# Analysis of the influence of sensitivity on the quality of regulation in digital control systems

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*Abstract:* The article describes the problem of the influence of sensitivity on the quality of control in digital control systems at the premier synthesis of approaches used in the theory of electrical circuits and the theory of automated control. The analysis of the amplitude-frequency characteristics of the classical system of automatic control, a mathematical model of this system. After that, the obtained model was used to analyze the sensitivity and influence of the initial system settings on the quality control of the industrial process.

*Key-Words:* mathematical modeling, sensitivity analysis, synthesis problems, feedback control, discrete-time systems, digital systems.

## 1 Introduction

In the synthesis of chains that meet stringent requirements, due to scatter element parameters there is the problem of determining their tolerances. This task is relevant when analyzing the parameters of the circuits, when choosing the appropriate equivalent circuit and optimizing in the process structural (iterative) synthesis, as well as in the synthesis of automatic control systems and circuits with adjustable parameters. [10]

There are many reasons why deviations occur between the nominal values of circuit functions found by the analytical (calculated) and valid (actual) values of these functions. Among the sets of these reasons let us single out one of them, the essence of which is that the production of chains uses components, parameters which (due to factors inherent in production) have other than nominal values. At the approximation stage, the specified characteristics are replaced by physically realizable functions, and at the implementation stage, structures (topologies) and parameters of nominal elements corresponding to the function found chains. Since the same function of a chain can correspond to several different equivalent chains, differing from each other in topology and the parameters of the elements, then ultimately the choice of a specific implementation is based, as a rule, on structural synthesis, taking into account the specific additional requirements that the final circuit of the designed circuit must satisfy.

Requirements for the designed circuit are formed either in time t or in frequency ω domains. They are satisfied by an appropriate choice of the elements of the circuit xi based on an analysis of the tolerances in the space of the parameters of the circuit

## 2 Problem Formulation

The system, in general, we understand as a purpose-defined, finite and bounded set of certain elements $\boldsymbol{A}=\{a_1, a_2, \ldots, a_k, \ldots, a_n\}$ and a set of heterogeneous (simple and multiple) mutual relations (relations, bonds) between them $\boldsymbol{R}\{r_{i,j}\}$ for i = 1.., n; j = 1, .., n. The physical nature of the elements, their essential characteristics, and the set of interrelationships between them, together create an internal structure of the system that determines its immanent qualities and also the modes (capabilities, abilities) of behavior in interaction with the essential existential environment. If a system so defined is capable of responding to time-varying environmental influences (stimuli, actions) also by variable response (s), then it is a dynamic system (DS). In this sense, we say that there is a dynamic process in the system.

It is clear that the course of DS reactions to the input influences (inputs) from the environment will depend, in addition to their immediate action (over time), also on the instantaneous state of the process in the system as well as on the state of its structure (elements, their parameters and links). If we define the initial state of DS (initial conditions), the state of the environment, a unique exciting function (input signal), and we assume that the linkages between the individual elements of the system do not change, then the reaction of the system will depend only on the properties of its elements. Any change in the system element's parameters is more or less reflected in its behavior. Therefore, it is necessary and useful to analyze the effects of changes in DS parameter parameters on its behavior, respectively. Changes in behavior of the system when predicting element parameter changes, and apply this knowledge when creating, when monitoring the operation (operation) of dynamic (or mechatronic) systems.

## 3 Problem Solution

The filtering properties of the control system are characterized by its ability not to miss the noise contained in the control action. The input of the control system receives not only useful signals, but also noise and interference. Interference is caused by random deviations of the control action, noise and errors in the measuring elements and other factors [5].

Interference and control arrive to different inputs of the system, but due to signals can be brought to the input of the control, we will assume in future that interference and control will always arrive at the input of the system.

Interference can be specified in the form of several time functions $\beta_p(t)$, but in most cases, the interference will have random characteristics, hereupon we will define their spectral density $S_p(\omega)$. Interference can cause additional changes in the output value, as a result of increasing the amount of error and reduce the quality of control.

The variance of the error of the control system caused by interference, which is a random function, is given by

$$\overrightarrow{\delta_p^2} = \frac{1}{\pi}\int_0^\infty S_p(\omega)[\Phi(j\omega)]^2\, d\omega, \qquad (1)$$

Where $\Phi(j\omega)$ is the amplitude-phase frequency characteristic of a system with a closed loop.

In practice, when calculating the control system, there are often cases where the noise is specified in white noise format, whose spectral density can be expressed by the formula: $S_p(\omega) = c^2$. Then

$$\overrightarrow{\delta_p^2} = \frac{1}{\pi} c^2 \int_0^\infty [\Phi(j\omega)]^2\, d\omega, \qquad (2)$$

From the preceding formula it follows that the value of the integral

$$I = \frac{1}{\pi}\int_0^\infty [\Phi(j\omega)]^2\, d\omega, \qquad (3)$$

It can serve as a measure of filtering properties of the control system, the value of I being directly proportional to the mean square error caused by interference. Accordingly, the smaller the value of the desired integral, the better the filtering properties of the system.

The value of the integral

$$\tilde{I} = \frac{1}{\pi}\int_0^\infty [\omega^2\Phi(j\omega)]^2\, d\omega, \qquad (4)$$

Can serve as an indicator of the power developed by the control system. When operating the control system, there are times when the root-mean-square error caused by interference does not exceed the permissible value, while the actuator's effective power is much higher than its nominal value. Which leads to an unacceptable load on the control system. Thus, the value of the integral I can also serve as an indicator of the quality of the control system.

The control system with respect to the control action is an astatic system of the first, second order, depending on the type of object and the type of regulator. Therefore, the low-frequency asymptote of the desired Bode diagram of the open control system has a slope of -20, -40, -60 dB/dek []. Asymptotes of the desired Bode diagram can be graphically divided into two parts: unchanged part and variable. The asymptotes of the Bode diagram of the control system elements that belong to the control object or cannot be changed belong to the unchanged part of the desired diagram. The variable part includes diagrams of Bode regulators and frequency correction elements (for example, sensor filters).

The position of the low-frequency asymptote belonging to the Bode diagram of the unchangeable part of the control system is determined by the value of the gain of the open system μ. Since the value of μ can only be found with allowance for the transfer function of the immutable part, the position of the low-frequency asymptote, which will be referred to below as the first low-frequency asymptote of the Bode diagram, will be determined separately for each case. In this case, the requirements for the component of the error caused by the perturbation applied to the control object must be taken into account.

The high-frequency part of the desired Bode diagram of the open control system is determined by the frequency response of the immovable part of the system. The high-frequency part of the Bode diagram of all considered types of control system elements can have the first asymptote with the second and third slope, the second asymptote with the fourth and fifth slope (Fig. 2)

The task of forming the desired Bode diagram of the open control system is reduced to determining the mid-frequency asymptote intersecting the frequency axis with the corresponding asymptotes of the Bode diagram of the unchanged part of the system. The point of intersection of the frequency axis by the Bode diagram is the cutoff frequency. The cutoff frequency determines the operating bandwidth (a bandwidth) of the control system and, together with the phase characteristic, makes it possible to evaluate its stability. [7]

High-frequency asymptotes of the unchanged part mainly affect the stability of the internal loop of the control system and do not have a significant effect on the stability stocks of the system. Therefore, it becomes possible to use a filter in the control system to suppress high-frequency interference. The above asymptotes should be taken into account when analyzing the stability of the internal contour. [8]

The position of the second low-frequency asymptote of the desired Bode diagram of the open control system can be determined from the predetermined accuracy of reproduction of the control system of the harmonic control component. The expression for the amplitude $\boldsymbol{\delta^a}$ of the harmonic component of the error of the control system in the case when the control action is characterized by the equality

$$(t) = \dot{\beta}_0 + \beta_a \sin \omega_p t \tag{5}$$

has the form:

$$\delta_a \approx |W\,(j\omega_p)|\beta_a\,. \tag{6}$$

Where: - $\beta_a$ is the amplitude of the harmonic component of the control action.

The total greatest dynamic error in the absence of a perturbing moment

$$\delta = \beta_v + \beta_a = \frac{\dot{\beta}_0}{\mu} + \beta_a |W(j\omega_p)| \qquad (7$$

Where: - $\beta_v$ is the speed component of the error.

As desired, we use the characteristics obtained above. The formation of these characteristics is made taking into account the minimization of integrals $I$ and $\tilde{I}$ and the provision of required reserves of stability.

In accordance with the above calculations, for the desired characteristic of the first series, we have:

$$\theta = \frac{p}{\omega_p} \tag{8}$$

$$W_1(\theta) = \frac{1}{\theta(0{,}63\theta + 1)[0{,}0625\theta^2 + 0{,}15\theta + 1]} \tag{9}$$

For the desired characteristic of the second type, we have:

$$W_2(\theta) = \frac{1{,}25\theta + 1}{\theta^2(0{,}25\theta + 1)[0{,}0256\theta^2 + 0{,}096\theta + 1]} \tag{10}$$

For the desired characteristic of the third type

$$W_3(\theta) = \frac{2{,}56\theta^2 + 1{,}6\theta + 1}{\theta^3(0{,}125\theta + 1)[0{,}0064\theta^2 + 0{,}048\theta + 1]} \tag{11}$$

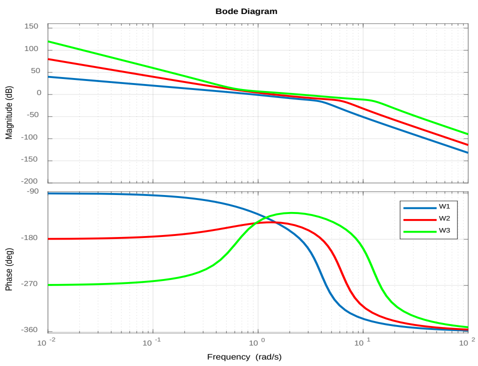


Fig. 1. The optimal form of the frequency response

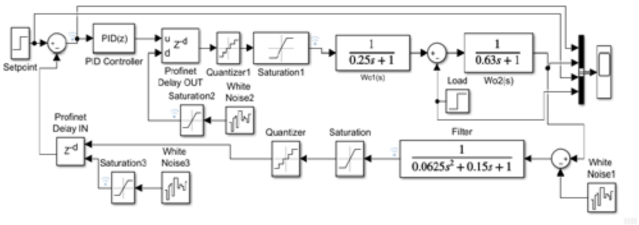


Fig. 2 Mathematical model of a distributed control system.

The central processor of the distributed control system, when performing the function of the controller of a large number of objects, introduces a time delay in the calculation of control signals. With a large overload of the signal transmission channels, there is also an additional time delay. Therefore, when setting up a robust regulator, it is necessary to take into account the sensitivity of the control system. To study the effect of random additional delays on the quality of regulation and the development of a robust controller tuning method, a single DCS channel model was developed with a setting corresponding to the transfer function given in formula (9). The model is implemented using Simulink Matlab (Fig. 3) and contains elements that mimic nonlinearity, discreteness of information processing, the random nature of measurement noise and time delays.

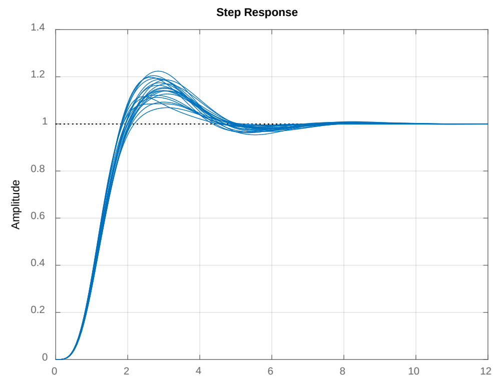


Fig.3. sensitivity analysis of control system

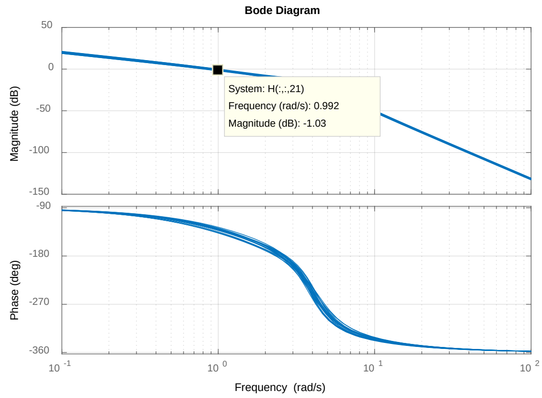


Fig. 4 Bode diagram for open-loop system

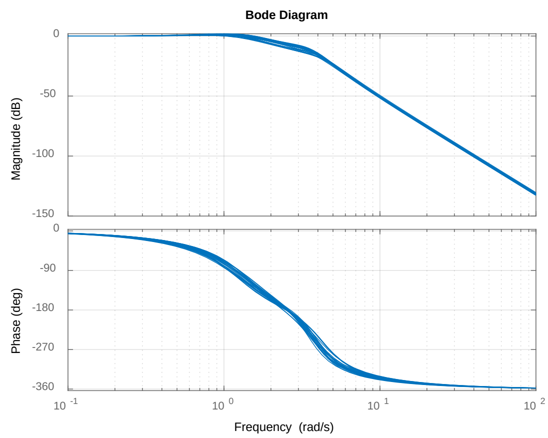


Fig. 5 Bode diagram for a closed-loop system

Below is a pragment of the development code for the robust controller from the matlab environment.

```
>> bw1 = ureal('bw1',
25.4,'Percentage',10)
```

```
>> Tw1 = ureal('Tw1', 1,'Percentage',10)

>> Tw2 = ureal('Tw2', 3.988,'Percentage',10)

>> Tw3 = ureal('Tw3', 19.76 ,'Percentage',10)

>> Tw4 = ureal('Tw4', 25.4,'Percentage',10)

>> H = tf(bw1,[Tw1 Tw2 Tw3 Tw4 bw1])


>> H = tf(bw1,[Tw1 Tw2 Tw3 Tw4 bw1])


  Uncertain continuous-time state-space model with 1 outputs, 1 inputs, 4 states.

  The model uncertainty consists of the following blocks:

    Tw1: Uncertain real, nominal = 1, variability = [-10,10]%, 1 occurrences

    Tw2: Uncertain real, nominal = 3.99, variability = [-10,10]%, 1 occurrences

    Tw3: Uncertain real, nominal = 19.8, variability = [-10,10]%, 1 occurrences

    Tw4: Uncertain real, nominal = 25.4, variability = [-10,10]%, 1 occurrences

    bw1: Uncertain real, nominal = 25.4, variability = [-10,10]%, 1 occurrences
```

## 4 Conclusion

In the article, within the permissible scope, attention was paid to some basic (immanent) properties of dynamic systems, which are manifested by their behavior towards the environment, respectively. In their responses to the input wake. The aim of the article was to point out the importance of this issue and also to present some original results of the work of the author presented at conferences and published in journals. The aim of this contribution is also to provoke a wider scientific community working in other fields such as cybernetics, automation and informatics to activate their scientific potential and to solve the problems they are discussing.[9]

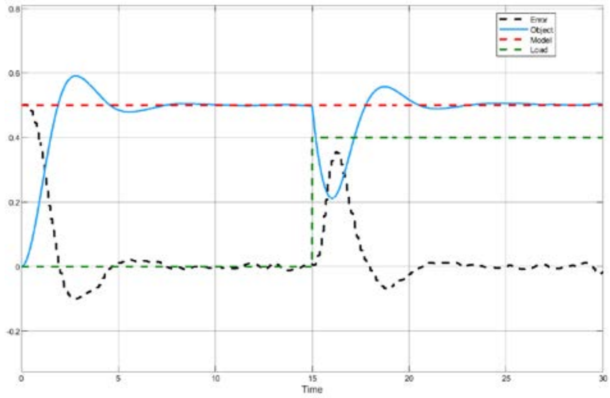


Fig. 6 Transient response graph with exact parameters of controller

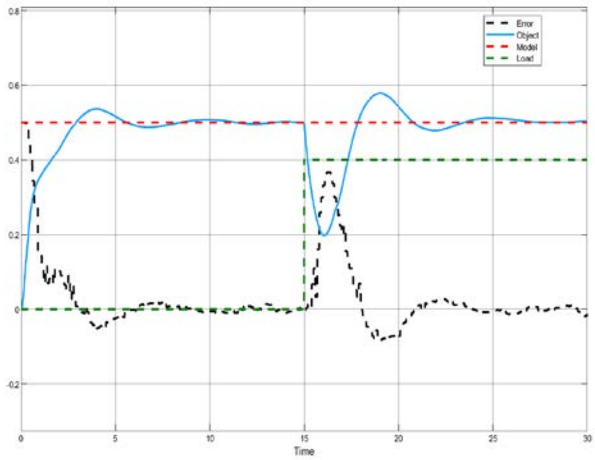


Fig. 7. Transient response graph with 10% variations of controller parameters

The transient response of the control system with 10% random changes in the controller parameters (Fig. 7) remains satisfactory and stable.

Thus, the use of optimal types of tuning regulators does not violate the robustness of the whole system. The use of typical universal frequency characteristics of control systems allows you to quickly adjust the optimal controller while maintaining the robustness of the whole system.

## 5 Acknowledgments

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