Decentralized model predictive control of urban drainage systems

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Abstract: Using real time control (RTC) techniques to improve urban drainage system performance is proven to be an effective solution for alleviating urban flooding. Many modeling methods of urban drainage systems have been introduced in existing literatures to accommodate different situations and scenarios. Nonlinear hydrologic models are useful for detailed simulations in pipes and sewers. However, to utilize RTC requires users to establish suitable models that both reflect physical characteristics of the system while not over complicating with unnecessary details. A mixed integer system was proposed where hybrid Model Predictive Control can be used to compute control actions in previous literature. However, the complexity of solving associated optimization problem grows exponentially with the size of the system and therefore, the computation time renders direct application of such method infeasible. This paper investigates the possibility of partitioning the system into several subsystems with communications and instead of computing solutions in centralized framework, the control actions are obtained distributedly by individual subsystems. The performance of decentralized schemes is demonstrated with numerical simulations on a fictional sewage system composed of 13 tanks and 12 control follows under 4 rain scenarios corresponding to different rain intensities. Decentralized Model Predictive Control is shown to have comparable performance compared with the centralized framework while having significantly improved computation time. Two methods are also presented to reduce pumping energy costs by harvesting rainfall energy.

Key–Words: Urban drainage systems, Decentralized Model predictive control, Cost reduction

1 Introduction

Various studies indicate that extreme weather conditions leading to increased flooding frequency and severity may continue in the near future [1,2]. A recent study estimated that the global cost of flooding in the worlds 136 largest cities could rise to $52 billion a year by 2050, a significant increase from $6 billion in 2005 [3]. Thus, it is of growing interest among both research and engineering communities to improve city water management measures. Combined urban sewage systems collect rainfall and waste water from all parts of the city using open canals and sewer pipes. The sewage is transported through interceptors, weirs and main sewer pipes into temporary storage tanks and waste water treatment plant before it is released to the environment. When severe regional rain storm occurs, large volume of water can easily overload parts of the system and excess sewage is released to the nearest receiving environment. The excess discharge of rainfall along with untreated waste water, known as Combined Sewage Overflow (CSO), endangers city infrastructures and contains biological and chemical contaminant which brings significant hazard to public health. The associated social, economical and environmental costs prompted several propositions. A prevalent solution to the CSO problem is to enhance existing drainage system by increasing storage volume and water treatment plant capacity. Examples include the Tunnel and Reservoir Plan (TARP) in Chicago [4] , Escola Industrial reservoir in Barcelona [5] and Quebec Urban Communitys Westerly sewer networks [6]. To take most profits of these expensive infrastructures, it is necessary to apply real-time control (RTC) techniques which can efficiently utilize the total storage volume and avoid overflowing in parts of sewage system while other parts operating under capacities. Optimal control techniques can also help operators to establish priorities among various objectives by associating different weight factors and constraints. Model Predictive Control (MPC) [7], also referred to as Receding Horizon Control, has proven to be one of the most effective and successful control schemes for large interconnected systems [7, 8]. Using rain prediction over prescribed horizon, MPC computes a set of decision variables over the horizon as set points for lower level control objectives ahead of time but the controller only implements control actions corresponding to the first sampling time and allows for further update of weather condition, thus providing built-in robustness compared to other methods. The ability to incorporate multi-objective and scalability to large systems makes MPC applicable to urban sewage system which is hierarchical and distributed in nature. Nonlinear hydrodynamic models are useful for detailed simulations and physical considerations of flow conditions in pipes and sewers. However, the partial differential equations involved often induce significant computational burden and provide unnecessary details for real-time control purposes. Thus, it is necessary to develop a model that captures system dynamics and is easily expandable using telemetry sensory information. Several modeling techniques have been presented in the literature [9]. The methods presented here follow closely the modeling principles introduced in [5, 8]. By introducing logical variables, nonlinear behaviors of the system, such as change of modes of operations and overflowing at specific locations etc, can be described with a set of linear equations and constraints thus preserving convexity with Mixed Logical Dynamic (MLD) [11] formulation. Under this framework, the model for urban drainage systems tends to have hundreds of decision variables, depending on the level of interest and desirable performance criteria, even for a neighborhood of urban area. The complexity of solving such Mixed Integer Linear Programming (MILP) problems grows exponentially with the number of variables which hinders its direct implementation. We therefore propose to apply decentralized MPC methods where several subsystems are formed and each of them only receives local rainfall predictions and computes local control actions in parallel. The total number of control actions remains the same but are computed distributedly and thus reducing the computation burden for each subsystem. Neighboring subsystems communicate with each other to exchange information on future actions, inter-subsystem coupling and states estimate to compensate for the lack of centralized information processing. We compare the performance of centralized MPC and decentralized MPC with several numerical simulations characterizing different rain scenarios. The advantage of decentralized MPC is demonstrated with significantly reduced computation time and minor performance loss compared with the centralized MPC.

2 Method

Mathematical description of physical flow conditions inside sewers and pipes can be achieved by solving a set of partial differential equations involving flow conditions and water levels in open sewers. However, the extent of details provided by computing such solution is unnecessary for our purpose of real time control and the computation costs render such approach unsuitable for obtaining control actions in a timely manner, especially for medium to large scale systems. There exist several modeling techniques in previous literature that use linear models to represent the system by considering a neighborhood of urban area as a virtual reservoir [12]. Such formulation preserves the convexity of the problem and therefore well-established optimization techniques can be used off the shelf. However, for urban drainage systems, there are inherent nonlinear dynamics that cannot be captured by such formulation. We here follow the work of [5, 8] by introducing continuous and binary logical variables to describe system dynamics that may exhibit a nonlinear mode of operation depending on system states.

2.1 MLD Systems and Modeling

The Mixed Logical Dynamic systems are first introduced by [11] where linear equations and inequalities involving continuous and binary variables are used to describe systems constrained by physical laws and logical conditions. In such framework, mixed-integer programming techniques can be used to develop Model Predictive control strategies. The following example system was taken from [11] to demonstrate its application. Consider the following system:

$$x(t+1) = \begin{cases} 0.8x(t) + u(t) & \text{if} x(t) \geq 0 \\ -0.8(t) + u(t) & \text{if} x(t) < 0 \end{cases} \tag{1}$$

where t is the time step of consideration and we associate the following condition with a binary variable:

$$\delta(t) = 1 \Leftrightarrow x(t) \geq 0. \tag{2}$$

This can be equivalently expressed with the following linear constraints:

$$\begin{cases} -m\delta(t) \leq x(t) - m \\ -(M+\epsilon) \leq -x - \epsilon \end{cases} \tag{3}$$

where M and m are the maximum and minimum of $x(t)$ respectively and ϵ is a small positive scalar depending computation accuracy. Let us introduce a new variable along with following constraints:

$$\begin{cases} z(t) \leq M\delta(t) \\ z(t) \geq m\delta(t) \\ z(t) \leq x(t) - m(1-\delta(t)) \\ z(t) \geq x(t) - M(1-\delta(t)). \end{cases} \tag{4}$$

The evolution of the system can then be expressed by the following linear difference equation :

$$x(t+1) = 1.6z(t) - 0.8x(t) + u(t) \tag{5}$$

We now present urban drainage systems model in the above frame work. The system is divided into several catchments according to geography and their coupling relationships with neighbors. Each one of them are represented as a virtual tank [12] which aggregates the total storage volume of a specified neighborhood of urban sewage system. The total volume can be computed by the mass balance of the inflows, outflows and stored volume of rainfall. Some other elements of sewage system can be incorporated into this frame work easily such as detention tanks, diversion gates, nodes and weirs.

Virtual and real tanks

The virtual tanks represent the primary storing element of each neighborhood. Considering the mass balance of inflow and outflow, virtual tank discrete-time dynamics can be expressed as follows:

$$\begin{aligned} v_n(t+1) = v_n(t) + \Delta t \varphi_n S_n P_n(t) + \\ \Delta t (q_n^{in}(t) - q_n^{out}(t) - q_n^d(t)) \end{aligned} \tag{6}$$

where we use subscript to denote location and t for time label. Furthermore, Δt is the sample time, φ_n is the ground absorption coefficient, S_n is the surface area, P_n is the rain intensity and q_n^{in} represents the combined input of manipulated flows and sewer flows into the corresponding tank. When more water than the storage volume of virtual tanks is sent to the tank, excess amounts are redirected to other parts of sewage system or to the nearest receiving environment. When such overflows occur, we denote the extra flow path created as $q_n^d(t)$ which can be expressed as follows:

$$q_n^d(t) = \begin{cases} \frac{v_n(t) - \overline{v}_n}{\Delta t} & \text{if } v_n(t) - \overline{v}_n \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

where $\overline{v}_n$ is the maximum capacity of n-th tank. Tank outflows $q_n^{out}(t)$ are assumed to be proportional to the tank volumes and represented as follows:

$$q_n^{out}(t) = \begin{cases} \beta_n \overline{v}_n & \text{if } v_n(t) > \overline{v}_n \\ \beta_n v_n(t) & \text{otherwise} \end{cases} \tag{8}$$

where β_n is defined as the volumetric flow coefficient suggested in [10]. Real Tanks mentioned above represent the actual storage elements in the sewage system such as reservoirs and detention gates where no external environment input is received,

$$v_{real}(t+1) = v_{real}(t) + \Delta t (q_{real}^{in}(t) - q_{real}^{out}(t)). \tag{9}$$

Since real tanks are considered to be without overflowing capabilities and the upper limit capacities are hard physical constraints in the sense that the inflow into real tanks, $q_{real}^{in}(t)$, has to be pre-manipulated to ensure the net input wont exceed the remaining capacity at every time instant.

Controlled gates, sewer pipes and canals

In sewer networks, diversion gates are used to divert flows to desired locations so that storage volumes are fully utilized to avoid partial overflows and detention gates are used to temporally stop the flows at certain locations such as input nodes at real tanks. When water is discharged from the tanks, it is transported through weirs, open sewer pipes, canals and interceptors to other parts of the system. It is possible that the amount of discharged water exceeds the flow limit of such elements and overflows occur. We distinguish outflows of tanks as active controlled flows and passive transit elements mentioned above with the following characterizations. Assuming there are m sewer paths and j controlled flows at n-th junction of outflows:

$$q_n^{out}(t) = \sum_{i=1}^{m} q_i^s + \sum_{i=1}^{j} q_i^u \tag{10}$$

where q_i^s denotes the i-th passive transit element as a default path for sewage flow, which can be further expressed as:

$$\begin{aligned} q_i^s(t) = q_n^{out}(t) - \sum q^u(t) - \sum_{j \neq i} q_j^s(t) \\ \text{if } q_n^{out} - \sum q^u(t) - \sum_{j \neq i} q_j^s(t) \leq \overline{q}_i^s \\ q_i^s(t) = \overline{q}_i^s \quad \text{otherwise} \end{aligned} \tag{11}$$

where $\overline{q}_i^s$ is the flow upper limits for i-th sewer pipe.

Using binary logical variables and linear inequality constraints, we collect system states into a column vector $v(t)$ and express the system evolution process as a linear matrix equation defined in terms of system states $v(t)$, control actions $u(t)$, continuous logical variables $z(t)$ and binary logical variables $\delta(t)$ as

$$v(t+1) = Av(t) + B_1u(t) + B_2\delta(t) + B_3z(t) + B_4d(t) \tag{12}$$

where A_i and B_i's are constant matrices with appropriate dimensions and $d(t)$ is the rainfall vector.

2.2 Optimization problem formulation

With above mixed integer dynamical system formulation, we can pose the optimization problem with desired objective functions where system state evolution is treated as constraints along with physical constraints and logical constraints associated with previously introduced logical variables. Many solvers can then be used off the shelf to derive a solution with finite horizon. Note that in this framework, we preserve the convexity of the problem which facilitates the computation process. We now briefly discuss the form of constraints and objective functions used.

Constraints

In the previous section, we presented the constraints associated with logical variables which are used to ensure exact correspondence between logical variables and logical events they represent. The nonlinear behavior of the system is thus expressed with linear equations along with linear inequality constraints. As we are manipulating the flow rates at various locations, physical constraints such as operation range of control variables and mass balance at nodes and junctions must be respected along with afore introduced logical constraints. Special attention should also be given to the manipulated flows which are directed into real tanks with hard physical constraints. Since the volume of the water inside a real tank can never exceed its physical capacity, we have to ensure that the net input into real tanks cannot be larger than the remaining available storage room. Note that all the constraints can be expressed linearly in terms of controlled flows q_i^u, logical variables δ_i and z_i, system states variables v_i and constant parameters as follows :

$$E_2\delta(t) + E_3z(t) \leq E_1u(t) + E_4v(t) + E_5 \tag{13}$$

where E_i's are matrices of appropriate dimension with entries of system parameters.

Cost functions

In most applications, the minimization of sewage overflows on the city streets is of primary interest. By redirecting sewage to other parts of the system, we can fully utilize the storage volume and evenly distribute the sewage among the system, thus avoiding situations when part of the system is operating under capacity while other parts suffer from overloading. This is usually achieved by manipulating flow rates using hydraulic pumps installed at various locations of the system. Along with telemetry sensors, minimizing installation and operation costs of hydraulic pumps is also commonly pursued by users. Due to limits on actuation range and the structure of the network of interest, the location of installing hydraulic pumps should be carefully considered. In cities where frequent rainfalls with large intensities are expected, operators also emphasize on maximizing available tank volumes at each time instant to accommodate future rainfall. This objective is consistent with reducing pollution to the environment as we should maximize the volume of sewage being sent to the treatment plant before it is released. Therefore, depending on the size of the system, local environment, city infrastructure and user objectives, cost functions with different prioritized goals can be formulated easily within the framework constructed earlier. In the following, we consider objectives commonly pursued in most situations: minimizing overflows in urban neighborhood (virtual tanks), minimizing overflows in connecting sewer paths and minimizing energy cost from pumping. Virtual tanks represent the total storage volume in certain neighborhoods of an urban area. These elements can easily be overloaded during extreme weather conditions and avoiding overflows in virtual tanks and sewer pipes is often the primary interest. The formulation for their cost functions are stated as below:

(1) Virtual tanks:

$$J_1 = \begin{cases} \sum(z_i - \overline{v}_i) & \text{if } \delta_i = 1 \\ 0 & \text{otherwise} \end{cases} \tag{14}$$

(2) Sewer pipes

$$J_2 = \begin{cases} \sum(q_i^{out} - q_i^u - \overline{q}_i^s) & \text{if } \delta_i^s = 1 \\ 0 & \text{otherwise} \end{cases} \tag{15}$$

where $\delta_i^s = 1$ represents the logical event of overflowing at i-th sewer path.

Electricity costs associated with manipulating sewage with hydraulic pumps can be expensive especially when large amounts of control actions are required. And it is often a trade-off between reducing energy costs and minimizing overflows. We now briefly discuss a potential approach [13] that can be used to reduce energy cost by introducing two devices that can harvest rainfall energy and generate electricity. The first approach is characterized as *Pressure forebay* (shown in Figure 1) which utilizes rainwater collected from high altitudes such as building roof top. The water collected will be transported through pipes into pressure regulating bay. When enough water is collected and reaches operating level, the valve is opened, and water pushes through hydraulic generator with constant pressure. The water will then be directed to reservoirs in lower altitudes. When collected water flow is not sufficient for generators operating conditions, the valve will be closed. *Pressure forebay* thus functions as both temporary storage element and as a pressure regulating device.

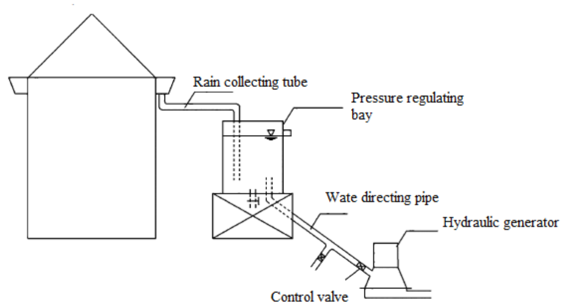


Figure 1: Pressure forebay schematic

The second approach is characterized as *Surface aqueduct* (shown in Figure 2) in which water with comparatively large kinetic energy is collected and directed through collecting tubes into surface aqueduct. Once it is regulated through valves, water enters the spiral case and pushes through turbine to generate electricity. The remaining water is then directed to near storage elements for further recycling use. When water exceeds the limit capacity of aqueducts and spiral cases, the excess water will be directly discharged through overflow holes into storage elements mentioned earlier. When the volumetric flow is deficient for operating, the valve will be closed, and the generator stops working.

Generating electricity using rain energy shares a lot of similarities as normal hydraulic power generating scheme. Normal hydraulic power generating requires terrain altitude difference but the approaches we presented here are more suitable for urban environments and can be implemented as an extension to the urban drainage systems. The amount of electricity generated is directly related to the waterhead in reservoirs. It is therefore most preferable to implement such methods at locations with high altitudes and with large available collecting areas. Networks

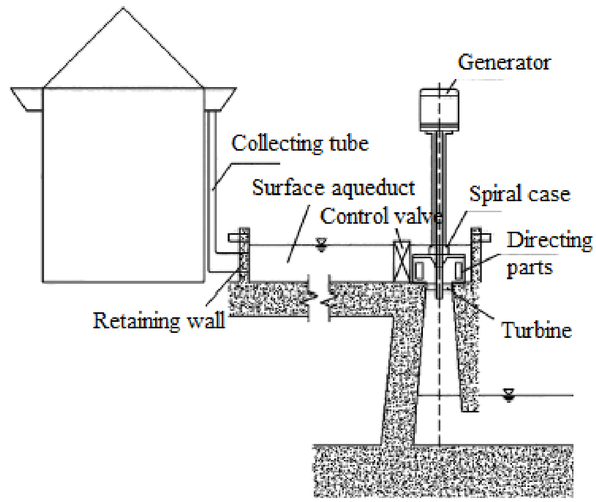


Figure 2: Surface aqueduct

of these two devices can be formulated by connecting them with water pipes to further increase power output. A physical implementation of pressure forebay is shown in Figure 3. It is installed on the roof of a warehouse in Jinhua city, Zhejiang province, China, as a prototype with collecting area of 1200 m^3 , maximum water head 4.6 , maximum power generating flow 2.5 L/s and with maximum power generation capability of 80 W. The rainfall is collected from the roof and transported through pressure regulating bay which provides stable flow for the hydraulic generator. Other than providing extra storage volume, it provides extra clean energy for daily use of the factory.

In this paper, we incorporate these devices into energy cost functions by associating a lower cost coefficient with control actions where such devices are implemented as follows:

$$J_3 = \sum C_i(d) q_i^u \Delta t \tag{16}$$

where $C_i(d_i)$ is a rainfall dependent cost coefficient for pumping. Locations with rainfall energy harvesting devices installed will have smaller coefficients. Depending on the level of priorities, operators can associate different weight factors to each objective to acquire desired performance,

$$J = a_1 J_1 + a_2 J_2 + a_3 J_3 \tag{17}$$

The sewage network management can now be posed as an optimization problem with manipulated flow and logical variables as decision variables constrained by logical constraints, physical constraints and system evolution dynamics. Using the state equation for discrete-time dynamical systems, we can express the states at arbitrary time instant of interest as a function of initial states, control actions and logical variables in previous sampling times as follow:

Figure 3: Rainfall harvesting device implemented at Zhejiang province

$$v(k) = A^t v_0 + \sum_{i=0}^{t-1} A^i [B_1 u(t-1-i) + B_2 \delta(t-1-i) + B_3 z(t-1-i) + B_4 d(t-1-i)]. \tag{18}$$

We collect all the decision variables in a single column vector as follows:

$$\begin{aligned} \alpha &\triangleq [u(0) \dots u(t)]^T \\ \beta &\triangleq [\delta(0) \dots \delta(t)]^T \\ \psi &\triangleq [z(0) \dots z(t)]^T \\ \gamma &\triangleq [\alpha, \beta, \psi]^T \end{aligned} \tag{19}$$

By reformulating the constraints inequalities and cost function, we obtain the following mixed integer linear programming problem :

$$\begin{aligned} \min \quad & L\gamma \\ \text{s.t} \quad & F_1 \gamma \leq F_2 + F_3 v_0 \\ & \gamma_l \leq \gamma \leq \gamma_u \end{aligned} \tag{20}$$

where L collects all the constant coefficients of cost functions, γ_l and γ_u corresponds to lower and upper limit of logical variables and controlled flows, and F_i's are matrices of appropriate dimension.

2.3 Control strategy

Model Predictive Control (MPC) [7] is often used in process control such as chemical plants, oil refineries and power electronics. With a dynamics model that is usually linear and empirically derived from system identification, MPC optimizes the prescribed cost function over some finite horizon constituted by several sampling time steps. Once the control strategies are computed for the whole horizon, only the first set of control actions are implemented and then the controller re-solves the optimization problem with a shifted horizon and it is therefore also known as Receding Horizon Control. MPC allows updated information to be incorporated at the start of each optimization iteration and therefore is able to accommodate for future events and take actions accordingly. This approachs ability to allow for system modeling error and the effectiveness of considering multi-objectives have led to great results in practice. For sewage systems, MPC is used to compute the decision variables (the controlled flows) ahead of time according to a set of control goals expressed as cost functions formulated earlier with possibly different priorities. These computed control goals can be achieved by local PID controllers at each part of the sewage system.

The mixed integer linear programming problem formulated in previous sections can be solved using MATLAB OPTI Toolbox [14]. However, in the framework of MPC where we specify a horizon constituted by several time steps, the optimization problem has to be solved repeatedly and the complexity of solving such problem grows exponentially with the number of variables. For urban drainage systems, it is typical to establish system models with tens or even hundreds of virtual tanks to achieve desirable performance. Consequently, the search space becomes too large to have the optimization problem solved in an efficient manner that suits our real time control purpose under a centralized MPC framework. It is also difficult to adjust the centralized model when modifications to the system are made, such as the aforementioned addition of rainfall energy harvesting devices. We therefore propose to partition the system into several sections according to geographical proximities and coupling relationships. Each subsystem receives local rainfall prediction and considers neighboring coupling as external disturbances. A controller solves for local actions in parallel and exchanges information with neighboring subsystems to update actions taken and states estimate. If a local measurement of the tank volume is available, the controller does not need neighboring controller information to estimate system states thus reducing communication load. Similar to the centralized standard MPC, only the first set of control actions are applied to the system and each subsystem will solve the optimization problem again with updated information and shifted horizon. Since each subsystem only receives partial information and makes local decisions, the performance of decentralized MPC is usually worse than centralized one. However, the computation time can be significantly reduced by breaking the large system into several small ones as shown in Figure 4. There are naturally different separation schemes depending on priorities on geographical consideration, coupling emphasis and information transmission. Under different rain scenarios we expect different separating schemes to have different performance. The decentralized MPC algorithm is summarized as below.

Algorithm Decentralized Model Predictive Control

Input: system parameters, initial states, cost priorities, horizon length and sampling time

1: Each subsystem i computes for γ_i by solving the subsystem optimization problem with local rainfall prediction and assumed neighboring coupling based on historical data.

$$\begin{aligned} \min \quad & L_i \gamma_i \\ \text{s.t} \quad & F_{1_i} \leq F_{2_i} + F_{3_i} v_{0_i} \\ & \gamma_{l_i} \leq \gamma_i \leq \gamma_{u_i} \end{aligned}$$

2: Each subsystem implements the first set of solutions and computes final states.

3: Neighboring subsystems communicate and update for coupling information and final states over the horizon.

4: Each subsystem receives new rainfall prediction over shifted horizon and repeats step 1 with updated coupling information from neighboring controllers.

2.4 Simulation setup and solver

We demonstrate the application of the above modeling method and compare the performances of centralized and decentralized setting with a fictional urban drainage system composed of 13 tanks where 4 tanks are real. The form of rainfall data input was employed as those in [12] where intensity is measured using a tipping bucket rain gauge. Each event of tipping corresponds to a of rainfall and after appropriate conversion, the rainfall prediction is provided to each subsystem in unit of at the beginning of the horizon and updated periodically before each optimization iteration. The system model has 12 control variables, 18 logical variables and 41 auxiliary variables with parameters, including tank volumes, sewer flow capacity and hydraulic pump power, taken from [12]. The decentralized system is partitioned into 4 subsystems, shown in Figure 4 with different colors, based mainly on geographical considerations. The system is designed to be able to implement negative inflows to real tanks to respect hard constraints on real tank capacity. Four real tanks are placed at locations to reflect geographical separation and subsystem independence. All sewage is transported through waste water treatment plant (WWTP) before it is released to the environment. The priorities are set on minimizing CSO volumes by considering a cost function constituted by overflow volumes (J_1 and J_2) with cost factor 1 and manipulated flow volumes (J_3) with cost factor 0.1. These cost factors can be adjusted depending on the level of emphasis placed on reducing overflows and saving electricity. The optimization horizon is set to 5 with sampling time equals 300 seconds and the performance of each system is compared with length of 75 minutes with total of 15 sets of control actions. Each horizon is ignorant of rainfall events in subsequent horizons. This simulation is implemented with MATLAB code and solved with OPTIBOX mixed integer linear programming on an Intel i7-4700 MQ 2.40 GHz machine. The performance of the two setup is compared under rain scenarios corresponding to small, medium and large rainfall intensity. We measure the effectiveness of each setup with total costs, overflow reduction, total controlled sewage volumes and computation time. The effect of geographical rainfall heterogeneity is also investigated by varying rainfall profile for each subsystem and we assume all control action consequences finish propagating in the prescribed sampling time.

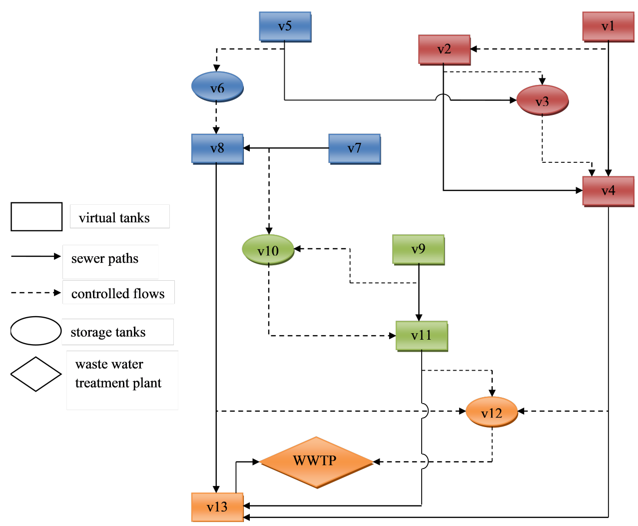


Figure 4: Simulation system

3 Results

The performance of decentralized and centralized MPC is summarized in Table 1 and Table 2 under several characteristic rain events. In all cases, decentralized MPC computes control actions that induce more costs than centralized MPC as expected. Since each subsystem of decentralized MPC only has local information where couplings between neighboring subsystems are considered as disturbances based on estimation without full control. The optimal control actions under the small rainfall event are simple to derive as the input quantity often does not exceed the capacity of tanks or sewers. Therefore, no control actions are required for most of the time during small rainfall scenario and the sewage is transported by the default sewer pipe. Since overflow is avoided in this case, most of the costs come from controlling actions. The improvement of computation time by using decentralized MPC is not significant as the optimization problem is not hard to solve.

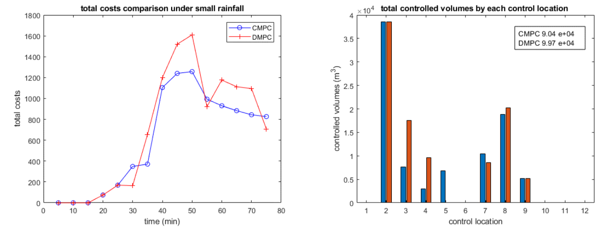
As rainfall inputs start increasing, the system dynamics become more complicated and computation time increases for both control strategies. Overflows also start occurring at locations where input exceeds elements capacities. In a medium rainfall scenario, the computation time of decentralized MPC is significantly reduced (7% of centralized MPC) with 22% more induced costs. The advantage of decentralized MPC is mostly pronounced in this rainfall scenario in which overflows can be kept small if optimal solution is implemented. For centralized MPC, the computation time can render physical implementation meaningless especially if large system is involved when computation time might exceed the sampling time. In a large rainfall event, significant overflow occurs for both control strategies and the advantage of decentralized MPC in terms of computation time is less obvious. We observe that the computation time of centralized MPC decreases for the storm scenario where large quantity of rainfall enters system in short amount of time. This is possibly due to the fact that the system reaches the saturation state and the only available action is to use maximum pumping power.

Table 1: Computation time (s) comparison

Rain profile	Decentralized	Centralized
Small rain	2.32	14.36
Medium rain	4.83	68.72
Large rain	14.21	69.76
Storm	39.04	64.89

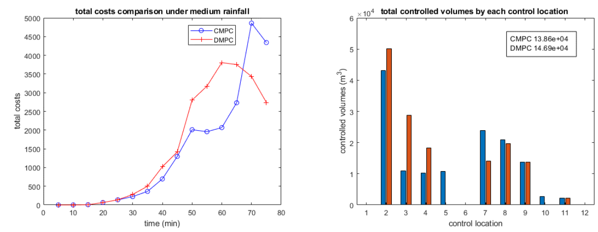
Table 2: Overflow volume ($10^3 m^3$) comparison

Rain profile	Decentralized	Centralized
Small rain	0.43	0
Medium rain	8.44	6.89
Large rain	53.82	38.96
Storm	192.87	174.91



(a) costs trajectory (b) controlled volume

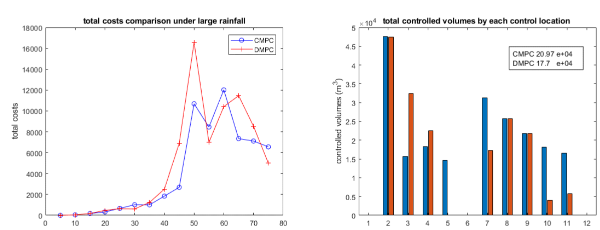
Figure 5: Small rainfall scenario



(a) costs trajectory (b) controlled volume

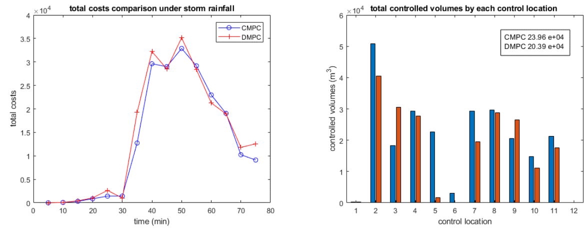
Figure 6: Medium rainfall scenario

The total costs (including overflows and operation costs) and total controlled flow volumes comparison is shown in Figures 5,6,7, and 8 for different rain events. It is obvious that more control actions are implemented as the rainfall inputs increase for both decentralized MPC and centralized MPC. Centralized strategy clearly computes more optimal control actions as it achieves the goal of minimizing overflows even with less control. The performance of decentralized MPC can be further improved by installing rainfall energy harvesting devices discussed in Section 2.2. Two control strategies can generate very different solutions under the same rainfall event as shown by system trajectories and control variable activations. Since during each horizon, the rainfall prediction of subsequent horizons is unavailable, it is possible that tank volumes are not drained with maximum capacity and results in overflow surge at the start of a new horizon when rainfall inputs suddenly increase. Such problems can be addressed by adjusting control horizon according to rainfall predictions data or by penalizing terminal state volumes in the objective function.



(a) costs trajectory (b) controlled volume

Figure 7: Large rainfall scenario



(a) costs trajectory (b) controlled volume

Figure 8: Storm scenario

In the storm rainfall scenario, system trajectories generated by both control strategy share significant similarities due to saturation of system states as mentioned before.

4 Conclusions

The need for application of real time control techniques in drainage systems is getting increased attention around the globe due to increased extreme weather conditions and growing concern for environmental and economical cost associated with urban overflows. We presented a modeling framework introduced in [5, 8] which can be used to pose the drainage system management problem as a mixed integer linear programming problem. Model predictive control was used so we were able to take advantage of periodically updated rainfall prediction to increase accuracy and robustness. The exponential increase in computation complexity associated with large systems was addressed by partitioning the overall system into several subsystems considering geographical proximities and coupling relationships. Neighboring subsystems in decentralized MPC were allowed to communicate system states (tank volumes) and coupling information (sewer pipes and controlled flows) to accommodate for lack of central processing node. We also discussed two approaches to harvest rainfall energy and thus decreasing control energy inputs. The locations of installation of discussed devices should be carefully selected to ensure maximum power output and system optimization. The advantage of decentralized MPC was demonstrated in several numerical simulations on a fictional system. Simulation were run in several rainfall events corresponding to different levels of rainfall inputs. It was shown that decentralized MPC suffer from performance loss in all rainfall scenarios and the degree of improvement in computation time closely depends on the rainfall intensity. In small rainfall scenarios where system dynamics are simple and optimal solution is easy to derive, simulations suggest that even though centralized MPC achieves the goal of avoiding overflows with longer computation time compared with decentralized MPC, but still remains applicable. In situations where overflows can be reduced only with appropriate management, decentralized MPC computes control actions significantly faster than the centralized one with minor performance losses. The advantage of decentralized MPC is not obvious as rainfall inputs further increase and the system approaches the saturation state. But in such cases, simply maximizing pump power can approximately achieve the optimal solution. In the framework of MPC, we were able to easily incorporate several objectives with different priorities in the cost function to reflect different goals pursued by the user, such as minimizing overflows, minimizing operation costs and maximizing available storage volumes. The general applicability of presented modeling methods and control schemes make it applicable to modern urban sewage systems with telemetry sensors installed and sufficient automatic control. The built-in robustness provided by MPC techniques makes it effective for managing drainage system especially for inaccurate rainfall data and model errors. Future research includes investigating time delay on control consequence propagating in the network and also a consideration of physical phenomena present in drainage systems, such as effect of built-up water pressure on the controllability of the system. The optimal placement of hydraulic pumps should also be studied for new drainage system implementation.

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