Continuous Time Generalized Predictive Control from a classical control perspective

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Abstract: - Continuous Time Generalized Predictive Control (CGPC) can deal with non-minimum linear systems and thanks to this property it restricts the predicted input to be constant in the future by choosing an appropriate control order so cancellation of non-minimum zeros is not performed. In this article the CGPC is analyzed by using classical control theory, the analysis is carried out through a numerical example.

Key-Words: - Linear control, Predictive Control, Feedback linearization, Relative Degree, Continuous system.

1 Introduction

The CGPC studied in this paper is a special case of NCGPC, [1, 2] which is an alternative nonlinear predictive controller; this controller was developed in a different way than conventional nonlinear controllers. The NCGPC is based in the prediction of the system output and due to the fact that it was not derived with the objective of cancelling nonlinearities, as feedback linearization techniques do, the NCGPC has two advantages: First, it can constrain the predicted control through -additionally the response becomes slow and the control is not very active-, and second, when , there is not zero dynamics cancellation and then the internal stability is preserved. Therefore, the NCGPC [2] provides a good way to handle systems with zero unstable dynamics. Another of the main advantages of NCGPC control schemes is that, when Nu= Ny-r they do not require on-line optimization and asymptotic tracking of the smooth reference signal is guaranteed. The CGPC, like its non-linear counterpart, retains these characteristics. One of the most important questions in MPC (Model Predictive Control) is if a finite horizon MPC strategy does guarantee stability of the closed-loop or not. In this paper the characteristics and the stability will be analysed through the classic control.

2 Development of the CGPC

The linear version of CGPC [4], originally was developed by using transfer function. In this section, the linear version of CGPC is recast in state space form following the same steps as the NCGPC presented in [1, 2]. Consider a linear SISO system

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t)\\ y(t) &= Cx(t)\end{aligned} \qquad (1)$$

Where $x \in R^n$, $y \in R$ and *A, B, C* are matrices of appropriate dimensions.

A. Prediction of the output

The output prediction is approximated for a Maclaurin series expansion of the system output as follows.

$$y(t+T) = y(t) + \dot{y}(t)T + y^{(2)}(t)\frac{T^2}{2!} + \cdots + y^{(N_y)}(t)\frac{T^{N_y}}{N_y!} \qquad (2)$$

or

$$y(t+T) = T_{N_y} Y_{N_y} \qquad (3)$$

where

$$Y_{N_y} = [y \quad \dot{y} \quad y^{(2)} \quad \cdots \quad y^{(N_y)}]^T \qquad (4)$$

The predictor order N_y is the number of the times that the output must be differentiated.

B. Derivative emulation

As in the nonlinear case the predictive control is based in taking the derivatives of the output, which are emulated by

$$\begin{aligned}\dot{y}(t) &= CAx(t)\\ y^{(2)}(t) &= CA^2x(t)\\ &\vdots\\ y^{(r)}(t) &= CA^r x(t) + CA^{r-1}Bu(t)\\ &\vdots\\ y^{(N_y)}(t) &= CA^{N_y}x(t) + \cdots + CA^{r-1}Bu^{N_y-r}(t)\end{aligned} \qquad (6)$$

These output derivatives are obtained from the system of equation (1), where r is the relative order.

Output and its derivatives can be rewritten by:

$$Y_{N_y} = Y^0(x(t)) + H(x(t))u_{N_u} \tag{7}$$

where

$$u_{N_u} = [u \quad \dot{u} \quad u^{(2)} \quad \cdots \quad u^{(N_y-r)}]^T$$

$$Y^0 = \begin{bmatrix} Cx(t) \\ CAx(t) \\ CA^2x(t) \\ \vdots \\ CA^r x(t) \\ CA^{r+1}x(t) \\ CA^{r+2}x(t) \\ \vdots \\ CA^{N_y}x(t) \end{bmatrix} \quad H = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ CA^{r-1}B & 0 & \cdots & 0 \\ CA^rB & CA^{r-1}B & \cdots & 0 \\ CA^{r+1}B & CA^rB & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ CA^{N_y-1}B & CA^{N_y-2}B & \cdots & CA^{r-1}B \end{bmatrix} \tag{8}$$

Thus, predicted output equation (3) is given by

$$y(t+T) = T_{N_y}Hu_{N_u} + T_{N_y}Y^0 \tag{9}$$

C. Prediction of the reference trajectory of the output

To drive the predicted output along a desired smooth path (reference trajectory) to a set point. A reference trajectory is chosen as the output of the following reference model [4].

$$W_r(t,s) = \frac{R_n(s)}{R_d(s)}\frac{w(t)-y(t)}{s} \tag{10}$$

The Laplace operator s represents the Laplace transform with respect to future variable T.

Considering the following approximation by using Markov parameters.

$$\frac{R_n(s)}{R_d(s)} \approx \sum_{i=0}^{N_y} r_i s^{-i} \tag{11}$$

The reference trajectory is given by:

$$w_r^*(t,T) = [r_0 + r_1T + r_2\frac{T^2}{2!} + \cdots + r_r\frac{T^{N_y}}{N_y!}][w - y(t)] \tag{12}$$

where w the set-point. Rewriting this equation.

$$w_r^*(t,T) = T_{N_y}w_r \tag{13}$$

where

$$w_r = [r_0 \quad r_1 \quad \cdots \quad r_r]^T(w - y(t))$$

and T_{N_y} is given by (5)

D. Cost function minimization

The cost function is not defined with respect current time, but respect a moving frame, which origin is in time t and T is the future variable. CGPC calculates the future controls from a predicted output over a time frame. The first element *u(t)* of the predicted controls is then applied to the system and the same procedure is repeated at the next time instant. Thus, predicted output depends on the input *u(t)* and its derivatives, and the future controls being function of *u(t)* and its N_u-derivatives. The cost function is:

$$J(u_{N_y}) = \int_{T_1}^{T_2} [y^*(t,T) - w_r^*(T,t)]^2 dT \tag{14}$$

where

$$y^*(t,T) = y(t+T) - y(t) \tag{15}$$

It can see that equation (15) is the same predicted output equation (9), except that the first element of Y^0 is set to zero. With the substitution of equations (3) and (7) the cost function becomes

$$J(u_{N_u}) = \int_{T_1}^{T_2} [T_{N_y}Y^0 + T_{N_y}Hu_{N_y} - T_{N_y}w_r)]^2 dT \tag{16}$$

and the minimization results in

$$u_{N_u} = K(w_r - Y^0) \tag{17}$$

where

$$T_y = \int_{T_1}^{T_2} T_{N_y}^T T_{N_y} dT \tag{18}$$

The ij^{th} element of T_y is:

$$T_{y_{ij}} = \frac{T_2^{i+j-1} - T_1^{i+j-1}}{(i-1)!(j-1)!(i+j-1)!}$$

and

$$K = [H^T T_y H]^{-1}[H^T T_y] \tag{19}$$

As explained above, just the first element of u_{N_u} is applied. The control law is given by

$$u(t) = k[w_r - Y^0] \tag{20}$$

3 Closed Loop for Stable and Minimum Phase Systems

In this section the case when $N_u = N_y - r$ is chosen for stable and minimum phase systems will be analyzed. It is considered that the following assumptions are satisfied

- The system given by equation (1) is stable
- It has LHZs
- $N_u = N_y - r$
- The system states must be measurable.

The control law given by equation (20) is analyzed; the matrix H equation (9) is decomposed as

$$H = \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} \tag{21}$$

H_1 is a zero matrix with dimension $r \times (N_y - r + 1)$, and H_2 is a lower triangular matrix with dimension $(N_y - r + 1) \times (N_y - r + 1)$ given by:

$$H_2 = \begin{bmatrix} CA^{r-1}B & 0 & \cdots & 0 \\ CA^{r}B & CA^{r-1}B & \cdots & 0 \\ CA^{r+1}B & CA^{r}B & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ CA^{N_y-1}B & CA^{N_y-2}B & \cdots & CA^{r-1}B \end{bmatrix} \tag{22}$$

The matrix T_y equation (5) is decomposed as

$$T_y = \begin{bmatrix} T_{y_{11}} & T_{y_{12}} \\ T_{y_{21}} & T_{y_{22}} \end{bmatrix}$$

where

$T_{y_{11}}$ is $r \times r$

$T_{y_{12}}$ is $r \times (N_y - r + 1)$

$T_{y_{21}}$ is $(N_y - r + 1) \times r$

$T_{y_{22}}$ is $(N_y - r + 1) \times (N_y - r + 1)$

Equation (22) can be written as

$$K = H_2^{-1}[T_{y_{22}}^{-1} T_{y_{21}} I] \tag{23}$$

With I unitary matrix with dimension $(N_y - r + 1) \times (N_y - r + 1)$. The first row of the inverse of H_2 is given by

$$h_2^{-1} = \begin{bmatrix} 1/CA^{r-1}B & 0 & \ldots & 0 \end{bmatrix} \tag{24}$$

Then, the first row of K, which will be called k

$$k = \frac{1}{CA^{r-1}B} \begin{bmatrix} t_1 & t_2 & \ldots & t_r & 1 & 0 & \ldots & 0 \end{bmatrix} \tag{25}$$

where t_1, t_2, t_3, … t_r are elements of the first row of $T_{y_{22}}^{-1} T_{y_{21}}$. The control law is given by:

$$u(t) = \frac{1/\beta_r (w - y(t)) - \sum_{i=1}^{r-1} \beta_i / \beta_r CA^i x(t) - CA^r x(t)}{CA^{r-1}B}$$

$$u(t) = \frac{(t_1 r_0 + \ldots + t_e r_{r-1})(w - y(t)) - \sum_{i=1}^{r-1} t_{i+1} CA^i x(t) - CA^r x(t)}{CA^{r-1}Bx(t)}$$

$$u(t) = \frac{(w - y(t)) - \sum_{i=1}^{r-1} \beta_i CA^i x(t) - CA^r x(t)}{CA^{r-1}B} \tag{26}$$

Where

$$\beta_r = 1/(t_1 r_0 + t_2 r_1 + \cdots + t_r r_{r-1})$$

$$\beta_i = t_{i+1}/(t_1 r_0 + t_2 r_1 + \cdots + t_r r_{r-1}) \quad i = 1, 2 \ldots, r-1 \tag{27}$$

We can notice, that incredible as it may seem, large N_y does not require a bigger computational effort, because as we can see from equation (26), the control depends just on the r-first derivatives, thus the rest of the derivatives only have influence in obtaining the parameters of t_i, which just depends on T. Moreover, N_y can be chosen as the smallest predictor order, which is such that the predicted output depends on *u(t)*. The relative degree r of the system is exactly equal to the number of times the output must be differentiated for the input to explicitly appear. Because of this, the relative degree r will be the smallest predictor order N_y.

We can conclude if $N_u = N_y - r$, the control law is independent of the last $N_y - r$ derivatives. Then it is possible to calculate the parameters β_i considering the largest N_y without the use of the remaining derivatives. We will consider this case, in all the process, except in the non-minimum phase systems.

Substituting equation (26) into the *rth* derivative given by equation (6) leads to:

$$y^r(t) = \frac{1}{\beta_r}(w - y) - \sum_{i=1}^{r-1} \frac{\beta_i}{\beta_r} y^i \tag{28}$$

Rearranging and taking Laplace transforms, the resulting closed-loop transfer function is given by:

$$Y(s) = G(s)W(s)$$

$$G(s) = \frac{1}{\beta_r s^r + \beta_{r-1} s^{r-1} + \ldots + \beta_1 s + 1} = \frac{N(s)}{D(s)} \tag{29}$$

Note that, by using the Routh-criterion, we can show that the systems are stable.

If the model is considered as a perfect model, this state feedback places the poles at the roots of the polynomial

$$\sum_{k=0}^{r} \beta_k s^k = D(s) \cdot$$

The predictive control does not require on-line optimization and asymptotic tracking of the smooth reference signal is guaranteed.

The state feedback of equation (26) cancels all the zeros of the process by placing poles at the same values. This fact is analogous with NCGPC when $N_u = N_y - r$, the control feedback cancels the zeros dynamics. Therefore, as with NCGPC, the process has to be minimum phase in order to preserve internal stability, unless $N_u < N_y - r$, in which the case the zero cancellation is not carried out.

4 Closed Loop for Stable and Non-Minimum Phase Systems: A Numerical Example

In this section the case when $N_u < N_y - r$ is chosen for stable and non-minimum phase systems will be analysed through a numerical example. It is considered that the following assumptions are satisfied.

- The system given by equation (1) is stable
- It has RHPZ's
- $N_u = N_y - r$

Consider the following system which is non-minimum phase, described by:

$$\begin{aligned} \dot{x}_1(t) &= -4x_1 - 4x_2 + u \\ \dot{x}_2(t) &= x_1 \\ y(t) &= x_1 - x_2 \end{aligned} \tag{30}$$

Which can be written as equation (1) being

$$A = \begin{bmatrix} -4 & -4 \\ 1 & 0 \end{bmatrix},\ B = \begin{bmatrix} 1 \\ 0 \end{bmatrix},\ C = \begin{bmatrix} 1 & -1 \end{bmatrix} \tag{31}$$

Its corresponding transfer function is:

$$G_P(s) = \frac{s-1}{s^2+4s+4} \tag{32}$$

The system has one zero at the right half plane. It can see that the relative degree of the system is 2. For the design of the predictive control it was considered $N_y = 3$ and $N_u = 0$ in order to not carry out the cancellation.

To obtain the prediction of the output, its derivatives are expressed as the equation (7)

$$y^*(t,T) = y(t+T) - y(t)$$

$$Y^0 = \begin{bmatrix} 0 \\ CAx(t) \\ CA^2x(t) \\ CA^3x(t) \end{bmatrix} = \begin{bmatrix} 0 \\ -5x_1 - 4x_2 \\ 16x_1 + 20x_2 \\ -44x_1 - 64x_2 \end{bmatrix},\ H = \begin{bmatrix} 0 \\ CB \\ CAB \\ CA^2B \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -5 \\ 16 \end{bmatrix} \tag{33}$$

The control signal is given by equation (16)

$$u_{N_u} = K(w_r - Y^0) \tag{34}$$

where

$$w_r = \begin{bmatrix} -r_0 \\ -r_1 \\ -r_2 \\ -r_3 \end{bmatrix} \tag{35}$$

and

$$K = [H^T T_y H]^{-1}[H^T T_y] = \begin{bmatrix} b_0 & b_1 & b_2 & b_3 \end{bmatrix} \tag{36}$$

The last equation only depends on T and the elements of the matrix H, so they will be constants by substituting the equations (33) - (36) the following expression is obtained.

$$\begin{aligned} u(t) &= (5b_1 - 16b_2 + 44b_3 + r_2b_2 + r_3b_3)x_1(t) + (4b_1 - 20b_2 + 64b_3 \\ &\quad - r_2b_2 - r_3b_3)x_2(t) - (r_2b_2 + r_3b_3)w \end{aligned} \tag{37}$$

To analyze the closed-loop response, the above equation is substituted into equation (30), obtaining:

$$\begin{aligned} \dot{x}_1(t) &= (-4 + 5b_1 - 16b_2 + 44b_3 + r_2b_2 + r_3b_3)x_1(t) + (-4 + 4b_1 - 20b_2 \\ &\quad + 64b_3 - r_2b_2 - r_3b_3)x_2(t) - (r_2b_2 + r_3b_3)w \\ \dot{x}_2(t) &= x_1(t) \\ y(t) &= x_1(t) - x_2(t) \end{aligned} \tag{38}$$

The above equation is rewritten as follows

$$\dot{x}_1(t) = a_{LC}x_1(t) + b_{LC}x_2(t) - c_{LC}w \tag{39}$$

Where

$$\begin{aligned} a_{LC} &= -4 + 5b_1 - 16b_2 + 44b_3 + r_2b_2 + r_3b_3 \\ b_{LC} &= -4 + 4b_1 - 20b_2 + 64b_3 - r_2b_2 - r_3b_3 \\ c_{LC} &= r_2b_2 + r_3b_3 \end{aligned}$$

The corresponding closed loop transfer function is as follows

$$\frac{Y(s)}{W(s)} = \frac{-c_{LC}s + c_{LC}}{s^2 - a_{LC}s - b_{LC}} \tag{40}$$

Where

$$\begin{aligned} a_{LC} &= a_{LC1} + a_{LC2} \\ a_{LC1} &= -4 + 5b_1 - 16b_2 + 44b_3 \\ a_{LC2} &= r_2b_2 + r_3b_3 \end{aligned}$$

and

$$b_{LC} = b_{LC1} + b_{LC2}$$
$$b_{LC1} = -4 + 4b_1 - 20b_2 + 64b_3 = 0$$
$$b_{LC2} = -r_2 b_2 - r_3 b_3$$

It is easy to see that

$$b_{LC} = b_{LC2} = -a_{LC2} \quad (43)$$

To make the system output asymptotically tracks the reference, it is necessary that

$$c_{LC} = -b_{LC} = a_{LC2} \quad (44)$$

so equation (40) becomes

$$\frac{Y(s)}{W(s)} = \frac{-a_{LC2}s + a_{LC2}}{s^2 - a_{LC}s + a_{LC2}} = G_{LC}(s) \quad (45)$$

The necessary and sufficient condition for stability is that all poles of $G_{LC}(s)$ have negative real parts. Therefore, the following conditions are necessary

$$\begin{aligned} &a_{LC} < 0 \\ &a_{LC2} > 0 \\ &|a_{LC1}| > a_{LC2} \\ &a_{LC1} < 0 \end{aligned} \quad (46)$$

Equation (37) is rewritten as follows

$$\begin{aligned} u(t) &= (5b_1 - 16b_2 + 44b_3)x_1(t) + (4b_1 - 20b_2 + 64b_3)x_2(t) \\ &-(r_2 b_2 + r_3 b_3)e(t) \end{aligned} \quad (47)$$

Where

$$e(t) = w - y(t)$$

To analyze the open loop response, the above equation is substituted into equation (30), obtaining:

$$\begin{aligned} \dot{x}_1(t) &= (-4 + 5b_1 - 16b_2 + 44b_3)x_1(t) + (-4 + 4b_1 - 20b_2 \\ &+ 64b_3)x_2(t) - (r_2 b_2 + r_3 b_3)e(t) \\ \dot{x}_2(t) &= x_1(t) \\ y(t) &= x_1(t) - x_2(t) \end{aligned} \quad (48)$$

The above equation is rewritten as follows

$$\dot{x}_1(t) = a_{LA}x_1(t) + b_{LA}x_2(t) - c_{LA}e(t)$$

where

$$\begin{aligned} a_{LA} &= -4 + 5b_1 - 16b_2 + 44b_3 = a_{LC1} \\ b_{LA} &= -4 + 4b_1 - 20b_2 + 64b_3 = b_{LC1} = 0 \\ c_{LA} &= r_2 b_2 + r_3 b_3 = a_{LC2} \end{aligned} \quad (49)$$

The corresponding open loop transfer function is as follows

$$\frac{Y(s)}{E(s)} = \frac{-c_{LA}s + c_{LA}}{s^2 - a_{LA}s - b_{LA}} \quad (50)$$

It is easy to see that

$$\frac{Y(s)}{E(s)} = \frac{-a_{LC2}s + a_{LC2}}{s^2 - a_{LC1}s} = G_{LA}(s) \quad (51)$$

Conditions of equation (46) are conserved:

$$\begin{aligned} &a_{LC2} > 0 \\ &|a_{LC1}| > a_{LC2} \\ &a_{LC1} < 0 \end{aligned} \quad (52)$$

It is possible to obtain the controller transfer function.

$$\frac{Y(s)}{E(s)} = \frac{-a_{LC2}s + a_{LC2}}{s^2 - a_{LC1}s} = G_{LA}(s) = G_C(s)G_P(s) \quad (53)$$

$$G_C(s) = \frac{G_{LA}(s)}{G_P(s)} = \frac{-a_{LC2}(s^2 + 4s + 4)}{s^2 - a_{LC1}s} \quad (54)$$

Figure (1) shown the LHP pole-zero cancelation, but there is no RHP pole-zero cancellation when the product G_cG_P formed, with this, internal stability is guaranteed. Also, the closed loop has all its poles in left half plane. Therefore, the closed loop is stable.

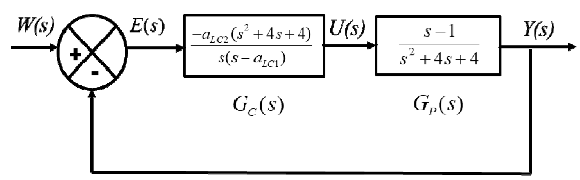

Fig. 1. Closed Loop System

5 Simulation

To show the effectiveness of controller CGPC simulation will be presented. The example used in the simulation is given by equation (30)

$$\begin{aligned} \dot{x}_1(t) &= -4x_1 - 4x_2 + u \\ \dot{x}_2(t) &= x_1 \\ y(t) &= x_1 - x_2 \end{aligned}$$

The control order has the function to constrain the predicted input. When the $N_u=0$ the predicted control, input is constrained to be constant in the future, i.e. derivatives of *u(t)* are taken equal zero. Small value of N_u gives less active control *u(t)* and slow response, which is good for the non-minimum phase systems.

The parameters are chosen as $N_u=0$, $N_y=3$, $T=3$, $r_0=0$, $r_1=0$, $r_2=-3$, $r_3=8$, obtaining:

$$b_0 = 0.0336; \quad b_1 = 0.082$$
$$b_2 = 0.1039; \quad b_3 = 0.0898$$
$$a_{LC} = -0.892; \quad a_{LC1} = -1.2991$$
$$a_{LC2} = 0.4071; \quad b_{LC} = -0.4071$$
$$b_{LC1} = 0; \quad b_{LC2} = -0.4071$$
$$c_{LC} = 0.4071; \quad a_{LA} = -1.2991 = a_{LC1}$$
$$b_{LA} = -0.4071 = b_{LC2}$$
$$c_{LA} = 0.4071 = a_{LC2}$$

The assumptions given by equation (46) are satisfied, therefore, the closed loop transfer function given by

$$\frac{Y(s)}{W(s)} = \frac{-0.4071(s-1)}{s^2 + 0.892s + 0.4071} = G_{LC}(s)$$

is stable as all its poles are Hurwitz.

The open loop transfer function is given by

$$\frac{Y(s)}{E(s)} = \frac{-.4071(s-1)}{s^2 + 1.2991s} = G_{LA}(s)$$

Finally, the controller transfer function is given by

$$\frac{U(s)}{E(s)} = \frac{-0.4071(s^2 + 4s + 4)}{s(s + 1.2991)} = G_C(s)$$

Just cancels the poles in the left half plane, due to the zero s=1 is not cancelled, the internal stability is ensured. Figure (2) shown the output response when the reference is a unit step.

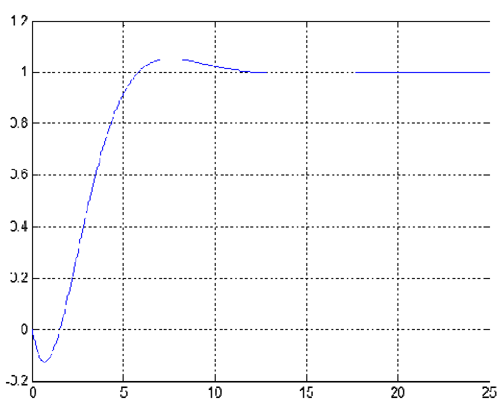

Fig. 2. System output response

6 Conclusion

One of the main advantages of CGPC control schemes is that when $N_u = N_y - r$, for minimum and stable systems, on-line optimization is not required, and asymptotic tracking of the smooth reference signal is guaranteed. It was shown that large N_y does not require a bigger computational effort, because the control depends only on the r-first derivatives; thus the rest of the derivatives only have influence in obtaining the parameters of t_i, which just depends on T. Then it is possible to calculate the parameters β_i considering the largest $N_{y.}$, without the use of the remaining derivatives. A closed-loop transfer function was found, the characteristic polynomial must be Hurwitz, in order to ensure closed-loop stability.

When the system is stable and non-minimum phase, there is LHP pole-zero cancellation, but there is no RHP pole-zero cancellation when the product G_cG_P formed, with this internal stability is guaranteed. Also, the closed loop has all its poles in left half plane. Therefore, the closed loop is stable. Condition for stability for the numerical example were given. The controller is composed by an integrator and a low pass filter, ensuring that the zero error tends to zero.

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