Delay-Dependent Stability Analysis and Design of Input-delayed Systems by Smooth Sliding Mode Control: Lagrange Theorem Approach

ELBROUS M. JAFAROV
Aeronautical and Astronautical Engineering Department
Istanbul Technical University
34469, Maslak, TURKEY

Abstract: - A new smooth sliding mode control design methodology based on Lagrange mean value theorem is proposed for stabilization of single input delayed systems. The Lagrange mean value theorem as a basic theorem of calculus is used for the design of linear sliding mode time-delay controller for the first time. This controller satisfies the sliding condition using a Zhou and Fisher type continuous control law eliminating the chattering effect. The constructive delay-dependent asymptotically stable sliding conditions are obtained by using the augmented Lyapunov-Krasovskii functionals and formulated in terms of simple (4x4)-matrix inequality with scalar elements. Developed design approach can be extended to robust stabilization of sliding system with unknown but bounded input delay. The maximum upper bounds of delay size can be found by using simple optimization algorithms. Helicopter hover control is considered as a design example for illustrating the usefulness of smooth sliding mode approach. Unstable helicopter dynamics are successfully stabilized by using linear sliding mode time-delay controller. For example, settling time is about 20 sec. Therefore, simulation results confirmed the effectiveness of proposed design methodology. Apparently, the proposed method has a great potential in design of time-delayed controllers.

Key-Words: - Input-delayed systems, Lagrange mean value theorem, sliding mode control, robust stabilization, Lyapunov-Krasovskii functional method.
Received: June 15, 2021. Revised: April 17, 2022. Accepted: May 18, 2022. Published: June 16, 2022.

1 Introduction

Time-delay effect is frequently encountered in oil-chemical systems, metallurgy and machine-tool process control, nuclear reactors, bio-technical systems missile-guidance and aircraft control systems, aerospace remote control and communication control systems, etc. The presence of delay effect complicates the analysis and design of control systems. Moreover, time delay effects in state vector, especially in control input degrades the control performances and make the closed–loop stabilization problem challenging. A common design method of input-delayed systems is well known Smith predictor control to cancel the effect of time-delay. Smith predictor is a popular and very effective long delay compensator for stable processes. The main advantage of the Smith predictor control method is that, the time-delay is eliminated from the characteristic equation of the closed-loop system. Classical Smith predictor was suggested by Smith [1], [2]. Modified Smith predictor scheme's have been advanced by Marshall [3], Aleviskas and Seborg [4], Watanabe and Ito [5], [6], Al-Sunni and Al-Neymer [7], Majhi and Atherton [8].

The other important control design method of input-delayed systems is the reduction method that was suggested by Kwon and Pearson [9]. Recently several new variable structure control design methods for stabilization of various classes of systems without time-delay are developed, for example by Wang, Lee and Juang [10], Lee and Xu [11], Cao and Xu [12], [13], Choi [14], Edwards, Spurgeon and Hebden [15], Sabanovic, Fridman and Spurgeon [16], Jafarov [17]-[19], Yeh, Chien and Fu [20], Singh, Steinberg and Page [21], Koshkouei and Zinober [22]. But, there is no a large number of papers concerning the problem of stabilization of time-delay systems by variable structure control, for example see Shyu and Yan [23], Yan [24], Luo, De La Sen and Rodellar [25], Gouaisbaut, Dambrine and Richard [26], Richard [27], Perruquetti and Barbot [28], Jafarov [29], [30], Li and De Carlo [31], Gouaisbaut, Blango and Richard [32], Koshkouei and Zinober [33] etc. In analysis and design of time-delay systems by sliding mode control the Lyapunov-Krasovskii functional method is commonly used. Recent advances in time-delay systems are presented by Richard [27], Fridman and Shaked [34], Jafarov [35], Niculescu and Gu [36], Niculescu [37], Mahmoud [38], Gu, Kharitonov and Chen [39], Boukas and Liu [40]. Some sufficient delay-dependent stability conditions for linear delay perturbed systems are derived using exact Lyapunov-Krasovskii functionals by Kharitonov and Niculescu [41]. Several new LMI delay-dependent robust stability results for linear time-delay systems with unknown time-invariant delays by using Padé approximation are presented by Zhang, Knospe and Tsiotras [42]. Both delay-independent and delay-dependent robust stability LMI's from conditions for linear time-delay systems with unknown delays by using appropriately selected Lyapunov-Krasovskii functionals are systematically investigated by Zhang, Knospe and Tsiotras in another paper [43]. Stability of the internet network rate control with diverse delays based on Nyquist criterion is considered by Tian and Yang [44]. Improved delay-dependent stability conditions for time-delay systems in terms of strict LMI's avoiding cross terms are developed by Xu and Lam [45]. A new state transformation is introduced to exhibit the delay-dependent stability condition for time-delay systems by Mahmoud and Ismail [46].

Variable structure control is often used to handle the worst-case control environment: parametric perturbations, external disturbances with knowledge of only the upper bounds etc. Sometimes we may come up with more appropriate control approaches such as incorporating VSC with linear control, time-delay control etc. It is well known that classical sliding mode control uses a discontinuous control action to drive the state to the origin along the reaching and sliding paths and is insensitive to parametric uncertainties and external disturbances. However, the control chattering due to the discontinuity in control law sometimes is undesirable. The continuous sliding mode control approach satisfies the sliding conditions using a continuous control law without requiring discontinuous switching in the controller. Therefore, it retains the advantages of sliding control but without the chattering phenomena. Such approach is used by Zhou and Fisher [47], Shtessel and Buffingtonn [48] etc. Continuous sliding mode control concept is discussed in details and its comparison analysis with the conventional discontinuous sliding mode control by Zhou and Fisher [47].

Note that VSC cannot be directly applied to the control of input-delayed system. Feng, Mian and Weibing [49], Hu, Basker and Crisalle [50] have been successfully used the reduction method combined with variable structure control for stabilization of certain and uncertain multivariable input-delayed systems with known delays.

In this paper, a new sliding mode control design methodology for the single input delayed systems with known or unknown but bounded delays is developed. This design method is based on the Lagrange mean value theorem, which is used for the first time for the stabilization of input-delayed systems. Proposed linear sliding mode time-delay controller also satisfies the sliding condition, but in contrast to classical variable structure control, uses Zhou and Fisher type of continuous control law without requiring discontinuous switching in the controller. Therefore, undesired control chattering in this case is avoided.

The constructive delay-dependent asymptotical stability and robustly stable sliding conditions are obtained by using the Lyapunov-Krasovskii functional method and formulated in terms of some matrix inequalities. Hence, it is possible also to compute the maximum upper bound of the allowable time-delay $\bar{h}$ using efficient convex optimization algorithms. Helicopter hover control is considered as a design example for illustrating the performances of smooth sliding mode approach. Unstable helicopter dynamics is successfully stabilized by using linear sliding mode time-delay controller. For example, settling time is about 20 sec. Therefore, simulation results confirmed the effectiveness of the proposed design methodology.

2 A New Design Method

Let us consider the following single input-delayed system

$$\dot{x}(t) = Ax(t) + bu(t-h) \tag{1}$$

where $x(t)$ is the measurable n-state vector, u(t) is the scalar control input, A is a constant real ($n \times n$)-matrix, b is the constant n-vector, $h > 0$ is a time-delay $h = const > 0$ or unknown but bounded delay $0 < h < \bar{h}$ and initial condition $u(t) = \phi(t)$ for $-h \le t \le 0$, where $\phi(t)$ is a known scalar function.

This design method is based on the Lagrange mean value theorem.

Remember that Lagrange mean value theorem [53], [54] is stated as follows

$$\frac{f(b) - f(a)}{b - a} = f'(\xi), \quad a < \xi < b \tag{2}$$

where $f(x)$ is a continuous at every point of the closed interval [a, b] and differentiable at every point of its interior (a, b) or in terms of delayed control input

$$u(t-h) = u(t) - h\dot{u}(\theta) \tag{3}$$

where θ is a point in $t - h < \theta < t$.

After introducing the θ parameter, the constructive delay-dependent asymptotical stability and robustly stable sliding conditions can be derived by using the augmented Lyapunov-Krasovskii functionals.

Now, after preparing the necessary background we can present a new continuous sliding mode control design methodology for input-delayed systems with known or unknown but bounded delays.

Select a Zhou and Fisher type of continuous sliding mode controller as

$$u(t) = -ks(t) \tag{4}$$

where k is a constant gain scalar to be designed. Assume that linear sliding mode is defined in n-dimensional state space by the following linear function:

$$s(t) = c^T x(t) \tag{5}$$

where c is a design n-vector to be selected. This linear control law must satisfy the sliding condition.

Using the Lagrange mean value theorem (3) let us represent input-delayed system (1) as follows

$$\begin{aligned}\dot{x}(t) &= Ax(t) + b[u(t) - h\dot{u}(\theta)]\\ &= Ax(t) - bks(t) - bh\dot{u}(\theta)\\ &= Ax(t) - bks(t) + kbh\dot{s}(\theta)\\ &= Ax(t) - bks(t) + kbhc^T\dot{x}(\theta)\\ &= Ax(t) - bks(t) + kbhc^T[Ax(\theta) - bks(\theta - h)]\\ &= Ax(t) - bks(t) + kbhc^T Ax(\theta) - k^2hbc^T bs(\theta - h)]\end{aligned} \tag{6}$$

From (6) it is obvious that full delay term h already appears in transformed system. Now, our goal is to organize an asymptotically stable linear sliding mode on defined hyper plane $s(t) = 0$ (5). Stable sliding mode conditions are formulated in the following theorem. But, we need to make the following assumption.

Assumption 1: Time-delay parameter θ is a time-dependent function and norm-bounded such that

$$0 < 1 - \eta \le \dot{\theta}(t) \le \eta < 1 \tag{7}$$

where η is a scalar.

Note that time-delay Assumption 1 is conventional and is commonly used by many authors, for example, by Ikeda and Ashida [55], Su and Chu [56], Su, Ji and Chu [57], Wu, He, She and Liu [58], Kim [59] etc.

Theorem 1: Suppose that Assumption 1 holds. Then the transformed time-delay system (6) driven by continuous sliding mode controller (4), (5), is delay-dependent asymptotically stable relative to the manifold $s(t) = 0$ (5), if there are design parameters k, c, α, β, γ and η such that the following sliding conditions are satisfied:

$$H = \begin{bmatrix} \lambda - kc^T\mathrm{b} + \beta + \gamma & \frac{1}{2}khc^Tb\lambda & -\frac{1}{2}k^2h(c^Tb)^2 & 0\\ \frac{1}{2}khc^Tb\lambda & \alpha\eta - (1-\eta)\beta & 0 & 0\\ -\frac{1}{2}k^2h(c^Tb)^2 & 0 & -\alpha(1-\eta) & 0\\ 0 & 0 & 0 & -\gamma \end{bmatrix}$$

$$H < 0 \tag{8}$$

or

$$k > \frac{\lambda + \beta + \gamma}{c^T b} \tag{9}$$

$$\alpha < \frac{1-\eta}{\eta}\beta \tag{10}$$

$$c^T A = \lambda c^T \tag{11}$$

where λ is any left or right eigenvalue of matrix A; α, β and γ are some positive adjustable scalars.

Note that, design of the manifold $s(t) = 0$ (5) does not imply assigning the eigenvalue λ of the matrix; it appears only in proof of the theorem and may take an arbitrary value as pointed by Ackermann and Utkin [52].

Proof: Choose an augmented Lyapunov-Krasovskii functionals as

$$V(s(t), s(\theta), s(\theta - h), s(t-h)) = \frac{1}{2}s^2(t) + \alpha\int_{\theta-h}^{\theta} s^2(\zeta)d\zeta + \beta\int_{\theta}^{t} s^2(\xi)d\xi + \gamma\int_{t-h}^{t} s^2(\varphi)d\varphi \tag{12}$$

where α, β and γ are some positive adjustable scalars.

The time derivative of (12) along the state trajectory of (6) can be calculated as follows:

$$\begin{aligned}\dot{V} &= s(t)\dot{s}(t) + \dot{\theta}(t)\alpha(s^2(\theta) - s^2(\theta-h)) + \beta s^2(t)\\ &\quad - \dot{\theta}(t)\beta s^2(\theta) + \gamma s^2(t) - \gamma s^2(t-h)\\ &= s(t)[c^T Ax(t) - kc^T bs(t) + khc^T bc^T Ax(\theta)\\ &\quad - k^2h(c^Tb)^2 s(\theta-h)] + (\beta+\gamma)s^2(t) - \gamma s^2(t-h)\\ &\quad + \alpha\dot{\theta}(t)s^2(\theta) - \alpha(\theta)\dot{\theta}(t)s^2(\theta-h) - \beta\dot{\theta}(t)s^2(\theta)\end{aligned} \tag{13}$$

Since

$$\dot{\theta}(t)s^2(\theta) \le \eta s^2(\theta) \tag{14}$$

$$-\dot{\theta}(t)s^2(\theta-h) \le -(1-\eta)s^2(\theta-h) \tag{15}$$

and (11) hold, then (13) reduces to

$$\begin{aligned}\dot{V} &\le \lambda s^2(t) - kc^T bs^2(t) + khc^T b\lambda s(\theta)s(t)\\ &\quad - k^2h(c^Tb)^2 s(\theta-h)s(t) + (\beta+\gamma)s^2(t)\\ &\quad - \gamma s^2(\theta-h) + \eta\alpha s^2(\theta) - (1-\eta)\alpha s^2(\theta-h)\\ &\quad - (1-\eta)\beta s^2(\theta) - \gamma s^2(t-h)\\ &= (\lambda - kc^T b + \beta + \gamma)s^2(t) + khc^T b\lambda s(\theta)s(t)\\ &\quad - k^2h(c^Tb)^2 s(t)s(\theta-h) + (\eta\alpha - (1-\eta)\beta)s^2(\theta)\\ &\quad - (1-\eta)\alpha s^2(\theta-h) - \gamma s^2(t-h)\\ &= \begin{bmatrix} s(t)\\ s(\theta)\\ s(\theta-h)\\ s(t-h)\end{bmatrix}^T \begin{bmatrix} \lambda - kc^T\mathrm{b} + \beta + \gamma & \frac{1}{2}khc^Tb\lambda & -\frac{1}{2}k^2h(c^Tb)^2 & 0\\ \frac{1}{2}khc^Tb\lambda & \alpha\eta - (1-\eta)\beta & 0 & 0\\ -\frac{1}{2}k^2h(c^Tb)^2 & 0 & -\alpha(1-\eta) & 0\\ 0 & 0 & 0 & -\gamma \end{bmatrix} \begin{bmatrix} s(t)\\ s(\theta)\\ s(\theta-h)\\ s(t-h)\end{bmatrix} = y^T(t)Hy(t) < -\lambda_{\min}(H)\|y(t)\|^2 < 0\end{aligned} \tag{16}$$

where $y(t) = \begin{bmatrix} s(t) & s(\theta) & s(\theta-h) & s(t-h)\end{bmatrix}^T$

Note that matrix H has its own quadratic structure $H = MH_1M^T$

where

$$H_1 = \begin{bmatrix} \lambda+\beta+\gamma-kc^Tb & 0 & 0 & 0 \\ 0 & \alpha\eta-(1-\eta)\beta & 0 & 0 \\ 0 & 0 & -\alpha(1-\eta) & 0 \\ 0 & 0 & 0 & -\gamma \end{bmatrix}$$

$$M = \begin{bmatrix} 1 & \dfrac{0.5kh\lambda c^Tb}{\alpha\eta-(1-\eta)\beta} & \dfrac{0.5k^2h(c^Tb)^2}{\alpha(1-\eta)} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Since M is a nonsingular and $H_1 < 0$ because its leading principle elements are always negative then $H < 0$. Therefore, condition (16) means that manifold $s(t) = 0$ is reached in finite time and the reaching time can be evaluated approximately as follows:

$$t_s \le \frac{\|y(0)\|}{\lambda_{\min}(H)} \tag{17}$$

Thus, the time-delay system (6) with known delay is delay-dependent asymptotically stable relative to the manifold $s(t) = 0$ (5).

If we consider a case where the delay term is unknown but bounded $0 < h < \bar{h}$ then we can solve the following convex optimization problem:

OP: maximize h
Subject to conditions (8) (18)
with $\alpha > 0$, $\beta > 0$, $\gamma > 0$

This ends the proof of Theorem 2.

Let us consider a simple analytical example to illustrate our design approach

Example 1: Consider the first order input-delayed system

$$\dot{x}(t) + ax(t) = bu(t-h) \tag{19}$$

where a and b are some constant scalars.

Define a continuous sliding mode controller as follows.

$$u(t) = -ks(t) \tag{20}$$

$$s(t) = cx(t) \tag{21}$$

where k and c are the design scalars.

Substituting (3) with (20) and (21) into (19) we have

$$\begin{aligned}\dot{x}(t) &= -ax(t) + bu(t-h) \\ &= -ax(t) + bu(t) - bh\dot{u}(\theta) \\ &= -ax(t) - bks(t) + bhk\dot{s}(\theta) \\ &= -ax(t) - bks(t) + bhkc\dot{x}(\theta) \\ &= -ax(t) - bks(t) - abhkcx(\theta) - b^2hk^2cs(\theta-h) \\ &= -ax(t) - bks(t) - abhks(\theta) - b^2hk^2cs(\theta-h)\end{aligned} \tag{22}$$

Then the time-derivative of (12) along (22) is given by

$$\begin{aligned}\dot{V} &= s(t)\dot{s}(t) + \dot{\theta}(t)\alpha\left[s^2(\theta) - s^2(\theta-h)\right] \\ &\quad + \beta s^2(t) - \dot{\theta}(t)\beta s^2(\theta) + \gamma s^2(t) - \gamma s^2(t-h) \\ &= s(t)[-acx(t) - bkcs(t) - abchks(\theta) \\ &\quad - b^2c^2hk^2 s(\theta-h)] + \dot{\theta}(t)\alpha s^2(\theta) - \dot{\theta}(t)\alpha s^2(\theta-h) \\ &\quad + \beta s^2(t) - \dot{\theta}(t)\beta s^2(\theta) + \gamma s^2(t) - \gamma s^2(t-h) \\ &\le \left(-a - bck + \beta + \gamma\right)s^2(t) - abchks(t)s(\theta) \\ &\quad - b^2c^2hk^2 s(t)s(\theta-h) + \left(\alpha\eta + \beta(1-\eta)\right)s^2(\theta) \\ &\quad - \alpha(1-\eta)s^2(\theta-h) - \gamma s^2(t-h)\end{aligned} \tag{23}$$

$$= \begin{bmatrix} s(t) \\ s(\theta) \\ s(\theta-h) \\ s(t-h) \end{bmatrix}^T \begin{bmatrix} -bck - a + \beta + \gamma & -\frac{1}{2}abchk & -\frac{1}{2}b^2c^2hk^2 & 0 \\ -\frac{1}{2}abchk & \alpha\eta-(1-\eta)\beta & 0 & 0 \\ -\frac{1}{2}b^2c^2hk^2 & 0 & -\alpha(1-\eta) & 0 \\ 0 & 0 & 0 & -\gamma \end{bmatrix} \begin{bmatrix} s(t) \\ s(\theta) \\ s(\theta-h) \\ s(t-h) \end{bmatrix} = y^T(t)\bar{H}y(t) < -\lambda_{\min}(\bar{H})\|y(t)\|^2 < 0 \tag{24}$$

or the following Sylvester's conditions hold:

$$|\bar{H}_1| = -bck - a + \beta + \gamma < 0 \text{ or } bck > -a + \beta + \gamma$$
$$\text{with } \beta + \gamma > 0 \tag{25}$$

$$|\bar{H}_2| = \left[-bck - a + \beta + \gamma\right]\left[\alpha\eta + \beta(1-\eta)\right] - \frac{1}{4}(abchk)^2 < 0 \tag{26}$$

$|\bar{H}_3| < 0$ and $|\bar{H}_4| < 0$ respectively.

Then, time-delay system (22) with known h is delay-dependent asymptotically stable relative to the $s(t) = 0$.

If we consider a case where h is unknown but bounded $0 < h < \bar{h}$ then the maximum upper bound can be calculated as follows.

From min $|H_2| < 0$ (26) we compute

$$\frac{\partial|\bar{H}_2|}{\partial k} = -abc\left[\alpha\eta + \beta(1-\eta)\right] - \frac{1}{2}(abch)^2 k = 0 \tag{27}$$

Hence, $$\bar{h} = \sqrt{\frac{-2bc\left[\alpha\eta + \beta(1-\eta)\right]}{k(abc)^2}} \tag{28}$$

with $\alpha\eta + \beta(1-\eta) < 0$, $bc > 0$.

Thus, time-delay system (22) with unknown but bounded delay term is robustly asymptotically stable relative to the $s(t) = 0$ with upper bound $\bar{h}$ (28).

3 Design example: Helicopter hover control

The linearized longitudinal motion of helicopter near hover (Fig.1) can be modeled by the normalized linear third order system [60] with introduced pilot time-delay h [61] as follows:

$$\begin{bmatrix}\dot{q}\\ \dot{\theta}\\ \dot{u}\end{bmatrix}=\begin{bmatrix}-0.4 & 0 & -0.01\\ 1 & 0 & 0\\ -1.4 & 9.8 & -0.02\end{bmatrix}\begin{bmatrix}q\\ \theta\\ u\end{bmatrix}+\begin{bmatrix}6.3\\ 0\\ 9.8\end{bmatrix}\delta(t-h) \qquad (29)$$

where
q is the pitch rate,
θ is the pitch angle of fuselage,
u is the horizontal velocity (standard aircraft notation),
δ is the rotor tilt angle (control variable),
h is the pilot's effective time- delay, for example, $h=0.43$ s.

Continuous sliding mode controller is formed as (4):

$$u(t)=-ks(t) \qquad (30)$$

where k is a scalar to be designed by (8), (9) and sliding function is defined as (5):

$$s(t)=c_1q+c_2\theta+c_3u \qquad (31)$$

where c_1,c_2,c_3 are design parameters to be determined.

Design procedure can be fulfilled with MATLAB programming (which is given in Appendix 1) by the following steps:

$$eig(A)=\begin{bmatrix}\text{-0.6565}\\ \text{0.1183 + 0.3678i}\\ \text{0.1183 - 0.3678i}\end{bmatrix}$$

A is unstable with one pear conjugate complex-roots.
Calculate matrix (8)

$$H=\begin{bmatrix}-0.0313 & 0.0177 & -0.0034 & 0\\ 0.0177 & -0.1640 & 0 & 0\\ -0.0034 & 0 & -0.1820 & 0\\ 0 & 0 & 0 & -0.3000\end{bmatrix}$$

$$eig(H)=\begin{bmatrix}-0.3000\\ -0.1821\\ -0.1663\\ -0.0289\end{bmatrix}$$

H is a negative definite matrix.
c1 = -0.0389
c2 = 0.0592
c3 = -0.9975
k = 0.0125
h = 0.4300
eta = 0.0900
alpha = 0.2000
beta =0.0200
gamma = 0.3000
hmax = 1.714
cTb = -10.0204

Thus all design parameters are calculated. Maximum upper bound of time delay, hmax = 1.714, is found from condition (8). A block diagram of continuous sliding mode controller for helicopter input-delayed system (1), (4), (5) or (29), (30), (31) is shown in Fig. 2. This system is simulated by using MATLAB-Simulink. Continuous sliding mode controller is performed by linear Simulink blocks $s(t)$ and $u(t)$. Note that, these are not variable structure blocks, but linear blocks satisfying the sliding condition (16). Helicopter control performances are shown in Fig. 3, from which can be seen that unstable helicopter dynamics is successfully stabilized by using linear sliding mode controller. For example, settling time is about 20 sec. Reaching time is also 20 sec. Therefore, simulation results confirmed the usefulness of the developed design methodology.

4 Conclusion

A new continuous sliding mode control design methodology based on Lagrange mean value theorem is proposed for stabilization of single input delayed systems. The Lagrange mean value theorem as a basic theorem of calculus is used for the design of linear sliding mode time-delay controller for the first time. This controller satisfies the sliding condition using a Zhou and Fisher type continuous control law eliminating the chattering effect. The constructive delay-dependent asymptotically stable sliding conditions are obtained by using the augmented Lyapunov-Krasovskii functionals and formulated in terms of simple (4×4)-matrix inequality with scalar elements. Developed design approach are extended to robust stabilization of sliding system with unknown but bounded input delay. The maximum upper bounds of delay size are found by using simple optimization algorithms. Helicopter hover control is considered as design example for illustrating the performances of smooth sliding mode approach. Unstable helicopter dynamics are successfully stabilized by using linear sliding mode time-delay controller. For example, settling time is about 20 sec. Therefore, simulation results confirmed the effectiveness of the proposed design methodology. Apparently, the proposed method has a great potential in design of time-delayed controllers.

References:

[1] Smith, O.J.M, 1957, Closer control of loops with dead time. *Chemical Engineering Progress*, vol. 53, pp.217-219.
[2] Smith, O.J.M, 1958, *Feedback Control Systems*, New York: McGraw-Hill.
[3] Marshall, J. E., 1974, Extensions of O. J. Smith's method to digital and other systems. *International Journal of Control*, vol. 19, no. 5 pp. 933-939.
[4] Aleviskas, G. and Seborg D.E., 1973, An extension of the Smith Predictor method to multivariable linear systems containing time delays. *International Journal of Control*, vol. 3., no. 17, pp. 541-551

[5] Watanabe, K., and Ito, M., 1984, A process-model control for the linear systems with delay. *IEEE Transactions on Automatic Control*, vol. AC-26, no. 6, pp. 1261-1269

[6] Watanabe, K., and Ishiyama Y., and Ito, M., 1983, Modified Smith Predictor control for the multivariable systems with delays and unmeasurable step disturbances. *International Journal of Control*, vol. 37, no. 5, pp.959-973.

[7] Al-Sunni, F.M. and Al-Nemer, T.M, 1997, A Smith Predictor FL-based controller for processes with long dead time. *International Journal of Control*, vol. 28, no. 12, pp. 1251-1258.

[8] Majhi, S., and Atherton, D. P, 1999, Modified Smith predictor and controller for processes with time-delay. *IEE Proceedings Control Theory and Applications*, vol. 146, No. 5, pp. 359-366.

[9] Kwon, W. H., and Pearson, A. E., 1980, Feedback stabilization of linear systems with delayed control. *IEEE Transactions on Automatic Control*, vol. AC-75, no. 2, pp. 266-269.

[10] Wang., J. D., Lee, T.L., and Juang, Y.T., 1996, New methods to design an integral variable structure controller. IEEE Transactions on Automatic Control, vol. 41, No 1, pp. 140-143.

[11] Lee, J.J., and Xu, Y., 1994, A new method of switching surface design for multivariable structure systems. *IEEE Transactions on Automatic Control*, vol. 39, No 2, pp. 414-419.

[12] Cao, W.J., and Xu, J. X., 2002, On functional approximation of the equivalent control using learning variable structure control. *IEEE Transactions on Automatic Control*, vol. 47, No 5, pp. 824-830.

[13] Cao, W.J., and Xu, J.X., 2004, Nonlinear integral type sliding surface for both matched and unmatched uncertain systems. *IEEE Transactions on Automatic Control*, vol. 49, no. 8, pp. 1355-1360.

[14] Choi, H.H., 2004, An analysis and design method for uncertain variable structure systems with bounded controllers. *IEEE Transactions on Automatic Control*, vol. 49, no. 4, pp. 602-607.

[15] Edwards, C., Spurgeon, S.K., and Hebden, R.G., 2003, On the design of sliding mode output feedback controllers. *International Journal of Control*, vol. 76, no. 9/10, pp. 893-905.

[16] Sabanovic, A., Fridman, L. M., and Spurgeon, S. (Editors), 2004, *Variable Structure Systems: from Principles to Implementation*, The IEE, London.

[17] Jafarov, E. M., and Tasaltin, R., 2000, Robust sliding-mode control for the uncertain MIMO aircraft model F-18, *IEEE Transactions on Aerospace and Electronic Systems*, vol.36, no.4, pp. 1127-1141.

[18] Jafarov, E. M, and Tasaltin, R., 2001, Design of robust autopilot-output integral sliding mode controllers for guided missile systems with parameter perturbations, *Aircraft Engineering and Aerospace Technology: An International Journal*, vol.73, no.1, pp. 16-25.

[19] Jafarov, E. M, Parlakci, M. N. A., and Istefanopulos, Y., 2005, A new variable structure PID-controller design for robot manipulators, *IEEE Transactions on Control Systems Technology*, vol.13, no.1, pp.122-130.

[20] Yeh, F. K., Chien, H. H., and Fu, L. C., 2003, Design of optimal midcourse guidance sliding-mode control for missiles with TVC, *IEEE Transactions on Aerospace and Electronic Systems*, vol. 39, no.3, pp.824-837.

[21] Singh, S. N., Steinberg, M. L., and Page A. B., 2003, Nonlinear adaptive and sliding-mode flight path control of F/A-18 model, *IEEE Transactions on Aerospace and Electronic Systems*, vol.39, no.4, pp.1250-1262.

[22] Koshkouei, A. J., and Zinober, A. S. I., 2004, Sliding mode state observation for nonlinear systems, *International Journal of Control*, vol.77, no.2, pp.118-127.

[23] Shyu, K.K., and Yan, J.J., 1993, Robust stability of uncertain time-delay systems and its stabilization by variable structure control, *International Journal of Control*, vol. 57, no. 1, pp. 237-246.

[24] Yan, J.J., 2003, Sliding mode control design for uncertain time-delay systems subjected to a class of nonlinear inputs. *International Journal of Robust and Nonlinear Control*, vol. 13, pp. 519-532.

[25] Luo, N., DE La Sen, N.L.M. and Rodellar, J., 1997, Robust stabilization of a class of uncertain time-delay systems in sliding mode. *International Journal of Robust and Non-linear Control*, no. 1, pp. 59-74.

[26] Gouaisbaut, F., Dambrine, M., and Richard, J.P., 2001, Robust control of delay systems: a sliding mode control design via LMI. *Systems and Control Letters*, vol. 46, pp. 219-230.

[27] Richard, J.P., 2003, Time-delay systems: an overview of some recent advances and open problems. *Automatica*, vol. 39, pp. 1667-1694.

[28] Perruquetti, W., and Barbot, J.P. (Eds), 2002, *Sliding Mode Control in Engineering*, Marcel Dekker, New York.

[29] Jafarov, E. M., 1998, State feedback sliding mode control for SISO systems with time-delay. Proceedings of *8th IFAC Symposium Large Scale Systems: Theory and Applications LSS'98*, Rio Patras, July 15-17, Greece, pp. 195-200.

[30] Jafarov, E.M., 2003, Robust sliding mode control of multivariable time-delay systems. *Systems Science Journal*, vol. 29, no. 3, pp. 17-36.

[31] Li, X., and DeCarlo, R. A., 2003, Robust sliding mode control of uncertain time-delay systems,

International Journal of Control, vol.76, no.13, pp.1296-1305.3

[32] Gouaisbaout, F., Blango, Y., and Richard, J. P., 2004, Robust sliding mode control of non-linear systems with delay: a design via polytopic formulation, *International Journal of Control*, vol.77, no.2, pp.206-215.

[33] Koshkouei, A. J., and Zinober, A. S., 1996, Sliding mode time-delay systems, *Proceedings of the International Workshop of VSS*, Tokyo, Japan, pp.97-101.

[34] Fridman, E., and Shaked, U., 2003, Delay-dependent stability and H_∞ control: constant and time-varying delays, *International Journal of Control*, vol.76, no.1, pp.48-60.

[35] Jafarov, E. M., 2003, Delay-dependent stability and α-stability criterions for linear time-delay systems, *WSEAS Transactions on Systems*, no. 4, vol. 2, pp.1138-1147.

[36] Niculescu, S. I., and Gu, K., 2004, *Advances in Time-Delay Systems*, Springer-Verlag, Berlin.

[37] Niculescu, S. I., 2001, *Delay Effects on Stability*, Springer-Verlag, London.

[38] Mahmoud, M. S., 2000, *Robust Control and Filtering for Time-Delay Systems*, Marcel Dekker, New York.

[39] Gu, K., Kharitonov, V. L., and Chen, J., 2003, *Stability of Time-Delay Systems*, Birkhauser, Boston.

[40] Boukas, E. K., and Liu, Z. K., 2002, *Deterministic and Stochastic Time Delay Systems*, Birkhauser, Boston.

[41] Kharitonov, V.L., and Niculescu, S.I, 2003, On the stability of linear systems with uncertain delay, *IEEE Transactions on Automatic Control*, vol. 18, no. 1, pp. 127-132.

[42] Zhang, X, Knospe, C.R., and Tsiotras, P, 2002, Stability analysis of LPV time-delayed systems, *International Journal of Control*, vol. 75, no. 7, pp. 538-558.

[43] Zhang, J, Knospe, C.R., and Tsiotras, P, 2003, New results for the analysis of linear systems with time-invariant delays, *International Journal of Robust and Nonlinear Control*, vol 13, pp.1149-1175.

[44] Yu-Ping Tian, Hong-Yong Yang, 2004, Stability of internet congestion control with diverse delays, *Automatica*, In Press, Corrected Proof, Available online 11 May 2004, 9 pages.

[45] Xu, S., and Lam, J., 2005, Improved delay-dependent stability criteria for time-delay systems, *IEEE Transactions on Automatic Control*, vol.50, no.3, pp.384-387.

[46] Mahmoud, M. S., and Ismail, A., 2005, New results on delay-dependent control of time-delay systems, *IEEE Transactions on Automatic Control*, vol.50, no.1, pp.95-100.

[47] Zhou, F., and Fisher, D. G., 1992, Continuous sliding mode control. *International Journal of Control*, vol.55, no. 2, pp. 313-327.

[48] Shtessel, Y. B., and Buffington, J. M., 1998, Continuous sliding mode control. *Proceedings of the 1998 American Control Conference*, Adam's Mark Hotel, Philadelphia, Pennsylvania USA, June 24-26, vol. 1, pp. 562-563.

[49] Feng, Z., Mian, C., and Weibing G., 1993, Variable structure control of systems with delays in control. *Journal of Control and Decision*, vol. 8, no. 2, pp. 95-100 (in Chinese).

[50] Hu, K.J., Basker, V. R., and Crisalle, O. D., 1998, Sliding mode control of uncertain input-delay systems. *Proceedings of the 1998 American Control Conference*, Adam's Mark Hotel, Philadelphia, Pennsylvania USA, June 24-26, vol. 1, pp. 564-568.

[51] Boid, S., Ghaouı, L. E., Feron, E., and V. Balakrishnan, 1994, *Linear Matrix Inequalities in System and Control Theory*. Philadelphia PA: SIAM.

[52] Ackermann, J., and Utkin, V., 1998, Sliding mode control design based on Ackermann's formula. *IEEE Transactions on Automatic Control*, vol. 43, no.2, pp. 234-237.

[53] Thomas, G. B., Jr. And R. L. Finney, 1992, *Calculus*. Addison-Wesley Publishing Company, 8th Edition.

[54] Behrmant, A. F., 1964, *Brief Course of Mathematical Analysis*, Moscow, Nauka (in Russian).

[55] Ikeda, M., and Ashida, T., 1979, Stabilization of linear systems with time-varying delay. *IEEE Transactions on Automatic Control*, vol. AC-24, no.2, pp. 369-370.

[56] Su, H., and Chu, J., 1999, Robust H_∞ control for linear time-varying uncertain time-delay systems via dynamic output feedback, *International Journal of Systems Science*, vol.30, no.10, pp.1093-1107.

[57] Su., H., Ji, X., and Chu, J., 2005, New results of robust quadratically stabilizing control for uncertain linear time-delay systems, *International Journal of Systems Science*, vol.36, no.1, pp.27-37.

[58] Wu, M., He, Y., She, J. H., and Liu, G. P., 2004, Delay-dependent criteria for robust stability of time-varying delay systems, Elsevier, *Automatica*, vol.40, pp.1435-1439.

[59] Kim, J. H., 2001, Delay and its time-derivative dependent robust stability of time-delayed linear systems with uncertainty, *IEEE Transactions on Automatic Control*, vol.46, no.5, pp.789-792.

[60] Franklin, G.F., Powell, J.D., and Emami-Naeini, A., 2002, *Feedback Control of Dynamic Systems, 4th Ed.*, New Jersey: Prentice Hall.

[61] Blakelock, J.H., 1991, *Automatic Control of Aircraft and Missiles*. 2nd Ed., New York: John Wiley.

Appendix:

```
clear; clc;
A = [-0.4 0 -0.01; 1 0 0;
    -1.4 9.8 -0.02];
[V,D] = eig(A);
D     = diag(D)
% selection according to case a):
lamda_L = D(1)
c1    = V(1,1)
c2    = V(2,1)
c3    = V(3,1)
h     = 0.43
eta   = 0.09
alpha = 0.2
beta  = 0.2
gamma = 0.3
c_T   = [c1 c2 c3];
b     = [6.3; 0; 9.8];
k     = 0.8*(lamda_L+beta+gamma)/(c_T*b)
h_max = 1.714 % delay
cTb   = c_T*b
h11 = lamda_L-k*c_T*b+beta+gamma
h22 = alpha*eta-(1-eta)*beta
h33 =-alpha*(1-eta)
h44 = -gamma
H1 = [ lamda_L-k*c_T*b+beta+gamma;
       0.5*k*h*c_T*b*lamda_L;
      -0.5*k^2*h*(c_T*b)^2; 0];
H2 = [0.5*k*h*c_T*b*lamda_L;
      alpha*eta-(1-eta)*beta; 0; 0];
H3 = [-0.5*k^2*h*(c_T*b)^2; 0;
      -alpha*(1-eta); 0];
H4 = [0; 0; 0; -gamma];
H  = [H1 H2 H3 H4];
```

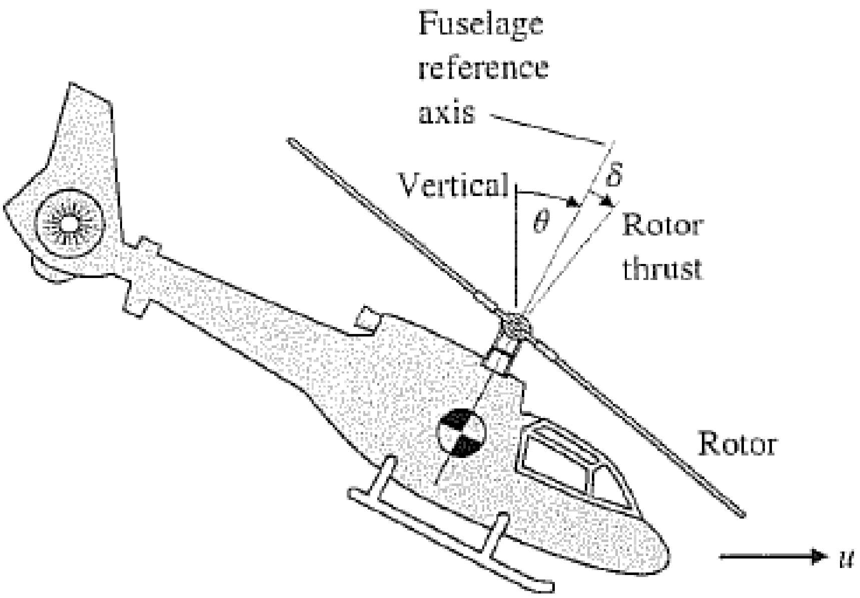


Fig.1 Helicopter.

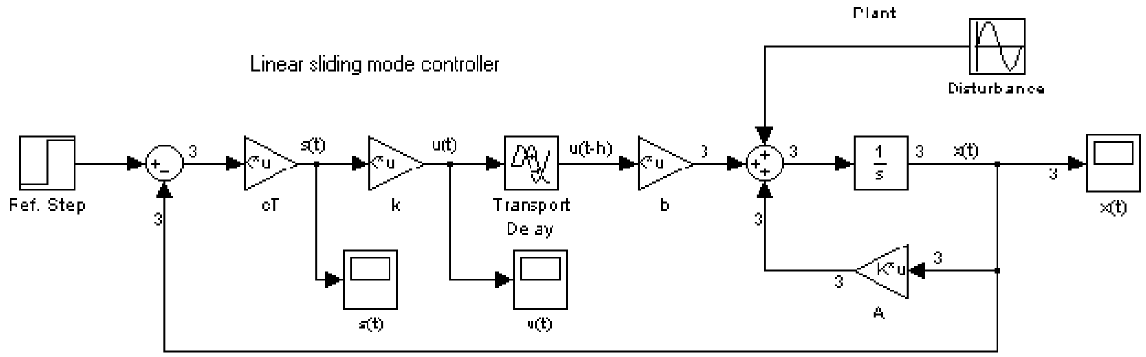
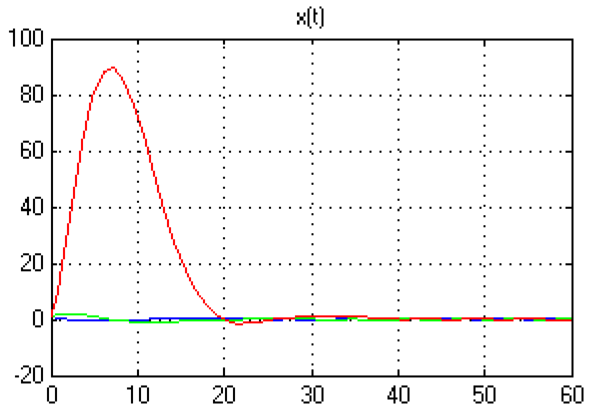
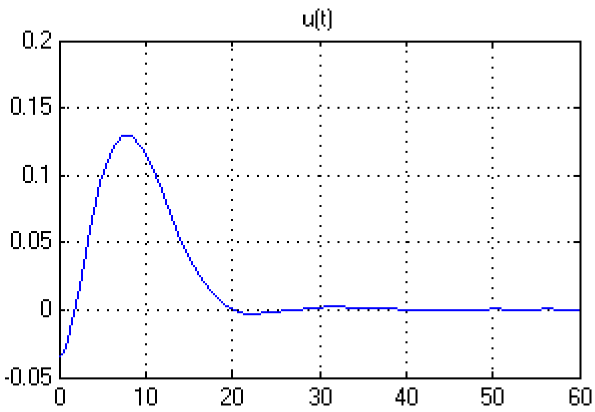


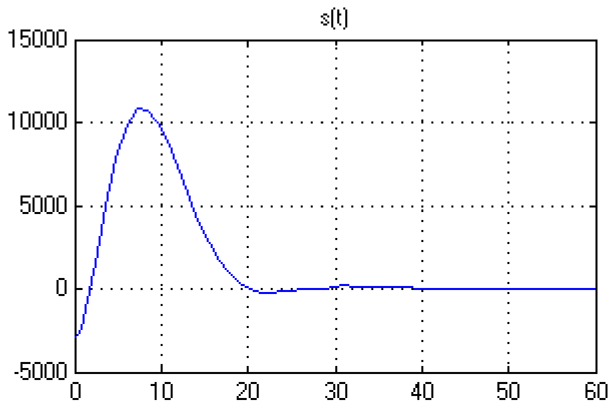
Fig.2 Block diagram of linear sliding mode controller for input-delayed system



a) State time responses



b) Linear sliding mode control function



c) Sliding function

Fig.3 Smooth sliding mode control