Optimized Genetic Algorithms Reduced Order Model Based RST Roll Control of Antiroll Bar Dedicated to Semi-active Suspension

SAAD BABESSE
Department of Electrical Engineering
University of Setif1 19000 Setif, ALGERIA

FOUAD INEL
Department of Mechanical Engineering, Automatic Laboratory,
University of Skikda, ALGERIA

Abstract: Working with high-order transfer functions needs a lot of work and leads to major difficulties in analysis, simulation, and control design. Model reduction studies the large-scale system properties and helps to reduce these difficulties. In this paper, the genetic algorithms (GA) optimization method is used to calculate the second reduced order model (ROM) of the original high order model (HOM) of the actuator. Here, the studied hydraulic actuator is a single input, single output (SISO), and linear time invariant (LTI) system that can be modeled by an eight-order transfer function with uncontrollable modes. The genetic algorithms are successfully applied to reduce the original model order using MATLAB software. Thus, the proposed approach is applied to both the original and suggested reduced order models to check the effectiveness of the reduction method. Finally, a digital RST roll control based on the robust pole placement is applied for the two models, and simulations are carried out to show the effectiveness of the control strategy

Key-Words: - Antiroll bar; Optimization; Genetic algorithms; Reduced model; Roll control; RST control.
Received: October 29, 2021. Revised: October 28, 2022. Accepted: November 29, 2022. Published: December 9, 2022.

1 Introduction

The mathematical modelling of most physical systems results in infinite dimensional models, order thus the complexity of the systems directed the researchers towards the reduction of order of these systems, not only to facilitate the analysis but too to find a suitable approximation of the high order systems while keeping the same important characteristics as closely as possible.

In the literature, several methods are available: some of them are based on original continued fraction expansion technique,[1], the disadvantage of these methods is the failure to retain the stability of the original systems in the reduced order systems, and for the improved suggested methods suffer from the possibility of not having a reduction of order but an increase in order. Modal-Padé methods, [2], the major disadvantage of such methods is the difficulty in deciding the dominant poles of the original system. However, most of the optimal techniques follow time-consuming, iterative procedures that usually result in non-robustly stable models with poor frequency response resemblance to the original high order model in some frequency ranges.

Genetic Algorithms (GA) method has proved to be excellent optimization tools in the past few years. The use of such search-based optimization algorithms in model reduction ensures that all the model reduction objectives are realized with minimal computational effort, [3].

MATLAB 7.9's embedded GA toolbox was used to build the GA model reduction approach based on L1 Norm.

In this paper, the high order system is a hydraulic actuator dedicated to a heavy vehicle anti-roll bar mechanism, which is modelling by a seven-order transfer function, and by adding a lead-lag Pre-filter, the overall system becomes with eight order transfer function, stable, but not fully controllable.

The reduced order model obtained has a second order transfer function, stable, and fully controllable, these properties are suitable for feedback controllers.

On the other side, we choose to control the system by applying a digital polynomial RST controller characterized by 2 D.O.F of control (one for the input and the other for the output), thus his robustness against the disturbances and noises.

2 Hydraulic Actuator Modeling

The studied system is a hydraulic actuator dedicated to semi-active suspension of single unit heavy vehicles. The suspension consists of two trailing arms free to rotate about their axis independently of each other. Each end of the anti-roll bar is attached to one trailing arm whose position is determined by the wheels and actuator positions. The actuators are mounted between the anti-roll bar and the frame of the trailer. By extending one actuator and retracting the other, the anti-roll bar is twisted and a torque is provided to counteract the moment generated by the lateral acceleration and tilt the vehicle into the turn. The different transfer functions are given in the following paragraph, [4], [5]:

First, the transfer function between the displacement transducer extension x and the actuator extension y_a is given by:

$$\bar{x} = \frac{L}{d_a}\frac{I_{arb}s^2+K_{arb}}{(I_p+I_{arb})s^2+2Cd_Td_ds+2k_sd_sd_T+K_{arb}-mgh}\overline{y_a} - \frac{F_chL}{(I_p+I_{arb})s^2+2Cd_Td_ds+2k_sd_sd_T+K_{arb}-mgh} \quad (1)$$

The transfer function between the actuator extension y_a and the servo-valve spool displacement x_v is given by:

$$y_a = \frac{K_x/A_p}{a_{11}s^3+a_{12}s^2+a_{13}s+a_{14}}.x_v \quad (2)$$

With:

$$a_{11} = \frac{V_tM_t}{4\beta_eA_p^2}, a_{12} = \left(\frac{K_{pe}M_t}{A_p^2}+\frac{V_tB_p}{4\beta_eA_p^2}\right), a_{13} = \left(1+\frac{K_{pe}B_p}{A_p^2}+\frac{V_tK_h}{4\beta_eA_p^2}\right), a_{14} = \frac{K_{pe}K_h}{A_p^2}$$

The servo-valve is modeled as a 2nd order Butterworth filter; it is a low-pass filter with a cut-off frequency of 15Hz, which is given by:

$$BW(s) = \frac{\omega_c^2(\alpha^2+\beta^2)}{s^2+2\alpha\omega_cs+\omega_c^2(\alpha^2+\beta^2)} \quad (3)$$

Where: $\omega_c = 30\pi\left[{rad}/{s}\right], \alpha = 0.5949, \beta = 0.2830$

To keep the entire system stable with a range of given references, we add a lead-lag pre-filter, given by:

$$PF(s) = \frac{\tau_1s+1}{\tau_2s+1} \quad (4)$$

Where: τ_1, τ_2 were chosen to enable a reasonable choice of the regulator parameters:

$$\tau_1 = 0.001, \tau_2 = 2$$

The open loop transfer function of the overall system is given by the following transfer function:

$$G(s) = \frac{b_3s^3+b_2s^2+b_1s+b_0}{s^8+a_7s^7+a_6s^6+a_5s^5+a_4s^4+a_3s^3+a_2s^2+a_1s+a_0} \quad (5)$$

Where: $b_3 = 1.135.10^4, b_2 = 1.135.10^7, b_1 = 1.897.10^9, b_0 = 1.897.10^{12}, a_7 = 122.1,$
$a_6 = 2.175.10^5, a_5 = 2.421.10^7, a_4 = 8.836.10^8, a_3 = 4.288.10^9, a_2 = 1.046.10^{11},$
$a_1 = 1.167.10^{11}, a_0 = 3.271.10^{10}$

All the actuator parameters are listed in Table 2.

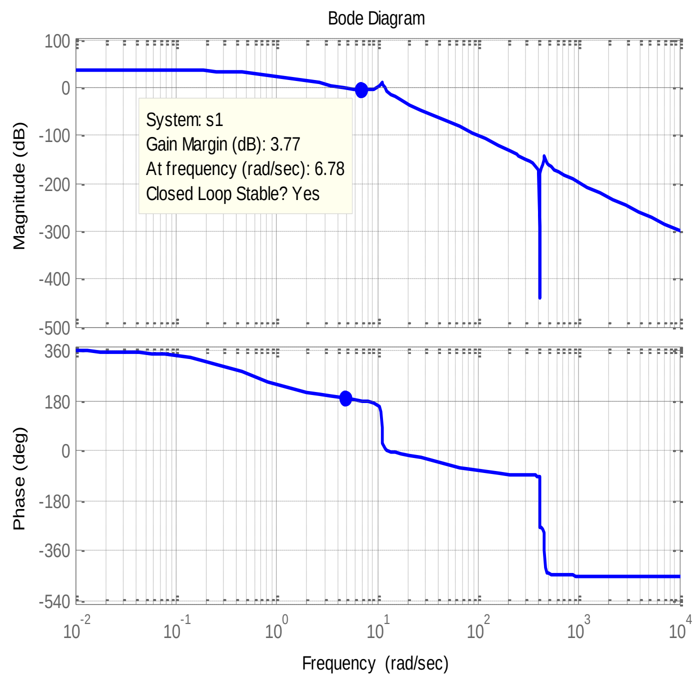


Figure 1. Bode plot of the overall hydraulic actuator.

From Fig.1, one can see that this system is stable. However, it's not fully controllable, since: rank(A)=8,rank(ctrb(A,B))=3

For a stabilization purpose, one can use a reduction method to control the system freely; this step is well described in the following paragraph.

3 Reduction Model Based Genetic Algorithms

Model reduction is a branch of systems and control theory, which studies properties of dynamical systems in order to reduce their complexity, while preserving (to the possible extent) their input-output behavior, [6]. And one can note that the use of low order models lead to a simple design and analysis, computational benefit, simplicity of simulation. On the other hand, the accuracy measure of the approximation should in some concrete way take into consideration the difference in behavior between the original system and the reduced order model, so that, different norms are used for the formulation of the model reduction problem : H∞, H2, L1-Norm and hybrid norm, [7].

In this section, we adopt to use the L1 Norm Model Reduction approach to reduce the 8th order hydraulic actuator of eq. (5) into a 2nd order reduced model, and GA's approach will be used to perform the model reduction.

3.1 Genetic algorithm theory

Genetic algorithm is a robust optimization technique based on natural selection. The basic goal of GAs is to optimize functions called fitness functions. GA-based angle approaches differ from conventional problem-solving methods in several ways, [8].
First, GAs work with a coding of the parameter set rather than the parameters themselves. Second, GAs search from a population of points rather than a single point. Third, GAs use payoff (objective function) information, not other auxiliary knowledge. Finally, GAs use probabilistic transition rules, not deterministic rules. These properties make GAs robust, powerful, and data-independent. Its basis in natural selection allows a GA to employ a "survival of the fittest" strategy when searching for optima. The use of a population of points helps the GA avoid converging to false peaks (local optima) in the search space. The following sections describe GAs in more detail. Most of the information presented here is based on:

- Chromosome: A simple GA requires the parameter set of the optimization problem to be encoded as a string (binary, real, etc.). These strings are known as chromosomes. They are manipulated by the GA in an attempt to obtain the string that represents the optimal solution to the problem.
- Genes: A character or symbol in a GA chromosome is called a gene. Genes are the basic building blocks of the solution and represent the properties which make one solution different from the other.
- Allele: The value of a gene in a GA is called an
allele, such as for eye color, the different possible 'settings' (e.g., blue, brown, hazel etc.) are called alleles.
- Selection: A genetic operator used to select individuals for reproduction.
- Crossover: A key operator used in the GA to create new individuals by combining portions of two parent strings.
- Crossover probability: Probability of performing crossover operation, denoted by pc, i.e., the ratio of number of offspring produced in each generation to the population size. This value of pc is chosen generally in the range of 0.7 to 0.9.
- Mutation: An incremental change to a member of the GA population.
- Mutation Probability: The probability of mutating each gene in a GA chromosome, denoted by pm. This value is chosen generally in the range of 0.01 to 0.03.

3.2 L1 Norm Model Reduction Approach

Starting in 1977, El-Attar and Vidyasagar presented new procedures for model reduction based on interpreting the system impulse response (or transfer function) as an input-output map, [8], [9].
The L1 norm of the system with transfer function G(s) and impulse response g(t) on the other hand is defined as, [6]:

$$\|g\|_1 = \int_0^\infty |g(t)|dt \tag{6}$$

on the other hand, the L1 norm is defined as:

$$\|e\|_1 = \int_0^\infty |g(t) - g_r(t)|dt \tag{7}$$

Where: e(t) is the impulse response difference between the original system and the reduced system:

$$e(t) = g(t) - g_r(t) \tag{8}$$

This last equation was implemented in MATLAB using trapezoidal numerical integration which computes an approximate integral of the error between the impulse response of the original system and the impulse response of the reduced order system with respect to time.

3.3 Reduction Model Using GAs

First, the settings of the GA used to perform the reduction for the hydraulic actuator were as in Table 1:

Table 1. GA's settings

Population size	150
Encoding Criteria	*Double Vector*
Crossover Fraction	*0.8*
Mutation Fraction	0.02
Elite Count	*10*
Stall Generations Limit	*1500*
Stall Time Limit	∞
Selection Function	*Roulette Wheel*
Crossover Function	*Crossover Scattered*
Mutation Function	*Mutation Gaussian*
Maximum Number of Generations	1500

We use the polynomials of the high order original model of eq.(5) as input data to the optimization algorithm. The reduced order model is obtained, after 1500 iterations, as:

$$G_r(s) = \frac{-1.848s+21.17}{s^2+1.293s+0.3648} \tag{9}$$

The steady state error is 0.050

The L1-Norm of the reduced model is 4.7147225

The impulse response, the step response, and Bode plot are shown in Fig.2, Fig.3 and Fig.4 respectively:

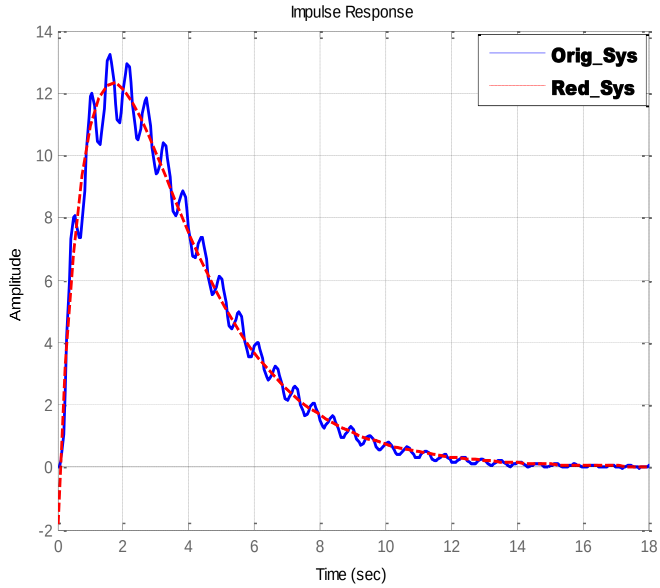


Figure 2. Impulse responses of original and reduced order models.

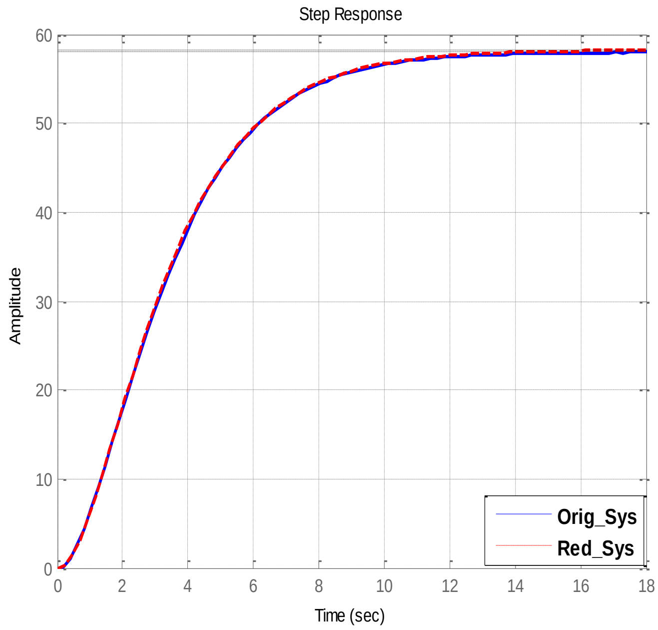


Figure3. Step responses of original and reduced order models.

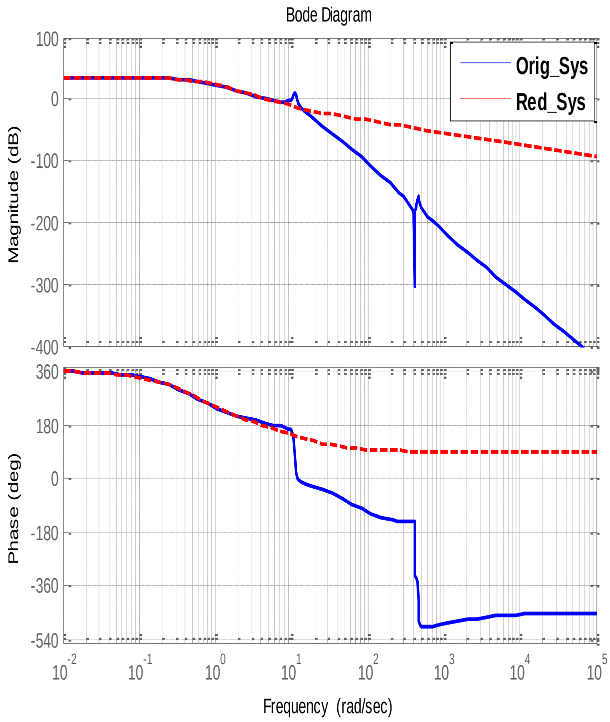


Figure 4. Bode plot of original and reduced order models.

Fig. 2 and Fig 3 show the good equivalence between the high order model and the optimized low order model, because of identical step responses, and quasi-identical impulse responses with minor errors. The impulse response of the original model presents naturally oscillations that can make some control difficulties.

In Fig.4, the frequency responses of the reduced order models highly resemble those of the original systems at low frequencies. The magnitude of the reduced order model shows some error at high frequencies due to the six missing states in the reduced order model. However, since most real-time physical systems operate at low frequencies, this error at high frequencies tends to be acceptable and can be ignored.

4 Digital RST Control of the Actuator

4.1 Models sampling

In the automatic control, the choice of the sampling frequency is based on the following formula, [10]:

$$6f_{BP}^{BF} < f_e < 25f_{BP}^{BF}$$

Where: f_{BP}^{BF} is the closed loop pass-band of the system.

So, we choose: $f_e = 10hz \rightarrow T_e = \frac{1}{f_e} = 0.1s$

The discrete-time original model is obtained by the discretization of the continuous-time model (Eq.5) with Zero Order Holder and the sampling time $T_e = 0.1s$:

$$G(z) = \frac{a_7z^7 + a_6z^6 + a_5z^5 + a_4z^4 + a_3z^3 + a_2z^2 + a_1z^1 + a_0}{z^8 + b_7z^7 + b_6z^6 + b_5z^5 + b_4z^4 + b_3z^3 + b_2z^2 + b_1z^1 + b_0}$$

$$G(z) = \frac{a_7z^{-1} + a_6z^{-2} + a_5z^{-3} + a_4z^{-4} + a_3z^{-5} + a_2z^{-6} + a_1z^{-7} + a_0z^{-8}}{1 + b_7z^{-1} + b_6z^{-2} + b_5z^{-3} + b_4z^{-4} + b_3z^{-5} + b_2z^{-6} + b_1z^{-7} + b_0z^{-8}}$$

With: $a_7 = 0{,}00442, a_6 = 0.07371, a_5 = 0.1494$

$a_4 = 0.1155, a_3 = 0.05587, a_2 = 0.008015$

$a_1 = 7.793.10^{-5}, a_0 = 8.359.10^{-8}$.

$b_7 = -2.093, b_6 = 2.088, b_5 = -1.453, b_4 = 0.6484$

$b_3 = -0.5521, b_2 = 0.3659, b_1 = 0.0024,$

$b_0 = 4.98.10^{-6}$

The discrete-time reduced order model is:

$$G_r(z) = \frac{-0.07184Z + 0.2704}{z^2 - 1.875Z + 0.8787} = \frac{-0.07184z^{-1} + 0.2704z^{-2}}{1 - 1.875z^{-1} + 0.8787z^{-2}}$$

The reduced model obtained has two stable poles at 0.9593 and 0.9160 because their modules are less than the unity, and it has a non-stable zero at: 3.7632.

4.2 RST Digital Control

The RST digital controllers have two degrees of freedom (one for tracking, the other for regulation). The design of such controller is done in two steps, [11], [12]:

1) Calculation of the polynomials R and S (regulation)
2) Calculation of T (tracking).

The general scheme of RST control is shown in Fig.5:

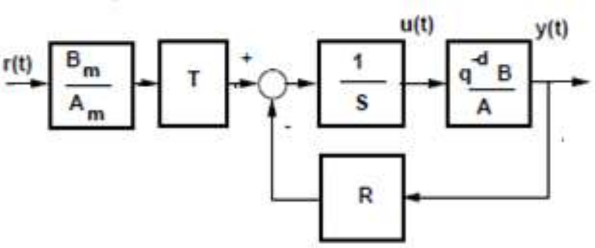


Figure 5. Structure of RST controller

In this scheme:
q^{-d}: is the time delay of the plant (in our model d=0).
$\frac{B_m}{A_m}$: is the tracking reference model.
Before calculating the three polynomials, we impose some specifications in time continuous to satisfy both tracking dynamics and regulation dynamics, and then in the next step, we do the discretization.
In this paper, we adopt the following specifications:

- Second order damped response in 15 samples (i.e 1,5 s).
- Desired dominant poles in continuous time: $p_{dom1} = \frac{1}{5}$, that gives in discrete time: $z_{dom1} = 0.8187$.

We choose another dominant pole at 0.5, we find:
$$A_m = (1 - 0.8187z^{-1})(1 - 0.5z^{-1})$$

- Imposing unit static gain equal to 1. This is done by choosing $B_m = A_m(1) = 0,0906$
Then, the tracking reference model is:
$$\frac{B_m}{A_m} = \frac{0,0906}{1 - 1,3187z^{-1} + 0.4093z^{-2}}$$

1) We can choose the regulation dynamics by imposing the poles of the closed loop polynomial $P(z^{-1})$, here:
$$P(z^{-1}) = (1 - 1.875z^{-1} + 0.8787z^{-2})(1 - 0.5z^{-1}) = 1 - 2.3818z^{-1} + 1.8260z^{-2} - 0.4425z^{-3}$$

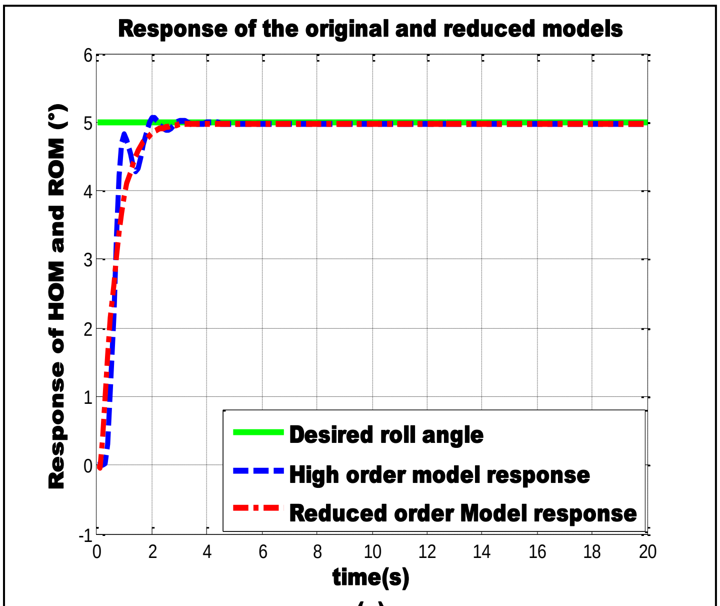
Then, we add a pre-specified fixed part to the $S(z^{-1})$ polynomial $(1 - z^{-1})$ to impose a null static error.
The different polynomial orders, to obtain a feasible controller, are:
$$deg(P) = 3 \rightarrow \begin{cases} deg(R) = 2 \\ deg(S) = 2 \\ deg(T) = deg(P) = 3 \end{cases}$$
2)

The resolution of the Equation of Bezzout gives:
$$\begin{cases} R(z^{-1}) = 2.6357 - 4.9599z^{-1} + 2.3328z^{-2} \\ S(z^{-1}) = 1 - 0.4539z^{-1} - 0.5461z^{-2} \end{cases}$$

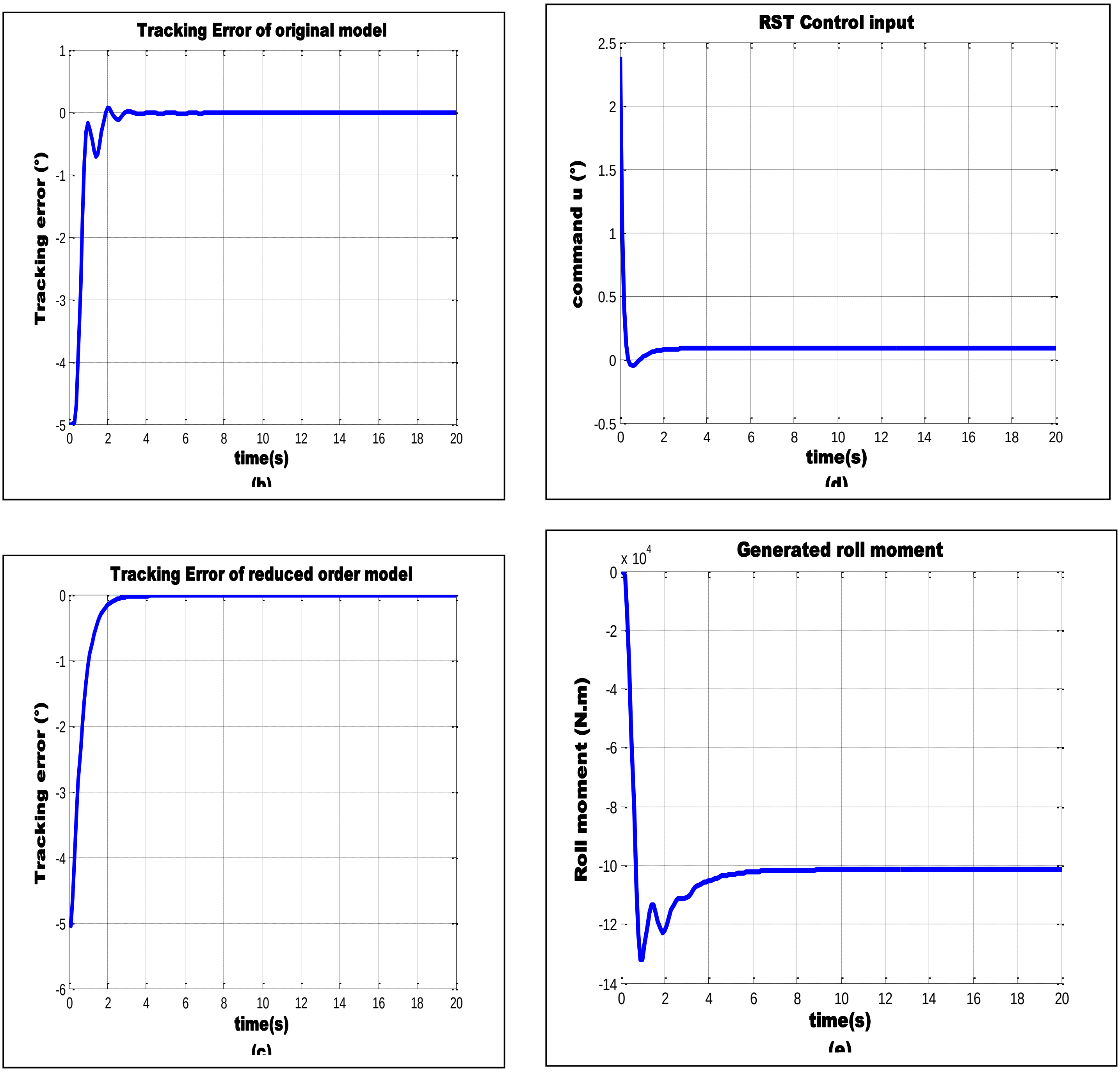
And, since the plant has a non-stable zero, the $T(z^{-1})$ polynomial is given as: $T(z^{-1}) = \frac{P(z^{-1})}{B(1)}$, so:
$$T(z^{-1}) = 5.2715 - 12.5556z^{-1} + 9.6255z^{-2} - 2.3328z^{-3}$$

5 Simulations Results and Discussions

First, we define the main input of the plant (hydraulic actuator) as the desired roll angle as mentioned in, [13],[14], after that, we apply the designed RST controller to the reduced order model(ROM), and next, to the original high order model (HOM) respectively.



(a)


Tracking Error of original model
Tracking error (°)
time(s)
(b)
RST Control input
command u (°)
time(s)
(d)
Tracking Error of reduced order model
Tracking error (°)
time(s)
(c)
Generated roll moment
x 10^4
Roll moment (N.m)
time(s)
(e)

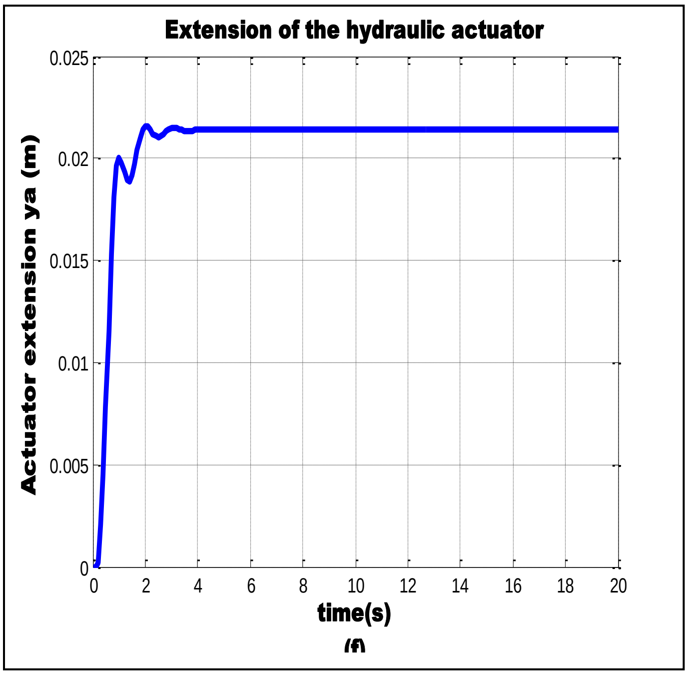


Figure 6. RST control results without external disturbance.

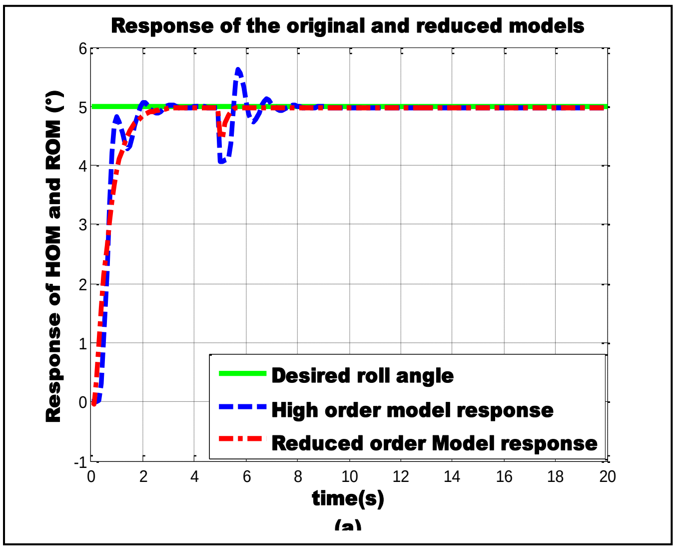
(a): Response of the two models –ROM and HOM-

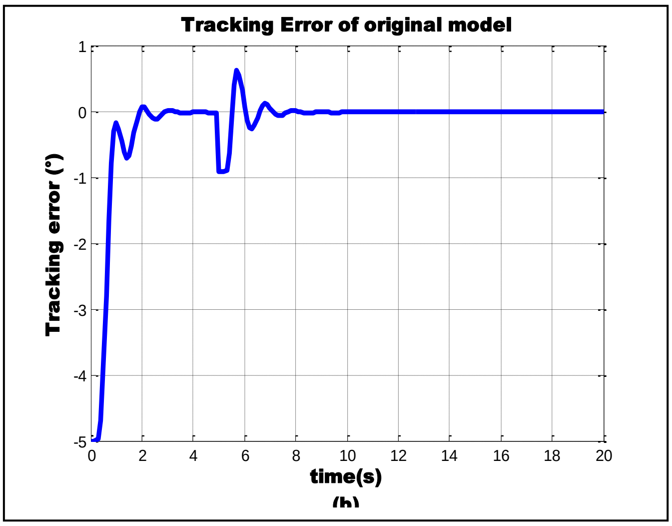
(b): Tracking error of the original model, (c): Tracking error of the reduced order model, (d): command input u (control input), (e): Resulting roll moment, (f): Actuator extension y_a

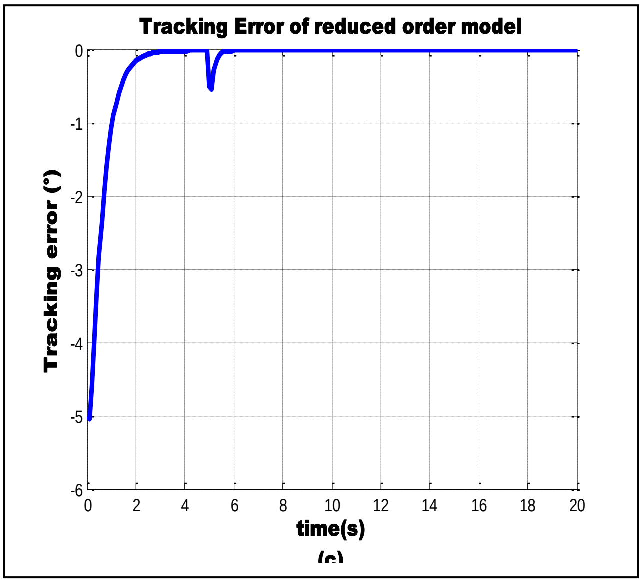
Figure 6(a) compares the responses of the original high order model of the hydraulic actuator to its optimized second order model. There is good approximation between the two models and the RST controller satisfies the underlined specifications (settling time and steady state error). One can see few fluctuations in the original model response due to the missing poles in the establishment of the reduced model.

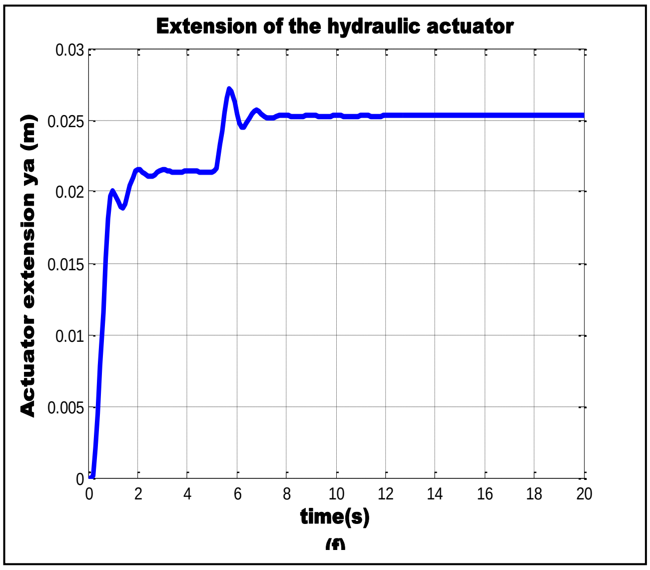
The tracking errors are shown in figure 6(b) and 6(c): the steady state errors are being nullified after a few seconds. In Fig 6(d), the roll angle demand in degrees is illustrated. The steady state value is 0.1°. The resulting roll moment of the actuators, left side and right-side actuators, is given in Fig 6(e). The steady state value is towards 105 N.m (this value must be less than the maximum tolerated moment of the actuator). The sign (-) indicates that the roll moment and the roll angle are in opposite directions.

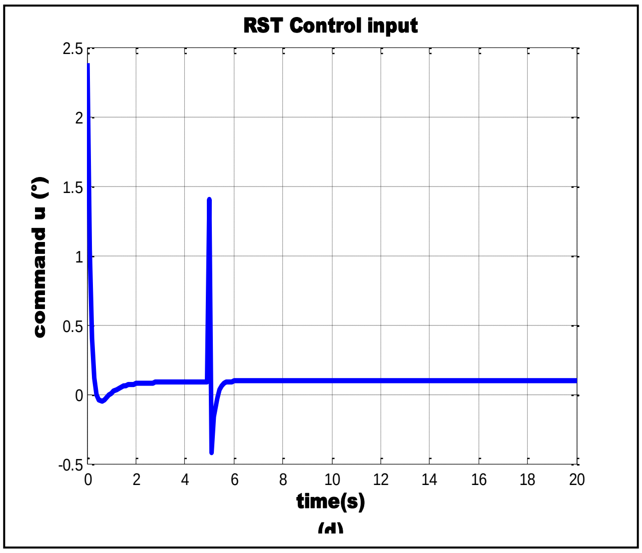
Finally, the actuator extension is shown in Fig 6(f), where the steady state value is 2,2 cm. Now, to check the robustness of the investigated RST controller, we introduce an external output disturbance, at t=5s, where its value is 10% of the plant input.











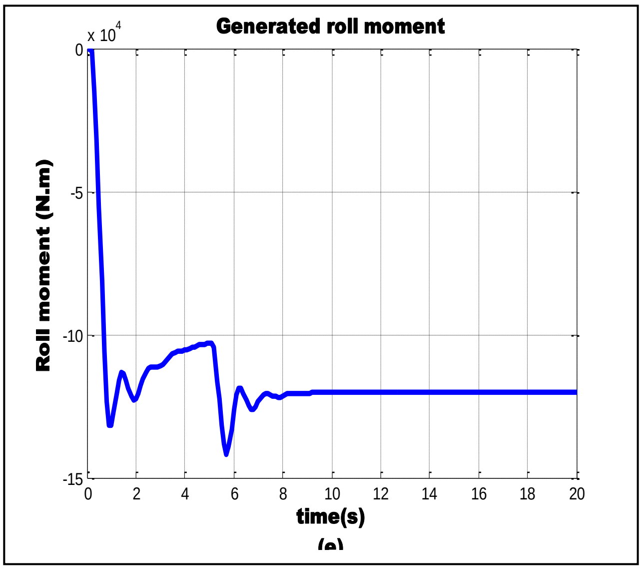


Figure7. RST control results with external output disturbance.

(a): Response of the two models –ROM and HOM- (b): Tracking error of the original model, (c): Tracking error of the reduced order model, (d): command input u (control input), (e): Resulting roll moment, (f): Actuator extension y_a

Fig 7(a) shows fast disturbance rejection of the reduced order model (in 0.6s towards) relatively to the high order model (in 2.5 s towards) this is due to the non-stable poles of the HOM transfer function. This observation is well seen in Fig 7(b) and Fig 7(c).

In Fig 7(d), we see that the controller compensates the disturbance effect in the control input (command).

The Roll moment generated is illustrated in Fig 7(e): the peak value becomes 14.10^4N.m, and the steady state value is towards 122000 N.m .

At last, the actuator extension is shown in Fig 7(f), its maximum becomes 2,75 cm, and its steady state value is towards 2,55 cm.

Table 2. Hydraulic actuator parameters,[4].

Parameter	Description	Value
$\beta_e \left(N/_{m^2}\right)$	Effective bulk modulus of hydraulic oil	$6.89.10^6$
$A_p(m^2)$	Area of piston of hydraulic actuator	0.0123
$C\left(Ns/_{m^2}\right)$	Damping force coefficient	10000
$d_a(m)$	Distance to the actuator from the centerline of suspension	0.215
$d_d(m)$	Distance to damper from centerline of suspension	0.23
$d_s(m)$	Distance to air spring from centerline of suspension	0.535
$d_T(m)$	Half-track width	0.93
$I_{arb}(Kg.m^2)$	Moment of inertia of anti-roll bar about roll center of suspension	6.10
$I_p(Kg.m^2)$	Moment of inertia of sprung mass about roll center of suspension	9500
$k_s(N/m)$	Air spring stiffness	$2.37.10^5$
$k_{arb}(Nm/rad)$	Roll stiffness of anti-roll bar	$1.02.10^6$
$k_h\left(\frac{N}{m.rad}\right)$	Spring stiffness in Merritt's valve-piston model	$1.1033.10^7$
$k_{pe}\left(\frac{m^3/_s}{N/_{m^2}}\right)$	Servo-valve total flow pressure coefficient	$4.2.10^{-11}$
$k_x\left(\frac{m^3/_s}{m}\right)$	Servo-valve flow gain coefficient	2.5
$L(m)$	Distance to displacement transducer from centerline of suspension	0.552
$M_t(kg)$	Mass of load in Merritt's valve-piston model	65.9816
$B_p\left(\left(\frac{kg.N}{m.rad}\right)^{0.5}\right)$	damping coefficient in Merritt's valve-piston model	539.6201
$P_s(bar)$	Supply pressure of hydraulic system	210
$V_t(m^3)$	Volume of 'trapped' oil at high pressure in the hydraulic system	0.0014

6 Conclusion

In this paper, two objectives were underlined. The first is that using the L1 norm, genetic algorithms can be used to find optimal reduced models for a complex high-order SISO model.Moreover, the second is the investigation of the RST controller to control the roll angle for the original model and the reduced order model of a hydraulic actuator.

From the different results, it has been clearly seen that the reduced model using genetic algorithms keeps the main properties of its original model, namely in low frequencies and steady state range. Second, the RST controller provides good tracking and moderate robustness in the face of external disturbances.

Further, the given RST controller has a discrete character and can be easily implemented on the real process (the experimental truck).

References:

[1] Sinha N.K, Pille W, "A new method for order reduction of dynamic systems", International Journal of Control 14(1), 1971 pp. 111-118.

[2] Marshall, S.A, "An approximate method for reducing the order of a linear system", International Journal of Control, Vol. 10, 1966, pp.642–643.

[3] Hsu, C.C. and Yu, C.H. Model Reduction of Uncertain Interval Systems Using Genetic Algorithms. SICE Annual Conference 2004, 1, 264-267, 2004.

[4] Arnaud J.P. Miége, "Development of Active Anti-Roll Control for Heavy Vehicles", First Year Report Submitted to the University of Cambridge, 2000.

[5] S. Babesse, D. Ameddah, "Neuronal active anti-roll control of a single unit heavy vehicle associated with RST control of the hydraulic actuator," International Journal of heavy vehicle Systems, IJHVS,Vol. 22, Issue. 3, 2015.

[6] Massachusetts Institute of Technology What is Model Order Reduction? Retrieved : March 21, 2009,from:http://scripts.mit.edu/~mor/wiki/index.php? title=What_is_Model_order_Reduction.

[7] D. E. Goldberg, Genetic Algorithms is Search, Optimization, and Machine learning, Reading MA: Addison-Wesley, 1989.

[8] Bettayeb, M. Approximation of Linear Systems: New Approaches based on Singular Value Decomposition. Ph.D. Thesis, University of Southern California, Los Angeles, 1981.

[9] El-Attar, R.A. and Vidyasagar, M," Order Reduction by l1 and l∞-Norm Minimization"., IEEE Transaction on Automatic Control, AC-23(4), 1978, 731-734.

[10] Doyle, J., Francis, B. and Tannenbaum, A, Feedback Control Theory, Macmillan Publishing Co, 1990.

[11] Carlos A.S. and Armando B.C, Principles and practice of automatic process control. John Wiley &Sons, 1997.

[12] Landau I. D, Commande des systèmes, conception, identification, et mise en œuvre. Lavoisiers Paris – France, 2002.

[13] Sampson D.J.M., McKevitt P.G., Cebon D, "The development of an active roll control system for heavy vehicles". Proc. 16th IAVSD Symposium on the Dynamics of Vehicles on Roads and Tracks, Pretoria, South Africa, 1999.

[14] Babesse S, Ameddah D and Inel F, "Comparison between RST and PID Controllers Performance of a Reduced Order Model and the Original Model of a Hydraulic Actuator dedicated to a Semi-active Suspension", World Journal of Engineering, 2016.

Creative Commons Attribution License 4.0 (Attribution 4.0 International, CC BY 4.0)

This article is published under the terms of the Creative Commons Attribution License 4.0
https://creativecommons.org/licenses/by/4.0/deed.en_US