Evaluation of Variable Parameter MCV Control Chart in Downward Process Shifts

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Abstract: - Previous studies revealed that the coefficient of variation (CV) is important in ensuring process quality, especially for monitoring a process where its process mean and variance are highly correlated. The fact that almost all industrial process monitoring involves a minimum of two or more related quality characteristics being monitored simultaneously. The existing adaptive charts for monitoring the multivariate CV are focused on detecting upward process shifts. The downward process monitoring is crucial since it shows process improvement. Very little research works are available on the downward adaptive multivariate CV chart. This makes it difficult for the quality engineer who wishes to implement the adaptive multivariate CV chart on the downward process monitoring. Therefore, this paper filled the research gap by proposing a downward variable parameter chart for the multivariate coefficient of variation. The performance measures of the proposed charts are derived based on the Markov-chain approach. Numerical comparisons between the proposed and existing charts have been made, in terms of the average time to signal criterion. The findings reveal that the proposed charts outperform the existing charts for detecting small and moderate downward process shifts.

Key-words: Statistics, downward shifts, Markov-chain, variable parameter, control charting technique

1. Introduction

Control charting techniques are commonly used and received great attention to their application in various industries, such as manufacturing healthcare, chemical, biological, agriculture and service industries. The traditional $\bar{X}$ and R charts could not be used for some processes especially when the mean and standard deviation of the processes are highly correlated. In this case, the traditional control charts may provide erroneous conclusions. To deal with this kind of process, the implementation of a coefficient of variation (CV) control chart is the best solution. In the past few decades, the application of CV is very common to be applied in various disciplines. This can be shown in some recent research works. Whelan and Siqueira [1] applied the CV in crop area classification across multiple climates. Weber [2] investigated the CV as the predictor of risk sensitivity. Chanda et al. [3] presented the use of CV on the sugarcane population and stalk production. Additionally, the CV can be implemented to evaluate the global solar radiation in terms of time scale separation [4] and chemical reactor process [5].

Kang et al [6] were the pioneer to introduce a standard CV chart. Subsequently, research works on CV charts have been extensively conducted to improve the sensitivity of the standard CV chart, in terms of detecting small and moderate CV shifts. such as those by [7 – 10], to name a few. Besides, Yeong et al. [11] introduced a multivariate CV (MCV) chart for multivariate process monitoring. Khaw et al. [12, 13] suggested the adaptive MCV and synthetic MCV charts enhance the statistical performance of the standard MCV chart of Yeong et al. [11]. Nguyen et al. [14] and Chew and Khaw [15] proposed the one-sided MCV charts by incorporating the variable sampling interval (VSI) and variable sample size and sampling interval (VSSI), respectively. Chew et al. [16] discussed the upward MCV chart with variable parameters. In addition, a fixed and variable exponentially weighted moving average (EWMA) chart for monitoring the MCV was presented by Giner-Bosch et al. [17] and Ayyoub et al. [18], respectively.

The adaptive scheme is a well-known scheme in the control charting technique, which consists of VSI, variable sample size (VSS), VSSI and variable parameter (VP) strategies. Among those strategies, VP is known as the most effective adaptive scheme. The VP strategy is continuously being investigated by many researchers. Yeong et al. [19] recommended a VP chart for monitoring the CV. Lee [20] and Lin and Chou [21] used a VP chart to monitor the autocorrelated and non-normality processes,

respectively. Moreover, the VP charts with correlated A&L switching rule [22], VP $\bar{X}$ chart [23] and VP S chart [24] were investigated in terms of economic criterion. More recently, Sabahno et al. [25] considered the VP chart for simultaneously monitoring the process mean and variability with measurement error while the VP auxiliary information based chart was developed using a Markov-chain approach [26].

The VP MCV chart of Chew et al. [16] showed an outstanding performance in the detection of MCV shifts when compared to other existing MCV charts. However, a setback of this study is the VP MCV chart was only considered for the detection of upward MCV shifts. In most of the scenarios, the detection of downward MCV shifts is important as it shows process improvement. With the intention to fill the research gap in downward process monitoring and the excellent feature of the VP scheme, this paper extends the VP MCV chart of Chew et al. [16] and proposes a one-sided downward VP (DVP MCV) chart for monitoring the downward MCV shifts. Note that the one-sided DVP MCV chart can avoid the biased average time to signal (ATS) performance. In the existing literature, only the Shewhart (SH) and VSSI MCV charts are available for monitoring the downward process shifts. The SH MCV chart can detect large process shifts quickly but it is rather slow in the detection of small and moderate process shifts. The statistical sensitivity of the SH MCV chart can be enhanced by using a variable parameter strategy. This paper proposes a one-sided downward VP MCV (called DVP MCV) chart. The performance measures of the proposed chart are derived using the Markov-chain approach. As it will be shown in Section 3, the DVP MCV chart generally outperforms the existing DSH MCV and DVSSI MCV charts, in the detection of small and moderate shifts, in terms of the ATS criterion.

The remainder sections are organized as follows: Section 2 discusses the design of the one-sided DVP MCV charts. The derivations of the formulae and algorithms to compute the ATS values, by adopting the Markov-chain method are also provided. Numerical comparisons among the DVP MCV, DVSSI MCV and DSH MCV charts are enumerated in Section 3. Lastly, the research findings and recommendations for future research are given in the last section.

2. One-sided Downward Variable Parameter MCV Chart

Let $X_1, X_2, ..., X_n$ refers to a multivariate n_0 from the p-variate normal distribution with μ and Σ. Voinov and Nikulin [27] derived the population MCV statistic as $\gamma = \left(\mu^T \Sigma^{-1} \mu \right)^{-\frac{1}{2}}$, where μ and Σ refer to the mean vector and covariance matrix, respectively. Note that γ can be estimated by the sample MCV, $\hat{\gamma}$ when μ and Σ are unspecified. Thus, $\hat{\gamma} = \left(\bar{X}^T S^{-1} \bar{X} \right)^{-\frac{1}{2}}$ by replacing μ and Σ with $\bar{X}$ and S, respectively. Here, $\bar{X}$ is the sample mean vector whereas S is the sample covariance matrix. The computations of $\bar{X}$ and S are $\bar{X} = 1/n \sum X$ and $S = 1/(n-1) \sum \left(X - \bar{X} \right) \left(X - \bar{X} \right)^T$, respectively, where $\bar{X}$ and S are independent of one another.

The DSH MCV chart of Yeong et al. [11] uses a fixed sampling interval, h_0, fixed sample size, n_0 and fixed-width constant. It consists of two regions, i.e. the safe region and the action region. Contrary to the DSH MCV chart, the DVP MCV chart contains three regions, which represent the safe, warning and action regions. The VP scheme varies the sampling interval, sample size and width constant to enhance the flaw of the DSH MCV chart in the detection of small and moderate downward MCV shifts. The sampling interval adopted for the proposed chart is varied between the short sampling interval, h_S and the long sampling interval, h_L, where $h_S < \text{ASI}_0 < h_L$ while the sample size is varied between the small sample size, n_S and the large sample size, n_L, where $n_S < \text{ASS}_0 < n_L$. Note that ASI_0 and ASS_0 refer to the in-control average sampling interval (ASI) and in-control average sample size, respectively, where ASI_0 and ASS_0 of the DVP MCV chart is equal to the fixed sampling interval and fixed sample size of the DSH MCV for ensuring a fair comparison can be made. The DVP MCV chart works as follows:

i)   When the t-th sample MCV, $\hat{\gamma}_t$ falls above the warning limit, the process shows no indication of trouble. Hence, h_L, n_S and loosened control and warning limits are taken to compute the $(t + 1)$-th sample MCV, $\hat{\gamma}_{t+1}$;

ii) When $\hat{\gamma}_t$ falls below the warning limit and above the control limit, the process still shows no indication of trouble but there is a higher tendency for it to become out-of-control. Hence, h_S, n_L and tightened control and warning limits are taken to compute $\hat{\gamma}_{t+1}$; and

iii) When $\hat{\gamma}_t$ falls below the control limit, the process shows an indication of trouble at the t-th sample due to the presence of

assignable causes. In this case, an immediate investigation should be taken.

Subsequently, the lower warning and control limits of the DVP MCV chart are computed as

$$\mathrm{LWL}_k = F_{\hat{\gamma}}^{-1}\left(\alpha_k' \mid n_r, p, \delta_0\right), \tag{1}$$

and

$$\mathrm{LCL}_k = F_{\hat{\gamma}}^{-1}\left(\alpha_k \mid n_r, p, \delta_0\right). \tag{2}$$

Note that in Eqns (1) and (2), when $k = S$, then $n_r = n_L$. Conversely, when $k = L$, then $n_r = n_S$. The Type-I error probability, α_0 is set based on the desired in-control ATS (ATS_0) and $\alpha_k' > \alpha_0$. The Type-I error probability is used in the computation of lower control limit, i.e. LCL. The LCL is computed according to the specified ATS_0 value, where the Type-I error probability is set to fulfil this constraint. LWL_S, LCL_S, LWL_L and LCL_L denote the tightened warning limit, tightened control limit, loosened warning limit and loosened control limit, respectively, where $\mathrm{LCL}_S < \mathrm{LCL}_0 < \mathrm{LCL}_L$. Here, LCL_0 is the lower control limit of the non-adaptive MCV chart and can be obtained using Eqn (2) by substituting α_k as α_0. Here, $\delta_0 = n/\gamma_0^2$ and p denotes the number of quality characteristics.

Equations (1) and (2) follow an inverse cumulative distribution function (cdf) of $\hat{\gamma}$, i.e. $F_{\hat{\gamma}}^{-1}\left(\alpha \mid n, p, \delta\right) =$ $\sqrt{[(n(n-p))/((n-1)p)] \times (1/F_F^{-1}\left(1-\alpha \mid p, n-p, \delta\right))}$. Note that $F_F^{-1}(\cdot)$ refers to an inverse cdf of a noncentral F distribution, where this distribution is only valid when $p < n$ due to the degree of freedom and δ is defined as $n/(\tau\gamma_0)^2$, where the shift size $\tau = 1$ when the process is in-control. Note that $\gamma_1 = \tau\gamma_0$ is the out-of-control MCV, where $\tau \neq 1$, with the values of $0 < \tau < 1$ refer to downward MCV shifts.

The Markov chain model of the DVP MCV chart has three states, which correspond to the safe, warning and action regions. States 1 and 2 are the transient states (i.e. A_{11}, A_{12}, A_{21} and A_{22}) while state 3 is the absorbing state, where A_{ij} is the transition probability from the previous state i to the current state j. Then, the transition probabilities of the transient states in the matrix can be obtained as

$$A_{11} = \Pr\left(\hat{\gamma} \geq \mathrm{LWL}_L \mid n_S, p, \delta_{11}\right) = 1 - F_{\hat{\gamma}}\left(\mathrm{LWL}_L \mid n_S, p, \delta_{11}\right)$$

$$\text{, where } \delta_{11} = \frac{n_S}{\gamma_1^2} \tag{3}$$

$$A_{12} = \Pr\left(\mathrm{LCL}_L < \hat{\gamma} \leq \mathrm{LWL}_L \mid n_S, p, \delta_{11}\right)$$

$$= F_{\hat{\gamma}}\left(\mathrm{LWL}_L \mid n_S, p, \delta_{11}\right) - F_{\hat{\gamma}}\left(\mathrm{LCL}_L \mid n_S, p, \delta_{11}\right) \tag{4}$$

$$A_{21} = \Pr\left(\hat{\gamma} \geq \mathrm{LWL}_S \mid n_L, p, \delta_{12}\right) = 1 - F_{\hat{\gamma}}\left(\mathrm{LWL}_S \mid n_L, p, \delta_{12}\right)$$

$$\text{, where } \delta_{12} = \frac{n_L}{\gamma_1^2} \tag{5}$$

$$A_{22} = \Pr\left(\mathrm{LCL}_S < \hat{\gamma} \leq \mathrm{LWL}_S \mid n_L, p, \delta_{12}\right)$$

$$= F_{\hat{\gamma}}\left(\mathrm{LWL}_S \mid n_L, p, \delta_{12}\right) - F_{\hat{\gamma}}\left(\mathrm{LCL}_S \mid n_L, p, \delta_{12}\right) \tag{6}$$

The ATS of the DVP MCV chart can be computed as

$$\mathrm{ATS} = b^T \left(I - Q\right)^{-1} t \tag{7}$$

I and Q are the identity and transient state transition probability matrices with 2×2 dimension, respectively, whereas $t^T = (h_2, h_1)$ is a sampling intervals vector. Subsequently, $b^T = (b_1, b_2)$ is the initial probability vector, subject to $b_1 + b_2 = 1$. Here, b_1 and b_2 are the time spent proportions in the safe and warning regions, respectively. Both b_1 and b_2 are obtained based on $\tau = 1$, where

$$b_1 = \frac{1 - F_{\hat{\gamma}}\left(\mathrm{LWL}_L \mid n_S, p, \delta_{11}\right)}{1 - F_{\hat{\gamma}}\left(\mathrm{LCL}_L \mid n_S, p, \delta_{11}\right)} \tag{8}$$

And

$$b_2 = \frac{F_{\hat{\gamma}}\left(\mathrm{LWL}_S \mid n_L, p, \delta_{12}\right) - F_{\hat{\gamma}}\left(\mathrm{LCL}_S \mid n_L, p, \delta_{12}\right)}{1 - F_{\hat{\gamma}}\left(\mathrm{LCL}_S \mid n_L, p, \delta_{12}\right)}, \tag{9}$$

subject to the specified values of ASS_0, ASI_0 and UCL_0, where $\mathrm{ASI}_0 = h_2 b_1 + h_1 b_2$, $\mathrm{ASS}_0 = n_1 b_1 + n_2 b_2$ and $\mathrm{UCL}_0 = \mathrm{UCL}_L b_1 + \mathrm{UCL}_S b_2$.

3. Numerical Comparison

The numerical performance of the proposed chart is evaluated in terms of the ATS criterion and the ATS_0 is specified as 370. According to Muhammad et al. [28], an ATS_0 of 370 ensures a low false alarm rate, Moreover, it also allows a fair comparison to be made with other MCV charts, i.e. DSH MCV, DVSI MCV, DVSS MCV and DVSSI MCV charts. The existing DSH MCV chart for monitoring the downward process has its fixed sampling interval set as, say $h_0 = 1$. As $\mathrm{ATS} = h_0 \times \mathrm{ATS}$, then $\mathrm{ATS} = \mathrm{ARL}$ for the

proposed DVP MCV chart. For ensuring a fair comparison with the DSH MCV chart in terms of the ATS criterion, the $\text{ASI}_0 (= h_0)$ of the DVP MCV chart is set as unity. Then, the h_S value is set to be at least 0.1 to minimize the ATS_1 values, for detecting the decrease of MCV shifts. Tables 1 – 2 present the comparison between the DVP MCV, DVSSI MCV [15], DVSS MCV [15], DVSI MCV [15] and DSH MCV [11] charts, in terms of the ATS_1 criterion, for $p \in \{2,3\}$, $\gamma_0 \in \{0.1, 0.3, 0.5\}$, $n \in \{5,10\}$ and $\tau \in \{0.50, 0.60, 0.70, 0.80, 0.90\}$. In the detection of downward MCV shifts, it is clear that the DVP MCV chart has the best performance, for small and moderate downward shift sizes. For example, from Tables 1 and 3, when $p = 2$, $n_0 = 5$, $\gamma_0 = 0.1$ and $\tau = 0.50$, the DVP MCV, DVSSI MCV, DVSS MCV, DVSI MCV and DSH MCV charts provide $\text{ATS}_1 = 2.09$, 2.56, 4.69, 6.18 and 33.43, respectively, where the DVP MCV chart has the smallest ATS_1 value. Another example can be shown from Tables 2 and 4, when $p = 3$, $n_0 = 10$, $\gamma_0 = 0.1$ and $\tau = 0.70$, the DVP MCV, DVSSI MCV and DSH MCV charts give $\text{ATS}_1 = 2.82$, 3.78, 6.92, 7.05 and 44.02, respectively, where the DVP MCV chart has the smallest ATS_1 value. In addition, from Tables 1 to 4, another notable trend observed is that the larger shift τ provides smaller TARL_1 values regardless of the sample size n values. When n and τ values are fixed, the TARL_1 values increase consistently by increasing the γ_0 value. When γ_0 and τ values are fixed while the n value increases, the TARL_1 value decreases. When the p value increases, the TARL_1 value increases as well, this shows that the p value is proportional to the TARL_1 value.

Table 1. ATS_1 values of the DVP MCV, DVSSI MCV and DSH MCV charts when $p = 2$, $n_0 \in \{5,10\}$, $\gamma_0 \in \{0.1, 0.3, 0.5\}$, $h_S = 0.1$ and $\text{ATS}_0 = 370$.

	DVP MCV	DVSSI MCV	DSH MCV
	$\gamma_0 = 0.1$		
τ	$n_0 = 5$		
0.5	2.09	2.56	33.43
0.6	3.90	5.41	64.03
0.7	9.41	15.52	103.67
0.8	25.80	80.80	129.98
0.9	102.31	203.13	237.24
	$n_0 = 10$		
0.5	1.13	1.18	5.28
0.6	1.40	1.58	14.50
0.7	2.40	3.12	36.56
0.8	8.01	16.70	80.33
0.9	64.37	88.68	187.53
	$\gamma_0 = 0.3$		
τ	$n_0 = 5$		
0.5	2.14	2.60	52.15
0.6	4.18	5.59	87.72
0.7	10.29	15.68	135.30
0.8	28.47	81.93	198.12
0.9	108.36	204.42	277.14
	$n_0 = 10$		
0.5	1.14	1.19	5.88
0.6	1.43	1.62	15.94
0.7	2.57	3.18	39.28
0.8	8.97	17.28	86.08
0.9	66.80	90.43	190.22
	$\gamma_0 = 0.5$		
τ	$n_0 = 5$		
0.5	2.23	2.84	55.84
0.6	4.55	6.70	93.57
0.7	13.62	19.89	142.81
0.8	10.88	98.25	204.75
0.9	115.21	214.12	282.74
	$n_0 = 10$		
0.5	1.16	1.22	7.08
0.6	1.52	1.71	18.55
0.7	2.71	3.78	44.45
0.8	9.08	20.17	96.26
0.9	69.43	98.72	197.42

Table 2. ATS_1 values of the DVP MCV, DVSSI MCV and DSH MCV charts when $p = 3$, $n_0 \in \{5,10\}$, $\gamma_0 \in \{0.1, 0.3, 0.5\}$, $h_S = 0.1$ and $\text{ATS}_0 = 370$.

	DVP MCV	DVSSI MCV	DSH MCV
	$\gamma_0 = 0.1$		
τ	$n_0 = 5$		
0.5	3.67	5.24	93.38
0.6	8.12	12.79	135.17
0.7	18.35	37.64	183.46
0.8	42.47	142.73	237.73
0.9	125.64	259.18	302.76
	$n_0 = 10$		
0.5	1.17	1.24	8.22
0.6	1.52	1.76	20.33
0.7	2.82	3.78	44.02
0.8	9.70	21.43	101.49
0.9	70.58	102.47	202.16
	$\gamma_0 = 0.3$		
τ	$n_0 = 5$		
0.5	4.53	5.42	96.66
0.6	9.34	13.29	139.23
0.7	19.98	38.57	186.39
0.8	44.66	145.04	241.67
0.9	128.21	260.56	304.71
	$n_0 = 10$		
0.5	1.19	1.27	8.46
0.6	1.61	1.93	21.05
0.7	2.97	3.94	48.60
0.8	10.26	23.21	103.33
0.9	73.62	110.39	203.71
	$\gamma_0 = 0.5$		
τ	$n_0 = 5$		
0.5	4.94	6.36	103.00

0.6	10.67	15.97	146.12
0.7	21.81	46.98	194.52
0.8	47.18	164.27	248.88
0.9	135.79	267.85	309.54
	$n_0 = 10$		
0.5	1.21	1.30	9.88
0.6	1.72	1.95	24.26
0.7	3.13	4.70	54.47
0.8	13.99	25.92	111.62
0.9	77.80	113.17	210.63

Table 3. ATS_1 values of the DVP MCV, DVSS MCV and DVSI MCV charts when $p = 2$, $n_0 \in \{5,10\}$, $\gamma_0 \in \{0.1, 0.3, 0.5\}$, $h_S = 0.1$ and $ATS_0 = 370$.

	DVP MCV	DVSS MCV	DVSI MCV
	$\gamma_0 = 0.1$		
τ	$n_0 = 5$		
0.5	2.09	4.69	6.18
0.6	3.90	7.11	12.11
0.7	9.41	17.71	32.55
0.8	25.80	83.49	85.43
0.9	102.31	227.32	205.96
	$n_0 = 10$		
0.5	1.13	2.02	1.46
0.6	1.40	2.74	2.47
0.7	2.40	5.86	5.31
0.8	8.01	26.15	23.11
0.9	64.37	147.58	105.16
	$\gamma_0 = 0.3$		
τ	$n_0 = 5$		
0.5	2.14	4.72	6.29
0.6	4.18	7.98	12.77
0.7	10.29	18.12	33.33
0.8	28.47	87.80	86.90
0.9	108.36	256.49	208.42
	$n_0 = 10$		
0.5	1.14	2.03	1.51
0.6	1.43	2.93	2.52
0.7	2.57	6.23	6.03
0.8	8.97	27.34	24.43
0.9	66.80	154.59	107.41
	$\gamma_0 = 0.5$		
τ	$n_0 = 5$		
0.5	2.23	5.11	6.86
0.6	4.55	8.98	14.71
0.7	13.62	22.74	38.87
0.8	10.88	104.09	96.18
0.9	115.21	266.05	224.61
	$n_0 = 10$		
0.5	1.16	2.13	1.62
0.6	1.52	3.23	2.83
0.7	2.71	7.46	7.39
0.8	9.08	33.90	30.01
0.9	69.43	166.49	118.35

Table 4. ATS_1 values of the DVP MCV, DVSS MCV and DVSI MCV charts when $p = 3$, $n_0 \in \{5,10\}$, $\gamma_0 \in \{0.1, 0.3, 0.5\}$, $h_S = 0.1$ and $ATS_0 = 370$.

	DVP MCV	DVSS MCV	DVSI MCV
	$\gamma_0 = 0.1$		
τ	$n_0 = 5$		
0.5	3.67	7.64	10.27
0.6	8.12	14.94	23.13
0.7	18.35	38.49	56.23
0.8	42.47	145.31	120.84
0.9	125.64	289.12	263.32
	$n_0 = 10$		
0.5	1.17	2.19	1.62
0.6	1.52	2.87	2.88
0.7	2.82	6.92	7.05
0.8	9.70	30.75	29.08
0.9	70.58	157.08	112.68
	$\gamma_0 = 0.3$		
τ	$n_0 = 5$		
0.5	4.53	7.82	11.97
0.6	9.34	15.10	25.34
0.7	19.98	39.54	58.83
0.8	44.66	148.52	124.45
0.9	128.21	293.75	268.24
	$n_0 = 10$		
0.5	1.19	2.29	1.74
0.6	1.61	3.28	3.07
0.7	2.97	7.07	7.73
0.8	10.26	32.21	30.27
0.9	73.62	169.74	119.34
	$\gamma_0 = 0.5$		
τ	$n_0 = 5$		
0.5	4.94	8.67	13.31
0.6	10.67	17.61	29.32
0.7	21.81	49.32	67.10
0.8	47.18	167.58	134.79
0.9	135.79	300.02	274.36
	$n_0 = 10$		
0.5	1.21	2.33	1.89
0.6	1.72	3.62	3.48
0.7	3.13	8.58	9.54
0.8	13.99	40.05	36.83
0.9	77.80	181.76	130.58

4. Conclusion

In this paper, a one-sided downward DVP MCV chart is proposed for enhancing downward multivariate process monitoring. The performances of the proposed chart are measured in terms of the ATS criterion based on the Markov-chain approach. The numerical comparisons reveal that the proposed DVP MCV chart outperforms the existing DVSSI MCV, DVSS MCV, DVSI MCV and DSH MCV charts, for detecting small or moderate downward MCV shifts. No attempt has been made to improve the efficiency

of the downward multivariate process monitoring using the VP strategy in the existing literature. The existing VP MCV chart only considered monitoring the upward MCV shifts. In certain scenarios, the detection of downward MCV shifts is very important as it shows process improvement. Moreover, the proposed DVP MCV chart also can resolve the problem of ATS-biased performances. In future research, the proposed chart can be extended to the DVP MCV charts with the presence of measurement errors and estimated parameters for monitoring the downward process shifts.

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