Comprehensive Analysis Dedicated to Three-Phase Power Transformer with Three-Phase Bridge in AC/DC Traction Substations

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Abstract: This study encompasses a comprehensive analysis of the three-phase transformer - three-phase rectifier assembly, and the establishment of the equivalent circuit of the AC / DC conversion group at the average DC components level. Since the efficiency standards can be expressed in terms of electrical efficiency, in an attempt to improve the transformer efficiency, in this study an enhancement of three-phase power transformer modelling with space phasors is presented. There are established the equations with space phasors of the three-phase transformer with symmetrical compact core. This equations system can be used to analyse the dynamic regimes of three-phase transformers. In this study the authors comprehensively analysed aspects of three-phase power transformer with Graetz three-phase bridge assembly operating in AC/DC traction substations.

Key-Words: - Power transformer, space phasor, three-phase bridge, traction substation

1 Introduction

Nowadays, because of economic and business growth, standards of life and development of civilization are too often interpreted in correlation with the use of electricity, and the demand of electricity is constantly rising. Three-phase power transformer is one of the most important elements in the electric power systems, and it plays a significant role in terms of energy savings [1-5]. The transmission and distribution of electricity through different voltage levels are possible due to the use of power transformers. The efficiency and sustainability of power transformers are in correlation with the reliability of the whole network, and could have considerable economic and environmental impact.

Forecast based on mathematical models enlarges our beliefs on the world functionality [6]. Although the mathematical modelling is a complex process and entails a large element of compromise the interacting systems in the real world can be studied identifying the most important interrelations of the systems [3-8]. Since the efficiency standards can be expressed in terms of electrical efficiency depending on load characteristics, in an attempt to improve the transformer efficiency, below an enhancement of three-phase power transformer modelling with space phasors is presented [1,4-5].

2 Three-Phase Transformer Modelling with Space Phasors

Basically, three-phase transformers are widely used since three phase power is the common way to produce, transmit and use the electrical energy. A three-phase transformer transfers electric power from the three-phase primary winding through inductively coupled three-phase secondary winding, changing values of three-phase RMS voltage and current [4-5]. Most common, the transformers windings are wound around a ferromagnetic core.

In this study we take into consideration a three-phase transformer with a non-saturated magnetic core, and in a symmetrical construction, as depicted in Fig.1.

The primary phase windings *(A-X), (B-Y)* and *(C-Z)* are identical, each of them having w_1 turns and the electric resistance R_1. Similarly, the secondary phase windings *(a-x), (b-y)* and *(c-z)* are identical each of them having w_2 turns and the electric resistance R_2. Moreover, the three-phase primary winding is connected in star (Y) being supplied by the RST power network, while the three-phase secondary winding is connected in star (y) and is supplying the three-phase load connected in star, with the parameters R_s-L_s-C_s on each phase, as shown in Fig.1.

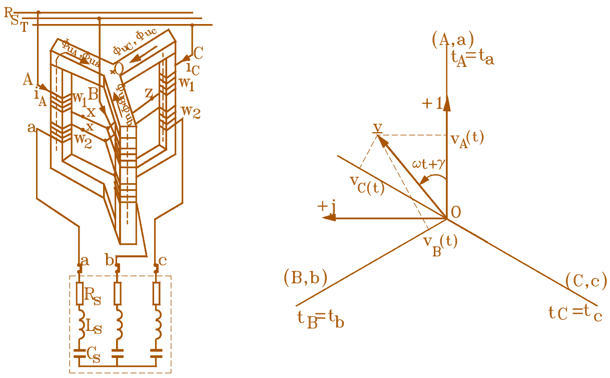

Fig.1. Three-phase power transformer

Varying currents flowing in the primary winding (due to the varying phase voltages u_A, u_B and u_C) create a varying magnetic flux in the transformer core, and thus a varying magnetic field through the secondary winding. This varying magnetic field induces a varying electromotive force in the secondary winding. Since a three-phase electric load is connected to the secondary winding, electrical energy will be transferred from the primary circuit through the transformer to the load [1-5, 7-8].

In this paper the three-phase electromagnetic phenomena will be described into the space phasors theory [1,4-5, 9-10].

In case of three-phase transformer all variables ($\underline{u}_1$ = primary voltage; $\underline{i}_1$ = primary current; $\underline{u}_2$ = secondary voltage and $\underline{i}_2$ = secondary current) are not real but complex mathematical quantities. In this context, in the study of the three-phase transformer one can use the space phasors method, highlighting that the time axes $t_A=t_a$, $t_B=t_b$ and $t_C=t_c$ are physically associated at the axes of the three-phase primary and secondary windings, which are symmetrically disposed in space.

In this framework we obtain the voltage equations of the primary phase windings as follows:

$$\begin{aligned} u_A &= R_1 \cdot i_A + \frac{d}{dt}\Psi_A \\ u_B &= R_1 \cdot i_B + \frac{d}{dt}\Psi_B \\ u_C &= R_1 \cdot i_C + \frac{d}{dt}\Psi_C \end{aligned} \tag{1}$$

By amplifying the equations (1) with *2/3, 2a/3, 2a²/3* and subsequently summing them will result the equation with space phasors as below:

$$\underline{u}_1 = R_1 \cdot \underline{i}_1 + \frac{d}{dt}\underline{\Psi}_1 \tag{2}$$

where:

$$\begin{aligned} \underline{u}_1 &= \frac{2}{3}\cdot(u_A + a\cdot u_B + a^2\cdot u_C) \\ \underline{i}_1 &= \frac{2}{3}\cdot(i_A + a\cdot i_B + a^2\cdot i_C) \\ \underline{\Psi}_1 &= \frac{2}{3}\cdot(\Psi_A + a\cdot\Psi_B + a^2\cdot\Psi_C) \end{aligned} \tag{3}$$

are the space phasors of voltages $\underline{u}_1$, currents $\underline{i}_1$ and total fluxes $\underline{\Psi}_1$ corresponding to primary phase windings of three-phase transformer.

Similarly, we obtain the voltage equations of the secondary phase windings as below:

$$\begin{aligned} u_a &= -R_2 \cdot i_a - \frac{d}{dt}\Psi_a \\ u_b &= -R_2 \cdot i_b - \frac{d}{dt}\Psi_b \\ u_c &= -R_2 \cdot i_c - \frac{d}{dt}\Psi_c \end{aligned} \tag{4}$$

By amplifying the equations (4) with *2/3, 2a/3, 2a²/3* and subsequently summing them will result the voltage equation with space phasors as follows:

$$\underline{u}_2 = -R_2 \cdot \underline{i}_2 - \frac{d}{dt}\underline{\Psi}_2 \quad (5)$$

In equation (5) $\underline{u}_2$, $\underline{i}_2$ and $\underline{\Psi}_2$ denote, respectively, the space phasors of voltages, currents and fluxes, corresponding to secondary phase windings of three-phase transformer.

One could note that the symmetrical three-phase transformer with compact ferromagnetic core has the phase windings magnetically coupled. Consequently, the total magnetic fluxes will be determined based on superposition principle. As example below there are presented the relationships for the fluxes through the total turns surface of windings A-X and a-x that are wound around the same column of ferromagnetic core.

$$\begin{aligned}\Psi_A &= \Psi_{\sigma A} + \Psi_{uA} + \Psi_{BA} + \Psi_{CA} + \Psi_{aA} + \Psi_{bA} + \Psi_{cA} \\ \Psi_a &= \Psi_{\sigma a} + \Psi_{ua} + \Psi_{ba} + \Psi_{ca} + \Psi_{Aa} + \Psi_{Ba} + \Psi_{Ca}\end{aligned} \quad (6)$$

Taking into consideration the magnetic core symmetry and in correlation with the positive sense of useful fascicular fluxes the relationships for the total magnetic coupling fluxes result as below:

$$\begin{aligned}&\Psi_{BA} = -\frac{1}{2}\cdot\Psi_{uB}; \Psi_{CA} = -\frac{1}{2}\cdot\Psi_{uC} \\ &\Psi_{aA} = w_1 \cdot \frac{\Psi_{ua}}{w_2}; \Psi_{bA} = w_1(-\frac{1}{2}\cdot\frac{\Psi_{ub}}{w_2}); \Psi_{cA} = w_1(-\frac{1}{2}\cdot\frac{\Psi_{uc}}{w_2})\end{aligned} \quad (7)$$

$$\begin{aligned}&\Psi_{ba} = -\frac{1}{2}\cdot\Psi_{ub}; \Psi_{Cca} = -\frac{1}{2}\cdot\Psi_{uc} \\ &\Psi_{Aa} = w_2 \cdot \frac{\Psi_{uA}}{w_1}; \Psi_{Ba} = w_2(-\frac{1}{2}\cdot\frac{\Psi_{uB}}{w_1}); \Psi_{Ca} = w_2(-\frac{1}{2}\cdot\frac{\Psi_{uC}}{w_1})\end{aligned}$$

Subsequently the expressions (6) can be rewritten as:

$$\begin{aligned}\Psi_A &= \Psi_{\sigma A} + \Psi_{uA} - \frac{1}{2}(\Psi_{uB} + \Psi_{uC}) + \frac{w_1}{w_2}[\Psi_{ua} - \frac{1}{2}(\Psi_{ub} + \Psi_{uc})] \\ \Psi_a &= \Psi_{\sigma a} + \Psi_{ua} - \frac{1}{2}(\Psi_{ub} + \Psi_{uc}) + \frac{w_1}{w_2}[\Psi_{uA} - \frac{1}{2}(\Psi_{uB} + \Psi_{uC})]\end{aligned} \quad (8)$$

Since the useful fascicular fluxes verify the equations:

$$\Phi_{uA} + \Phi_{uB} + \Phi_{uC} = 0; \Phi_{ua} + \Phi_{ub} + \Phi_{uc} = 0 \quad (9)$$

by amplifying with the turn numbers will result the relations for the total useful magnetic fluxes as follows:

$$\Psi_{uB} + \Psi_{uC} = -\Psi_{uA}; \Psi_{ub} + \Psi_{uc} = \Psi_{ua} \quad (10)$$

Based on expressions (10) the relationships (8) will become:

$$\begin{aligned}\Psi_A &= \Psi_{\sigma A} + \frac{3}{2}\cdot\Psi_{uA} + \frac{w_1}{w_2}\cdot\frac{3}{2}\cdot\Psi_{ua} \\ \Psi_a &= \Psi_{\sigma a} + \frac{3}{2}\Psi_{ua} + \frac{w_1}{w_2}\cdot\frac{3}{2}\cdot\Psi_{uA}\end{aligned} \quad (11)$$

Since $\Psi_{uA} = w_1 \cdot \phi_{uA}$, respectively $\Psi_{ua} = w_2 \cdot \phi_{ua}$, and $\phi_{uA} = \frac{w_1 \cdot i_A}{R_{mu}} = w_1 \cdot i_A \cdot \Lambda_u$ respectively $\phi_{ua} = \frac{w_2 \cdot i_a}{R_{mu}} = w_{21} \cdot i_a \cdot \Lambda_u$ will result the relationships for the total magnetic fluxes Ψ_A and Ψ_a as below:

$$\begin{aligned}\Psi_A &= \Psi_{\sigma A} + (w_1 \cdot i_A + w_2 i_a)\cdot\frac{3}{2}\cdot w_1 \cdot \Lambda_u \\ \Psi_a &= \Psi_{\sigma a} + \frac{w_2}{w_1}\cdot(w_1 \cdot i_A + w_2 \cdot i_a)\cdot\frac{3}{2}\cdot w_1 \cdot \Lambda_u\end{aligned} \quad (12)$$

Further one could remind that:

$$\theta_{A\mu} = w_1 \cdot i_A + w_2 \cdot i_a = w_1 \cdot i_{A\mu}$$

and: $L_{u1} = w_1^2 \cdot \Lambda_u$

Consequently, the fluxes relationships (12) can be expressed as follows:

$$\Psi_A = L_{\sigma 1} \cdot i_A + \frac{3}{2}\cdot L_{u1} \cdot i_{A\mu} \quad (13)$$

$$\Psi_a = L_{\sigma 2} \cdot i_a + \frac{w_2}{w_1}\cdot\frac{3}{2}\cdot L_{u1} \cdot i_{A\mu}$$

in which: $i_{A\mu} = i_A + \frac{w_2}{w_1}\cdot i_a$

Due to the symmetrical construction for the other two windings pairs (B and b, respectively C and c) can be written the following equations:

$$\begin{aligned}\Psi_B &= L_{\sigma 1} \cdot i_B + \frac{3}{2}\cdot L_{u1} \cdot i_{B\mu}; \\ \Psi_b &= L_{\sigma 2} \cdot i_b + \frac{w_2}{w_1}\cdot\frac{3}{2}\cdot L_{u1} \cdot i_{B\mu};\end{aligned} \quad (14)$$

$$\begin{aligned}\Psi_C &= L_{\sigma 1} \cdot i_C + \frac{3}{2} L_{u1} \cdot i_{C\mu} \\ \Psi_c &= L_{\sigma 2} \cdot i_c + \frac{w_2}{w_1}\cdot\frac{3}{2}\cdot L_{u1} \cdot i_{C\mu}\end{aligned}$$

in which: $i_{B\mu} = i_B + \frac{w_2}{w_1} \cdot i_b$

respectively $i_{C\mu} = i_C + \frac{w_2}{w_1} \cdot i_c$

Subsequently, based on relationships (13) and (14) one could build the space phasors of magnetic fluxes of three-phase transformer as below:

$$\underline{\Psi}_1 = \frac{2}{3} \cdot (\Psi_A + a \cdot \Psi_B + a^2 \cdot \Psi_C) = L_{\sigma 1} \cdot \underline{i}_1 + \underline{\Psi}_{u1} \quad (15)$$

$$\underline{\Psi}_2 = \frac{2}{3} \cdot (\Psi_a + a \cdot \Psi_b + a^2 \cdot \Psi_c) = L_{\sigma 2} \cdot \underline{i}_2 + \underline{\Psi}_{u2}$$

In relationships (15) have been introduced the notations:

$$\underline{\Psi}_{u1} = L \cdot \underline{i}_{1\mu}; \underline{\Psi}_{u2} = \frac{w_2}{w_1} \cdot L \cdot \underline{i}_{1\mu}$$

$$\underline{i}_{1\mu} = \frac{2}{3} \cdot (i_{A\mu} + a \cdot i_{B\mu} + a^2 \cdot i_{C\mu}) = \underline{i}_1 + \frac{w_2}{w_1} \cdot \underline{i}_2 \quad (16)$$

where: $L = \frac{3}{2} L_{u1}$ denotes the cyclical inductance of primary and $\underline{i}_{1\mu}$ denotes the space phasor of magnetization currents.

With respect to the three-phase load connected at the transformer secondary terminals, the phase voltage equations are as follows:

$$u_a = R_s \cdot i_a + L_s \cdot \frac{di_a}{dt} + \frac{1}{C_s} \cdot \int i_a dt$$

$$u_b = R_s \cdot i_b + L_s \cdot \frac{di_b}{dt} + \frac{1}{C_s} \cdot \int i_b dt \quad (17)$$

$$u_c = R_s \cdot i_c + L_s \cdot \frac{di_c}{dt} + \frac{1}{C_s} \cdot \int i_c dt$$

By amplifying the equations (17) with *2/3, 2a/3, 2a²/3* respectively, and subsequently summing them will result the voltage equation with space phasors of the three-phase load circuit, as below:

$$\underline{u}_2 = R_s \cdot \underline{i}_2 + L_s \cdot \frac{d}{dt} \underline{i}_2 + \frac{1}{C_s} \cdot \int \underline{i}_2 dt \quad (18)$$

Subsequently, by introducing the space phasors of the electromotive forces induced in primary and secondary windings, the equations of the three-phase transformer with symmetrical compact core can be ordered in the following space phasors system:

$$\underline{u}_1 = -\underline{e}_1 + R_1 \cdot \underline{i}_1 + L_{\sigma 1} \cdot \frac{d}{dt} \underline{i}_1$$

$$\underline{u}_2 = \underline{e}_2 - R_2 \cdot \underline{i}_2 - L_{\sigma 2} \cdot \frac{d}{dt} \underline{i}_2$$

$$w_1 \underline{i}_1 + w_2 \underline{i}_2 = w_1 \underline{i}_{1\mu}$$

$$\underline{e}_1 = -\frac{d}{dt} \underline{\Psi}_{u1}; \underline{e}_2 = -\frac{d}{dt} \underline{\Psi}_{u2} \quad (19)$$

$$\underline{\Psi}_{u1} = L \cdot \underline{i}_{1\mu}; \underline{\Psi}_{u2} = \frac{w_2}{w_1} \cdot \underline{\Psi}_{u1}$$

$$L = \frac{3}{2} \cdot L_{u1}$$

$$\underline{u}_2 = R_s \cdot \underline{i}_2 + L_s \cdot \frac{d}{dt} \underline{i}_2 + \frac{1}{C_s} \cdot \int \underline{i}_2 dt$$

Moreover, one could proceed to secondary reported to primary, with the secondary space phasors:

$$\underline{i}'_2 = \frac{w_2}{w_1} \cdot \underline{i}_2; \underline{u}'_2 = \frac{w_1}{w_2} \cdot \underline{u}_2; R'_2 = (\frac{w_1}{w_2})^2 \cdot R_2; L'_2 = (\frac{w_1}{w_2})^2 \cdot L_2 \quad (20)$$

further obtaining the equations of three-phase transformer with secondary reduced to primary:

$$\underline{u}_1 = -\underline{e}_1 + R_1 \cdot \underline{i}_1 + L_{\sigma 1} \cdot \frac{d}{dt} \underline{i}_1$$

$$\underline{u}'_2 = \underline{e}'_2 - R'_2 \cdot \underline{i}'_2 - L'_{\sigma 2} \cdot \frac{d}{dt} \underline{i}'_2$$

$$\underline{i}_1 + \underline{i}' = \underline{i}_{1\mu} \quad (21)$$

$$\underline{e}_1 = \underline{e}'_2 = -\frac{d}{dt} \underline{\Psi}_{u1}$$

$$\underline{\Psi}_{u1} = \underline{\Psi}'_{u2} = -L \cdot \underline{i}_{1\mu}; L = \frac{3}{2} L_{u1}$$

$$\underline{u}'_2 = R'_s \cdot \underline{i}'_2 + L'_s \cdot \frac{d}{dt} \underline{i}'_2 + \frac{1}{C'_s} \cdot \int \underline{i}'_2 dt$$

The equations system (21) are on the whole conclusive.

Particularly, in a permanent harmonic regime all space phasors of the three-phase symmetrical systems of sinusoidal quantities take the form $\underline{v} = SQRT2 \cdot \underline{V} \cdot e^{j\omega t}$. Taking into account the space phasors derivation and integration relationships:

$$\frac{d}{dt}\underline{v} = j \cdot \omega \cdot \underline{v} \qquad \int \underline{v}dt = \frac{1}{j \cdot \omega} \cdot \underline{v} \tag{22}$$

one could find the space phasors equations of system (21) rewritten in the classic form:

$$\begin{aligned}
&\underline{u}_1 = -\underline{e}_1 + R_1 \cdot \underline{i}_1 + j \cdot X_{\sigma 1} \cdot \underline{i}_1 \\
&\underline{u}_2' = \underline{e}_2' - R_2' \cdot \underline{i}_2' - j \cdot X_{\sigma 2} \cdot \underline{i}_2' \\
&\underline{i}_1 + \underline{i}_2' = \underline{i}_{1\mu} + \underline{i}_{10a}; \underline{i}_{10a} = -\underline{e}_1 / R_{Fe} \\
&\underline{e}_1 = \underline{e}_2' = -j \cdot \omega \cdot \underline{\Psi}_{u1} = -j \cdot 3/2 \cdot X_{u1} \cdot \underline{i}_{1\mu} \\
&\underline{u}_2' = R_s' \cdot \underline{i}_2' + jX_s \cdot \underline{i}_2'
\end{aligned} \tag{23}$$

One could emphasize that the equations system (21) can be used to analyse the dynamic regimes of three-phase transformers, being successfully applied, for instance, in the method of structural diagrams in analysing the power transformer operation [9]. Subsequently we will analyse some aspects of three-phase power transformer operation in an AC/DC traction substation.

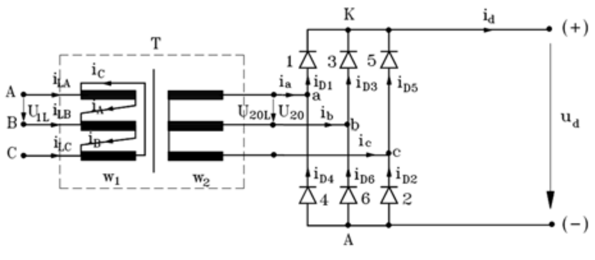

Fig.2. Rectifier with Graetz three-phase bridge

3 Power Transformer in an AC/DC Traction Substation

The DC electrical traction substations are those fixed traction installations that receive electricity (in three-phase AC) from the national power system (at high voltage), reduce the voltage level and modify the current type (from AC to DC) and, finally, distributes the electric power to contact line sections in order to supply the non-autonomous electric railway vehicles [1,10-12].

As spreading, the DC substations are used both in urban (surface and underground) electrical traction and in DC electrified railway traction. As a location, they are "indoor" installations, most of the equipment being arranged in a "cellular" structure (in sideboards).

The basics of the DC traction substations are the AC-DC conversion group.. Over time, the AC-DCconversion groups have made significant progress in terms of performance, efficiency, maintenance and reliability [1,10-11].

Nowadays, all substations are equipped with static rectifiers with diodes [1,10-12].

Optionally, reversible DC substations (with anti-parallel transformer- thyristorized inverter groups) can also be used to recover electrical energy in case of electric vehicle recuperative braking.

In principle, any DC traction substation consists of a high-voltage alternating current system (comprising: the three-phase primary line, three-phase high-voltage bars, the tripolar protective circuit breakers of the transformer rectifier group and the power transformers) and a DC power system with U_{LC} rated voltage (consisting of rectifier bridges, DC breakers and ultrafast DC switches).

The AC-DC conversion groups are made up of:

- a three phase transformer in order to reduce the actual voltage value (from U_1 of the three-phase primary line to the U_2 for supplying the rectifier) in close correlation with the continuous voltage U_{LC} magnitude across the contact line, and

- a three-phase rectifier, usually with diodes (connected in three-phase bridge and mounted in "cabinets").

The basic structure of a rectifier system in a DC traction substation consists of a three-phase bridge of type Graetz bridge [10-11], depicted in Fig.2.

This bridge is powered from the secondary of a three-phase transformer (T), usually with a Delta-Star (Dy) connection scheme, having the transformation ratio K (of the line voltages) given as follows:

$$K = \frac{U_{1L}}{U_{20L}} = \frac{1}{\sqrt{3}} \cdot \frac{w_1}{w_2} \quad (24)$$

where:

w_1 = the phase turns number of the primary winding (connected in Δ), and

w_2 = the phase turns number of the secondary winding (connected în star).

3.1 The Rectified Voltage (Idealized)

In order to study the idealized operation of the three-phase rectifier bridge, the following hypotheses are accepted [1,10]:

1. The inductance L_d (of the DC circuit) can be considered as infinitely high ($L_d \rightarrow \infty$); consequently the DC current i_d will be perfectly smooth, and constant over time $i_d = I_d$.

2. It is considered perfect magnetic coupling between the rectifier transformer windings. This means neglecting the transformer leakages ($L_{\sigma 1} \rightarrow 0$ and $L_{\sigma 2} \rightarrow 0$), and consequently the neglect of the inductance L_k of the swithching circuit ($L_k \rightarrow 0$). Therefore, sudden variations in currents are admitted, which is equivalent to neglecting the natural switching phenomenon.

3. There are neglected the ohmic resistance (primary $R_1 \rightarrow 0$ and secondary $R_2 \rightarrow 0$) of the rectifier transformer windings.

Under these conditions, the three-phase transformer T (fed into the primary) and seen on the secondary terminals will appear as a three phase (ideal) source with sinusoidal phase voltages e_{a0}, e_{b0} şi e_{c0} (symmetric, by direct sequence, with efffective values E_{20}) so that the composed voltages (line voltages) can preserve their effective value $U_{20L} = U_{20}$. Accordingly one can write:

$$U_{20} = \frac{U_{1L}}{K}; \quad E_{20} = \frac{1}{\sqrt{3}} U_{20} \quad (25)$$

If the three-phase diode bridge is fed from the ideal three-phase source (equivalent to the transformer) with the symmetrical sinusoidal phase voltages:

$$e_{a0} = \sqrt{2}\, E_{20} \sin \omega t$$
$$e_{b0} = \sqrt{2}\, E_{20} \sin(\omega t - \frac{2\pi}{3}) \quad (26)$$
$$e_{c0} = \sqrt{2}\, E_{20} \sin(\omega t - \frac{4\pi}{3})$$

then, at any time moment ωt>0, there will be only two diodes in conduction, namely:

a) only the diode in the cathodic group (1, 3, 5) with the anode connected to that phase of the source with the highest positive instantaneous phase voltaget, and

b) only the diode in the anode group (2, 4, 6) with the cathode connected to that phase of the source with the lowest negative instantaneous phase voltage.

All other diodes being momentarily subjected to inverse voltages are locked.

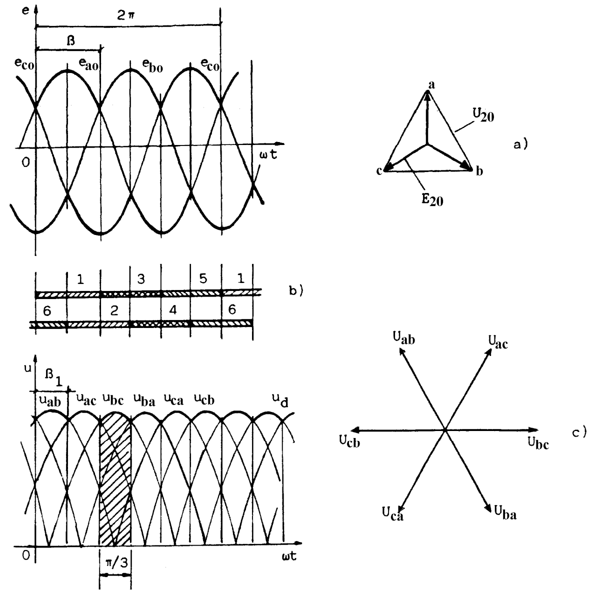

Fig.3. Diagrams of three-phase bridge

a) diagrams of phase voltages; b) conduction intervals of diodes; c) rectified voltage $u_d(\omega t)$ and line voltage phasors

As an example, with the temporary origin ωt = 0 (in the phase voltages diagram) when $e_{c0} = e_{a0}$ (see Fig.3, pos.a), left) it is noted that in the interval [0,π/3] only lead diodes 1 and 6, and the rectified voltage u_d results as:

$$u_d = e_{a0} - e_{b0} - 2 \cdot U_D = u_{ab} - 2 \cdot U_D \quad (27)$$

Here $u_{ab}=e_{a0}-e_{b0}$ represents the line voltage, and U_D is the direct voltage drop at the terminals of any diode in conduction.

The situation analyzed above is repeated (with other pairs of diodes) six times in each T period. The sequence of conduction intervals of the three-phase bridge diodes is shown in Fig.3, pos.b). In addition, if direct voltage drops are also neglected on diodes temporarily in conduction (meaning if $U_D \rightarrow 0$), then the rectified voltage u_d will be given (in each period T=1/f) only by the „positive elevations" of line voltages, exactly as is depicted in Fig.3, pos.c) (in left side, where the thickened curve represents the diagram $u_d=f(\omega t)$).

Consequently, the rectified voltage u_d is not constant over time (since it has p=6 "elevations") but is is periodical, with the main period $T_1=T/p$ or, in angular magnitude, with the angular period β_1 as below:

$$\beta_1 = \frac{2\pi}{p} = \frac{2\pi}{s \cdot q} = \frac{\beta}{s} = \frac{\pi}{3} \quad (28)$$

The average value U_{d0} of the rectified voltage u_d calculated on the inteval of a main period β_1 when the voltage $u_d \approx u_{bc}$ (see Fig.3, pos.c)) has the analytical expression:

$$u_d(\omega t) = u_d \approx u_{bc} = \sqrt{2} U_{20} \cdot \cos \omega t \quad (29)$$

is determined (according to the first theorem of average) with the formula:

$$U_{d0} = \frac{1}{\beta_1} \int_{-\beta_1/2}^{+\beta_1/2} u_d(\omega t) \cdot d(\omega t) = \frac{1}{\beta_1} \int_{-\beta_1/2}^{+\beta_1/2} \sqrt{2} U_{20} \cdot \cos \omega t \cdot d(\omega t) = \quad (30)$$

$$= \frac{\sqrt{2} U_{20}}{\beta_1} (\sin \omega t) \Big|_{-\beta_1/2}^{+\beta_1/2} = \sqrt{2} \cdot U_{20} \cdot \frac{\sin \beta_1/2}{\beta_1/2}$$

Concretely, for $\beta_1=\pi/3$ the average value U_{d0} (of the rectified voltage u_d) becomes:

$$U_{d0} = \sqrt{2} U_{20} \cdot \frac{\sin \frac{\pi}{6}}{\frac{\pi}{6}} = \frac{3\sqrt{2}}{\pi} \cdot U_{20} \approx 1{,}35 \cdot U_{20} \quad (31)$$

3.2 Diagram of Currents

As previously assumed, the inductance L_d (of the DC circuit) can be considered as infinitely high ($L_d \rightarrow \infty$). Consequently the DC current i_d will be perfectly smooth, and constant over time $i_d=I_d$. Subsequently, under these assumptions will be determined the diagrams of currents.

3.2.1 Currents through Diodes

If the diode switching phenomenon (in each switching group) is neglected, it can be admitted that through each semiconductor diode (of the three-phase bridge) will flow the constant current:

$$i_D = i_d = I_d \quad (32)$$

during each conduction interval $\beta=2\pi/3$ of each variation period $\omega T=2\pi$ of the supply voltage.

Outside of the conduction interval, the current through the respective diode is null ($i_D=0$).

Taking into consideration the sequence of the conduction intervals (see Fig.3, pos.b)), in Fig. 4 there are depicted (through „rectangular blocks") the currents i_{D1}, i_{D3} şi i_{D5}, and respectively i_{D2}, i_{D4} şi i_{D6} corresponding to the valves (diodes) of the two switching groups of the three-phase bridge.

The average value (on a period interval $\omega T=2\pi$) of the currents through the three-phase bridge is calculated with the formula:

$$I_{Dmed} = \frac{1}{2\pi} \int_0^{2\pi} i_D \cdot d(\omega t) = \frac{1}{2\pi} \int_0^{2\pi/3} I_d \cdot d(\omega t) = \frac{I_d}{3} \quad (33)$$

The effective value of the currents through the diodes I_D is given by:

$$I_D = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} i_D^2 \cdot d(\omega t)} = \sqrt{\frac{1}{2\pi} \int_0^{\frac{2\pi}{3}} I_D^2 \cdot d(\omega t)} = \frac{I_d}{\sqrt{3}} \quad (34)$$

3.2.2 Currents through Secondary Windings

To the Star configuration (Y) of the secondary phase windings, the currents i_a, i_b and i_c in the three secondary phases of the rectifier transformer (see Fig.2) result as follows:

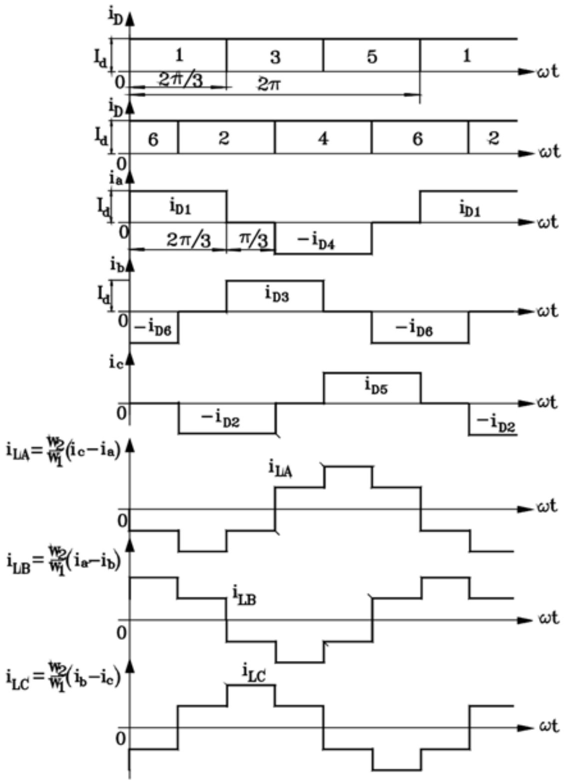

Fig.4. Diagrams of currents through diodes (i_{D1}, i_{D3}, i_{D5} and i_{D2}, i_{D4}, i_{D6}), through the secondary windings (i_a, i_b and i_c) and the primary line currents i_{LA}, i_{LB} and i_{LC}

$$\begin{aligned} i_a &= i_{D1} - i_{D4} \\ i_b &= i_{D3} - i_{D6} \\ i_c &= i_{D5} - i_{D2} \end{aligned} \qquad (35)$$

Graphically, the diagrams of secondary currents i_a, i_b and i_c depending on ωt are depicted in Fig.4. In the neglect of the switching, they are formed (on each phase) of "rectangular blocks" of amplitude $\pm I_d$ and duration $2\pi/3$ separated by pauses (of null value) of duration $\pi/3$.

The average value of these alternating currents (non-sinusoidal) is null. Instead the effective value I_2 of the secondary phase currents is given by:

$$I_2 = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i_a^2 \cdot d(\omega t)} = \sqrt{\frac{1}{2\pi}[I_d^2 \cdot \frac{2\pi}{3} + I_d^2 \cdot \frac{2\pi}{3}]} = \sqrt{\frac{2}{3}} \cdot I_d \qquad (36)$$

3.2.3 Currents through Primary Windings

Let i_A, i_B and i_C be the three-phase system of the currents passing through the transformer primary phase windings (see Fig.2).

If w_1 and w_2 represent the phase turns numbers of the primary and secondary, respectively from the condition of neglecting the magnetization currents in the total currents equations corresponding to each column of the core of the three-phase transformer (so in the hypothesis $\mu_{Fe} \rightarrow \infty$) we obtain the expressions of the primary currents i_A, i_B and i_C:

$$\begin{aligned} w_1 \cdot i_A + w_2 \cdot i_a &\approx 0 \qquad i_A \approx -\frac{w_2}{w_1} \cdot i_a \\ w_1 \cdot i_B + w_2 \cdot i_b &\approx 0 \qquad i_B \approx -\frac{w_2}{w_1} \cdot i_b \\ w_1 \cdot i_C + w_2 \cdot i_c &\approx 0 \qquad i_C \approx -\frac{w_2}{w_1} \cdot i_c \end{aligned} \qquad (37)$$

Consequently, the primary phase currents i_A, i_B and i_C vary (with the time) vary proportionately (being in phase opposition and having the amplitudes w_2/w_1 times increased) with the secondary phase currents i_a, i_b and i_c. So their effective values I_1 will be proportional with I_2, meaning:

$$I_1 = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i_A^2 \cdot d[\omega t]} = \frac{w2}{w1} \cdot I_2 = \frac{w2}{w1} \cdot \sqrt{\frac{2}{3}} \cdot I_d \qquad (38)$$

3.2.4 Line Currents in the Primary

To the Delta configuration (Δ) of the primary phase windings of the power transformer (see Fig.2), the three-phase system of the line currents i_{LA}, i_{LB} and i_{LC} is determined with the relationships:

$$\begin{aligned} i_{LA} &= i_A - i_C = \frac{w2}{w1} \cdot (i_c - i_a) \\ i_{LB} &= i_B - i_A = \frac{w2}{w1} \cdot (i_a - i_b) \\ i_{LC} &= i_C - i_B = \frac{w_2}{w1} \cdot (i_b - i_c) \end{aligned} \qquad (39)$$

Graphically, the diagrams of line currents i_{LA}, i_{LB} and i_{LC} (depending on ωt) are depicted in the bottom side of Fig.4 [1,10].

The effective value I_{1L} of the line currents is given by:

$$I_{1L}=\sqrt{\frac{1}{2\pi}\int_0^{2\pi} i_{LA}^2\cdot d(\omega t)}=\sqrt{\frac{2}{2\pi}\int_0^{\pi} i_{LA}^2\cdot d(\omega t)}= \tag{40}$$

$$=\sqrt{\frac{1}{\pi}\cdot(\frac{w2}{w1})^2[\frac{\pi}{3}I_d^2+\frac{\pi}{3}(2I_d)^2+\frac{\pi}{3}I_d^2]}=\frac{w2}{w1}\cdot I_d\cdot\sqrt{2}$$

One can highlight that although the line and phase voltages vary sinusoid over time, both primary currents and secondary currents (phase and line) vary non-sinusoid over time. This way results explicitly in the deforming (non-sinusoidal) regime in which the rectifier transformer of traction substation is operating.

4 Rectifier Transformer Calculation Power

Because of operation in a deforming regime, any transformer intended to feed the rectifier from the traction substation must be rated to a calculation power S_c higher than the apparent power corresponding to harmonic operation (when all currents would vary sinusoidally over time) [10]. Usually, when dimensioning the three-phase rectifier transformer, one could start from the following calculation quantities (previously known):

1. $P_{d0}=U_{d0}\bullet I_d$= the ideal power on DC side of rectifier;

2. U_l= the effective value of the line voltage at the high voltage supply network;

3. U_{LC}= the rated value of contact line voltage;

4. The connection configuration Dy (or Yd) of the transformer windings.

If we take into account both the voltage drops in load on the rectifier bridge (ie the direct voltage drops on the diodes in conduction, the inductive voltage drop due to the switching phenomenon and the ohmic falls $R\bullet I_d$) as well as the fact that the rectifier bridge feeds the contact line, it can be appreciated that:

$U_{do} \approx (1{,}15 \div 1{,}20)\bullet U_{LC}$

With this value for U_{d0}, from the expression (31) can be determined the effective value of the secondary line voltage U_{20} (and in the case of the star connection also the value of the effective phase voltage E_{20}) using the relationships below:

$$U_{20}=\frac{\pi}{3\sqrt{2}}\cdot U_{do}\,;\quad E_{20}=\frac{1}{\sqrt{3}}\cdot U_{20} \tag{41}$$

Therefore, the secondary winding of the three-phase transformer (consisting of three identical phases, with w_2 turns each) will be dimensioned (in the case of the star connection) at voltage E_{20} and current I_2, ie at the apparent power S_2:

$$S_2=3\cdot E_{20}\cdot I_2=3\cdot\frac{1}{\sqrt{3}}\cdot\frac{\pi}{3\sqrt{2}}U_{do}\cdot\sqrt{\frac{2}{3}}\cdot I_d= \tag{42}$$

$$=\frac{\pi}{3}U_{do}\cdot I_d\approx 1{,}047\cdot P_{do}$$

Absolutely similar, the primary winding of the three-phase transformer (connected in Δ) consisting of three identical phases (with w_1 turns each) will be dimensioned at voltage $E_1=U_1$ and current I_1, so at the apparent power S_1:

$$S_1=3\cdot E_1\cdot I_1=3\cdot E_1\cdot\frac{w2}{w1}\cdot\sqrt{\frac{2}{3}}\cdot I_d \tag{43}$$

However, since $E_1/w_1 = E_{20}/w_2$ even represents the turn voltage (of the transformer windings), the equality of apparent powers S_1 and S_2 immediately results:

$$S_1=S_2=\frac{\pi}{3}\cdot U_{d0}\cdot I_d\approx 1{,}047\cdot P_{d0} \tag{44}$$

In contrast, the core (or magnetic circuit) of the transformer is dimensioned to the "S_T" type power (defined as the half-sum of the S_1 and S_2 powers):

$$S_T=\frac{S_1+S_2}{2}=\frac{\pi}{3}U_{d0}\cdot I_d\approx 1{,}047\cdot P_{d0} \tag{45}$$

Consequently, for the three-phase transformer with the Dy connection scheme used for the three-phase bridge rectifier, the following equals are true:

$$S_1=S_2=S_T=\frac{\pi}{3}\cdot U_{d0}\cdot I_d\approx 1{,}047\cdot P_{d0} \tag{46}$$

The common value of the apparent powers of dimensioning the primary S_1 and secondary S_2 electrical windings and the magnetic core S_T (corresponding to the rectifier transformer) is denoted with S_c and is called "calculation power".

If S_c is the calculation power, and $P_{d0}=U_{d0}\bullet I_d$ is the "ideal power" (on the DC side of the rectifier) then the "utilization coefficient" of the rectifier transformer is defined by the subunit ratio y, given by: $y = \frac{P_{d0}}{S_C} < 1$ (47)

For the bridge rectifier scheme (see Fig. 2) the following results: $y = \frac{3}{\pi} \approx 0{,}955$ 1.

The case of rectifier schemes with p = 12 pulses is absolutely similar.

5 Voltage Drops, External Characteristic, and Equivalent Circuit of Conversion Group

One could note that the average value of the U_{do} rectified voltage given by the relationship (31) is constant and does not take into account (in any way) the voltage drops on load operation. For this reason, during the load operation the average value of the rectified voltage U_d can be expressed by subtracting from U_{d0} (the average value, at no-load operation) all the voltage drops that accompany the on-load operation of the rectifier from the DC traction substations [10-11].

5.1 Voltage Drops

Voltage drops in load can be grouped into the following three categories:

1. Direct voltage drop u_D (on all diodes in conduction).

From the configuration of the three-phase rectifier (with diodes) one could identify the numerical values of the following quantities:

n_s = the number of diodes connected in series, on each current path, and

s = the number of commutation groups connected in series;

If (1,5 ÷ 2)V is the direct voltage drop on a single diode then the direct voltage drop "u_D" on all diodes currently in conduction is calculated with the formula:

$u_D \approx s \cdot n_s \cdot (1.5 \div 2)$ V (48)

For the three-phase bridge from Fig.2 we have s = 2 and n_s = 1.

Basically, u_D is independent of the magnitude of the load DC current I_d.

2. Inductive voltage drop ΔU_L

This voltage drop is caused by the commutation phenomenon, and is depending on the magnitude of DC current I_d according to relation: $\Delta U_L = \frac{3}{\pi} \omega L_k \cdot I_d$. If the internal resistance R_i is introduce by the relationship: $R_i = \frac{3}{\pi} \omega L_k$, then the inductive voltage drop ΔU_L can be classically expressed with the formula:

$$\Delta U_L = R_i \cdot I_d \quad (49)$$

One should note that the internal resistance R_i is a fictitious quantity (a computational one) that flowed by the DC current I_d determines (at its terminals) a voltage drop $R_i \bullet I_d$ numerically equal to the inductive voltage drop ΔU_L (produced by the commutation phenomenon). Being a fictitious size, on the internal resistance R_i it is not dissipated active power ($p_R = R_i \bullet I_d^2 = 0$) when it is crossed by the load current $I_d \neq 0$.

3. Resistive voltage drop ΔU_R

It is present and manifested during the conduction intervals, it is proportional to the load DC current intensity I_d and is caused by the presence the non-zero resistances $R_k = R_2 + R_1 (w_2/w_1)^2 \neq 0$ on each phase in the equivalent circuit of the rectifier transformer.

Since on the conduction intervals the load current $i_d = I_d$ flows (alternatively) through two (of the three) phases of the equivalent scheme (of the rectifier transformer), the resistive voltage drop ΔU_R can be evaluated by the relationship:

$$\Delta U_R = 2 R_k \cdot I_d \quad (50)$$

5.2 Conversion Group External Characteristic $U_d=f(I_d)$

If U_{d0} represents the ideal rectified voltage (the average value at no-load operation), then at on-load operation the rectified voltage U_d will be calculated with the relationship [10,13-14]:

$$U_d = U_{do} - u_D - \Delta U_L - \Delta U_R = U_{do} - u_D - (R_i + 2R_k) \cdot I_d \qquad (51)$$

Since $u_D \approx$ constant $\approx 3 \div 20$ V (regardless of the load current magnitude), through the graphical representation of the relationship (51) the external characteristic $U_d = f(I_d)$ of the conversion group of the DC traction substations will be obtained, as in Fig.5.

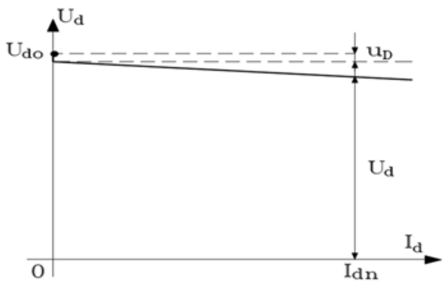

Fig. 5 External characteristic $U_d = f(I_d)$

Concretely, for the three-phase bridge rectifier, the inductive voltage drop ΔU_L can reach the value $\Delta U_L = R_i \bullet I_d = (10 \div 12)\%$ from U_{d0}, while $\Delta U_R \approx (0{,}1 \ldots 0{,}5)\%$ from U_{d0} (at rated load $I_d = I_{dn}$).

5.3 Conversion Group Equivalent Circuit

The relationships established in the preceding paragraphs allow (for the average DC components) the introduction of an equivalent electrical circuit of the conversion group, as in Fig.6 [10]. It contains three elements connected in series, namely:

- a constant voltage source with the voltage at the terminals: $U_{d0}' = U_{d0} - u_D \approx U_{d0}$

- an internal resistance with the magnitude: $R_i + 2R_k \approx R_i$, and

- an internal inductance with the magnitude $L_i = 2L_k$

The internal inductance $L_i=2L_k=2\{L_{\sigma 2}+L_{\sigma 1}\cdot(w_2/w_1)^2\}$ allows to consider the influence of the transient phenomena on the DC side (at the variation of the current I_d) in the magnitude of the voltage U_d.

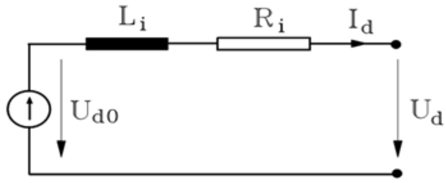

Fig. 6 Equivalent circuit of the diode conversion group at the average component level

Based on the equivalent circuit depicted in Fig.6 one can write the differential equation corresponding to average components of the conversion group, as below:

$$U_d = U_{d0} - R_i \cdot I_d - L_i \cdot \frac{dI_d}{dt} \qquad (52)$$

In a steady-state regime (when I_d = ct.), the equation (52) degenerates (with the particularities $u_D \approx 0$ and $R_k = 0$) into the relationship (51) used in calculating the average voltage (in load) U_d.

One could also note that often times, in the equivalent scheme of Fig. 6, the internal inductance $L_i = 2L_k$ is very small in relation to the inductance L_d of the load circuit ($L_i << L_d$) and, consequently, it can be neglected ($L_i \approx 0$). On the contrary, the internal resistance $R_i = (3/\pi)\omega L_k$ generally has the same order of magnitude as the resistance of the load circuit and, therefore, it must necessarily be taken into account (when evaluating the U_d voltage).

6 Discussion and Conclusion

The transmission and distribution of electricity through different voltage levels are possible due to the use of power transformers. In this article the authors carried on a comprehensive analysis of the three-phase transformer - three-phase rectifier assembly in traction substations, and established the equivalent circuit of the AC / DC conversion group at the average DC components level.

Since the efficiency standards can be expressed in terms of electrical efficiency, in order to enhance the transformer efficiency, in this study the authors carried out the three-phase power transformer modelling with space phasors. The equations system obtained with space phasors can be used to analyse the dynamic regimes of three-phase transformers, being successfully applied, for instance, in the method of structural diagrams for the power transformer operation.

Subsequently we have analysed some operation aspects of three-phase power transformers in AC/DC traction substations, concluding that although the line and phase voltages vary sinusoid over time, both primary currents and secondary phase and line currents vary non-sinusoid over time. This way results explicitly in the deforming (non-sinusoidal) regime in which the rectifier transformer of traction substation is operating.

In this article the authors also established and analysed the external characteristic and the equivalent circuit of the conversion group in AC/DC traction substations.

Looking forward the authors of this study intend to analyse the currents' harmonics in the three-phase power transformer of an AC/DC traction substation.

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