Fractional-Order Model Parameter Identification of BLDC Motor by Flower Pollination Algorithm

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Abstract: - The brushless DC (BLDC) motor has been increasingly used in industrial automation, automotive, aerospace, instrumentation and appliances. Analysis and design of the BLDC motor control system efficiently require its accurate model parameters. Based on fractional calculus, the fractional-order model provides more accurate than the conventional integer-order one. In this paper, the optimal fractional-order model parameter identification of the BLDC motor via the flower pollination algorithm (FPA) is proposed. The FPA, one of the newest and most efficient metaheuristic optimization methods, is applied to identify the fractional-order model parameters of the BLDC motor. As simulation results, the FPA can optimally provide the BLDC model parameters of both integer-order and fractional-order models. However, the fractional-order model obtained by the FPA performs more accurate than the integer-order model obtained by the FPA.

Key-Words: - Fractional-Order Model, Brushless DC Motor, Flower Pollination Algorithm, Metaheuristics

1 Introduction

The brushless DC (BLDC) motor is one of the motor types rapidly gaining popularity. The BLDC motor have been used in industries since 1970's such as appliances, automotive, aerospace, consumer, medical, industrial automation equipment and instrumentation [1]. The BLDC motors do not use brushes for commutation; instead, they are electronically commutated. The BLDC motors have many advantages over brushed DC motors and induction motors. A few of these are: better speed versus torque characteristics, high dynamic response, high efficiency, long operating life, noiseless operation, higher speed range and high power-to-weight ratio [2–5]. According to the control context, analysis and design of the BLDC motor control system efficiently require its accurate model. Regarding to the conventional identification, the model parameters can be obtained by the regression analysis [6, 7] that are widely used to linear and nonlinear system [6–8]. Although their closed-form formulae provide fast computation, its major drawback is the restriction of the class of difference equation models. This leads it cannot possible to be applied for identifying models of other forms.

Moving toward new era of control synthesis, the fractional calculus has been applied to modeling and design schemes [9–10]. It claimed that the fractional-order model can provide more accurate dynamic behaviour of the system than the conventional integer-order model. Associated with modern optimization, the control synthesis recently has been adapted to the constrained optimization problem that can be effectively solved by metaheuristics [11, 12]. One of the newest and most powerful population-based metaheuristic optimization techniques is the flower pollination algorithm (FPA) proposed by Yang in 2012 [13]. Performance evaluation of the FPA against many standard test functions was proposed [13]. The FPA outperformed well-known metaheuristic algorithms including genetic algorithm (GA) and particle swarm optimization (PSO) [13, 14]. In addition, the convergence properties of the FPA algorithm have been proven by Markov chain theory [15]. Moreover, the FPA was successfully applied to solve many real-world optimization problems such as pressure vessels design [13], disc break design [14], traveling transportation problem [16], control system design [17, 18] and integer-order model identification [19].

In this paper, the FPA is applied to optimal fractional-order model parameter identification of the BLDC motor to be compared with the integer-order one. The rest of the paper is organized as follows. Fractional-order dynamic system and mathematical model of BLDC motor are briefly described in section 2. Problem formulation and FPA algorithm are performed in section 3. Results and discussions are illustrated in section 4, while conclusions are followed in section 5.

2 Fractional-Order Dynamic Systems

In this section, fractional calculus and fractional-order dynamic systems are briefly reviewed. Then, the mathematical model of BLDC motor is followed.

2.1 Fractional Calculus

Fractional calculus is a more than 300 years old topic. It can be defined as the generalization of classical calculus to orders of integration and differentiation not necessarily integer. In recent years, fractional calculus has been applied in the modeling and control. Many real dynamic systems are better characterized using a non-integer order dynamic model based on fractional calculus [9–10]. In fractional calculus, a generalization of integration and differentiation can be represented by the non-integer order fundamental operator ${}_a\mathcal{D}_t^{\alpha}$, where a and t are the limits of the operator. The continuous integro-differential operator is defined as expressed in (1), where $\alpha \in \Re$ stands for the order of operation.

$$ {}_a\mathcal{D}_t^{\alpha} = \begin{cases} \dfrac{d^{\alpha}}{dt^{\alpha}} & \Re(\alpha) > 0 \\ 1 & \Re(\alpha) = 0 \\ \displaystyle\int_a^t (d\tau)^{-\alpha} & \Re(\alpha) < 0 \end{cases} \tag{1} $$

Well-known definitions used for the generally fractional differintegral are Grunwald-Letnikov (GL) as stated in (2), where $[\cdot]$ is integer part, n is an integer satisfying the condition $n-1 < \alpha < n$, binomial coefficient is stated in (3) and the Euler's gamma function $\Gamma(\cdot)$ is defined by (4), and Riemann-Liouville (RL) as expressed in (5), for $n-1 < \alpha < n$.

$$ {}_a\mathcal{D}_t^{\alpha} f(t) = \lim_{h \to 0} h^{-\alpha} \sum_{r=0}^{\left[\frac{t-a}{h}\right]} (-1)^r \binom{n}{r} f(t-rh) \tag{2} $$

$$ \binom{n}{r} = \frac{\Gamma(n+1)}{\Gamma(r+1)\Gamma(n-r+1)} \tag{3} $$

$$ \Gamma(x) = \int_0^{\infty} t^{x-1} e^{-t} dt \tag{4} $$

$$ {}_a\mathcal{D}_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \left(\frac{d}{dt}\right)^n \int_a^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau \tag{5} $$

For solving engineering problems, the Laplace transform is routinely conducted. The formula of the Laplace transform of the RL fractional derivative in (5) is stated in (6), for $n-1 < \alpha \le n$, where $s \equiv j\omega$ denotes the Laplace transform (complex) variable. Under zero initial conditions for order α ($0 < \alpha < 1$), the Laplace transform of the RL fractional derivative in (5) can be expressed in (7).

$$ \begin{aligned} \mathcal{L}\{{}_a\mathcal{D}_t^{\alpha} f(t)\} &= \int_0^{\infty} e^{-st} {}_0\mathcal{D}_t^{\alpha} f(t) dt \\ &= s^{\alpha} F(s) - \sum_{k=0}^{n-1} s^k {}_0\mathcal{D}_t^{\alpha-k-1} f(t)\Big|_{t=0} \end{aligned} \tag{6} $$

$$ \mathcal{L}\{{}_a\mathcal{D}_t^{\pm\alpha} f(t)\} = s^{\pm\alpha} F(s) \tag{7} $$

2.2 Fractional-Order Systems

A fractional-order dynamic system can be described by a fractional differential equation as stated in (8), where $\mathcal{D}^{\alpha} \equiv {}_a\mathcal{D}_t^{\alpha}$; a_k $(k = 0,\ldots,n)$ and b_k $(k = 0, \ldots, m)$ are constant, α_k $(k = 0,\ldots,n)$ and β_k $(k = 0, \ldots, m)$ are arbitrary real numbers, $u(t)$ is input variable and $y(t)$ is output variable, respectively. Without loss of generality, it can be assumed that $\alpha_n > \alpha_{n-1} > \cdots > \alpha_0$ and $\beta_m > \beta_{m-1} > \cdots > \beta_0$.

$$ \begin{aligned} & a_n \mathcal{D}^{\alpha_n} y(t) + a_{n-1} \mathcal{D}^{\alpha_{n-1}} y(t) + \cdots + a_0 \mathcal{D}^{\alpha_0} y(t) \\ & = b_m \mathcal{D}^{\beta_m} u(t) + b_{m-1} \mathcal{D}^{\beta_{m-1}} u(t) + \cdots + b_0 \mathcal{D}^{\beta_0} u(t) \end{aligned} \tag{8} $$

By taking the Laplace transform to (8) with zero initial conditions, a fractional-order system can be represented by a transfer function in (9).

$$ \frac{Y(s)}{U(s)} = \frac{b_m s^{\beta_m} + b_{m-1} s^{\beta_{m-1}} + \cdots + b_0 s^{\beta_0}}{a_n s^{\alpha_n} + a_{n-1} s^{\alpha_{n-1}} + \cdots + a_0 s^{\alpha_0}} \tag{9} $$

2.3 Model of BLDC Motor

The mathematical model of the BLDC motor can be developed in the similar manner of brushed DC motor. Based on the principal of Hall effect, the

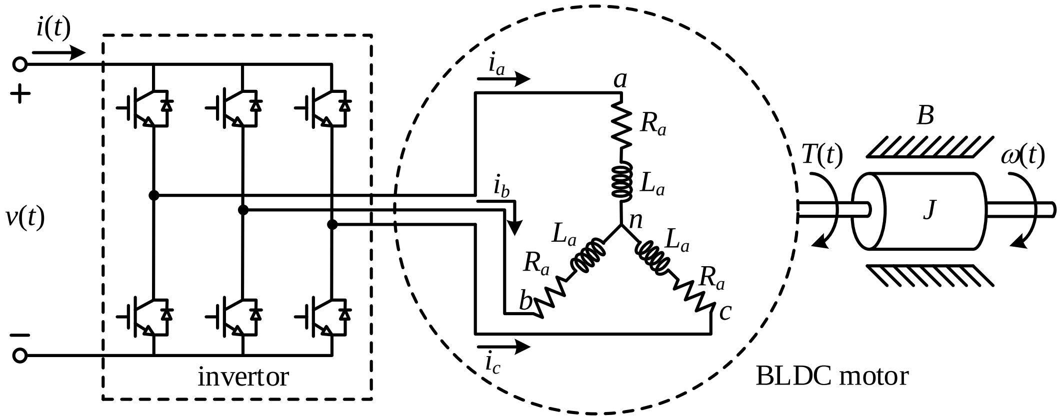

Fig. 1 BLDC motor schematic diagram with symmetrical 3-phase wye (Y) connection.

BLDC motor can be performed in many modes (or phase connections). However, the 3-phase has a very good precision in control. In general, a simple arrangement of a symmetrical (balanced) 3-phase wye (Y) connection as depicted in Fig. 1 could provide a brief illustration of all phase concepts. With symmetrical arrangement, the mechanical time constant τ_m and the electrical time constant τ_e of the BLDC can be written in (10) [20], where R_a is armature resistance, L_a is armature inductance, J is moment of inertia, K_t is torque constant and K_e is back emf constant. Commonly, the BLDC motor will be driven by DC input voltage via the particular invertor (or power amplifier). Such the amplifier can be approximated by the first order model. Therefore, the mathematical model of the BLDC motor used in this work can be formulated as stated in (11), where K_A is amplifier constant and τ_A is amplifier time constant.

$$\left.\begin{aligned} \tau_m &= \sum \frac{R_a J}{K_t K_e} = \frac{J \Sigma R_a}{K_t K_e} = \frac{J(3R_a)}{K_t K_e} \\ \tau_e &= \sum \frac{L_a}{R_a} = \frac{L_a}{\Sigma R_a} = \frac{L_a}{3R_a} \end{aligned}\right\} \tag{10}$$

$$\frac{\Omega(s)}{V(s)} = \left(\frac{K_A}{\tau_A s + 1}\right)\left(\frac{1/K_e}{\tau_m \tau_e s^2 + \tau_m s + 1}\right) \tag{11}$$

3 Problem Formulation

Regarding to the modern optimization context, the model parameter identification can be considered as one of the constrained optimization problems. In this work, the parameter identification of BLDC motor by the FPA can be formulated as shown in Fig. 2, where $v(t)$ is exciting input voltage, $\omega(t)$ and $\omega^*(t)$ are actual speed from sensory data and simulated speed from model and identified parameters, respectively. The objective function, f, is set as the sum-squared error between $\omega(t)$ and $\omega^*(t)$. Referring to Fig. 2, f will be fed back to the FPA in order to be minimized by searching for the appropriate values of the model parameters. According to (11), the integer-order (IO) and fractional-order (FO) models of the BLDC motor are set as (12) and (13), respectively. This identification problem of the BLDC motor is searching problem the appropriate values of a_0, $b_0,\ldots,b_3$ in (12) and a_0, $b_0,\ldots,b_3$, $\alpha_1,\ldots,\alpha_3$ in (13) by the FPA within their correspondingly search spaces.

$$G(s)\big|_{IO} = \frac{a_0}{b_3 s^3 + b_2 s^2 + b_1 s + b_0} \tag{12}$$

$$G(s)\big|_{FO} = \frac{a_0}{b_3 s^{\alpha_3} + b_2 s^{\alpha_2} + b_1 s^{\alpha_1} + b_0} \tag{13}$$

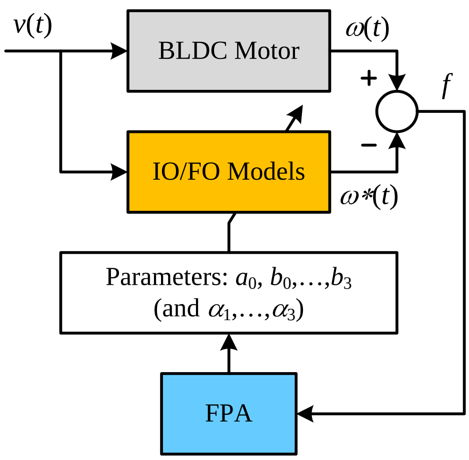

Fig. 2 FPA-based BLDC motor IO/FO-model parameter identification.

The FPA algorithm proposed by Yang [13] is based on four particular rules as follows:

- Biotic and cross-pollination are global pollination process via Lévy flight (Rule-1) by using (14) and (15), where $\boldsymbol{x}$ is solution, $\boldsymbol{g}^*$ is current best solution, L is Lévy flight and $\Gamma(\lambda)$ is the standard gamma function.
- Abiotic and self-pollination are local pollination process with random walk (Rule-2) by using (16) and (17), where $\varepsilon \in [0, 1]$ is a random draw a uniform distribution.
- Pollinators such as insects can develop flower constancy, which is equivalent to a reproduction probability that is proportional to the similarity of two flowers involved (Rule-3).
- Local pollination and global pollination can be controlled by a switch probability $p \in [0, 1]$ (Rule-4).

$$\boldsymbol{x}_i^{t+1} = \boldsymbol{x}_i^t + L(\boldsymbol{x}_i^t - \boldsymbol{g}^*) \tag{14}$$

$$L \approx \frac{\lambda \Gamma(\lambda) \sin(\pi\lambda/2)}{\pi} \frac{1}{s^{1+\lambda}}, \quad (s >> s_0 > 0) \tag{15}$$

$$\boldsymbol{x}_i^{t+1} = \boldsymbol{x}_i^t + \varepsilon(\boldsymbol{x}_j^t - \boldsymbol{x}_k^t) \tag{16}$$

$$\varepsilon(\rho) = \begin{cases} 1/(b-a), & a \le \rho \le b \\ 0, & \rho < a \text{ or } \rho > b \end{cases} \tag{17}$$

The algorithm of the FPA can be summarized by the pseudo code as shown in Fig. 3 [13].

```
- Objective function f(x), x = (x1, x2,…, xd)
- Initialize a population of n flowers/pollen gametes
  with random solutions
- Find the best solution g* in the initial population
- Define a switch probability p ∈ [0, 1]
while (t < MaxGeneration)
    for i = 1 : n (all n flowers in the population)
        if rand < p,
            - Draw a step vector L via Lévy flight
            - Activate global pollination
        else
            - Draw ε from a uniform distribution in [0, 1]
            - Randomly choose j and k among all the solutions
            - Invoke local pollination
        end if
        - Evaluate new solutions
        - If new solutions are better, update them in the population
        - Update t
    end for
    - Find the current best solution g*
end while
```

Fig. 3 Pseudo code of FPA algorithms.

The FPA algorithm shown in Fig. 3 is adapted to BLDC motor IO/FO model parameter identification as follows:

Step-0 Initialize the objective function, f, as stated in (18) and constraint functions in (19). Randomly generate a population of n flowers. Fine the best solution $\boldsymbol{g}^*$ among initial population. Define a switch probability p = 0.8 (or 80%). Set MaxGen as the termination criteria (TC) and Gen = 1 as a generation counter.

Step-1 If Gen ≤ MaxGen, go to Step-2. Otherwise go to Step-4.

Step-2 If `rand` < p, draw a step vector L via Lévy flight in (15) and activate global pollination in (14) to generate a new solution $\boldsymbol{x}$. Otherwise draw a uniform distribution $\varepsilon \in [0, 1]$ in (17). Randomly select j and k among all solutions. Invoke local pollination in (16) to generate a new solution $\boldsymbol{x}$.

Step-3 If $f(\boldsymbol{x}) < f(\boldsymbol{g}^*)$, update solution $\boldsymbol{g}^* = \boldsymbol{x}$ and update Gen = Gen+1. Otherwise update Gen = Gen+1. Go to Step-1 to proceed next generation.

Step-4 Report the best solution found and stop the search process.

$$\textbf{Min} \quad f = \sum_{i=1}^{N} (\omega_i - \omega_i^*)^2 \tag{18}$$

$$\text{Subject to} \quad \left.\begin{aligned} & a_{0\_\min} \le a_0 \le a_{0\_\max}, \\ & b_{0\_\min} \le b_0 \le b_{0\_\max}, \\ & b_{1\_\min} \le b_1 \le b_{1\_\max}, \\ & b_{2\_\min} \le b_2 \le b_{2\_\max}, \\ & b_{3\_\min} \le b_3 \le b_{3\_\max}, \\ & \alpha_{1\_\min} \le \alpha_1 \le \alpha_{1\_\max}, \\ & \alpha_{2\_\min} \le \alpha_2 \le \alpha_{2\_\max}, \\ & \alpha_{3\_\min} \le \alpha_3 \le \alpha_{3\_\max} \end{aligned}\right\} \tag{19}$$

4 Results and Discussions

The testing rig of the BLDC motor identification is depicted in Fig. 4. The blushless DC motor of 350 W, 24 VDC, 0.7 A, 300 rpm in laboratory is conducted. The experimental speed data at 305 rpm and 290 rpm are tested and recorded by a GW-INSTEK GDS-2104 100-MHz digital storage oscilloscope for identification and validation as depicted in Fig. 5 and Fig. 6, respectively.

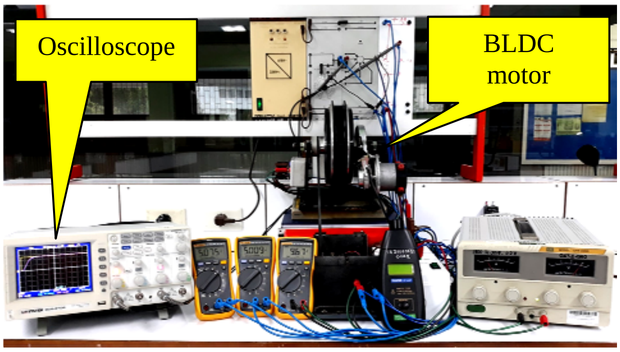

Fig. 4 BLDC motor testing rig.

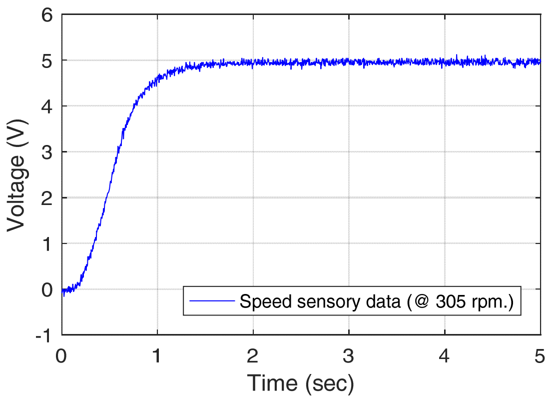

Fig. 5 Speed sensory data (305 rpm).

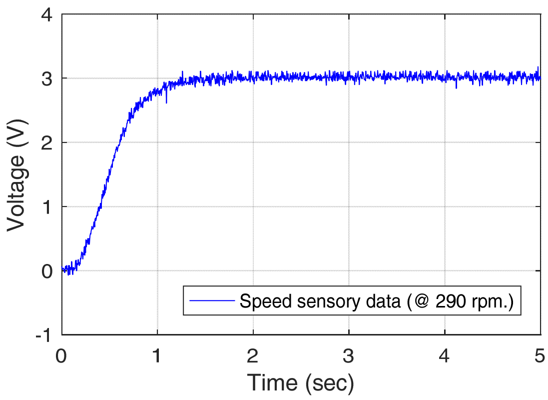

Fig. 6 Speed sensory data (290 rpm).

In order to fractional-order model parameter identification of the BLDC motor, the FPA algorithms were coded by MATLAB version 2017b (License No.#40637337) run on Intel(R) Core(TM) i5-3470 CPU@3.60GHz, 4.0GB-RAM. The FPA's parameters, i.e. number of flower (pollen gametes) n = 40 and a probability p = 0.2 (20%) for switching between local pollination and global pollination, are set according to recommendations of Yang [13, 14]. Search spaces in (19) are performed as given in (20). The maximum generation *MaxGeneration* = 100 is then set as the termination criteria (TC). 50 trials are conducted to find the optimal parameters of fractional-order (FO) BLDC model. For comparison with the integer-order (IO) BLDC model, α_1, α_2 and α_3 in (20) will be set as integer numbers: 1, 2 and 3, respectively.

$$\left.\begin{aligned}
&[a_{0_min},\ a_{0_max}] = [0.1,\ 150],\\
&[b_{0_min},\ b_{0_max}] = [0.1,\ 150],\\
&[b_{1_min},\ b_{1_max}] = [0.1,\ 100],\\
&[b_{2_min},\ b_{2_max}] = [0.1,\ 20],\\
&[b_{3_min},\ b_{3_max}] = [0.1,\ 10],\\
&[\alpha_{1_min},\ \alpha_{1_max}] = [0,\ 1.5],\\
&[\alpha_{2_min},\ \alpha_{2_max}] = [0,\ 2.5],\\
&[\alpha_{3_min},\ \alpha_{3_max}] = [0,\ 3.5]
\end{aligned}\right\} \tag{20}$$

Once 50 trials of the search process were completed within the average search time consumed of 165.9047 sec., the FPA can successfully provide the optimal parameters of both IO and FO models of BLDC motor as expressed in (21) and (22).

$$G(s)\big|_{IO} = \frac{148.80}{1.328s^3 + 13.05s^2 + 77.81s + 149.4} \tag{21}$$

$$G(s)\big|_{FO} = \frac{1.0}{0.029043s^{2.6582} + 0.47836s^{1.2376} + 1.1075s^{0.044262}} \tag{22}$$

Using MATLAB with FOMCON toolbox [21, 22] where Oustaloup's approximation is realized for fractional-order numerical simulation, the dynamic behaviours of the BLDC motor by IO and FO models associated with optimal parameters identified by the FPA are depicted in Fig. 7 against sensory data as actual speed at 305 rpm. For parameters validation, the sensory data of 290 rpm are employed to verify the model parameters obtained from those of 305 rpm as plotted in Fig. 8.

From Fig. 7, it was found that both IO and FO models show very good agreement to actual dynamics of BLDC motor. This means that The FPA can optimally provide the BLDC model parameters of IO and FO models. Once considering the values of the objective function, f, indicating the sum-squared error between $\omega(t)$ and $\omega^*(t)$, the IO model is with f = 3.7840, while the FO is with f = 3.3531. This can be noticed that the FO model performs more accurate than the IO one.

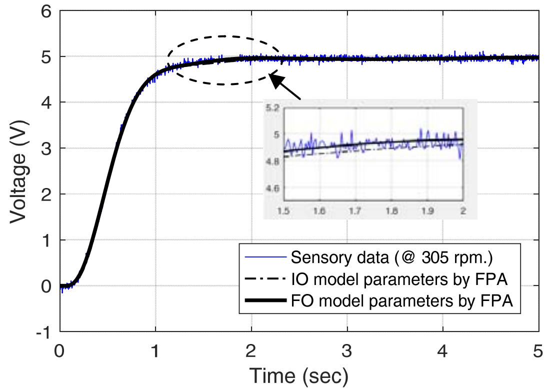

Fig. 7 Results of BLDC motor IO/FO-model parameter identification by FPA.

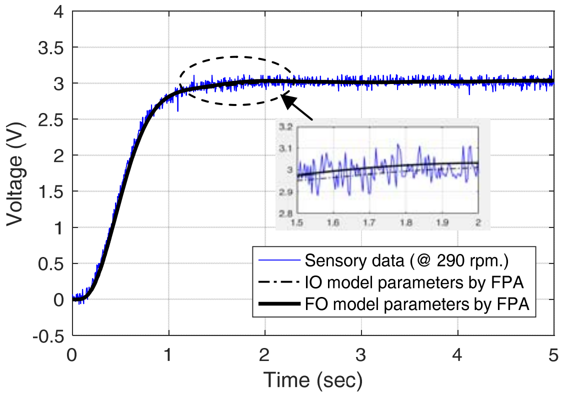

Fig. 8 Results of BLDC motor IO/FO-model parameter validation by FPA.

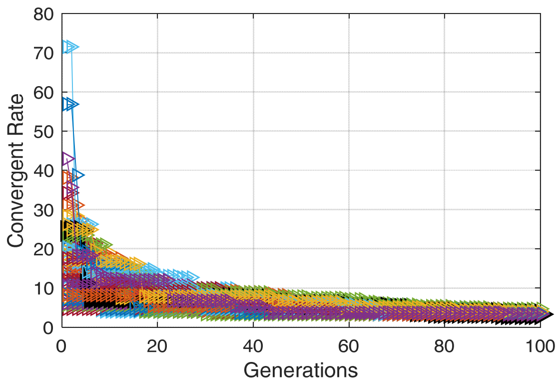

Fig. 9 Convergent rates proceeded by FPA.

Referring to Fig. 8, very good dynamic agreement between speed sensory data and model plots can be observed. This implies that optimal parameters of both IO and FO models obtained by the FPA can satisfactory describe the actual dynamics of the BLDC motor. However, in sense of values of the objective function, the FO model provides more accurate than the IO with less value.

The convergent rates of FO model parameter identification proceeded by the FPA over the objective function in (18) are depicted in Fig. 9. It can be observed that the proposed identification approach is averagely completed within the 92 iteration.

5 Conclusions

Fractional-order (FO) model parameter identification of BLDC motor by the flower pollination algorithm (FPA) has been presented in this paper. In order to compare with the integer-order (IO) model, the BLDC motor in laboratory has been particular tested. Sensory data of speed dynamics at 305 rpm and 290 rpm have been carefully recorded for identification and validation purposes. As results of identification and validation, it was found that the FPA could provide the optimal parameters of both IO and FO model parameters of BLDC motor. The IO and FO models associated with optimal parameters identified by the FPA have shown very good agreement to actual dynamics of BLDC motor. By considering the values of the objective function, it was cleared that the FO model parameters obtained by the FPA has performed more accurate than the IO model parameters obtained by the FPA. For the future research, the optimal FO-controller designed by the FPA for FO-model of BLDC motor will be elaborately extended based on fractional-order system and control context.

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