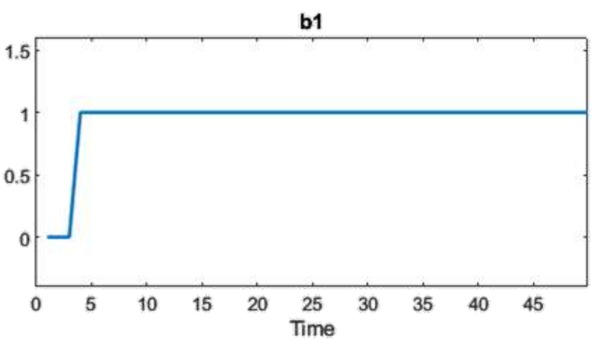


Fig. 6: The estimate of parameter $b_1$

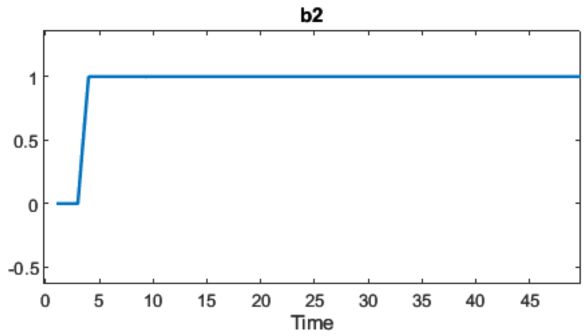


Fig. 7: The estimate of parameter $b_2$

The obtained results given by Figure 4, Figure 5, Figure 6 and Figure 7 show that the estimate parameters converge to their true values.

# 4 Conclusion

In this paper, the identification problem of nonlinear systems having a more general structure is discussed. Most studied nonlinear systems has been focused on Hammerstein and Wiener ones. It is shown that this nonlinear structure is more general than the Hammerstein and Wiener models. Then, the latter can be viewed as special cases of this nonlinear system. This approach is easy and converges quickly. Firstly, an input of a set of step signals is used. In the second stage, sine signal input is used to estimate the linear block parameters. Simulation examples show that the obtained parameter estimates are very close to the true nonlinear system parameters.

*References:*

[1] F. Giri, Y. Rochdi, A. Brouri, A. Radouane, F.Z. Chaoui, Frequency identification of nonparametric Wiener systems containing backlash nonlinearities, *Automatica,* Vol. 49, pp. 124-137, 2013. DOI: 10.1016/j.automatica.2012.08.043.

[2] A. Brouri, Wiener-Hammerstein nonlinear system identification using spectral analysis, *International Journal of Robust and Nonlinear Control,* Vol. 32, No. 10, pp. 6184-6204, 2022. DOI: 10.1002/rnc.6135.

[3] G. Bottegal, R. Castro-Garcia. J. A.K. Suykens, A two-experiment approach to Wiener system identification, *Automatica,* Vol. 93, pp. 282-289, 2018. DOI: 10.1016/j.automatica.2018.03.069.

[4] Q. Jin, Z. Wang, W. Cai, Y. Zhang, Parameter identification for Wiener-finite impulse response system with output data of missing completely at random mechanism and

time delay, *International Journal of Adaptive Control and Signal Processing*, Vol. 35 (5), pp. 811-827, 2021. DOI: 10.1002/acs.3227.

[5] H. Oubouaddi, A. Brouri, A. Ouannou, F.E. El Mansouri, A. Bouklata, L. Gadi, Switched Reluctance Motor Speed and Torque Control using ACO and PSO Algorithms, *WSEAS Transactions on Systems and Control*, Vol. 18, pp. 362-369, 2023. https://doi.org/10.37394/23203.2023.18.38.

[6] M. Schoukens and K. Tiels. Identification of block-oriented nonlinear systems starting from linear approximations: A survey, *Automatica*, Vol. 85, pp. 272-292, 2017. DOI: 10.1016/j.automatica.2017.06.044.

[7] H. Xua, L. Xua, S. Shen, Online identification methods for a class of Hammerstein nonlinear systems using the adaptive particle filtering. *Chaos, Solitons and Fractals,* Vol. 186, 115181, 2024. DOI: 10.1016/j.chaos.2024.115181.

[8] W. Yanjiao, L. Yitinga, C. Jiehaoa, T. Shihuaa, D. Muqing, Online identification of Hammerstein systems with B-spline networks, *International Journal of Adaptive Control and Signal Processing,* Vol. 38, pp. 2074-2092, 2024. DOI: 10.1002/acs.3792.

[9] Z. Tong, K. Gaëtan, Identification of secondary resonances of nonlinear systems using phase-locked loop testing, *Journal of Sound and Vibration,* Vol. 59010, 118549, 2024. DOI: 10.1016/j.jsv.2024.118549.

[10] L. Xu-Long, W. Sha, D. Hu, C. Li-Qun, A subspace parameter identification method for nonlinear structures under oversampling conditions, *Journal of Sound and Vibration*, Vol. 58924, 118590, 2024. DOI: 10.1016/j.jsv.2024.118590.

[11] F. Giri, Y. Rochdi, F.Z. Chaoui, A. Brouri, Identification of Hammerstein systems in presence of hysteresis-backlash and hysteresis-relay nonlinearities, *Automatica*, Vol. 44 (3), 767-775, 2008. DOI: 10.1016/j.automatica.2007.07.005.

[12] A. Brouri and L. Kadi, Identification of nonlinear systems, *Proceedings of the Conference on Control and its Applications - SIAM CT'19*, Jun 19-21, Chengdu, China, pp. 22-24,2019. DOI: 10.1137/1.9781611975758.4.

[13] A. Brouri, F.Z. Chaoui, F. Giri, Identification of Hammerstein-Wiener Models with Hysteresis Front Nonlinearities, *International Journal of Control,* Vol. 95, No. 10, pp. 3353-3367, 2022a. DOI: 10.1080/00207179.2021.1972160.

[14] H. Oubouaddi, F.E. El Mansouri, A. Brouri, Parameter Estimation of Linear and Nonlinear Systems Connected in Parallel, *WSEAS Transactions on Mathematics*, Vol. 22, pp. 373-377, 2023. https://doi.org/10.37394/23206.2023.22.44.

[15] F. Giri, A. Radouane, A. Brouri, and F.Z. Chaoui, Combined frequency-prediction error identification approach for Wiener systems with backlash and backlash-inverse operators, *Automatica,* Vol. 50, pp. 768-783, 2014. DOI: 10.1016/j.automatica.2013.12.030.

[16] R.S. Risuleo, F. Lindsten, H. Hjalmarsson, Bayesian nonparametric identification of Wiener systems, *Automatica,* pp. 1-8, 2019. DOI: 10.1016/j.automatica.2019.06.032.

[17] S. Jing, T. Pan, Q. Zhu, Identification of Wiener systems based on the variable forgetting factor multierror stochastic gradient and the key term separation, *International Journal of Adaptive Control and Signal Processing*, Vol. 35 (12), pp. 2537-2549, 2021. DOI: 10.1002/acs.3336.

[18] G. Mzyk, P. Wachel, Wiener system identification by input injection method, *International Journal of Adaptive Control and Signal Processing,* Vol. 34 (8), pp. 1105-1119, 2020. DOI: 10.1002/acs.3124.

[19] A. Ouannou, A. Brouri, L. Kadi, H. Oubouaddi, Identification of switched reluctance machine using fuzzy model, *International Journal of System Assurance Engineering and Management*, Vol. 13 (6), pp. 2833-2846, 2022. DOI: 10.1007/s13198-022-01749-4.

[20] A. Ouannou, A. Brouri, F. Giri, L. Kadi, H. Oubouaddi, S.A. Hussien, N. Alwadai, M.I. Mosaad, Parameter Identification of Switched Reluctance Motor SRM Using Exponential Swept-Sine Signal, *Machines*, Vol. 11(6), 625, 2023. DOI: 10.3390/machines11060625.

[21] H. Oubouaddi, F.E. El Mansouri, A. Bouklata, R. Larhouti, A. Ouannou, A. Brouri, Parameter Estimation of Electrical Vehicle Motor, *WSEAS Transactions on Systems and Control*, Vol. 18, pp. 430-436, 2023. DOI: https://doi.org/10.37394/23203.2023.18.46.

[22] A.G. Wu, F.Z. Fu, R.Q. Dong, Convergence Analysis of Weighted Stochastic Gradient Identification Algorithms Based on Latest-Estimation for ARX Models, *Asian Journal of*

*Control*, Vol. 21, No. 1, pp. 509-519, 2019. DOI: 10.1002/asjc.1747.

[23] S. Cheng, Y. Wei, D. Sheng, and Y. Wang, Identification for Hammerstein nonlinear systems based on universal spline fractional order LMS algorithm, *Communications in Nonlinear Science and Numerical Simulation*. Vol. 79, pp. 1-13, 2019. DOI: 10.1016/j.cnsns.2019.104901.

[24] A. Brouri, F. Giri, F. Ikhouane, F.Z. Chaoui, O. Amdouri, Identification of Hammerstein-Wiener Systems with Backlash Input Nonlinearity Bordered By Straight Lines, *19th IFAC World Congress, Cape Town, South Africa*, Vol. 47, No:3, August 24-29, pp. 475-480, 2014a. https://doi.org/10.3182/20140824-6-ZA-1003.00678.

[25] A. Brouri, F.Z. Chaoui, O. Amdouri, F. Giri, Frequency Identification of Hammerstein-Wiener Systems with Piecewise Affine Input Nonlinearity, *19th IFAC World Congress, Cape Town*, Vol. 47, No.3, August 24-29, pp. 10030-10035, 2014b. https://doi.org/10.3182/20140824-6-ZA-1003.00303.

[26] H. Shumin, C. Liangyun, L. Dong, L. Heng, Command Filtered Adaptive Fuzzy Control of Fractional-Order Nonlinear Systems With Unknown Dead Zones, *IEEE Transactions on Systems, Man, and Cybernetics: Systems*. Vol. 54, pp. 3705 – 37161, 2024. DOI: 10.1109/TSMC.2024.3370003.

[27] W. Huanqing, T. Miao, Fuzzy finite-time adaptive control of switched nonlinear systems with input nonlinearities, *International Journal of Adaptive Control and Signal Processing*. Vol. 38, pp. 2570-2587, 2024. DOI:10.1002/acs.3819.

[28] W. Yibing, Z. Manli, T. Shengnan, L. Chengda, W. Min, S. Daiki, An Adaptive Integral Sliding Mode Control-Based Amplitude and Phase Compensation Repetitive Control Method, *IEEE Transactions on Industrial Electronics*. pp. 1-10, 2024. DOI:10.1109/TIE.2024.3401183.

[29] L. Yan, Z. Xuan, Fuzzy Fixed-Time Control for Nonaffine Nonlinear Systems with Unknown Hysteresis Inputs, *International Journal of Fuzzy Systems*. Vol. 25, pp. 3036 – 3048, 2023. DOI: 10.1007/s40815-023-01556-4.

[30] B. Ni, Z. Yuyi, L. Xiaoyong, C. Wei, J. Changan, Passive robust control for uncertain Hamiltonian systems by using operator theory, *CAAI Transactions on Intelligence Technology*, Vol. 7, pp 594 – 605, 2022. DOI: 10.1049/cit2.12142.

[31] A. Brouri, L. Kadi, K. Lahdachi, Identification of nonlinear system composed of parallel coupling of Wiener and Hammerstein, *Asian Journal of Control*, Vol. 24, No. 3, pp. 1152-1164, 2022b. DOI: 10.1002/asjc.2533.

[32] A. Brouri, F. Giri, Identification of series-parallel systems composed of linear and nonlinear blocks, *International Journal Of Adaptive Control And Signal Processing*, Vol: 37, No: 8, pp. 2021-2040, 2023. DOI: 10.1002/acs3624.

[33] H. Oubouaddi, A. Brouri, A. Ouannou, F.E. El Mansouri, A. Bouklata, L. Gadi, Parameter Estimation of Linear and Nonlinear Systems Connected in Parallel, *WSEAS Transactions on Systems and Control*, Vol. 18, pp. 362-369, 2023. DOI: https://doi.org/10.37394/23206.2023.22.44.

[34] D. Westwick, M. Ishteva, P. Dreesen, and J. Schoukens. Tensor Factorization based Estimates of Parallel Wiener Hammerstein Models. *IFAC Papers Online*, Vol. 50 (1), pp. 9468-9473, 2017. DOI: 10.1016/j.ifacol.2017.08.1470.

**Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)**

- Chaimae Abdelaali and Ali Bouklata carried out the simulation, writing, software, and the optimization.
- Adil Brouri and Mohamed Benyassi project supervision

**Sources of Funding for Research Presented in a Scientific Article or Scientific Article Itself**

No funding was received for conducting this study.

**Conflict of Interest**

The authors have no conflicts of interest to declare.