

ultimate gain can be determined by the following equation.

$$K_u = 10^{(|G(j\omega_\pi)|/20)} \quad (7)$$

Where

ω_π is the frequency at the phase curve crosses the -180° line.

The frequency corresponding to the ultimate point is known as the oscillation frequency ω_π . This point is characterized by two parameters defined as the ultimate gain K_u and ultimate period T_u . The ultimate period T_u can be determined as

$$T_u = \frac{2\pi}{\omega_\pi} \quad (8)$$

On the other hand, the Ziegler-Nichols tuning method is based on adjusting a closed loop until steady oscillations occur. This requires increasing the proportional gain K_p until the oscillation response occurs with constant amplitude, while derivative and integral controller gains are set to zero. The value of the proportional gain that yields constant oscillations is called the ultimate gain K_u and the period of this oscillation is called the ultimate period T_u . Using the value of K_u and T_u Ziegler - Nichols prescribes the following values of K_c , T_i , and T_d of the controller as shown in Table 1.

Table 1. PID controller parameters using Ziegler-Nichols

Controller Type	Proportional Gain K_c	Integral Time T_i	Derivative Time T_d
PID	$K_u/1.7$	$T_u/2$	$T_u/8$

The frequency response data (Bode plot) of the open-loop transfer function of the coupled tank system $h_2(s)/q_c(s)$ is shown in Figure 2.

It is clear that, from the Bode plot, the magnitude is -20.7 dB and the oscillation frequency is 0.657 rad/sec . Using Eq (7) and (8), the ultimate gain and ultimate period are 10.839 and 9.563 sec respectively.

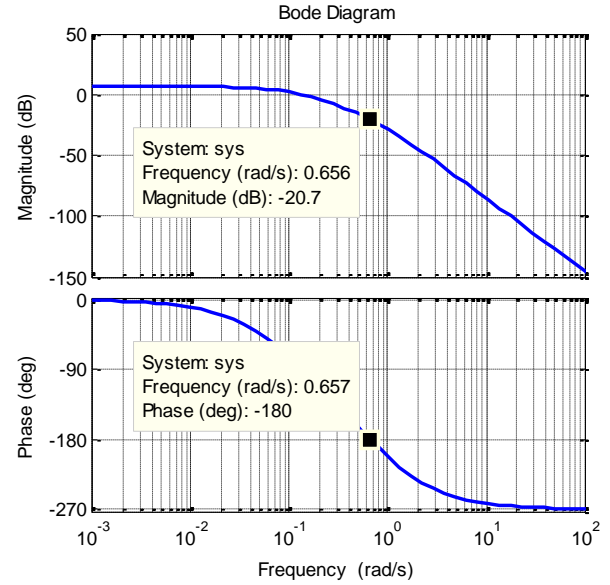


Fig. 2: Bode plot response of coupled tank system.

The PID controller parameters are determined based on formulas in Table 1. The obtained output response using these parameters has a high oscillation, and high overshoot, and takes a long time to reach a steady state. Therefore, a few refined tunes were done on these parameters to enhance the system response and the best PID controller parameters were $K_p = 6.4$, $K_i = 1.43$ and $K_d = 7.623$.

3.2 Fuzzy PID Controller Design

A general fuzzy controller model or a well-known fuzzy inference system (FIS) structure is shown in Figure 3. The fuzzification involves the conversion of the input numbers into several fuzzy variables with appropriate fuzzy sets, memberships function, and universe of discourse. Then, the inference mechanism provides the mechanism for referring to the rule base such that the appropriate rules are fired. The core of the knowledge base is the definition of the linguistic if-then rules of the Mamadani type, [20]. After that, the aggregation process is done by aggregating the outputs of all rules and combining them into a single fuzzy set. In the end, the defuzzification process converts an output fuzzy set into a crisp value. Many defuzzification methods have been proposed in recent years. Such as centroid, mean-max, max membership, and weight average methods. Centroid is the most prevalent and physically appealing of all the defuzzification methods.

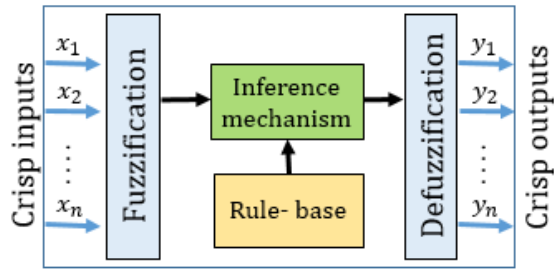


Fig. 3: Fuzzy controller.

In this paper, the suggested fuzzy controller consists of two input variables: error (E) and rate of change of error (CE). These variables are quantized into five fuzzy sets NL, N, Z, P, and PL that respectively represent Negative Large, Negative, Zero, Positive, and Positive Large. The fuzzy controller contains three output variables which are proportional fuzzy (K_{pf}), integral fuzzy (K_{if}), and derivative fuzzy (K_{df}). Each output variable is quantized to five fuzzy sets labeled as QS (Quite Small), S (Small), M (Medium), L (Large), and QL (Quite Large). In the universe of discourse of the first input, the error variable is chosen based on measuring the maximum and minimum values of the error signal and for the second input, the change of error is selected to be one-tenth of the error input. The universe of discourse of each output variable is chosen based on the best values of the PID controller K_{pf} from 0 to 6.4, K_{if} from 0 to 1.43, and K_{df} from 0 to 7.623. Figure 4 shows the membership functions of input variables. While the universe discourse of input variables is scaled as follows: $E = 30$ and $CE = 3$. The membership functions of the output variables are shown in Figure 5.

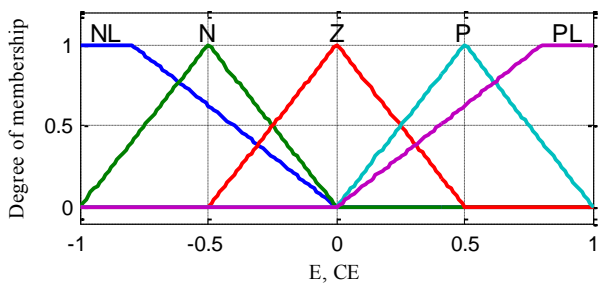
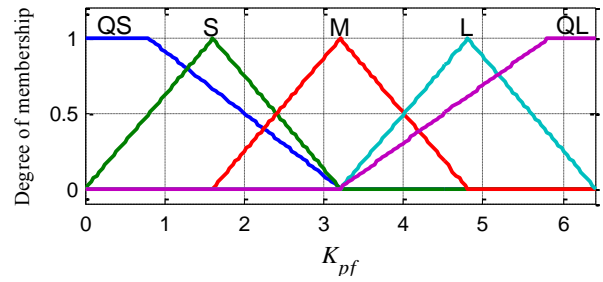
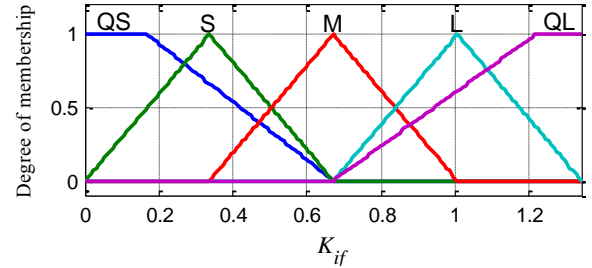


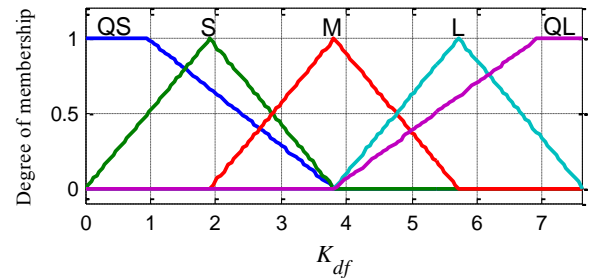
Fig. 4: Membership functions of input variables E and CE.



(a)



(b)



(c)

Fig. 5: Membership functions of output variables (a) K_{pf} , (b) K_{if} , and (c) K_{df} .

The most widely used structure in the formulation of fuzzy inference rules is based on experience and control engineering knowledge. In this paper, Mamadani-type FIS is used. Table 2 illustrates the suggested fuzzy rules, which are 25 rules for each output variable. The fuzzy output for each output variable is obtained using Max-Min composition and by applying the centroid defuzzification method, the crisp output values of the fuzzy controller K_{pf} , K_{if} , and K_{df} are calculated.

Table 2. Fuzzy controller rule base of the system.

		CE				
		NL	N	Z	P	PL
E	NL	QS	QS	QS	S	M
	N	QS	QS	S	M	L
	Z	QS	S	M	L	QL
	P	S	M	L	QL	QL
	PL	M	L	QL	QL	QL

The suggested Fuzzy PID controller as shown in Figure 6 is basic PID and the FLC structure. The basic PID controller is designed based on the linear model of the liquid-level process of the coupled-tank system. Hence, the best parameters of the PID controller are used as initial parameters K_{p0} , K_{i0} , and K_{d0} of the basic PID controller. Hence, the fuzzy PID controller parameters are calculated by the following equations:

$$K_p = K_{pf} + K_{p0}$$

$$K_i = K_{if} + K_{i0}$$

$$K_d = K_{df} + K_{d0}$$

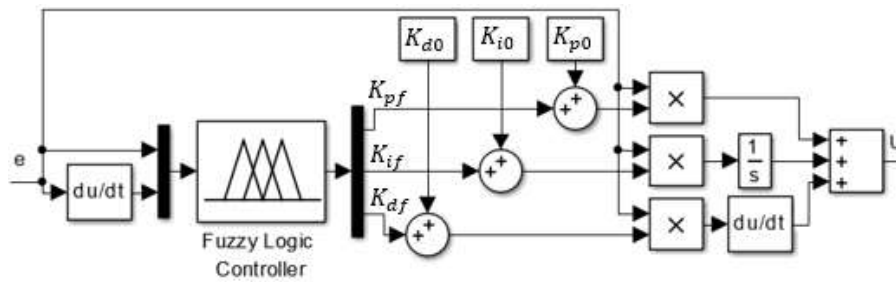


Fig. 6: Structure of fuzzy PID controller.

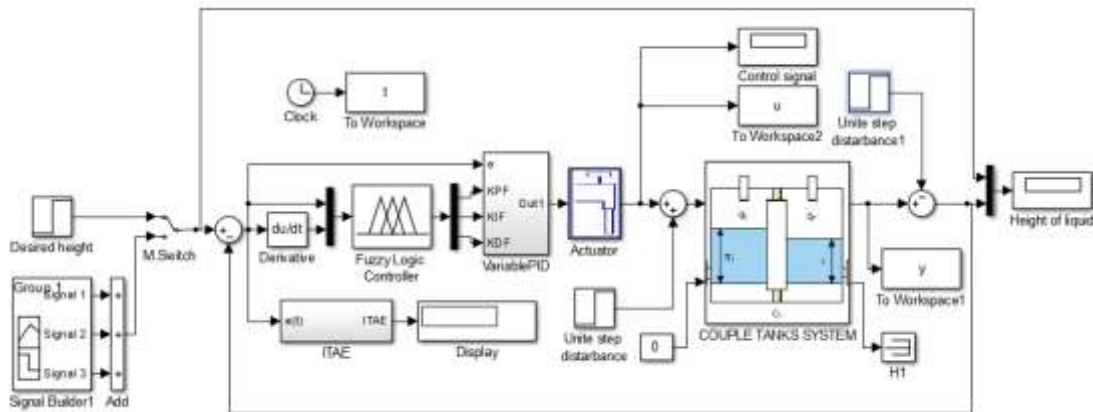


Fig. 7: Simulink block diagram to control the level height of tank 2 using a Fuzzy PID controller.

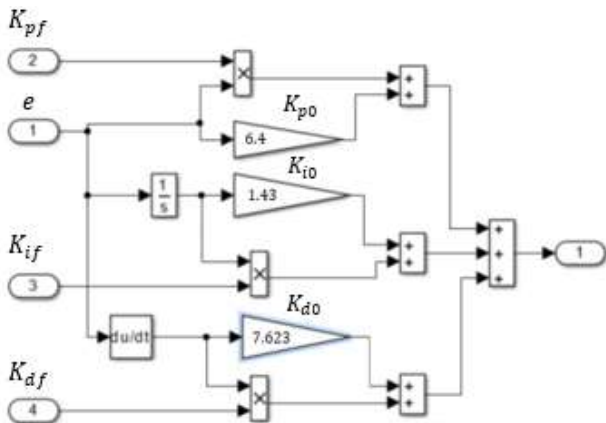


Fig. 8: Simulink of variable PID.

4 Simulation Results and Discussions

The output results of the designed control techniques that have been proposed in this paper are analyzed and compared with each other in this section. Specifically, the Simulink block diagram to control the level height of tank 2 using a Fuzzy PID controller is presented in Figure 7. Similarly, the Simulink depiction of variable PID is presented in Figure 8. Moreover, the Step responses of liquid level in tank2 are plotted for two controllers on the same window, to see more comparison between controllers' performance. Figure 9 shows the output responses using PID and Fuzzy PID controller for the 9 cm desired level height in tank 2.

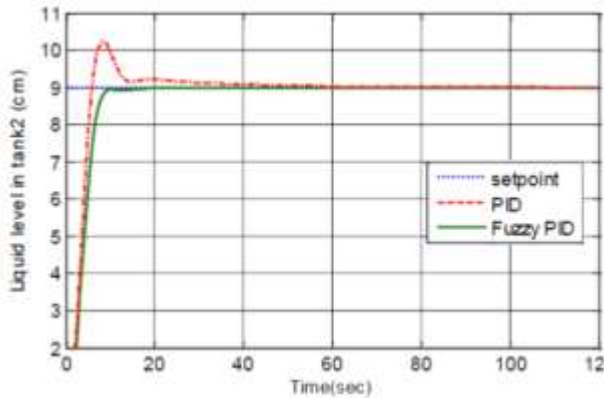


Fig. 9: Output responses using PID and Fuzzy PID controller.

The response with the Fuzzy PID controller has no overshoot compared to the PID controller response, which has a 12% overshoot. However, both responses have zero steady-state error, the response using Fuzzy PID has a smaller settling time of 8.56 seconds compared with 23.28 seconds using the PID controller. Even, the response using the PID controller is faster than the response using Fuzzy PID. The ITAE criteria of the Fuzzy PID controller were smaller than the PID controller. The performance characteristics of the level height in tank 2 using PID and Fuzzy PID controllers are summarized in Table 3.

Table 3. Performance specification using PID and Fuzzy PID controllers.

Performance specifications	PID	Fuzzy PID
Settling time (second)	23.28	8.56
Overshoot (%)	12	0
Peak time (second)	8.45	-
Steady-state error	0	0
ITAE	315.1	139.4

The setpoint change was performed at 100 seconds by a magnitude of 15 cm height in water level and at 200 seconds is changed again by a magnitude of 6 cm in water level height.

The setpoint-tracking test involved changing the setpoint through the operation as shown in Figure 10. Both approaches exactly tracked the reference input.

The controlled system is tested with disturbance in liquid deficiency. The disturbance is introduced by decreasing the amount of liquid flow rate for tank 2 by an amount of 2 cm³ /sec at a time of 80 sec. The responses of the coupled tank system under the effect of disturbance are shown in Figure 11.

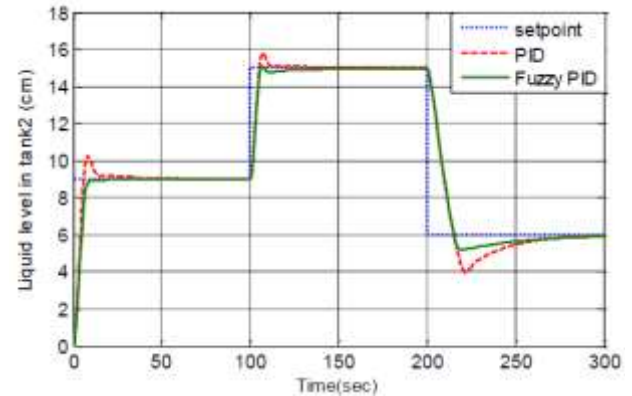


Fig. 10: Setpoint tracking the performance of the system

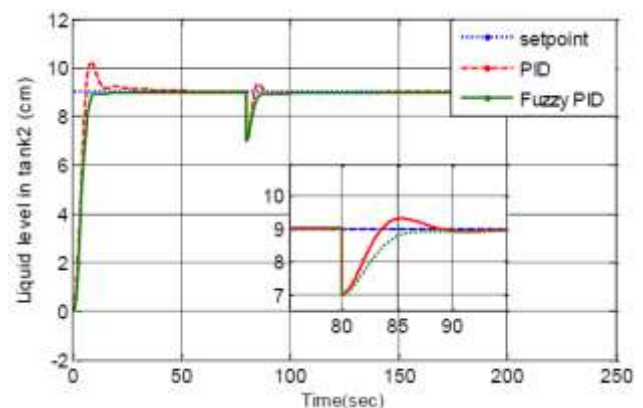


Fig. 11: Responses of PID and FPID controllers in the presence of disturbance with -2 cm³ /sec.

The PID controller takes approximately 145 seconds for the system to return to the commanded setpoint. The Fuzzy PID Controller takes a minimum effort to reject the effect of disturbance compared to the PID controller and a shorter time with 105 seconds in the presence of disturbance. The performance index using a Fuzzy PID controller is 769 compared to 941.3 using a PID controller.

5 Conclusions

In this paper. The PID controller is designed properly for the linearized model of the coupled tank system. Where, the controller parameters are tuned using the Ziegler-Nichols method, which gives oscillation in the response shape and high overshoot for the first trial. Then, the trial and error method is used, until the best responses are obtained by using this controller. The PID controller is tested for step-tracking input and disturbance input. It can be concluded that the PID controller has tracked the setpoint change for the linear model and reached the steady-state value in each change. For the

disturbance rejection, the designed PID controller approved its ability to reject this effect a few times.

The designed Fuzzy PID controller combines the advantages of the fuzzy logic controller and PID controller. In this research, the outputs of fuzzy controllers become a function of the PID controllers. Hence, any change in the error will cause a change in the controller parameters. Therefore, this method resulted in a zero overshoot and the fastest.

Simulation results showed the success of the proposed method, where the system responses using Fuzzy PID and conventional PID controller have good performance in transient response, steady-state response, step tracking signal, and disturbance rejection. So, the performance measure ITAE is used in this research to assist in choosing the best controller performance. In conclusion, it can be concluded that the Fuzzy PID controller is robust and attains excellent control performance as compared to the PID controller.

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Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

- Mustafa Saad and Muad Alshara carried out the simulation and the optimization.
- Mustafa Saad and Muad Alshara have organized and executed the controller design and simulation of sections 3 and 4.
- Mustafa Saad and Khaled Mustafa were responsible for the text writing.

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Conflict of Interest

The authors have no conflicts of interest to declare that are relevant to the content of this article.

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