Design of Fractional Calculus Free Controllers with Fractional Behaviors

JOCELYN SABATIER
IMS Laboratory,
Bordeaux University,
351, Cours de la Liberation 33400 Talence,
FRANCE

Abstract: - Faced with the complexity and drawbacks of fractional calculus highlighted in the literature, this paper proposes simple solutions to avoid its use in the field of feedback control and especially to define fractional PID- and CRONE-like controllers. It shows that it is possible to generate fractional behaviors, which are known since the work of Bode to be useful in the field of control, without invoking fractional calculus and fractional models. Fractional calculus based models and fractional behaviors are indeed two different concepts: one denotes a particular class of models and the other a class of dynamical behaviors that can be generated and modelled by a wide variety of mathematical tools other than fractional calculus. Solutions to tune the fractional PID- and Crone-like controllers defined in this paper are proposed.

Key-Words: - Fractional dynamical behaviors, Fractional PID Controllers, CRONE controllers, Fractional Calculus, Fractional differentiation, Robust control.

Received: April 8, 2023. Revised: December 12, 2023. Accepted: December 26, 2023. Published: December 31, 2023.

1 Introduction

An intensively studied application area of fractional calculus is automatic control, achieved primarily through fractional PID controllers or Crone Control. Fractional PID controllers are extensions of classical integer ones, [1], in which integral and derivative parts are replaced by fractional integral and derivative operators. In the Laplace domain these operators are respectively defined by $1/s^{\lambda}$ and s^{μ}, and a fractional PID controller is defined by the transfer function:

$$C_F(s) = K_p + \frac{K_I}{s^{\lambda}} + K_I s^{\mu} \tag{1}$$

with $\lambda \geq 0$ and $\mu \geq 0$.

Different design and tuning methods have been proposed for this class of controller, [2], [3], [4], [5], [6], [7], [8], [9] and some industrial process control applications now exist, [10].

Crone controllers are mainly dedicated to solving robustness issues in control loops, [11]. Assuming a unity feedback loop in which the controller (with transfer function $C_C(s)$) and the plant (with transfer function $H(s)$) are connected in series in the direct chain, the Crone controller is deduced (using a fitting algorithm) from the frequency response of the ratio

$$C_C(s) = \frac{\beta(s)}{H_0(s)}, \tag{2}$$

where $H_0(s)$ is the nominal plant. The transfer function $\beta(s)$ is defined by:

$$\beta(s) = K\left(\frac{\frac{s}{\omega_l}+1}{\frac{s}{\omega_l}}\right)^{n_l} \underbrace{\left(\Upsilon(s)\right)^a\left(cos\left(b\,log\left(C_0\Upsilon(s)\right)\right)\right)^{-sign(b)}}_{central\ term} \frac{1}{\left(1+\frac{s}{\omega_h}\right)^{n_h}} \tag{3}$$

where

$$\Upsilon(s) = \frac{1+\frac{s}{\omega_h}}{1+\frac{s}{\omega_l}} \qquad C_0 = \left(\frac{1+\left(\frac{\omega_{cg}}{\omega_l}\right)^2}{1+\left(\frac{\omega_{cg}}{\omega_h}\right)^2}\right)^{1/2} \tag{4}$$

ω_{cg} being the crossover gain frequency.

In the strict sense, we cannot say that this transfer function uses fractional operators for its definition (it is for example not possible to deduce a fractional differential equation from it), but it admits a fractional behaviour in the frequency range $[\omega_l, \omega_h]$, analogous to that of the complex fractional integrator

$$\frac{K_f}{s^{a+ib}}, \tag{5}$$

and exactly of the real part in relation to i of this complex fractional integrator defined by

$$\left(\cosh\left(b\frac{\pi}{2}\right)\right)^{sign(b)}\left(\frac{K_f}{s}\right)^{a}\left(\cos\left(b\log\left(\frac{s}{K_f}\right)\right)\right)^{-sign(b)} \quad (6)$$

These two classes of controller exhibit drawbacks and in particular manipulate a complex mathematical tool which undoubtedly contributes to limiting their use: fractional calculus. Keeping in mind that fractional calculus based models and fractional behaviors are two different concepts:

- the first one denotes a particular class of models,
- the second is a class of dynamical behaviors that can be generated and modelled by a wide variety of mathematical tools other than fractional calculus, [12],

the goal of this paper is to propose a new formulation of these controllers so that they maintain fractional behaviors without having to manipulate fractional calculus.

The paper is organized as follows. The limitations and drawbacks of fractional PID and Crone controllers are first described. Then a gain function that exhibits a fractional behavior but without involving fractional calculus is introduced. It is shown that such a function can be used to define a fractional PID-like or Crone-like controller. An algorithm is proposed to deduce the minimum phase corresponding to this gain function and its efficiency is demonstrated. Finally, solutions to tune these fractional PID-and Crone-like controllers are proposed.

2 Limitations and Drawbacks of Fractional PID and Crone Controllers

2.1 Fractional PID controllers

If the fractional PID controller (1) is used to solve a control problem, several drawbacks arise.

1 – The definition of a fractional PID controller does not take into account the fact that the fractional differentiation or integration operators are doubly infinite operators, [12] and that it is necessary to approximate them, or more precisely to truncate their frequency behavior at low and high frequencies. Many methodologies have been developed for the implementation of fractional operators but all lead to the above-mentioned truncation. Often, the high and low frequency asymptotic behaviors which result from these approximations are poorly controlled. This is for instance the case using the Grünwald-Letnikov definition to approximate a fractional integrator $1/s^{\lambda}$ with $0<\lambda<1$. If $u(t)$ and $y(t)$ are respectively the input and the output of this fractional integrator, the approximation of the sampled output is defined by:

$$y(kT_s) = \\ -\sum_{j=1}^{N}(-1)^j\binom{\lambda}{j}y\left((k-j)T_s\right)+{T_s}^{\lambda}u(kT_s). \quad (7)$$

A discrete transfer function approximation of a fractional integrator is thus

$$\frac{y(kT_s)}{u(kT_s)}=\frac{{T_s}^{\lambda}}{1+\sum_{j=1}^{N}(-1)^j\binom{\lambda}{j}z^{-j}}. \quad (8)$$

As z tends towards 1 (to evaluate the steady state behavior), this transfer function no longer tends to infinity. The effect of the integration is lost in the approximation/discretisation process.

2 - Although it is interesting to use a fractional differentiation transfer function for a lead effect, a fractional integral transfer function is of low interest. A fractional integrator of order $k+\lambda$, $k\in\mathbb{N}$, $0<\lambda<1$, is no more efficient for steady-state error cancellation than an integer integrator of order k (as can be proved using the final value theorem).

3 - The use of a non-band-limited differentiator leads to an infinite control effort value and to a great sensitivity to measurement noise. We consider, therefore, that the fractional differentiation part of the controller needs to be band-limited before being tuned, thus leading to an additional tuning parameter. Only then will the controller be proper.

4 – The above-mentioned approximations lead to discrete time or continuous-time approximations that require computer resources much greater than those necessary for a classic PID.

5 – Several papers in the field claim a greater efficiency of fractional PID controllers in comparison to classic PID, but they forget to mention that a fractional PID controller has 5 tuning parameters while a classic PID has only 3. Wouldn't a classical controller with 5 parameters defined by (ω_{cg} being the gain crossover frequency)

$$C(s)=K\left(\frac{T_i s+1}{T_i s}\right)\left(\frac{\frac{s}{a\omega_{cg}}+1}{\frac{as}{\omega_{cg}}+1}\right)\left(\frac{\frac{s}{a\omega_{cg}}+1}{\frac{as}{\omega_{cg}}+1}\right)\frac{1}{T_f s+1} \quad (9)$$

be just as effective?

As mentioned in [13], to take into account the previous two drawbacks, it would be better to define a fractional PID controller, with the same number of tuning parameters, by the transfer function:

$$C(s) = K\left(1+\frac{\omega_i}{s}\right)\left(\frac{\frac{s}{a\omega_{cg}}+1}{\frac{as}{\omega_{cg}}+1}\right)^{A_1}\frac{1}{\frac{s}{\omega_f}+1}, \qquad (10)$$

where ω_{cg} is the desired open loop crossover gain frequency (which is part of the specifications as it controls the loop rapidity). This form allows a fractional behaviour where it is necessary, i.e. around the corner frequency ω_{cg}. The tuning parameters are then the gain K, the corner frequencies ω_i and ω_f, the fractional order A_1 and the parameter a.

2.2 CRONE Controllers

Despite its 30 years of existence, CRONE control has had difficulty establishing itself in the industry. Applications of the resulting controllers very often remain at the level of research and development departments. This control methodology uses a mathematical tool, complex fractional differentiation, which is seldom taught in higher education, which may explain this situation. It is precisely the use of fractional differentiation that allows the parameterization of the open-loop transfer function with a small number of parameters (4 independent parameters), which is necessary because the search for the optimal value of these parameters in a control robustness problem is carried out using a non-linear optimization algorithm. Even if in practice this control methodology gives good results, it remains suboptimal concerning a given robustness problem as the structure of the open-loop transfer function for the nominal behavior of the process to be controlled is imposed (not the case with H_∞ control for instance). We can ask the following questions for the same problem: would a different structure of this open loop among the existing infinity not have led to a better result? Or again: would not the choice of another nominal behaviour for the calculation of the controller have led to better results? Aware of this sub-optimality, the authors of [14], [15], [16] proposed to introduce in the definitions of the open loop transfer function a curvilinear template, that is to say roughly, to add central terms such as the one which appears in relation (3) in this same relation. However the interest of using fractional differentiation is then lost, since as shown in [14], the search for 10 independent parameters is then required for the resolution of a robustness problem.

3 Introduction of a New Controller and Open Loop with Fractional Behaviors but without Involving Fractional Calculus

This section gives the definition of a new controller and/or new open loop in a unity feedback loop context
- that exhibit fractional behaviors
- without requiring fractional calculus.

The definition is first given for the controller and the diversity of frequency response shapes that can be obtained is illustrated. Some of these shapes are similar to those obtained with complex fractional calculus in CRONE control. The idea used to define the controller is then applied to the definition of a new open loop which allows shapes similar to those provided by the third generation CRONE control with and without an extra template (central term in relation (3)).

3.1 Definition of the New Controller

To propose a new controller with fractional behavior and without involving fractional calculus, let us start from a classical filtered PID controller of the form:

$$C(s) = K\underbrace{\left(\frac{\frac{as}{\omega_{cg}}+1}{\frac{as}{\omega_{cg}}s}\right)}_{Integral\ part}\underbrace{\left(\frac{\frac{as}{\omega_{cg}}+1}{\frac{s}{a\omega_{cg}}+1}\right)}_{Lead-lag\ part}\underbrace{\frac{1}{\left(\frac{s}{a\omega_{cg}}+1\right)^{n_h}}}_{Lowpass\ filter}. \qquad (11)$$

This form is classically taught to students as it permits an easy computation of its parameters using two specifications that model the closed loop response:

- the gain crossover frequency of the open loop ω_{cg}, that impacts the closed loop rapidity,
- the phase margin MP that impacts the closed loop damping.

To satisfy these two specifications it is necessary to impose the following equalities

- $a = tan\left(\frac{MP+n_h 90-\varphi\left(H(j\omega_{cg})\right)}{3+n_h}\right)$
- $K = \frac{1}{a^{n_h}(1+a^2)^{\frac{1-n_h}{2}}\left|H(j\omega_{cg})\right|}$

where $\left|H(j\omega_{cg})\right|$ and $\varphi\left(H(j\omega_{cg})\right)$ denote respectively the magnitude and the phase of the plant to be controlled (defined by the transfer function $H(s)$) at the frequency ω_{cg}.

The magnitude in decibels of the controller is defined by:

$$|C(j\omega)| = 20log\left(K\frac{\sqrt{\left(\frac{a\omega}{\omega_{cg}}\right)^2+1}}{\frac{a\omega}{\omega_{cg}}}\right) + \underbrace{20log\left(\frac{\sqrt{\left(\frac{a\omega}{\omega_{cg}}\right)^2+1}}{\sqrt{\left(\frac{\omega}{a\omega_{cg}}\right)^2+1}}\right)}_{Lead-lag\ part} - 20n_h log\left(\sqrt{\left(\frac{\omega}{a\omega_{cg}}\right)^2+1}\right) \quad (12)$$

To generalise such a controller, the idea is to make a kind of series expansion of the lead-lag part gain. It is proposed to replace relation (12) by an expression of the form:

$$|C^*(j\omega)| = 20log\left(K\frac{\sqrt{\left(\frac{a\omega}{\omega_{cg}}\right)^2+1}}{\frac{a\omega}{\omega_{cg}}}\right) + \chi(\omega) - 20n_h log\left(\sqrt{\left(\frac{\omega}{a\omega_{cg}}\right)^2+1}\right) \quad (13)$$

with

$$\chi(\omega) = \sum_{k=1}^{N} \frac{20}{ln(10)} A_k \left[ln\left(K_0\frac{\sqrt{\left(\frac{\omega}{a\omega_{cg}}\right)^2+1}}{\sqrt{\left(\frac{a\omega}{\omega_{cg}}\right)^2+1}}\right)\right]^k \quad (14)$$

and $K_0 = \frac{\sqrt{a^2+1}}{\sqrt{\left(\frac{1}{a}\right)^2+1}}$.

3.2 Phase Computation

For implementation purposes, the phase associated to relation (13) is required. To compute the phase, Bode relationships will be used as described in [17]. Under the assumption that $C^*(s)$ is analytic and has no zeros for $\mathcal{R}e(s) \geq 0$ (minimum-phase systems), then:

$$ln\left(C^*(j\omega)\right) = \alpha(\omega) + j\gamma(\omega). \quad (15)$$

In relation (15), phase $\gamma(\omega)$ is uniquely determined from the gain (in nepers) $\alpha(\omega)$ from the following relation, [17]:

$$\gamma(\omega) = \frac{1}{\pi}\int_{-\infty}^{+\infty}\frac{\alpha(y)}{y-\omega}dy. \quad (16)$$

For an easier numerical computation using Gaussian quadrature, the previous integral is split into two parts:

$$\gamma(\omega) = \frac{1}{\pi}\int_{-\infty}^{0}\frac{\alpha(y)}{y-\omega}dy + \frac{1}{\pi}\int_{0}^{+\infty}\frac{\alpha(y)}{y-\omega}dy = \gamma_1(\omega) + \gamma_2(\omega). \quad (17)$$

Using the change of variable $x = ln(y)/ln(10)$ and thus $dy = ln(10)e^{xln(10)}dx$, integral $\gamma_1(\omega)$ becomes:

$$\gamma_1(\omega) = \frac{1}{\pi}\int_{-\infty}^{+\infty}\frac{\alpha\left(e^{xln(10)}\right)}{e^{xln(10)}-\omega} ln(10)e^{xln(10)}dx. \quad (18)$$

Using the changes of variables $z = -y$ and then $x = ln(z)/ln(10)$, integral $\gamma_2(\omega)$ becomes:

$$\gamma_2(\omega) = \frac{1}{\pi}\int_{-\infty}^{+\infty}\frac{\alpha\left(e^{xln(10)}\right)}{-e^{xln(10)}-\omega} ln(10)e^{xln(10)}dx. \quad (19)$$

Numerically, considering $\frac{ln(\omega)}{ln(10)} - A < x < \frac{ln(\omega)}{ln(10)} + A$ and using the trapezoidal rule, the phase $\gamma(\omega)$ can be approximated by the following sums:

$$\gamma_1(\omega) \approx \frac{ln(10)\Delta x}{\pi}\left(\sum_{k=-N}^{N}\frac{\rho(\omega,k)\alpha(\rho(\omega,k))}{e^{\rho(\omega,k)}-\omega} + \frac{1}{2}\frac{\rho(\omega,-N)\alpha(\rho(\omega,-N))}{\left(e^{\rho(\omega,-N)}-\omega\right)} + \frac{1}{2}\frac{\rho(\omega,N)\alpha(\rho(\omega,N))}{\left(e^{\rho(\omega,N)}-\omega\right)}\right), \quad (20)$$

$$\gamma_2(\omega) \approx \frac{ln(10)\Delta x}{\pi}\left(\sum_{k=-N}^{N}\frac{\rho(\omega,k)\alpha(\rho(\omega,k))}{-e^{\rho(\omega,k)}-\omega} + \frac{1}{2}\frac{\rho(\omega,-N)\alpha(\rho(\omega,-N))}{\left(-e^{\rho(\omega,-N)}-\omega\right)} + \frac{1}{2}\frac{\rho(\omega,N)\alpha(\rho(\omega,N))}{\left(-e^{\rho(\omega,N)}-\omega\right)}\right), \quad (21)$$

with $\rho(\omega,k) = e^{\left(\frac{ln(\omega)}{ln(10)}+k\Delta x\right)ln(10)}$, $\Delta x = \frac{A}{N}$.

As an example, this method is applied to a low pass filter whose gain is defined by (in nepers):

$$G_{LF}(\omega) = -ln\left(\sqrt{\left(\frac{\omega}{\omega_0}\right)^2+1}\right). \quad (22)$$

To compute the phase $P_{LF}(\omega)$, the following parameters were chosen: $\omega_0 = 10\ rd/s$, $A = 6$, $N = 2500$. The estimated and the exact phases are compared in Figure 1, which shows that the estimation is accurate.

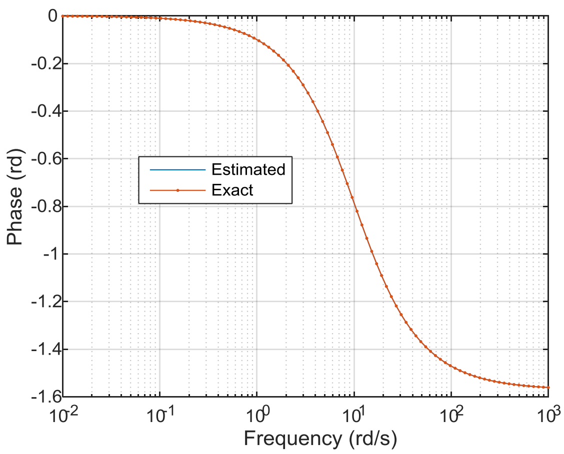

Fig. 1: Comparison of the exact and the estimated phase for the low pass filter

As another example, the method is applied to the fractional transfer function:

$$H_F(s) = K\frac{1}{s}\frac{\left(\frac{s}{\omega_l}+1\right)^{\nu}}{\left(\frac{s}{\omega_h}+1\right)^{\nu}}. \quad (23)$$

The gain (in nepers) of this transfer function is defined by

$$|H_F(j\omega)| = 20ln(K) - ln(\omega) + \nu ln\left(\frac{\sqrt{\left(\frac{\omega}{\omega_l}\right)^2+1}}{\sqrt{\left(\frac{\omega}{\omega_h}\right)^2+1}}\right). \quad (24)$$

The phase (in degrees) defined analytically by

$$\varphi\left(H_f(j\omega)\right) = -90 + \nu\left(\arctan\left(\frac{\omega}{\omega_l}\right) - \arctan\left(\frac{\omega}{\omega_h}\right)\right) \quad (25)$$

is compared in Figure 2 with the phase estimated by the algorithm described at the beginning of this section with $K = 10$, $\omega_l = 0.002\, rd/s$, $\omega_h = 500\, rd/s$ and $\nu = 1.5$. This comparison again reveals a very good accuracy of the estimated phase.

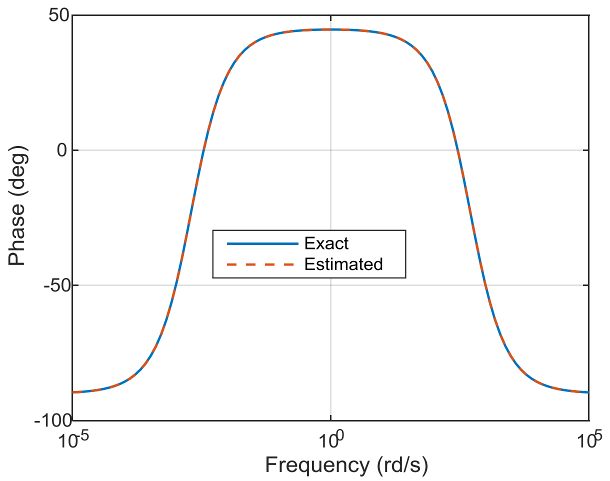

Fig. 2: Comparison of the exact and the estimated phase for the $H_F(s)$ transfer function

3.3 Frequency Behaviors Generated

To illustrate the impact of the parameters A_k and the terms of relations (13) and (14), the Nichols diagrams of the functions whose gain (in decibels) are

$$F_2(\omega) = 20log\left(\frac{\omega}{\omega_{cg}}\right) + \frac{20}{ln(10)}A_2\left[ln\left(K_0\frac{\sqrt{\left(\frac{\omega}{a\omega_{cg}}\right)^2+1}}{\sqrt{\left(\frac{a\omega}{\omega_{cg}}\right)^2+1}}\right)\right]^2 \quad (26)$$

with $K_0 = \frac{\sqrt{a^2+1}}{\sqrt{\left(\frac{1}{a}\right)^2+1}}$ and

$$F_3(\omega) = 20log\left(\frac{\omega}{\omega_{cg}}\right) + \frac{20}{ln(10)}A_3\left[ln\left(K_0\frac{\sqrt{\left(\frac{\omega}{a\omega_{cg}}\right)^2+1}}{\sqrt{\left(\frac{a\omega}{\omega_{cg}}\right)^2+1}}\right)\right]^3. \quad (27)$$

with $K_0 = \frac{\sqrt{a^2+1}}{\sqrt{\left(\frac{1}{a}\right)^2+1}}$, are represented respectively by Figure 3 and Figure 4. The phase is computed using the algorithm described in section 3.2.

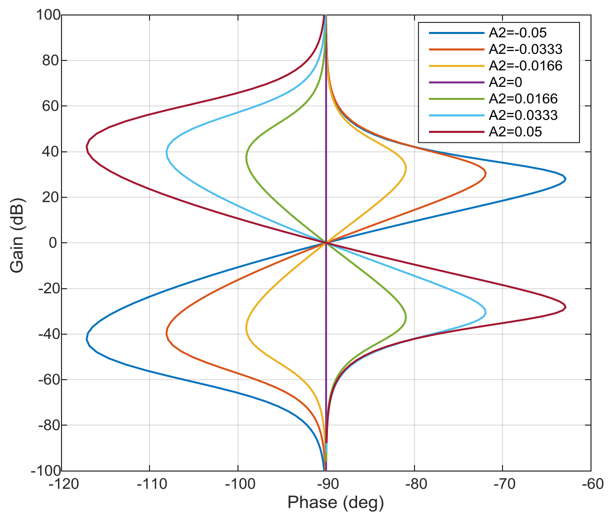

Fig. 3: Nichols diagram of the function whose gain is $F_2(\omega)$ showing the impact of parameter A_2

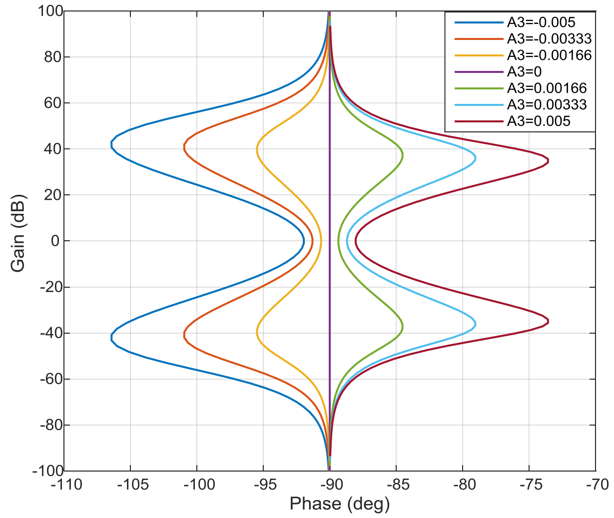

Fig. 4: Nichols diagram of the function whose gain is $F_3(\omega)$ showing the impact of parameter A_3

Figure 3 and Figure 4 illustrate the diversity of shapes that can be obtained with only two parameters to model the frequency response of the controller around the crossover gain frequency, without involving fractional calculus. They also highlight the low number of parameters required to obtain this diversity of shapes:

- with only two or three free additional parameters (A_1, A_2 and/or A_3) in relation to the controller (11) which has two free parameters (K, a),
- with only one or two additional free parameters (A_2, A_3) in relation to the fractional controller (10) which has three free parameters (K, a, A_1),
- with the same number of parameters as a fractional PID controller (relation (1)).

This diversity offers a large number of degrees of freedom to solve several regulation problems simultaneously. Beyond phase margin and bandwidth specifications, many other constraints can be taken into account as shown in section 3.6.

3.4 New Open Loop Definition

Figure 5 shows the Nichols diagram of the transfer function

$$H(s) = \frac{\omega_{cg}}{s}\left(cos\left(b\, ln\left(C_0\frac{1+\frac{s}{\omega_h}}{1+\frac{s}{\omega_b}}\right)\right)\right)^{-sign(b)} \quad (28)$$

with $C_0 = \left(\frac{1+\left(\frac{\omega_{cg}}{\omega_b}\right)^2}{1+\left(\frac{\omega_{cg}}{\omega_h}\right)^2}\right)^{1/2}$,

which contains the part that defines the "generalised template" in the CRONE open loop transfer function given by relation (3). This figure shows that the shapes obtained are similar to those in Figure 4. Relation (13) dedicated to the definition of a new controller can thus be used to define a new open loop without fractional calculus in a control design strategy similar to CRONE control. The new open loop gain definition (in nepers) is:

$$|\beta^*(j\omega)| = 20ln(K) + 20n_l ln\left(\frac{\sqrt{\left(\frac{\omega}{\omega_l}\right)^2+1}}{\frac{\omega}{\omega_l}}\right) + \psi(\omega) - 20n_h ln\left(\sqrt{\left(\frac{\omega}{\omega_h}\right)^2+1}\right)$$

with

$$\psi(\omega) = \sum_{k=1}^{N}\frac{20}{ln(10)}A_k\left[ln\left(K_0\frac{\sqrt{\left(\frac{\omega}{\omega_h}\right)^2+1}}{\sqrt{\left(\frac{\omega}{\omega_l}\right)^2+1}}\right)\right]^k$$

$$K = \frac{\left(\frac{\omega_{cg}}{\omega_b}\right)^{n_b}\left(\left(\frac{\omega_{cg}}{\omega_h}\right)^2+1\right)^{\frac{n_h}{2}}}{\left(\left(\frac{\omega_{cg}}{\omega_b}\right)^2+1\right)^{\frac{n_b}{2}}} \quad K_0 = \frac{\sqrt{\left(\frac{\omega_{cg}}{\omega_l}\right)^2+1}}{\sqrt{\left(\frac{\omega_{cg}}{\omega_h}\right)^2+1}}.$$

The corresponding phase can be computed using the algorithm defined in section 3.2. In relation (29), the parameters n_l and n_h can be defined as in CRONE Control, [11], [15]:

- if n_{pl} denotes the order of the asymptotic bbehaviorof the plant at low frequency ($\omega < \omega_l$), n_l is defined by $n_l \geq 1$ if $n_{pl} = 0$ and $n_l \geq n_{pl}$ if $n_{pl} \geq 1$, as $n_l = 1$ cancels the position error, and $n_l = 2$ cancels the hauling error;
- if n_{ph} denotes the order of the asymptotic bbehaviorof the plant at high frequency $\omega > \omega_h$, order n_h is given by $n_l \geq n_{ph}$.

The definition of this new open loop means that the following free parameters: ω_l, ω_h, A_k with $k \in [1..N]$ have to be defined. According to the comparison of Figure 3 and Figure 5, open loop shapes similar to those obtained with the third generation CRONE control can be obtained with $N = 2$ and thus four independent parameters. More complex shapes, similar to those obtained with the third generation CRONE control with a curvilinear template, can be obtained with $N > 2$.

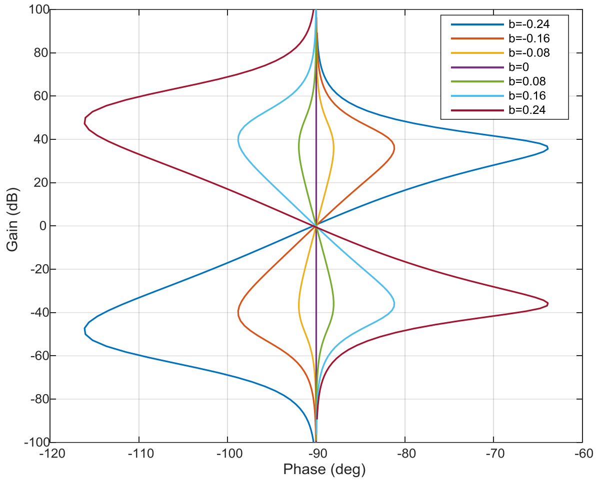

Fig. 5: Nichols diagram of the function whose gain is $F_3(\omega)$ to show the impact of parameter A_3

3.5 Implementable Controller

The controller $C^*(j\omega)$ and the open loop behaviors $\beta^*(j\omega)$ proposed in the previous sections are defined only by their frequency responses: the gains are respectively defined by relations (13) and (29) and the corresponding phases are computed using the algorithm described in section 3.2.

For the fractional PID-like controller (13), the implementable controller $C^*(s)$ can be obtained by using a frequency identification method such as the ones described in [18]. These frequency identification methods permits to fit the frequency response $C^*(j\omega)$ in the form of a rational transfer function to get $C^*(s)$.

From the Crone-like open loop of relation (29), the frequency response of the controller is defined from the relation (as in the CRONE control strategy):

$$C^*(j\omega) = \frac{\beta^*(j\omega)}{H_0(j\omega)}, \qquad (31)$$

where $H_0(j\omega)$ denotes the nominal frequency response of the plant. Then, again, the implementable controller $C^*(s)$ in the form of a rational transfer function can also be obtained by using a frequency identification method such as the ones described in [18].

For a minimum phase and stable plant $H_0(s)$, the resulting controller $C^*(s)$ is also stable and minimum phase as it is possible to constrain le pole and zeros location of $C^*(s)$ in the complex plane with the identification method used in [19]. For non minimum phase or unstable plant $H_0(s)$, terms can be added in relation (29) to unsure the stability of the loop and of the controller $C^*(s)$, as it is done in CRONE control, [18].

3.6 Parameters Tuning

To tune the parameters of the fractional-like controller $C^*(j\omega)$ or the Crone-like open loop behaviour $\beta^*(j\omega)$, the following specifications can be used, where $H(j\omega)$ denotes the frequency response of the plant to be controlled.

- ***Steady-state errors in relation to reference unit step or ramp signal***. To ensure steady state error cancellation relation to reference unit step or ramp signal, conditions on parameter n_l must be chosen as described just after relation (30).

- ***Gain crossover frequency***. To ensure a specified gain crossover frequency ω_{cg} the following equality must be met:

$$\left|C^*(j\omega_{cg})H(j\omega_{cg})\right|_{dB} = 0dB. \qquad (32)$$

- ***Phase margin***. To ensure a specified phase margin PM, the following equality must be met:

$$Arg\left(C^*(j\omega_{cg})H(j\omega_{cg})\right) = -180° + PM. \qquad (33)$$

- ***Phase crossover frequency***. To ensure a specified phase crossover frequency ω_{cp} the following equality must be met:

$$\varphi\left(C^*(j\omega_{cp})H(j\omega_{cp})\right) = -180°. \qquad (34)$$

- ***Gain margin***. To ensure a specified gain margin GM, the following equality must be met:

$$\left|\frac{1}{C^*(j\omega_{cg})H(j\omega_{cg})}\right| = GM. \qquad (35)$$

- ***Rejection of high frequency noise***. Measurement noise rejection can be adjusted using the following condition:

$$\left|T(j\omega) = \frac{C^*(j\omega)H(j\omega)}{1+C^*(j\omega)H(j\omega)}\right|_{dB} \leq A_T \qquad \forall\omega \geq \omega_T. \quad (36)$$

- ***Rejection of output disturbance***. Output disturbance rejection can be adjusted using the following condition:

$$\left|S(j\omega) = \frac{1}{1+C^*(j\omega)H(j\omega)}\right|_{dB} \leq A_S \qquad \forall\omega \leq \omega_S. \quad (37)$$

- ***Robustness to the plant phase variation***. For the control loop to ensure robustness to the plant phase variation, the following constraint can be imposed:

$$\left(\frac{d\left(\varphi\left(C^*(j\omega)H(j\omega)\right)\right)}{d\omega}\right)_{\omega=\omega_{cg}} = 0 \qquad (38)$$

in which $\varphi\left(C^*(j\omega)H(j\omega)\right)$ denotes the phase of the open loop function. Through this condition, the phase is forced to be almost constant around the frequency ω_{cg} and thus the closed loop system is more robust to gain changes.

- ***Robustness to any plant variation***. To ensure robustness to plant variation, a solution consists in using the stochastic robustness concept introduced in [19]. The stochastic robustness of a closed loop system can be evaluated using Monte Carlo simulations to draw a statistical portrait of parameter variations and their effect on the closed loop. In this work, the uncertain parameters are assumed to have a bounded, continuous, uncorrelated and uniform probability distribution. To ensure closed loop stability degree robustness while guaranteeing a given loop damping level, it is proposed to minimize the probability P_S of the first overshoot value $Q_{S,}$ of the sensitivity function to be out of a given interval $\left[Q_{S_{min}}, Q_{S_{max}}\right]$ due to plant variation. The cost function to be minimized can thus be defined by

$$\min_M \left(1 - P_S\left(Q_S \in \left[Q_{S_{min}}, Q_{S_{max}}\right]\right)\right). \quad (39)$$

M denotes the number of times the sensitivity function is evaluated in the Monte Carlo simulation to sweep uncertainty intervals. Minimization of such a criterion reduces the variations of the first overshoot value of the sensitivity function and thus minimizes the impact of the plant variation on the loop stability degree. Many other criteria can be defined on the same principle.

Using the previous specifications, algorithms that enable the fractional behaviour controller $C^*(j\omega)$ or the Crone-like open loop behaviour $\beta^*(j\omega)$ to be tuned can be defined as follows.

Algorithm for $C^*(j\omega)$ parameters tuning

1 – Impose parameters n_l and n_h as described just after relation (30).

2 – Impose the gain crossover frequency ω_{cg}

3 – Impose the phase margin PM

4 – Minimise the criterion $\left|Arg\left(C^*(j\omega_{cg})H(j\omega_{cg})\right) + 180° - PM\right|$ under the constraints (36) and (37), and thus, at each optimisation algorithm step with a new set of parameters $\{K, a, A_k \;\; k \in [1..N]\}$:

4.1 - Compute K to ensure ω_{cg}

4.2 - Compute $Arg\left(C^*(j\omega)\right)$ using the algorithm described in section 3.2

4.3 – Compute $|S(j\omega)| = \left|\frac{1}{1+C^*(j\omega)H(j\omega)}\right|_{dB}$ and $|T(j\omega)| = \left|\frac{C^*(j\omega)H(j\omega)}{1+C^*(j\omega)H(j\omega)}\right|_{dB}$

4.4 – Compute the cost function and check the constraints (36) and (37).

5 – For the obtained optimal values of parameters $\{K, a,, A_k \;\; k \in [1..N]\}$, fit the rational controller $C^*(s)$ according to the comments in section 3.5 using the frequency response of $C^*(j\omega)$.

Algorithm for $\beta^*(j\omega)$ parameters optimisation and $C^*(j\omega)$ computation

1 – Impose parameters n_l and n_h as described just after relation (30).

2 – Impose the gain crossover frequency ω_{cg}.

3 – Minimise the cost function (40) $\min_M \left(1 - P_S\left(Q_S \in \left[Q_{S_{min}}, Q_{S_{max}}\right]\right)\right)$ under the constraints (37) and (38), and thus, at each optimisation algorithm step with a new set of parameters $\{\omega_l, \omega_h, A_k \;\; k \in [1..N]\}$:

3.1 - Compute K using relation (30)

3.2 - Compute $|\beta^*(j\omega)|$

3.3 - Compute $Arg\left(\beta^*(j\omega)\right)$ using the algorithm described in section 3.2

3.4 – Compute $|C^*(j\omega)|$ and $Arg\left(C^*(j\omega)\right)$ using relation (31)

3.5 – Compute $|S(j\omega)| = \left|\frac{1}{1+C^*(j\omega)H(j\omega)}\right|_{dB}$ and $|T(j\omega)| = \left|\frac{C^*(j\omega)H(j\omega)}{1+C^*(j\omega)H(j\omega)}\right|_{dB}$

3.6 – Compute the cost function (39) and check the constraints (36) and (37).

4 – For the obtained optimal values of parameters $\{\omega_l, \omega_h, A_k \;\; k \in [1..N]\}$:

4.1 - Compute K using relation (30)

4.2 - Compute $|\beta^*(j\omega)|$

4.3 - Compute $Arg\left(\beta^*(j\omega)\right)$ using the algorithm described in section 3.2

4.4 – Compute $|C^*(j\omega)|$ and $Arg\left(C^*(j\omega)\right)$ using relation (31)

4.5 - Fit the rational controller $C^*(s)$ according to the comments in section 3.5 using the frequency response of $C^*(j\omega)$.

4 Conclusion

This paper proposes alternative solutions to fractional PID controllers and Crone controllers. Without resorting to fractional differentiation or integration notions, it introduces a fractional PID-like controller and a Crone-like open loop function (that is then used to define a Crone-like controller) with the same restricted number of parameters and similar generated frequency responses. First, the gains of the fractional PID-like controller and a Crone-like open loop function are defined in the form of a kind of series expansion of the lead-lag part of the classical fractional PID controller and Crone open loop function. The corresponding phase is computed with an algorithm based on Bode phase relationships and specially developed in this work.

Some solutions are also presented for parameter tuning of the proposed fractional PID-like controller and a Crone-like open loop function. This work makes it possible to overcome the limits and drawbacks inherent to fractional PID and Crone controllers, in particular the use of fractional calculus, which is a limiting factor in the diffusion of these control strategies.
This work reinforces the idea already mentioned by the author that fractional calculus based models and fractional behaviors are two different concepts:

- the first one denotes a particular class of models
- the second is a class of dynamical behaviors that can be generated and modeled by a wide variety of mathematical tools other than fractional calculus, [12].

Due to space constraints, applications of these new control strategies are not presented here and will be described in coming papers. However, beyond the topic of controller synthesis, considering fractional behaviors without being limited to fractional models opens up countless avenues of research in the field of model analysis and identification, and more generally in the understanding of the physical phenomena that induce fractional behaviors.

References:

[1] Podlubny I., *Fractional-order systems and fractional-order controllers*. Inst Exp Phys Slovak Acad Sci, Kosice, 1994.
[2] Ashu Ahuja, Sanjeev Kumar Aggarwal, "Design of Fractional Order PID Controller for DC Motor Using Evolutionary Optimization Techniques," *WSEAS Transactions on Systems and Control*, vol. 9, pp. 171-182, 2014
[3] Pritesh S., Sudhir A., Review of fractional PID controller, *Mechatronics*, Vol. 38, pp 29-41, 2016.
[4] Tarek A. Boghdady, Ali J. Alamer, M. A. Moustafa Hassan, A. A. Seif, "Improving the Fractional Order PID Controller Performance with an Energy Storage System for Photovoltaics," *WSEAS Transactions on Systems and Control*, vol. 17, pp. 123-132, 2022, https://doi.org/10.37394/23203.2022.17.14.
[5] Luis Angel, Jairo Viola, "Control Performance Assessment of Fractional-Order PID Controllers Applied to Tracking Trajectory Control of Robotic Systems," *WSEAS Transactions on Systems and Control*, vol. 17, pp. 62-73, 2022, https://doi.org/10.37394/23203.2022.17.8.
[6] Vinagre B. M, Monje C. A., Calderón A. J., Suárez J. I., Fractional PID Controllers for Industry Application. A Brief Introduction. *Journal of Vibration and Control*, Vol. 13, n° 9-10, pp. 1419-1429, 2007.
[7] Sondhi S., Yogesh V. Hote Y. V. Hote, Fractional Order Controller and its Applications: A Review, Conference: Proceedings of the *2nd IASTED Asian Conference on Modelling, Identification, and Control, AsiaMIC*, 2012.
[8] Subeekrishna M P, Aseem K, Comparitive Study of PID and Fractional Order PID Controllers for Industrial Applications, *International Journal of Engineering Research and Technology*, Vol. 7, n°1, 2019.
[9] Chen Y., Petras I., Xue D., Fractional order control - A tutorial, *2009 American Control Conference*, St. Louis, MO, pp. 1397-1411, USA, 2009.
[10] Tepljakov A., Baykant Alagoz B., Yeroglu C., Gonzalez E., Hassan HosseinNia S., Petlenkov E., FOPID Controllers and Their Industrial Applications: A Survey of Recent Results, *IFAC-PapersOnLine*, Vol. 51, n° 4, pp 25-30, 2018
[11] Sabatier J., Oustaloup A., Garcia Iturricha A., Lanusse P., CRONE control: Principles and extension to time-variant plants with asymptotically constant coefficients - *Nonlinear Dynamics*, Kluwer Academic Publishers, Vol. 29, N° 1-4, pp. 363-385, July 2002.
[12] Sabatier J., Farges C., Tartaglione V., Fractional behaviors modelling Analysis and application of several unusual tools, In *Intelligent Systems, Control and Automation: Science and Engineering series*, Vol 101, Springer, Switzerland, 2022.
[13] Lanusse P., Sabatier J., Oustaloup A., Extension of PID to fractional orders controllers: a frequency-domain tutorial presentation, *IFAC Proceedings Volumes*, Vol. 47, N° 3, pp 7436-7442, 2014
[14] Mathieu B., Oustaloup A, Lanusse P., Third generation CRONE control: generalized template and curvilinear template, *Proceedings of Computational engineering in systems applications conference CESA '96, Symposium on modelling, analysis and simulation*, Lille, France, 1996.
[15] Oustaloup A., Cois O., Lanusse P., Melchior P., Moreau X., The Crone approach:

theoretical developments and major applications, *IFAC Proceedings Volumes*, Vol. 39, N° 11, 2006.
[16] Lanusse P., Oustaloup A., Control of Time-Delay Systems Using Robust Fractional-Order Control and Robust Smith Predictor Based Control, Proceedings of the ASME 2005 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference. Volume 6: 5th *International Conference on Multibody Systems, Nonlinear Dynamics, and Control, Parts A, B, and C*. Long Beach, California, USA. September 24–28, 2005. pp. 1475-1483. ASME.
[17] Rusu C., Grama L., Gain-phase relationships evaluation by Gaussian quadrature, *2008 International Conference on Signals and Electronic Systems*, Krakow, Poland, 2008, pp. 23-26,.
[18] Pintelon R., Guillaume P., Rolain Y., Schoukens J., Van Hamme H., Parametric identification of transfer functions in the frequency domain-a survey, in *IEEE Transactions on Automatic Control*, vol. 39, no. 11, pp. 2245-2260, Nov. 1994.
[19] Stengel R. F., Ryan L. E., Stochastic robustness of linear time-invariant control systems, in *IEEE Transactions on Automatic Control*, vol. 36, no. 1, pp. 82-87, Jan. 1991.

Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

Jocelyn Sabatier has contributed to the conceptualization, le methodology development, simulations, writing of the original draft, review and editing.

Sources of Funding for Research Presented in a Scientific Article or Scientific Article Itself

No funding was received for conducting this study.

Conflict of Interest

The author has no conflicts of interest to declare that are relevant to the content of this article.