Optimization Models for Urban Traffic Management

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Abstract: - The main control tool for traffic management in urban areas is traffic light settings. The goal is to decrease the queue lengths at intersections. Usually, the duration of the green light of the traffic light is used for control. The control approach is based on the so-called "store-and-forward" model. However, this model does not reflect the stochastic nature of traffic dynamics. This study presents a model with some probabilistic conditions approximating real traffic behavior. An additional contribution concerns the definition of a bi-level optimization model that simultaneously optimizes the green light and traffic light cycle duration of an urban network of four intersections. Three traffic management optimization problems are defined and solved. Their solutions are graphically illustrated and commented on. Bi-level optimization outperforms by giving lower values of queue lengths compared to classical and stochastic nonlinear optimization problems in the considered network.

Key-Words: - modeling, traffic control, traffic light settings, store-and-forward model, optimization, bi-level optimization

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1 Introduction

Urban traffic management has been a topical research problem for many years. It is obvious that the number of vehicles in the cities is increasing significantly and traffic control is a very difficult problem. This research formalizes several problems for reducing congestion in cities. The main control influence for urban traffic management is the duration of traffic light settings, [1], [2], [3]. Most of the control problems estimate the duration of the green light, [4], [5], [6]. Intensively, these problems are formalized based on the simple, so-called "store-and-forward" model, [7]. This model is used extensively in [8], [9], [10]. The traffic signal optimization is implemented by various approaches such as fuzzy traffic control, [11], particle swarm optimization, [12], metamodeling and optimization, [13], a type of reinforcement learning, so-called deep Q-network, [14], [15]. In this study, we are trying to improve the traffic behavior not only at one or two intersections, which is the usual practice but a network of intersections. Due to the interrelationships between traffic flows in the network, formalization becomes difficult for online implementation. Our intention is to improve urban traffic through models that can be applied in real-time. A good tool for this intention is the store-and-forward model that minimizes the number of vehicles on a given road section for some time. The disadvantage of this model is that it is useful for cases of deterministic estimations of traffic parameters. The traffic has a stochastic nature, but this is not taken into account in the store-and-forward model. In this study, we define a model with relations formalizing the probabilistic traffic behavior. This defined management problem has a green light duration as a solution. The next goal of this study is to increase the closeness of the control model to the real traffic behavior by extending the control environment not only to the green light but also to the traffic light cycle length. Existing studies typically apply only one control variable, [4], [5], [6], [9], [10]. In this paper, the control variables become the green light of the traffic light and the cycle duration. The implementation of this objective is based on the methodology of bi-level optimization. This control approach is implemented by the authors in [10], but here probabilistic requirements are added to reflect the stochastic nature of the traffic. In this regard, a third optimization problem is developed by applying bi-level optimization together with an algorithm reflecting stochastic traffic behavior. To verify the proposed models, this study makes comparisons between the three optimization problems: classical nonlinear optimization based on store-and-forward, extensions of probabilistic constraints, and bi-level optimization for traffic light control with optimal green signal and traffic light cycle in an intersection network. Their solutions are graphically illustrated and discussed.

2 Traffic Management Methodology

The most popular way to model traffic is based on the store-and-forward model. It determines the number of vehicles on a given road section, for example at a traffic light. The vehicle value is calculated with the vehicles from the previous control cycle, adding the difference between inbound $x_{in}(t)$ and outbound traffic $x_{out}(t)$. The exiting vehicles depend on the duration of the green light $u(t)$ and the capacity s of the street. The store-and-forward model has the following mathematical formalization, [7]:

$$x(t+1) = x(t) + x_{in}(t) - s\,u(t) \quad , \qquad (1)$$

$x(t+1)$ - the number of vehicles at time t +1,

$x(t)$ – the number of vehicles at time t,

$x_{in}(t)$ – oncoming vehicles to the intersection,

$x_{out}(t)$ = $su(t)$ – outgoing vehicles.

The store-and-forward model (1) is applied to each street of the transport network and is included in the set of constraints of the optimization problem. The relationship between green, red, and yellow lights is according to (2), where y is the cycle of the traffic light.

$$u_{green} + u_{red} + u_{yellow} = y \qquad (2)$$

We consider the yellow light to be 10% of the cycle length (this part can be different without changing the model)

$$u_{green} + u_{red} = 0.9y \;. \qquad (3)$$

Relations (1) and (3) represent constraints of the proposed traffic management optimization problem (4). In our model, we want the outflow to be greater than the inflow and the current volume of vehicles. This requirement modifies the relation (3) to become the inequality in the classical optimization problem (4). The objective function is quadratic in form and aims to minimize the queue lengths and green light durations of the entire transport network. The control variables x and u can vary between the lower L and upper U limits chosen for technological reasons. The nonlinear quadratic optimization problem for traffic management is of the form:

$$\min_{x,u} \sum_t^T [\boldsymbol{x}(t)^{\mathrm{T}}\mathbf{Q}\boldsymbol{x}(t) + \boldsymbol{u}(t)^{\mathrm{T}}\mathbf{R}\boldsymbol{u}(t)] \qquad (4)$$

$$\boldsymbol{x}(t) + \boldsymbol{x}_{in}(t) \le \boldsymbol{s}\,\boldsymbol{u}(t)$$

$$u_{green} + u_{red} = 0.9y$$

$$\boldsymbol{x}_L \le \boldsymbol{x} \le \boldsymbol{x}_U, \quad \boldsymbol{u}_L \le \boldsymbol{u} \le \boldsymbol{u}_U \;.$$

This problem is applied to an urban network containing four intersections, Fig.1, where vehicles x_i, i =1, …, 16 can turn right (a_1), left (a_2), or forward (a_3). These coefficients depend on the capacity of the network, but for the case of normalization, their sum must have a value of 1:

$$a_1 + a_2 + a_3 = 1\,.$$

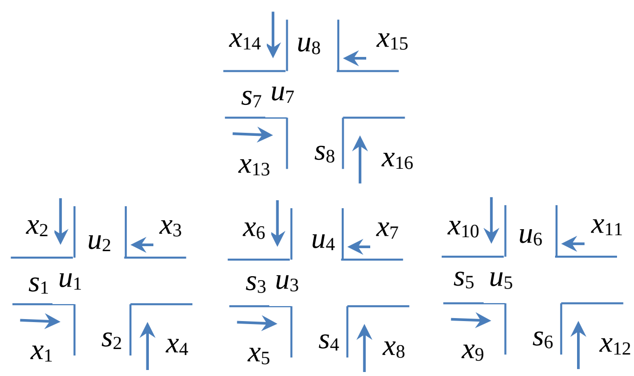

Fig. 1: Transport network architecture

3 Stochastic Optimization Model

The classical optimization problem (4) does not take into account the stochastic nature of the number of vehicles in a given segment of the urban network. This number is stochastic due to random events such as parking, stopping, and turning/entering of cars from side streets per section. The store-and-forward model can be modified with an additional value $\varepsilon(t)$ according to the stochastic traffic behavior

$$x(t+1) = x(t) + x_{in}(t) - s\,u(t) + \Sigma(\tau). \qquad (5)$$

The duration from t to t+1 represents one control cycle. We denote the maximum volume of vehicles that the street section can accommodate by x_{max} . The number of vehicles x on a given section is in practice a random variable. We require that the probability of this value be higher than x_{max} must be lower than the value of $\alpha< 1$

$$P(x \geq x_{max}) \leq \alpha, \tag{6}$$

where P is a probability density function (PDF). The probability value α is subjectively chosen, but in this study, our choice is $\alpha = 90\%$. Constraint (6) is added and the classical problem (4) is modified. Our approach is to approximate (6) in an algebraic relation to incorporate this approximation into the control problem (4). This modified problem is called a stochastic optimization problem (SP). For the approximation of (6), we assume that the stochastic variable ε (respectively x) has a normal distribution. We apply some statistics transformations to (6). We multiply by (-1) the inner inequality of (6)

$$P(-x \leq -x_{max}) \leq \alpha \tag{7}$$

Both sides of the inner inequality are normalized to a random process with mean E_x = 0 and standard deviation $\sigma_x = 1$ or

$$P\left(\frac{-x-E_x}{\sigma_x^2} \geq \frac{-x_{max}-E_x}{\sigma_x^2}\right) \geq \alpha . \tag{8}$$

We use a cumulative density function (CDF) F, taking into account the relationships between P and F

$$1-F(x)=P(x>x_{max}) \leq 1-\alpha. \tag{9}$$

The relation (8) can be written as

$$P\left(\frac{-x-E_x}{\sigma_x^2} \geq \frac{-x_{max}-E_x}{\sigma_x^2}\right) = 1 - \mathrm{F}\left(\frac{-x_{max}-E_x}{\sigma_x^2}\right) \geq \alpha$$

or

$$\mathrm{F}\left(\frac{-x_{max}-E_x}{\sigma_x^2}\right) \leq 1-\alpha$$

which can be rearranged as

$$x_{max}^{section} = -E_x + F^{-1}(1-\alpha)\sigma_x^2 \leq x_{max} \tag{10}$$

The expression $F^{-1}(1-\alpha)$ is the Z-score value of a normalized stochastic function with a normal distribution. The F^{-1} value for $(1-\alpha)$ is found in the available pre-calculated tables, [21]. The values E_x and σ_X^2 represent functions of the arguments of the control problem x and u: $E_x = E_x(x,u)$, $\sigma_x = \sigma_x(x,u)$. The mean of x and its standard deviation depends on the time horizon for their estimates. For real-time considerations, we limit the time horizon to two control cycles (initial t=0 and current t=1). The control policy consists of defining and solving a modified problem (4) for each control step. For t=0, the vehicles in the section are x_0. This value is the result after the implementation of the previous control loop $x(0) = x_0$. For the current control cycle t=1: $x(1) = x_0 + x_{in} - s\,u$. Here we consider the inflow x_{in} as a deterministic value. The average E_x value for these two-time cycles is

$$E_x(u) = \frac{1}{2}\left(x(0)+x(1)\right) = \frac{1}{2}(x_0+x_0+x_{in} - s\,u) = x_0 + \frac{1}{2}x_{in} - \frac{1}{2}s\,u . \tag{11}$$

The standard deviation of x for t=0 and t=1 is evaluated analytically for the two control cycles or

$$\sigma_x^2 = \frac{1}{2}[(x(0)-E_x)^2 + (x(1)-E_x)^2] = \frac{1}{4}(x_{in}-su)^2 \tag{12}$$

Relations (11) and (12) depend on the control variable $\boldsymbol{u}$. Substituting (11) and (12) into (10)

$$x_{max}^{section} = -\left(x_0 + \frac{1}{2}x_{in} - \frac{1}{2}\,s\,u\right) + F^{-1}(1-\alpha)\frac{1}{4}(x_{in} - s\,u)^2 \leq x_{max} \tag{13}$$

The last inequality (13) is used as the objective function for the stochastic optimization problem. The values of x_0, x_{in}, x_{max}, and α are input parameters for (13) where x_{max} and α are constant. The value x_{max} is determined by the capacity of the infrastructure and data on the possible maximum traffic flow Q_{max}. The value of the probability α is chosen subjectively by the control engineer to prevent congestion. The control argument $\boldsymbol{u}$ are the solution to the stochastic optimization problem (SP).

The problem (SP) is defined by modifying (4) with relation (13), which is used as a new objective function for the classical problem (4). The problem (SP) is aimed at minimizing the values $x_{3,max}^{section}(u)$ by choosing optimal values of the green lights u. For the case of the network of Fig.1, the objective function is analytic

$$\min_{\boldsymbol{u}} \sum_{i=1}^{n} x_{i,max}^{section}(\boldsymbol{u}) \tag{14}$$

The stochastic traffic light optimization problem (SP) is defined as a quadratic problem. The critical values $\boldsymbol{x}_{i,max}$ are included in the objective function, which aims to minimize their values. The upper index *P* is used to notify the application of the probabilistic constraints (7).

$$\min_{u} \sum_{i=1}^{n} \boldsymbol{u}^T \boldsymbol{Q}^P \boldsymbol{u} + \boldsymbol{R}^{PT} \boldsymbol{u} \tag{15}$$

$$\boldsymbol{x}_{i,0} + \boldsymbol{x}_{i,in} \leq \boldsymbol{s}\, \boldsymbol{u} \quad \text{for } i=1,\dots,n$$

$$\boldsymbol{LB}_{\boldsymbol{u}} \leq \boldsymbol{u} \leq \boldsymbol{UB}_{\boldsymbol{u}}.$$

The matrices $\boldsymbol{Q}^P$ and $\boldsymbol{R}^P$ are derived analytically according to relation (13) for each road section $i=1,\dots,n$ (n=16). This problem has a smaller set of arguments because the variables $\boldsymbol{x}$ are not explicitly represented in (15).

4 Bi-level Traffic Light Optimization Model

The classical optimization problem (4) has arguments $\boldsymbol{x}$ (the number of vehicles) and the duration of green lights $\boldsymbol{u}$, while the traffic light cycle $\boldsymbol{y}$ is a fixed-value parameter. The modified stochastic problem, (15) also implements the cycle length $\boldsymbol{y}$ as a parameter. This is the usual optimization methodology where the solution/control is of one type. Since our object of control – urban traffic passes through interconnected intersections, this result in a distributed system with a stochastic nature. In this system, in addition to the duration of green light, an important control variable is the duration of the traffic light cycle. To incorporate these two control variables, we propose a hierarchical optimization method. This method is of interest to many researchers because of the positive results such as in public transportation, [16]; determining pricing strategies of intermodal transport, [17]; urban traffic control for autonomous vehicles, [18]; for gas transportation, [19]; for bus lane optimization, [20]. This methodology is not a simple combination of two or more optimization problems, but hierarchically coordinated interactions of interrelated optimization problems. The extended set of control variables leads to the achievement of more goals in the control process, and accordingly, to the satisfaction of more constraints. When the hierarchical system has two levels, we can have two types of variables in vector form involved in the two optimization problems. These optimization problems are not given in explicit analytical form and cannot be solved separately. This is the reason for using the hierarchical approach. The hierarchical principle of operation states that the solution of a higher-level problem depends on the solution of a lower-level problem and vice versa. In our case, the control variables are the cycle *y* and green light duration *u* of the considered network. These variables are vectors and their sizes depend on the architecture of the transport network. Bi-level optimization has the following principle of operation. The higher-level optimization problem initially assumes that the solution to the lower-level optimization problem is known, its problem becomes analytically defined, and thus it can solve its problem with two types of variables. The solution of the higher-level problem is sent to the lower-level problem, so the lower-level problem has an analytical formalization and can be solved. The solution of the lower-level problem is sent to the higher-level problem so that the higher-level problem (which is new according to the lower-level solution) can be solved. This iterative process of solving the two optimization problems continues until the optimal solution is found. By formalizing the bi-level control problem, we aim for cycle *y* to be the argument/solution of the optimization as well as the duration of the green traffic signal *u*. Thus, the bi-level problem has two arguments $\boldsymbol{u}$, and $\boldsymbol{y}$ as solutions. The upper-level problem is defined to optimize the traffic lights $\boldsymbol{u}$ for a given $\boldsymbol{y}$

$$\min_{\mathbf{u}} f_u(\boldsymbol{u},\boldsymbol{y}) \,, \quad \boldsymbol{u} \in \mathbf{S}_{\boldsymbol{u}}(\boldsymbol{u},\boldsymbol{y}) \,.$$

The low-level optimization problem aims to optimize $\boldsymbol{y}$ for a given $\boldsymbol{u}$

$$\min_{\boldsymbol{y}} f_y(\boldsymbol{u},\boldsymbol{y}) \,, \boldsymbol{y} \in \mathbf{S}_{\boldsymbol{y}}(\boldsymbol{u},\boldsymbol{y}) \,.$$

The bi-level methodology hierarchically integrates these two optimization problems into a common form

$$\min_{\boldsymbol{u}} f_u(\boldsymbol{u},\boldsymbol{y}) \,, \quad \boldsymbol{u} \in \mathbf{S}_{\boldsymbol{u}}(\boldsymbol{u},\boldsymbol{y})$$

$$\boldsymbol{y} \equiv arg \equiv \begin{cases} \min_{\boldsymbol{y}} f_y(\boldsymbol{u},\boldsymbol{y}) \\ \boldsymbol{y} \in \mathbf{S}_{\boldsymbol{y}}(\boldsymbol{u},\boldsymbol{y}) \end{cases}.$$

This problem has an extended argument set ($\boldsymbol{u}$, $\boldsymbol{y}$). We apply a quadratic objective function for the upper level. Minimizing the cycle length $\boldsymbol{y}$ leads to maximizing the traffic flows in the considered network. The minimization includes a set of constraints on technological, operational, and administrative requirements leading to lower and upper bounds on the cycle length $\boldsymbol{LB_y}, \boldsymbol{UB_y}$. Both the upper and lower problems have mutual interaction because the solution of the former is used as a parameter for the latter and vice versa. The bi-level problem is analytically defined as

$$\min_{u} \sum_{i=1}^{n} [\boldsymbol{u}^T \boldsymbol{Q}^P \boldsymbol{u} + \boldsymbol{R}^{PT} \boldsymbol{u}]$$

$$\boldsymbol{x}_{i,0} + \boldsymbol{x}_{i,in} \le \boldsymbol{s}\, \boldsymbol{u}_{i,green} \text{ for } i = 1, \dots, n$$

$$\boldsymbol{u}_{v,green} + \boldsymbol{u}_{v,red} + \boldsymbol{u}_{v,amber} = \boldsymbol{y}_v, v = 1, \dots, j \quad \forall \mathrm{j}$$

$$\boldsymbol{LB}_u \le \boldsymbol{u} \le \boldsymbol{UB}_u ,$$

$$\boldsymbol{y} \equiv \boldsymbol{arg} \left\{ \begin{array}{c} \min_{\boldsymbol{y}} \sum_{v=1}^{j} [\boldsymbol{y}^T \boldsymbol{Q}^y \boldsymbol{y} + \boldsymbol{R}^{yT} \boldsymbol{y}], \\ \boldsymbol{u}_{v,green} + \boldsymbol{u}_{v,red} + \boldsymbol{u}_{v,amber} = \boldsymbol{y}_v, \\ \boldsymbol{LB}_y \le \boldsymbol{y} \le \boldsymbol{UB}_y \end{array} \right\} . \quad (17)$$

$$v = 1, \dots, j$$

The bi-level formulation (17) has an extended set of arguments (***u, y***), more constraints, and optimizes both objective functions. In this way, optimization is achieved for the traffic behavior through the cycle length (***y***) and the green light (***u***). In this way, the number of vehicles ***x*** in the entire urban network is minimized. Moreover, the optimization of the traffic light cycle ***y*** gives a further reduction in the waiting time of the vehicles at the traffic light.

5 Simulations and Comparisons of Results

The generated optimization problems are solved in the MATLAB environment. The store-and-forward model (1) allows for determining the number of vehicles in the transport network. In this way, network queue lengths can be calculated, which are an indicator of the traffic status. These queue lengths result from the three models and optimization problems, respectively: the so-called classical deterministic optimization problem (4), the modified stochastic optimization problem (15), and the bi-level problem (17).

After applying the solutions to these three problems (4), (15), and (17) with several control cycles, some of the resulting queue lengths x_3, x_{14}, and x_{16} are graphically illustrated in Fig. 2, Fig. 3 and Fig.4. With the dashed black line is the solution of the Deterministic Problem (4), denoted by the index DP. The results of the probability problem (PP) (15) are in red. In solid blue lines are the results for the queue lengths after solutions of the bi-level (BL) problem (17). These solutions show that after the 6th control cycle, the number of vehicles at the intersection does not change or the control process is established in a steady state. These estimated values are almost equal for the deterministic problem (DP) (4) and the probabilistic problem (PP) (15). The bi-level solution of (17) significantly reduces the number of vehicles in queues. The same nature of traffic dynamics applies to most queues on the network.

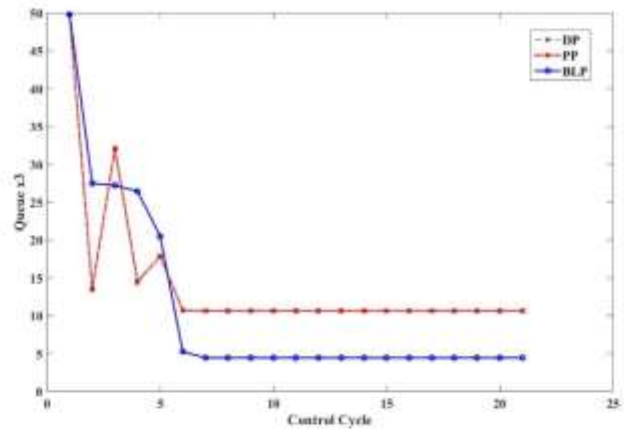

Fig. 2: Dynamics of queue length x_3

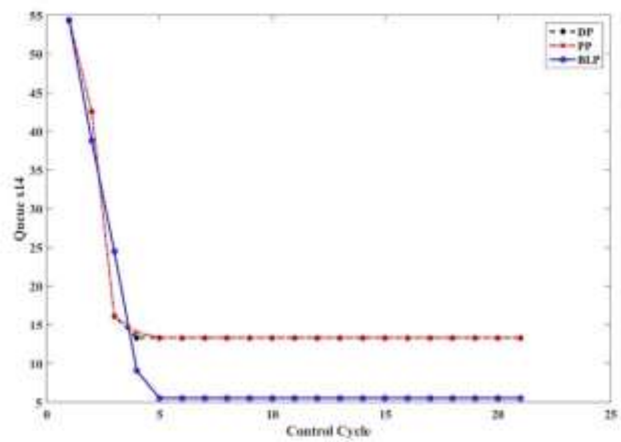

Fig. 3: Dynamics of queue length x_{14}

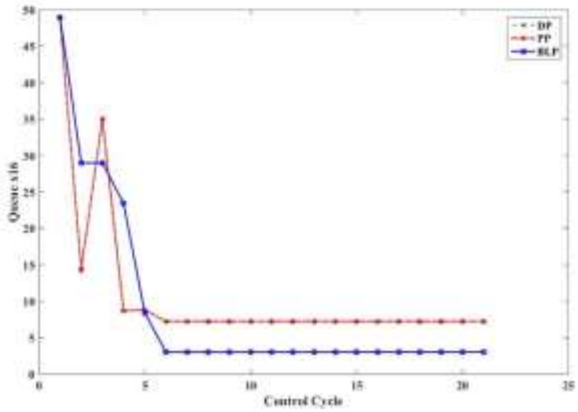

Fig. 4: Dynamics of queue length x_{16}

To estimate the traffic in the network, we sum the number of vehicles in it. The total number of vehicles in all queues in the network is presented in Fig.5.

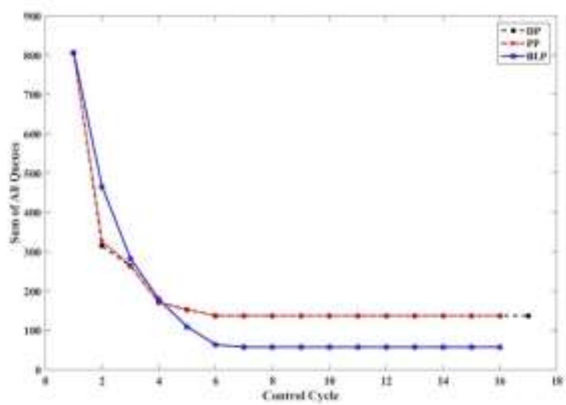

Fig. 5: Sum of all queues in the network

It can be seen that the results of the bi-level optimization are better compared to the deterministic and probabilistic cases. The reason for this reduction is the expanded set of control variables and constraints in the optimization problems and the applied hierarchical methodology that reflects the interrelationship of traffic between network intersections.

6 Conclusion

In this study, three optimization problems for urban traffic control are defined and solved: the classical quadratic optimization problem (DP) that minimizes the number of vehicles in queues. This model reflects the requirement that the outflow be greater than the inflow in the street section based on store-and-forward modeling. The second optimization problem (15) has an objective function that formalizes capacity constraints with probabilistic inequalities. This is an innovative approach in this research, reflecting the stochastic behavior of traffic. Both optimization problems (4) and (15) have as solutions the duration of the green light. The third optimization problem (17) has a bi-level formalization. It hierarchically optimizes two optimization problems that simultaneously minimize green light durations and cycles, reflecting stochastic traffic dynamics, which is another novelty of the paper. The solutions of these three optimization problems are compared and the results are presented graphically. The comparison of the results shows that the number of vehicles for the deterministic and probabilistic problems is close to each other. But the bi-level solutions lead to much smaller queues (Fig.2, Fig. 3, and Fig.4). It is important to estimate the sum of all the queues on the streets of the network. Fig. 5 shows a significant reduction in the total length of queues in the urban network when applying the bi-level optimization. This is the result of expanding the control space with the simultaneous application of two control tools: green lights and cycle length, satisfying an increased set of constraints. This result shows that the traffic dynamics are improved by increasing traffic intensity, reducing drivers' waiting time and the corresponding environmental pollution. Another positive result is the rapid convergence to the steady state (only six iterations). This is a prerequisite for real-time deployment. The limitation of the models is related to the greater preliminary work on the analytical formalization of the optimization problems when the network consists of more intersections. This limitation is not significant because this work is done offline. However, for management in this case, more powerful computing devices are needed for real-time implementation. Future extensions of these models could be by incorporating additional constraints on infrastructure considerations, for example matching traffic to the capacity of urban sections.

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Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

The authors equally contributed to the present research, at all stages from the formulation of the problem to the final findings and solution.

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Conflict of Interest

The authors have no conflicts of interest to declare.