Nonlinear Control of the Permanent Magnet Synchronous Motor PMSM using Backstepping Method

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Abstract: - This paper presents a nonlinear control of (PMSM) using backstepping. We will study the different performances and robustness of each type of control, by introducing a new Lyapunov function candidate with a large possibility of parameter choice. Simulation results clearly show that the speed and current tracking errors asymptotically converge to zeros. Compared with neural networks control schemes, we do not require the unknown parameters to be linear parametrizable. No regression matrices are needed, so no preliminary dynamical analysis is needed.

Key-Words: - Permanent magnet synchronous motor (PMSM), Backstepping control, Lyapunov fonction

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1 Introduction

Permanent magnet synchronous motor (PMSM) is widely used in industrial applications compared to other electric motors. Mainly, due to its compact design, high efficiency, high torque-inertia ratio, excellent reliability, high robustness, and low maintenance [1, 2] PMSM is used in wind and photovoltaic renewable energy, transportation (electric cars), railway traction and ship propulsion also make extensive use of these machines and in other fields. On the other hand, the non-linearity of the PMSM dynamic model produces a great difficulty of specific control. The parameters and load torque variations also the coupling between motor speed and electrical quantities, such as d-q axis currents, making this system obviously difficult to control [1, 3]. This motor can be controlled by the conventional PI controller but cannot guarantee satisfactory performance such as stability and control against disturbances [4]. To solve this problem, various nonlinear control methods have been developed and proposed to control and command the PMSM, such as input-output linearization control [5], sliding mode control [6], backstepping control [7] and DTC [8]...etc. Recently, Backstepping control is developed a technique for controlling uncertain nonlinear systems, in particular systems that do not satisfy the adaptation conditions [9, 10]. The most interesting point is to use the virtual control variable to simplify the original high-order system, so that the final control outputs can be derived systematically by appropriate Lyapunov functions. A robust and adaptive nonlinear controller, directly derived from this control method, is proposed for the speed control of PMSMs [11, 12]. The controller is robust against stator resistance, viscous friction, load torque uncertainties and unknown disturbances. However, this approach uses feedback linearization, the use of which can cancel out some useful non-linearity [13]. Of all the nonlinear adaptive control methods in the literature, the backstepping design on the control of highly nonlinear and uncertain systems has excellent performance in terms of its ability to adapt to parameter uncertainties, transient and steady-state performance, disturbance rejection capability, and suitability for real time implementation [10].

2 Mathematical Model of the PMSM

The PMSM model in the reference frame (d-q) is shown as follows:

$$\begin{cases} \frac{di_d}{dt} = -\frac{R_s}{L_d} i_d + \frac{L_q}{L_d} \omega i_q + \frac{1}{L_d} v_d \\ \frac{di_q}{dt} = -\frac{R_s}{L_q} i_d - \frac{L_d}{L_q} \omega i_d - \frac{\varphi_f}{L_q} \omega + \frac{1}{L_q} v_q \\ \frac{d\Omega}{dt} = \frac{P}{J} [(L_d - L_q) i_d + \varphi_f) i_q] - \frac{f}{J} \Omega - \frac{1}{J} C_r \end{cases} \tag{1}$$

In the above equations, we denote by:

- v_d, v_q is the stator voltages in (d-q) reference frame,
- i_d, i_q is the stator currents in (d-q) reference frame,
- L_d, L_q is the d-axes and q-axes stator inductance,
- ω Electrical pulse,
- Ω Rotor speed,
- J is rotor inertia,
- C_r Torque resistance,
- φ_f is the magnet flux,
- R_s is the stator resistance,
- f Viscous friction coefficient,
- P Number of pole paire of the PMSM,

2.1 The General Equations of State of the PMSM in the (d-q) Reference Frame

The state writing depends on the chosen reference frame, we see that the state representation is not unique. Any linear combination of the components of a state vector is called state variables. By developing the system of equations (1) we can deduce the final form of the PMSM equations in the reference frame (d-q):

$$\begin{bmatrix} \frac{di_d}{dt} \\ \frac{di_q}{dt} \\ \frac{d\Omega}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{R_s}{L_d} i_d + \frac{L_q}{L_d} \omega i_q \\ -\frac{R_s}{L_q} i_d - \frac{L_d}{L_q} \omega i_d - \frac{\varphi_f}{L_q} \omega \\ P \frac{\varphi_f}{J} i_d - \frac{P(L_q - L_d)}{J} i_d i_d - \frac{f}{J} \Omega \end{bmatrix} + \begin{bmatrix} \frac{1}{L_d} & 0 & 0 \\ 0 & \frac{1}{L_q} & 0 \\ 0 & 0 & \frac{-1}{J} \end{bmatrix} \begin{bmatrix} v_d \\ v_d \\ C_r \end{bmatrix} \tag{2}$$

The equation (2) represents the dynamic model of a nonlinear system whose general form is the following:

$$\dot{X} = F(X) + G(X)U \tag{3}$$

3 Backstepping Control Design

The backstepping controller is considered a very useful tool when some states are controlled by other states. This technique uses one state as a virtual controller to another state since the system is in triangular feedback form. It also overcomes the problem of finding a Lyapunov control function as a design tool. The design of backstepping control, nonlinear systems or subsystems of the form (4).

$$\begin{cases} \dot{x}_1 = f_1(x_1) + g_1(x_1) x_2 \\ \dot{x}_2 = f_2(x_1, x_2) + g_2(x_1, x_2) x_3 \\ \vdots \\ \dot{x}_n = f_n(x_1, \dots, x_n) + g_n(x_1, \dots, x_n) u \end{cases} \tag{4}$$

Where: $x = [x_1, x_2, \dots, x_n]^T \in \Re^n$, $u \in \Re$

We wish to make the output $y = x$ follow the reference signal y_{ref} supposed to be known. The system being of order n, the design is done in n steps.

4 Designed of Backstepping Controller

The basic idea of the backstepping control is to make the looped system into cascaded subsystems of order one stable in the Lyapunov sense, which gives it robustness qualities and an asymptotic global stability. The objective is to control the speed by choosing as subsystems the expressions of $\frac{di_d}{dt}$, $\frac{di_q}{dt}$ and as intermediate variables the stator currents (i_d,i_q). These last variables are considered as virtual commands, from these variables (i_d,i_q), we calculate the voltage commands (v_d and v_q) necessary to ensure the speed control of the PMSM and the stability of the global system.

4.1 Step 1: Control of i_d

Define the error as:

$$e_1 = i_{dref} - i_d \tag{5}$$

From the equations (1) and (5), the dynamic equations of the error are:

$$\dot{e}_1 = \dot{i}_{dref} - \dot{i}_d \tag{6}$$

$$\dot{e}_1 = \dot{i}_{dref} + \frac{R_s}{L_d} i_d - \frac{L_q}{L_d}\omega i_q - \frac{1}{L_d} v_d \tag{7}$$

Consider the first Lyapunov function as:

$$V_1 = \frac{1}{2} e_1^2 \tag{8}$$

And the derivative of V_1 is:

$$\dot{V}_1 = e_1 \dot{e}_1$$
$$\dot{V}_1 = e_1(\dot{i}_{dref} - \dot{i}_d) \tag{9}$$

We choose

$$\dot{e}_1 = -K_1 e_1 = \dot{i}_{dref} + \frac{R_s}{L_d} i_d - \frac{L_q}{L_d}\omega i_q - \frac{1}{L_d} v_d \tag{10}$$

Where K_1 is a positive scalar,

Then $\dot{V}_1 = -K_1 e_1^2 \le 0$ and the backstepping control law v_{dref} is designed as :

$$v_{dref} = L_d[K_1 e_1 + \dot{i}_{dref} + \frac{R_s}{L_d} i_d - \frac{\omega L_q}{L_d} i_q] \tag{11}$$

4.2 Step 2 : Control of Rotor Speed

As the rotor speed is the main control variable, its trajectory is defined as the reference value and the control error as :

$$e_2 = \Omega_{ref} - \Omega$$
$$\dot{e}_2 = \dot{\Omega}_{dref} - \dot{\Omega} \tag{12}$$

$$\dot{e}_2 = \dot{\Omega}_{dref} - \frac{P}{J} i_q[(L_d - L_q)i_d + \varphi_f] + \frac{f}{J}\Omega + \frac{1}{J}C_r \tag{13}$$

Define the second Lyapunov function as:

$$V_2 = V_1 + \frac{1}{2} e_2^2 \tag{14}$$

And $\dot{V}_2 = \dot{V}_1 + e_2\dot{e}_2 = -K_1 e_1^2 + e_2 \dot{e}_2$

In order to obtain $\dot{V}_2 \le 0$, we can choose

$$\dot{e}_2 = -K_2 e_2$$

Where K_2 is a positive scalar

Then

$$i_{qref} = (\dot{\Omega}_{dref} + k_2 e_2 + \frac{f}{J}\Omega + \frac{1}{J}C_r)\frac{J}{P[(L_d - L_q)i_d + \varphi_f]} \tag{15}$$

Considering that $i_{dref} = 0$ this leads to define i_{qref} the command necessary to determine the v_{qref} voltage

$$i_{qref} = (k_2 e_2 + \dot{\omega}_{dref} + \frac{f}{J}\omega + \frac{1}{J}C_r)(\frac{J}{P\varphi_f}) \tag{16}$$

4.3 Step 3: Control of i_q

Define the error as:

$$e_3 = i_{qref} - i_q \tag{17}$$

Then

$$\dot{e}_3 = \dot{i}_{qref} - \dot{i}_q$$

$$\dot{e}_3 = i_{qref} + \frac{R_s}{L_q} i_d - \frac{PL_d}{L_q}\Omega\, i_d + P\frac{\varphi_f}{L_q}\Omega - \frac{1}{L_q} v_q \tag{18}$$

The Lyapunov function can be defined as:

$$V_3 = V_1 + V_2 + \frac{1}{2} e_3^2 \tag{19}$$

Then $\dot{V}_3 = -K_1 e_1^2 - K_2 e_2^2 + e_3 \dot{e}_3$

In order to obtain $\dot{V}_3 \le 0$, we can choose

$$\dot{e}_3 = -K_3 e_3$$

Where K_3 is a positive scalar

$$\dot{V}_3 = \dot{V}_1 + \dot{V}_2 + e_3 \dot{e}_n$$

$$\dot{e}_3 = -K_3 e_3 = \dot{i}_{qref} + \frac{R_s}{L_q} i_q - P\frac{L_d}{L_q}\Omega\, i_d + P\frac{\varphi_f}{L_q}\Omega - \frac{1}{L_q} v_q \tag{20}$$

We deduce v_{qref} the final backstepping control law is designed as:

$$v_{qref} = L_q[K_3 e_3 + \dot{i}_{qref} + \frac{R_s}{L_q} i_q + \frac{P\Omega}{L_q}(L_d i_d + \varphi_f)] \tag{21}$$

Finally, we have defined from the backstepping control, the reference variables necessary to control the speed of the PMSM, while requiring a stability of the cascaded subsystems to ensure an asymptotic stability of the overall system.

5 Simulation Results and Discussion

5.1 Simulations Results

The adopted control is based on the Backstepping method applied to a PMSM, whose model is non-linear and multi-variable, is tested by numerical simulation for the following parameter values:

$K_1 = K_2 = 1000$ and $K_3 = 100$.

The motor parameters used for the simulations are given in Table 1.

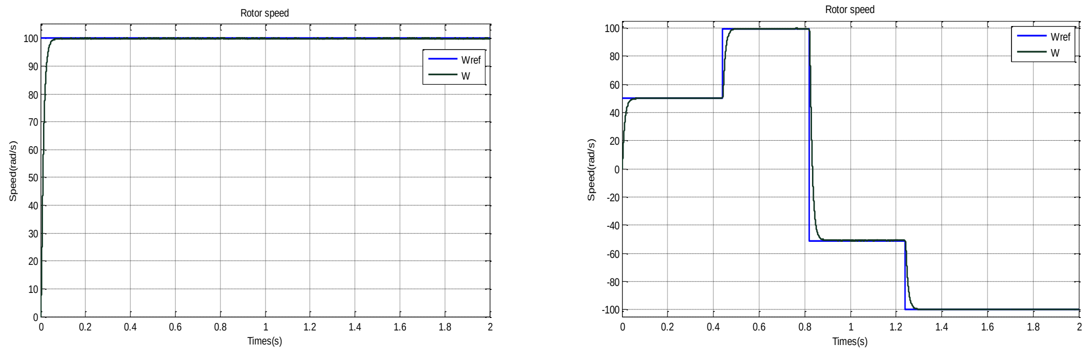
Table 1. The motor parameters.

P	**4**
R_s	**0.6**$[\Omega]$

L_d	**0.0014**$[\boldsymbol{mH}]$
L_q	**0.0028**$[\boldsymbol{mH}]$
φ_f	**0.2**$[\boldsymbol{Web}]$
J	**0.02**$[\boldsymbol{N.mS^2/rad}]$
f	**0.0014**

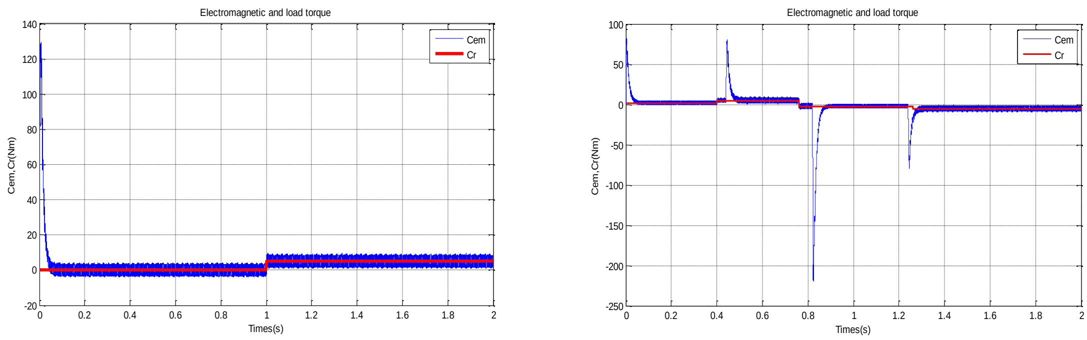
5.2 Results

The simulation tests are carried out during a simulation time of 2 seconds. In the first test, a load torque of 5 Nm was applied at time t = 1 second at constant speed. In contrast, in the second test, a variable load torque was applied at variable speed, in order to test and simulate the tracking of the reference speed variation under load torque disturbance variations.



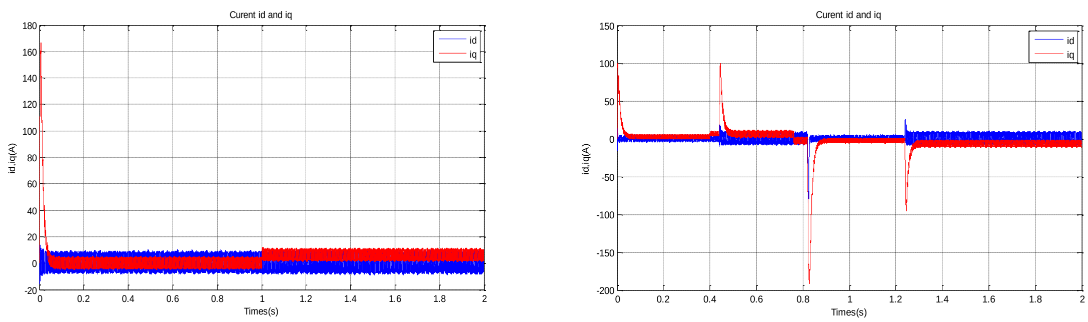
(a) (b)

Fig. 1: Speed tracking response for reference under load torque disturbance variations for (a) constant speed , (b) variable speed



(a) (b)

Fig. 2: Electromagnetique torque tracking a laod torque variation for (a) load torque 5N.m at time t = 1 second at constant speed, (b) variable load torque and variable speed



(a) (b)

Fig. 3: Current i_d and i_q for (a) constant speed, (b) variable speed

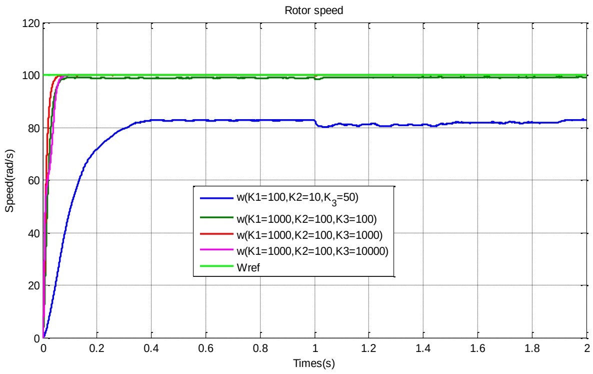


Fig. 4: Speed tracking response for reference under different values of K1, K2 and K3

5.3 Discussion of the Results

The figure 1 shows the results of the simulation of the speed control by backstepping, in figure 1(a) the curves show that during the no-load start-up, the quantities stabilise after a response time of 0.02 sec, the rotation speed is the reference speed without any overshoot. Also in The figure 1(b) shows the results of the simulation with a change of set point and a speed reversal, we notice that this control presents very satisfactory results with good tracking dynamics and a relatively acceptable rejection of the disturbance. On the other hand, we notice that the speed is established at its nominal value with good dynamics and without static error, at the moment when the load torque is applied, the speed is reduced but it is re-established again without static error.

The figure 2 shows the behaviour of load torque and electromagnetic torque. The latter oscillates during power-up reaching a maximum value and disappears once the steady state is reached. When the load is applied, the electromagnetic torque increases so as to instantly compensate the load torque with some additional ripples in the electromagnetic torque.

The figure 3 shows the characteristics of the stator currents id and iq at start-up the machine draws a large current afterwards we notice a decrease as the machine has the normal operating regime. The stator current components id and iq show the decoupling introduced by the PMSM Backstepping control (id= 0). The electromagnetic torque follows well the current Iq as shown in figure 2(b) and figure 3(b) with a peak related to the start-up, which is reached in the steady state, which shows the objective of the Backstepping control the stabilization of PMSM operation with presence of disturbances.

In order to test the robustness against parametric variations, the simulation results of the dynamic behaviour are presented as shown in Figure 4 for different values of K1, K2 and K3. The table 2 below gives the minimum, maximum and optimum values of K1, K2 and K3, it can be seen that the variation of these parameters influence the dynamics of the velocity ordered by Backstepping. This is mainly due to the recursive nature of the latter, which makes it possible to this is mainly due to the recursive nature of the latter, which allows the global system to be considered in cascaded subsystems, to guarantee the stabilisation of the measurements.

Table 2. The minimum, maximum and optimum values of K1,K2 and K3 .

minimum value	**optimum value**	**maximum value**
K1=300 K2=300 K3=20	K1=1000 K2=1000 K3=100	K1=2000 K2=2000 K3=300

6 Conclusions

The permanent magnet synchronous motor PMSM is an electric actuator of great industrial interest, due to its compactness, low inertia, efficiency, robustness and high power density, but its non-linear structure makes its control more complex, which led us to use the non-linear control model that can provide good performance. Thus, the work presented in this paper is essentially a contribution to the backstepping control. The results of the simulation show that the backstepping controller was successfully designed a good response of the PMSM, in pursuit the response time is low and a high control performance regarding the rapidity, the stability and robustness in relation to applied loads and parametric variations vis-à-vis. this way, she presents very satisfactory results with a good tracking dynamics as well as a good rejection of the disturbance. On the other hand, we notice a very good dynamics when applying the load torque.

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