Analysis Process Dispersion Variation Tracked using a Mixed MA-EWMA Control Chart

SUGANYA PHANTU[1], YUPAPORN AREEPONG[2], SAOWANIT SUKPARUNGSEE[2,*]
[1]Faulty of Science, Energy, and Environment,
King's Mongkut University of Technology North Bangkok,
Rayong, 21120,
THAILAND

[2]Department of Applied Statistics, Faculty of Applied Science,
King Mongkut's University of Technology, North Bangkok,
Bangkok, 10800,
THAILAND

[*]*Corresponding Author*

Abstract: - This study aims to develop a combined Moving Average - Exponentially Weighted Moving Average Control Chart with standard deviation based (MA-EWMA$_S$ chart) that can be used to identify changes in standard deviation in processes under a normal distribution. The average run length (ARL), standard deviation of run length (SRL), and median run length (MRL) are used to compare the performance of the proposed control chart with S, MA$_S$, and EWMA$_S$ control charts. This benchmark is assessed using Monte Carlo (MC) simulations. Furthermore, actual data is used to apply the suggested control charts. For all levels of variation, the recommended control chart outperforms S, MA$_S$, and EWMA$_S$ control charts in terms of detection performance, as indicated by the performance comparison results. Additionally, the MA-EWMA$_S$ chart demonstrates superior performance in managing process variability for moderate and large subgroup sizes across all magnitudes of shift parameters. One way to assess the control chart's effectiveness is to apply the suggested chart to track the fruit juice and wafer coating production process and verify that it complies with standards. The results of the simulations were found to align with the actual data.

Key-Words: - Dispersion, Performance, Detection, Mixed control chart, Change, Average run length

Received: March 2, 2024. Revised: May 24, 2024. Accepted: June 25, 2024. Published: July 29, 2024.

1 Introduction

Control charts are the primary tools used in statistical process control (SPC). Control charts have practical applications and are used extensively in a variety of industries, including the healthcare sector. Methods of production used in environmental science, etc. Shewhart initially developed the control chart, which is thought to be the main tool of SPC, using statistical concepts, [1]. The production process is ascertained by scatter plotting data from previous production processes in what is commonly known as a Shewhart control chart. Therefore, the plot distribution pattern cannot be determined if the production process remains largely unchanged. Therefore, using a Shewhart control chart, it is possible to detect more significant changes in a process. [2] and [3] subsequently created control charts to detect modifications in the production process, regardless of how minor. Compared to Shewhart control chart also known as cumulative sum (CUSUM) and exponentially weighted moving average (EWMA) control charts are more sensitive to identifying slight to moderate changes in a process. [4], developed a moving average (MA) control chart in 2004 to ascertain the percentage of inconsistent observations. The outcomes show that the MA control chart outperforms the other charts. Making EWMA and MA control charts for various scenarios is a common research focus. In order to detect process averages in the case of a lognormal distribution using integral equations and verify the accuracy of the results obtained from simulation techniques, [5], presented algorithms to design CUSUM control chart with time series observations. It was discovered that the integral equation outperformed the EWMA control chart, [6]. The EWMA-MA control chart was proposed in 2019 combined the EWMA and MA control charts, [7]. The performance of the EWMA-MA control chart and the MA-EWMA control chart are compared to detect changes in the process average. Upon comparing the suggested control chart with the MA-EWMA control chart for both symmetric and asymmetric distributions under all shift magnitudes, it is discovered that the MA-EWMA control chart outperforms the EWMA-MA control chart regarding parameter change detection.

On the other hand, the comparison results show that the ARL_1 value of the EWMA-MA and MA-EWMA control charts depend on the parameter of the control chart mentioned above; that is, if the values of λ change, ARL_1 will not change, but the MA-EWMA chart will vary ARL_1 when the range size changes. [8], looked into the use of moving average standard deviation (MA_S) control charts for process variation detection. Process variation is tracked by comparing the S control chart with the MA_S control chart using a moving average of the sample standard deviation. S control chart is found to be less effective than MA_S control chart through comparisons. Using a MA_S control chart, small and large process variation changes can be rapidly recognized as out-of-control events.

This research aims to propose a new control chart, called the MA-$EWMA_S$ control chart, which combines the $EWMA_S$ and MA_S control charts and can be used to track a change in the standard deviation process by comparing the chart performance. To manage MA-$EWMA_S$ for process dispersion change, use S, $EWMA_S$, and MA_S control charts. A control chart with the lowest ARL_1 performs best regarding change detection. Verifying the inter-thread thickness on real data can also be done with it. In addition, information on fruit juice bottle packaging and wafer surface coating quality inspection have been used to compare the performance of the presented chart with the original control chart.

2 Research Methodology

Let $S_1, S_2, ..., S_i, ...$ represent the standard deviation from a set of subgroups following a normal distribution ($Normal(\mu,\sigma^2)$) [9], which S_i signifies the standard deviation of the sample of subgroups i. Establishing a control limit is imperative to ascertain the value of the actual standard deviation (σ).

A value σ is unknown in most cases and needs to be estimated from prior data. The average sample standard deviation $\bar{S}$ is typically determined from an initial set of k subgroup standard deviations provided by the unbiased estimator $\hat{\sigma} = \bar{S} / c_4$, , where

$$\bar{S} = \frac{1}{k}\sum_{i=1}^{k} S_i, \tag{1}$$

where $S_i = \sqrt{\frac{1}{k-1}\sum_{j=1}^{k}(X_{ij} - \bar{X}_i)^2}, \bar{X}_i = \frac{1}{k}\sum_{j=1}^{k} X_{ij},$ and $c_4 = (\frac{2}{n-1})^{1/2}(\Gamma(\frac{n}{2}) / \Gamma(\frac{n-1}{2}))$ is a constant that depends on the sample size k, including the following chats in this research.

2.1 Control Chart for Standard Deviation (S Chart)

The standard deviation control limits are usually computed based on the 3σ approach, i.e., $\mu_S \pm 3\sigma_S$ where μ_S and σ_S are the mean and standard deviation of the process [9], respectively. Thus, the upper and lower control limits of the chart, when σ is unknown, are given as:

$$UCL / LCL = \hat{\sigma} \pm B_1 \hat{\sigma} \sqrt{1 - c_4^2} \tag{2}$$

where B_1 is the factor of control limit of the S chart. A sample point plot that deviates from the control limit indicates instability in the process.

2.2 Moving Average for Standard Deviation Control Chart (MA_S chart)

The moving average standard deviation (MA_S) control chart was examined by [8] in order to look into the control chart's middle movement for identifying process variability which original presented from [10]. For minor and large process variability changes, the MA_S chart can quickly identify departures from control. The statistical values of the MA_S chart can be categorized into two cases as follows:

$$MA_i = \begin{cases} \dfrac{S_i + S_{i-1} + S_{i-2} + ... + S_1}{i} & ; i < \omega \\ \dfrac{S_i + S_{i-1} + ... + S_{i-\omega+1}}{\omega} & ; i \geq \omega. \end{cases} \quad (3)$$

Where ω is the width of the MA_S chart. The average of the MA_S statistic is denoted as

$$E(MA_S) = c_4\hat{\sigma}. \quad (4)$$

Furthermore, the variance of the MA_S statistic can be split into the following two scenarios:

$$Var(MA_S) = \begin{cases} \dfrac{\hat{\sigma}^2(1-c_4^2)}{i}, & i < \omega \\ \dfrac{\hat{\sigma}^2(1-c_4^2)}{\omega}, & i \geq \omega. \end{cases} \quad (5)$$

As a result, the following is the upper and lower control limit,

$$UCL/LCL = \begin{cases} c_4\hat{\sigma} + B_2\sqrt{\dfrac{\hat{\sigma}^2(1-c_4^2)}{i}}, & i < \omega \\ c_4\hat{\sigma} - B_2\sqrt{\dfrac{\hat{\sigma}^2(1-c_4^2)}{\omega}}, & i \geq \omega \end{cases} \quad (6)$$

where B_2 is the factor of control limit of the MA_S chart.

2.3 Exponentially Weighted Moving Average for Standard Deviation Control Chart ($EWMA_S$ chart)

[3], introduced the EWMA chart to track minute parameter variations like process mean and standard deviation. EWMA statistics for detecting the variation of a process [9] are as follows

$$EWMA_{S_i} = \lambda\bar{S}_i + (1-\lambda)EWMA_{S_{i-1}},\ i = 1,2,... \quad (7)$$

where λ the weighting parameter of the past data has a value from 0 to 1, and $\bar{S}_i$ the average standard deviation at the time *i*. The expectation and variance of $EWMA_S$ are:

$$E\left(EWMA_{S_i}\right) = c_4\hat{\sigma}$$

and

$$Var\left(EWMA_{S_i}\right) = \hat{\sigma}\left(1-c_4^2\right)(\lambda/\lambda-2). \quad (8)$$

Therefore, the $EWMA_S$ chart's control limits are

$$UCL/LCL = c_4\hat{\sigma} \pm B_3\sqrt{\left(1-c_4^2\right)(\lambda/\lambda-2)} \quad (9)$$

where B_3 is the factor of control limit of the $EWMA_S$ chart.

2.4 Moving Average - Exponentially Weighted Moving Average for Standard Deviation Control Chart ($MA\text{-}EWMA_S$ chart)

The MA_S chart and the $EWMA_S$ chart are combined to create the $MA\text{-}EWMA_S$ chart. As an input to the MA_S chart, let us consider the statistical data of the $EWMA_S$ chart. Thus, the statistics of the $MA\text{-}EWMA_S$ chart are as follows:

$$MA-EWMA_{S_i} = \begin{cases} \dfrac{Z_i + Z_{i-1} + Z_{i-2} + ... + Z_1}{i} & ; i < \omega \\ \dfrac{Z_i + Z_{i-1} + ... + Z_{i-\omega+1}}{\omega} & ; i \geq \omega \end{cases} \quad (10)$$

where ω is the span of the moving average of the $MA\text{-}EWMA_S$ chart. The expectation and variance of statistics $MA\text{-}EWMA_S$ are:

$$E(MA-EWMA_S) = c_4\hat{\sigma}$$

and

$$Var(MA-EWMA_S) = \begin{cases} \dfrac{\hat{\sigma}^2(1-c_4^2)}{i}\left(\dfrac{\lambda}{\lambda-2}\right), & i < \omega \\ \dfrac{\hat{\sigma}^2(1-c_4^2)}{\omega}\left(\dfrac{\lambda}{\lambda-2}\right), & i \geq \omega. \end{cases} \quad (11)$$

Therefore, the $MA\text{-}EWMA_S$ chart's control limits are

$$UCL/LCL = \begin{cases} c_4\hat{\sigma} + B_4\sqrt{\dfrac{\hat{\sigma}^2(1-c_4^2)}{i}\left(\dfrac{\lambda}{\lambda-2}\right)}, & i < \omega \\ c_4\hat{\sigma} - B_4\sqrt{\dfrac{\hat{\sigma}^2(1-c_4^2)}{\omega}\left(\dfrac{\lambda}{\lambda-2}\right)}, & i \geq \omega \end{cases} \quad (12)$$

where B_4 is the factor of control limit of the $MA\text{-}EWMA_S$ chart.

3 The Performance of the Control Chart

The effectiveness of control charts can be evaluated using a variety of techniques. This research uses three values to consider the tracking performance of control charts with Monte Carlo (MC) simulation techniques, including the following methods.

3.1 Average Run Length (ARL)

The Average Run Length (ARL) is a commonly used metric to evaluate a control chart's effectiveness. ARL measures the control chart's effectiveness in identifying outliers in the production process. It measures how quickly the process parameter shifts are identified on the chart. The average number of data points (ARL) that must be plotted before one point indicates an out-of-control condition, which is represented by ARL_0 and ARL_1, respectively, representing the in-control and out-of-control processes. The ARL can be determined as follows:

$$ARL = \sum_{j=1}^{T} RL_j \Big/ T. \tag{13}$$

In this case, the sample being examined before the process surpasses the control limits for the first time is indicated by RL_j. T is set to 200,000, is the number of experiment repetitions in the simulation during round j.

3.2 Standard Deviation of Run Length (SRL)

The standard deviation of the run length (SRL) can be computed as follows:

$$SDRL = \sqrt{E\left(RL_j\right)^2 - ARL^2}. \tag{14}$$

3.3 Median Run Length (MRL)

The middle of RL_j points plotted on a chart before an out-of-control signal is given is called the median run length, or MRL. Thus, the MRL is calculated as follows:

$$MRL = Median(RL_j). \tag{15}$$

4 Numerical Results

The numerical results of this research are divided into three parts. Part 1 focuses on assessing the efficiency of the MA-EWMA$_S$ chart. Part 2 involves comparing the efficiency of control charts, and Part 3 explores the application of these charts to real-world data in that order.

4.1 Average Run Length of MA-EWMA$_S$ Chart

A performance metric is the average run length (ARL). The estimated number of samples needed until a control chart indicates an out-of-control condition is indicated by the ARL_0. A large ARL_0 is desirable when there is no change in the process variability. However, in the scenario where the process variability shifts from σ_0 to σ_1, $\sigma_1 = \kappa\sigma_0$, a small ARL_1 value is preferred. Monte Carlo simulations estimate the average run lengths of the MA-EWMA$_S$ chart that are in control and out-of-control, considering different shifts in the process standard deviation. The in-control process is assumed to follow a normal distribution with parameters Normal(μ,σ^2), while the out-of-control process is supposed to be normally distributed as Normal$(\mu,\kappa\sigma^2)$. The shift values are represented as $\kappa = \sigma_1 / \sigma_0$ where κ takes on values in the set {1.01, 1.025, 1.05, 1.10, 1.20, 1.50, 2.00}. It is assumed that $\mu = 0$ and $\sigma_0 = 1$, ensuring that the chart's in-control average run length (ARL_0) is approximately 370.

Table 1, Table 2 and Table 3 display the ARL of the MA- EWMA$_S$ chart for sample sizes of k = 5, 10, and 15, respectively, with the weighting parameter of the data (λ) as 0.5 and the width (ω) of the MA-EWMA$_S$ chart are 2, 5, 10 and 15. The results showed that in Table 1, when the number of subgroups is 5, the optimal width parameter (ω) for the MA-EWMA$_S$ chart, when the shift value is set to 1.01, is 2. Next, in Table 2, the optimal width parameter (ω) for the MA-EWMA$_S$ chart, when the shift value is set to 1.025, is determined to be 5. Finally, in Table 3, the optimal width parameter (ω) for the MA-EWMA$_S$ chart, when the shift value is greater than 1.05, the value of width (ω) is determined to be 15, resulting in the lowest ARL_1. Additionally, the number of subgroups is 10 and 15. The result indicates that the optimal width parameter (ω) for the MA-EWMAs chart, when the shift value is equal to 1.01, is found to be 2, and when the shift value starts from 1.025 and upwards, it is found to be 15.

Additionally, the efficiency of the MA-EWMA$_S$ chart increases performance as the width (ω) is augmented across all levels of parameter changes. At the same time, the subgroup size (k) does not impact the proposed chart's performance.

Table 4, Table 5 and Table 6 displays the performance of the MA-EWMA$_S$ chart for sample sizes of k = 5, 10, and 15, respectively, with the weighting factor of the past data (λ) based on the value in the set {0.05, 0.2, 0.25, 0.50} and the width

(ω) of the MA-EWMA$_S$ chart is 2. The results demonstrate that in Table 4, if the number of subgroups is 5, the optimal parameter of weighting (λ) for the MA-EWMA$_S$ chart, when the shift value is set from 1.01 to 1.20, is given to be 0.25. Next, in Table 5, when the shift value is set from 1.50 to 2.00, it is shown to be 0.2 and 0.25. In the scenario of the number of subgroups being 10, the optimal parameter of weighting (λ) for the MA-EWMA$_S$ chart, when the shift value is set from 1.01 to 1.20, is given to be 0.05. Furthermore, when the shift value is set from 1.50 to 2.00, it is shown to be 0.05 to 0.52. In Table 6, the number of subgroups is 15, and the optimal parameter of weighting (λ) for the MA-EWMA$_S$ chart, when the shift value is set from 1.01 to 1.20, is given to be 0.25. Finally, in Table 6, when the shift value is set from 1.50 to 2.00, it is found to be 0.05 to 0.52. Furthermore, the MA-EWMA chart's efficiency increases as weighting (λ) decreases, where the subgroup size affects the proposed chart's performance. When the subgroup size (k) is small, the weighting (λ) tends to increase; conversely, when the subgroup size (k) is large, the weighting value (λ) tends to decrease.

Table 1. Comparative ARL$_1$ of MA-EWMA$_S$ chart when ARL$_0$=370, k=5 and λ=0.2.

Shift sizes (κ)	ω=2	ω=5	ω=10	ω=15
	B_4=5.316	B_4=5.052	B_4=4.808	B_4=4.634
1.01	**323.274**	325.565	331.352	334.431
1.025	262.444	**261.825**	265.748	267.274
1.05	184.347	175.965	171.945	**166.776**
1.10	95.228	82.614	73.846	**67.939**
1.20	33.602	26.313	22.295	**20.394**
1.50	5.442	4.418	3.991	**3.744**
2.00	1.273	1.128	1.036	**0.974**

****bold is a minimum of ARL$_1$***

Table 2. Comparative ARL$_1$ of MA-EWMA$_S$ chart when ARL$_0$=370, k=10 and λ=0.2.

Shift sizes (κ)	ω=2	ω=5	ω=10	ω=15
	B_4=5.236	B_4=5.047	B_4=4.815	B_4=4.641
1.01	**317.787**	321.606	324.275	324.551
1.025	245.814	241.454	235.952	**228.462**
1.05	155.473	139.378	123.287	**111.568**
1.10	66.598	50.517	40.566	**35.603**
1.20	18.091	12.730	10.483	**9.556**
1.50	2.186	1.817	1.646	**1.537**
2.00	0.368	0.335	0.302	**0.279**

****bold is a minimum of ARL1***

Table 3. Comparative ARL$_1$ of MA-EWMA$_S$ chart when ARL$_0$=370, k=15 and λ=0.2.

Shift sizes (κ)	ω=2	ω=5	ω=10	ω=15
	B_4=5.219	B_4=5.043	B_4=4.809	B_4=4.639
1.01	**314.979**	317.512	316.863	315.912
1.025	235.683	225.311	210.330	**198.089**
1.05	137.286	114.375	93.817	**82.508**
1.10	51.077	35.467	27.113	**23.802**
1.20	11.917	8.090	6.704	**6.175**
1.50	1.228	1.051	0.952	**0.885**
2.00	0.141	0.127	0.112	**0.101**

****bold is a minimum of ARL1***

Table 4. Comparative ARL_1 of MA-EWMA$_S$ chart when ARL_0=370, k=5, and ω=2.

Shift sizes (κ)	$\lambda = 0.05$	$\lambda = 0.2$	$\lambda = 0.25$	$\lambda = 0.50$
	B_4=19.166	B_4=9.207	B_4=8.120	B_4=5.316
1.01	323.256	323.216	**323.204**	323.274
1.025	262.435	262.394	**262.386**	262.444
1.05	184.346	184.315	**184.312**	184.347
1.10	95.228	95.219	**95.217**	95.228
1.20	33.601	33.599	**33.598**	33.602
1.50	5.442	**5.441**	**5.441**	5.442
2.00	1.273	**1.272**	**1.272**	1.273

**bold is a minimum of ARL_1*

Table 5. Comparative ARL_1 of MA-EWMA$_S$ chart when ARL_0=370, k=10, and ω=2.

Shift sizes (κ)	$\lambda = 0.05$	$\lambda = 0.2$	$\lambda = 0.25$	$\lambda = 0.50$
	B_4=18.874	B_4=9.068	B_4=7.997	B_4=5.236
1.01	**317.048**	317.268	317.103	317.787
1.025	**245.307**	245.509	245.347	245.814
1.05	**155.144**	155.283	155.163	155.473
1.10	**66.465**	66.515	66.476	66.598
1.20	**18.066**	18.076	18.069	18.091
1.50	**2.185**	**2.185**	**2.185**	**2.185**
2.00	**0.368**	**0.368**	**0.368**	**0.368**

**bold is a minimum of ARL_1*

Table 6. Comparative ARL_1 of MA-EWMA$_S$ chart when ARL_0=370, k=15, and ω=2.

Shift sizes (κ)	$\lambda = 0.05$	$\lambda = 0.2$	$\lambda = 0.25$	$\lambda = 0.50$
	B_4=18.815	B_4=9.042	B_4=7.971	B_4=5.219
1.01	314.872	315.643	**314.866**	314.979
1.025	235.578	236.139	**235.576**	235.683
1.05	137.235	137.538	**137.234**	137.286
1.10	51.068	51.156	**51.062**	51.077
1.20	11.917	11.932	**11.916**	11.917
1.50	**1.228**	**1.228**	**1.228**	**1.228**
2.00	**0.141**	**0.141**	**0.141**	**0.141**

**bold is a minimum of ARL_1*

4.2 Comparison Performance of the Control Chart

This section compares the MA-EWMA$_S$ chart's efficiency to that of the S, MA$_S$, and EWMA$_S$ charts. The standard deviation of run length (SRL), median run length (MRL), and average run length (ARL) were among the metrics used to assess the effectiveness of control charts. The control chart exhibiting the lowest values for ARL_1, SRL, and MRL was deemed the most efficient. When the process is under control, it is given that ARL_0 = 370, the width (ω) for the MA-EWMA$_S$ chart is two, and the weighting factor of the data is 0.2. Table 7 shows that the number of subgroups is 5, indicating that the MA$_S$ chart performs best when the shift parameter is 1.01 to 1.02. Next, the shift parameter (κ) is 1.05 to 1.20, and the EWMA$_S$ chart achieves the most. Finally, if the shift parameter exceeds 1.50, the MA-EWMA$_S$ chart detects change most effectively. Table 8 for the number subgroup is 10, showing that when the shift parameter is 1.01, the MA$_S$ chart is the best performing. Next, the shift parameter is 1.02, and the EWMA$_S$ chart performs the most. Furthermore, if the shift parameter is greater than 1.05, the MA-EWMA$_S$ chart detects change most effectively. Additionally, Table 9 for the number of subgroups is 10, demonstrating that when the shift parameter is greater than 1.01, the MA-EWMA$_S$ chart is the best performing.

Table 7. The comparison of control chart when $ARL_0 = 370$, k = 5, $\lambda = 0.2$, and $\omega = 2$.

Shift sizes (κ)	S			MA_S			$EWMA_S$			MA-$EWMA_S$		
	$B_1 = 3.19$			$B_2 = 3.07$			$B_3 = 3.19$			$B_4 = 5.32$		
	ARL_1	SRL	MRL	ARL_1	SRL	MRL	ARL_1	SRL	MRL	ARL_1	SRL	MRL
1.01	330	0.83	243	**323**	**0.82**	**237**	324	0.94	219	323	0.82	237
1.02	276	0.73	196	**262**	**0.70**	**186**	263	0.81	171	262	0.70	186
1.05	203	0.57	142	184	0.52	128	**181**	**0.59**	**115**	184	0.52	128
1.10	114	0.33	79	95	0.28	66	**90**	**0.29**	**58**	95	0.28	95
1.20	44	0.13	30	33	0.10	23	**31**	**0.09**	**22**	33	0.10	23
1.50	7	0.23	5	6	0.02	4	6	0.02	5	**5**	**0.01**	**4**
2.00	1	0.01	1	1	0.01	1	2	0.01	2	**1**	**0.01**	**1**

**bold is a minimum of ARL_1, SRL, and MRL*

Table 8. The comparison of control chart when $ARL_0 = 370$, k = 10, $\lambda = 0.2$ and $\omega = 2$.

Shift sizes (κ)	S			MA_S			$EWMA_S$			MA-$EWMA_S$		
	$B_1 = 3.07$			$B_2 = 3.02$			$B_3 = 3.58$			$B_4 = 5.24$		
	ARL_1	SRL	MRL	ARL_1	SRL	MRL	ARL_1	SRL	MRL	ARL_1	SRL	MRL
1.01	319	0.81	232	**317**	**0.81**	**231**	355	1.27	232	317	0.81	232
1.02	253	0.69	179	245	0.67	174	**321**	**0.67**	**173**	245	0.67	174
1.05	170	0.49	119	155	0.44	108	239	0.45	45	**155**	**0.45**	**108**
1.10	82	0.24	57	66	0.19	46	100	0.42	37	**66**	**0.19**	**46**
1.20	25	0.07	17	18	0.05	14	24	0.09	15	**18**	**0.05**	**12**
1.50	2	0.01	2	2	0.01	1	4	0.01	4	**2**	**0.01**	**1**
2.00	0.5	0.01	0	0.5	0.01	0	1	0.01	1	**0.3**	**0.01**	**0**

**bold is a minimum of ARL_1, SRL, and MRL*

Table 9. The comparison of control chart when $ARL_0 = 370$, k = 15, $\lambda = 0.2$, and $\omega = 2$.

Shift sizes (κ)	S			MA_S			$EWMA_S$			MA-$EWMA_S$		
	$B_1 = 3.047$			$B_2 = 3.014$			$B_3 = 4.291$			$B_4 = 5.219$		
	ARL_1	SRL	MRL	ARL_1	SRL	MRL	ARL_1	SRL	MRL	ARL_1	SRL	MRL
1.01	317	0.81	230	315	0.80	230	385	1.39	232	**314**	**0.80**	**230**
1.02	244	0.67	172	235	0.65	166	404	1.36	171	**235**	**0.65**	**166**
1.05	155	0.45	108	137	0.39	96	431	1.36	96	**137**	**0.39**	**96**
1.10	66	0.19	46	51	0.15	36	51	1.03	84	**51**	**0.15**	**36**
1.20	17	0.05	12	11	0.03	8	41	0.15	24	**11**	**0.04**	**8**
1.50	1.6	0.01	1	1.2	0.01	1	4.3	0.01	4	**1.2**	**0.01**	**1**
2.00	0.2	0.01	0	0.1	0.01	0	1.5	0.01	0	**0.1**	**0.01**	**0**

**bold is a minimum of ARL_1, SRL, and MRL*

4.3 Comparison Performance of the Control Chart

This section describes how the control chart is applied to real data to inspect and manage production process quality and meet predetermined standards. The standard deviation is the value used to assess the performance of both data sets. The data is divided into two sets as follows:

4.3.1 Application I: Fruit Juice Volume

Data from the production process will be collected to inspect the quantity of fruit juice packages, which will involve 15 sample groups, each containing ten bottles. The measurement consists of assessing the height of the fruit juice level in the bottles compared to standard quality. The interpretation is as follows: if the measured height is 0, it indicates that the packaging quantity is equal to the normal amount. A positive or negative estimated height signifies that the packaging quantity is higher or lower than the standard quantity, [9].

The efficiency of detecting changes in the standard deviation values of S, MA_S, $EWMA_S$, and MA-$EWMA_S$ charts can be illustrated as follows. In Figure 1, the change detection performance for the S chart reveals no data from the sample group of fruit juice bottles exceeding the control limits. Next, Figure 2, Figure 3 and Figure 4 illustrate the change detection performance of the MA_S, $EWMA_S$, and MA-$EWMA_S$ charts, respectively. The results of the control chart performance measurements reveal that sample group no. At Sixth the fruit juice bottles exceed the control limits of the chart. Therefore, these charts detect changes faster than the S chart. Additionally, the comparative results indicate that MA_S, $EWMA_S$, and $MA\text{-}EWMA_S$ control charts exhibit comparable efficiency in detecting changes in the data's standard deviation.

Fig. 1: The performance of the S chart for fruit juice

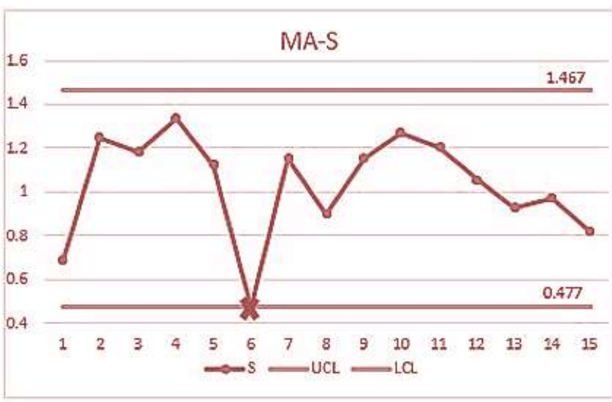

Fig. 2: The performance of the MAS chart for fruit juice

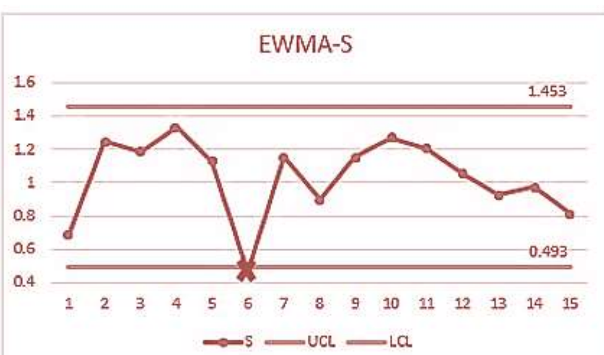

Fig. 3: The performance of the EWMAS chart for fruit juice

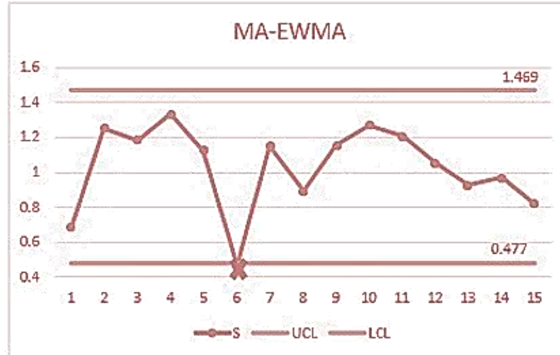

Fig. 4: The performance of the MA-EWMAS chart for fruit juice

4.3.2 Application II: The Coating on Wafers

Quality control is essential in semiconductor manufacturing, which involves hardback processes. The thickness of the surface coating on wafers is examined by sampling five units from each of the 20 groups to observe whether the production process is under control. The sampling intervals are set to be 1 hour apart for each instance, [9].

The efficiency of detecting changes in the standard deviation values of S, MA_S, $EWMA_S$, and $MA\text{-}EWMA_S$ charts can be explained as follows. Figure 5, Figure 6 and Figure 7 present the performance evaluation of the S, MA_S, and $EWMA_S$ control charts. The results indicate that these charts cannot detect changes in the data because no data from the sample group exceeds the control limits. The performance of the $MA\text{-}EWMA_S$ chart is finally displayed in Figure 8. The outcomes demonstrated that the MA-EWMAs chart can detect data changes immediately. For this reason, compared to the S, MA_S, and $EWMA_S$ charts, the MA-EWMAs chart is better at tracking changes in the data's standard deviation.

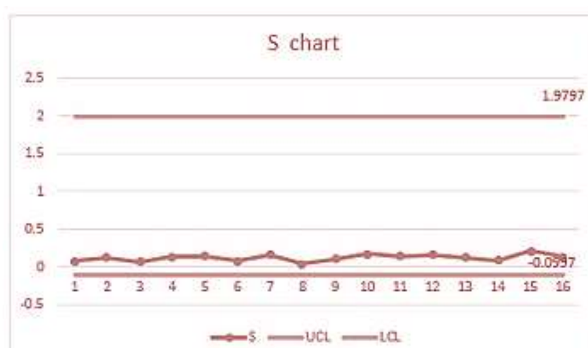

Fig. 5: The performance of the S chart for the coating on wafers

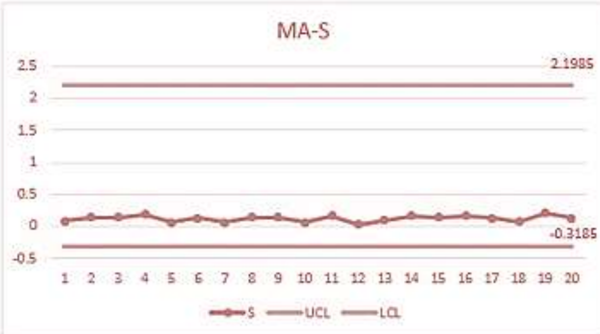

Fig. 6: The performance of the MA_S chart for the coating on wafers

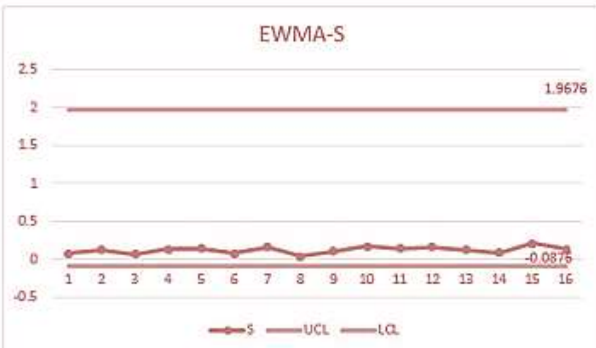

Fig. 7: The performance of the EWMAS chart for the coating on wafers

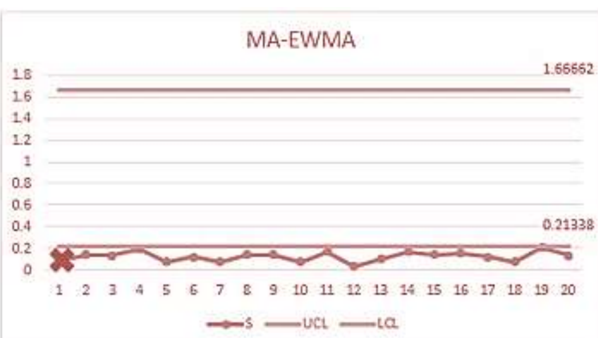

Fig. 8: The performance of the MA-EWMAS chart for the coating on wafers

5 Conclusion and Future Work

In order to monitor process variability, this study substitutes the moving average statistic for the MA_S and $EWMA_S$ charts. For the chart, control limit factors are given for a range of sample sizes and width parameters. Through simulation procedures, the average run length (ARL), standard deviation of run length (SRL), and median run length (MRL) values are used to assess the performance of the MA-$EWMA_S$ chart. The S, MA_S, and $EWMA_S$ charts for process variability monitoring are compared with the ARL_1, SRL, and MRL values. The comparison shows that the MA-$EWMA_S$ chart is superior to all charts when the shift parameter is significant, and the number of subgroups is small. The MA-$EWMA_S$ chart also performs best in process variability for moderate and large subgroup sizes (k) of all shift parameters. In all the above research, the MA-$EWMA_S$ chart primarily excels in rapidly identifying signals of process variability shift, even slight ones that could be crucial. Organizations value this capability as early detection of such variability can enhance overall process quality. The aspiration is for the MA-$EWMA_S$ chart to be seen as a compelling substitute for the traditional S chart, particularly among quality control operators dealing with minor to moderate shifts in process variability.

Acknowledgment:
The authors would like to express the Department of Applied Statistics for supporting materials and superhigh-performance computers. Besides, we would like to thank King Mongkut's University of Technology North Bangkok and Thailand Science Research and Innovation, Ministry of Higher Education, Science, and Research for supporting the research fund with contract no. KMUTNB-FF-67-B-18.

References:

[1] W. A. Shewhart, *“Statistical control,” in Economic Control of Quality Manufactured Product,”* NY: D. Van Nostrand Company, (1931).
[2] E. S. Page, "Continuous Inspection Scheme," *Biometrika,* 41(1-2), 100–115, (1954).
[3] S. W. Roberts, “Control Chart Tests Based on Geometric Moving Average,” *Techmometrics*, 1, 239–250, (1959).
[4] A. A. Olatunde, A. A. Abayomi and B. A. Femi, “Monitoring Process Variability Using Double Moving Average Control Chart,” *Communications in Statistics – Theory and Methods*, 46, 6980–6993, (2017), doi: 10.1080/03610926.2016.1139132.
[5] W. Peerajit, “Computation of ARL on a CUSUM Control Chart for a Long-Memory

Seasonal Autoregressive Fractionally Integrated Process with Exogenous Variables," *Journal of Physics: Conference Series*, 1, 012003, (2014), doi: 10.1088/1742-6596/2014/1/012003.

[6] N. Khan, M. Aslam and C-H. Jun, "An EWMA Control Chart for Exponential Distributed Quality Based on Moving Average Statistics," *Quality and Reliability Engineering International*, 32, 1179–1190, (2015), doi.org/10.1002/qre.1825.

[7] S. Sukparungsee, Y. Areepong and R. Taboran, "Exponentially weighted moving average-Moving average charts for monitoring the process mean," *PLoS ONE*, 15, 1–24, (2020), doi: 10.1371/journal.pone.0228208.

[8] A. A. Olatunde and J. O. Olaomi, "A moving average S control chart for monitoring process variability," *Quality Engineering*, 28, 212–219, (2016), doi: 10.1080/08982112.2015.1104542.

[9] D. C. Montgomery, *"Introduction to Statistical Quality Control Case Study*," NY: John Wiley and Sons, (2009).

[10] M. B. C. Khoo. "Moving average control chart for monitoring the fraction non-conforming," *Quality and Reliability Engineering International*, 20, 617–635, (2004), doi: 10.1002/qre.576.

Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

- P. S.: writing an original draft, software, data analysis, data curation, proof, and validate.
- Y. A.: investigation, methodology, validate, reviewing
- S. S.: conceptualization, investigation, funding acquisition, project administration, reviewing and editing.

Sources of Funding for Research Presented in a Scientific Article or Scientific Article Itself

Besides, we would like to thank King Mongkut's University of Technology North Bangkok and Thailand Science Research and Innovation, Ministry of Higher Education, Science, and Research for supporting the research fund with contract no. KMUTNB-FF-67-B-18.

Conflict of Interest

The authors have no conflict of interest to declare.