Frequency Analysis of the Estimated signals by Kalman Filter using Fast Fourier Transform

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Abstract: - Kalman filter is widely used in Power system for harmonics estimation, where the performance of Kalman filter depends on having an accurate model of the system harmonics to predict the next state variables based on the current states. The model of each harmonic signal is easy and only two state variables are required, but if the harmonic model doesn't include all the harmonics in the measured signal, this will cause error in the estimated states by Kalman filter, in most of previous researches the Mean Square Error (MSE) was calculated to show how much the estimated signal is closed to the actual signal, but the MSE is not enough to determine the source of the error whether it is resulted from an error in the estimated signal frequency or its amplitude. In this paper a frequency analysis using Fast Fourier Transform (FFT) for the estimated states of the Kalman filter was performed to understand the source of the error, after that a modification in Kalman filter calculation is proposed to reduce the error based on the frequency analysis.

Key-Words: - Kalman filter, FFT, Harmonics, Power System

1 Introduction

Power system has many of nonlinear devices that produce the system harmonics such as power electronics devices, nonlinear loads, dynamic loads and Photo Voltaic (PV) plants[1]..etc,. A lot of active filters have been used to estimate the fundemental signal and the harmonics in power system, where Kalman filter is one of the main of these filters [2]-[8].

Kalman filter is an optimal recursive linear filter[8], where, it needs an accurate model of the system to predict the next states and the output based on the current states, if this model is un accurate, this will cause an error of the estimated output. For the harmonics modeling, it is required to create a model for the fundamental and harmonics signal, where each harmonic signal can be modeled by two variable states, and they are independent from the other harmonic state variables.

The problem here is not the harmonics modeling but the number and orders of harmonics that must be modeled in the Kalman filter, where this problem has been discussed in details in [7]. And it was concluded that the Kalman model should have all the harmonics component in the input signal in Kalman filter to get an accurate estimation of the harmonic signals, even when only an estimation of the harmonic signal is required, and it was also concluded that, modeling extra harmonics, that are not existed in the input signal, will not affect the Kalman filter performance.

This paper will analyze the error in different way, where FFT will be used to determine all the frequencies and their amplitude in all the state variables, then based on the results, a modification in Kalman filter calculation is proposed to enhance the Kalman filter performance.

2 Kalman Filter

Kalman filter is a recursive linear optimal filter, the estimating state variables X_k and the output Y_K for a system can be modelled as follows [8]:

$$\begin{aligned} X_{k/k} &= A_k X_{k/k-1} + B_k U_k + W_k \\ Y_k &= C_k X_{k/k} + D_k U_k + V_k \end{aligned} \tag{1}$$

Where:

A_k : transition matrix.

B_k : input control vector.

W_k : process noise.

Y_k : observation state vector.

C_k : observation matrix.

V_k : observation noise.

the Kalman filter calculation can be divided into two stages; predicted stage and updating stage. in predicted stage, the predicted values of the state variables are calculated based on the system model as follows[8]:

$$\begin{aligned} X_{k/k-1} &= A_k X_{k-1/k-1} + B_{k-1} U_{k-1} \\ P_{k/k-1} &= A_k P_{k-1/k-1} + A_k^T + Q_k \end{aligned} \tag{2}$$

then, in the updating stage, the state variables and the output are updated based on the measurement signal as follows:

$$\begin{aligned} Y_k &= Z_k - C_k X_{k/k-1} \\ K_k &= P_{k/k-1} C_k^T [C_k P_{k/k-1} C_k^T + R_k]^{-1} \\ X_{k/k} &= X_{k/k-1} + K_k Y_k \\ P_{k/k} &= [1 - K_k C_k] P_{k/k-1} \end{aligned} \tag{3}$$

Where:

Q_k : process noise covariance matrix

R_k : observation noise covariance matrix.

For harmonics modelling in power system, it is required two state variables to model one harmonic's order, the state variables for each harmonic component are decoupled from the other states variables, the model will be as follows:

$$A_k = \begin{bmatrix} A_{1k} & 0 & 0 & \dots & 0 \\ 0 & A_{2k} & 0 & \dots & 0 \\ 0 & 0 & A_{3k} & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & A_{nk} \end{bmatrix}, B_k = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{4}$$

$$C_k = \begin{bmatrix} C_{1k} & C_{2k} & C_{3k} & \dots & C_{nk} \end{bmatrix}, D = [0]$$

Where:

$$A_{nk} = \begin{bmatrix} \cos(n\omega kT) & -\sin(n\omega kT) \\ \sin(n\omega kT & \cos(n\omega kT \end{bmatrix},$$
$$C_{nk} = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

As it is mentioned earlier, modelling one harmonic component required two state variables, one of them will appear in the output signal based on *C* matrix.

3 Frequency Analysis Using FFT

Let the first input signal as follows:

$$\begin{aligned} y_1(t) = 10\cos(wt) + 10\cos(2wt) + 10\cos(3wt) \\ + 10\cos(4wt) + 10\cos(5wt) \end{aligned} \tag{5}$$

The input signal is shown in Fig.1, it has fundamental, second, third, fourth and fifth harmonics components. The amplitude of all the harmonics components are equal, practically, the amplitude of the harmonics signal is decreased as the order of harmonics is increased, but here, the amplitude is assumed to be equal to focus on the order of the harmonics not in the amplitude of them,

Now a Kalman filter that can model up to 8 harmonics will be used, it will model the fundamental signal but it will not model all the harmonics components in the input signal. Table.1 shows all the possible cases that can be modelled by Kalman filter , except modelling all the harmonics signal since this case will produce zero error[7], For example, case 1 means that Kalman filter will model all the input signal frequencies except the second harmonic and since the Kalman filter can model up to 8 harmonics, so extra harmonics that are not existed in the input signal will be modelled (i.e. 6^{th}, 7^{th}, 8^{th} and 9^{th}). In case 2, the Kalman filter will model all the input signal frequencies except the 3rd harmonic and it will model extra harmonics that are not existed in the input signal (6th, 7th, 8th and 9th). For case 5, the Kalman filter will model the fundemental, 4th , and 5th harmonics and it will not model the 2nd and 3rd harmonics, and again since the Kalman filter can model up to 8 harmonics, 5 harmonic signals that are not existed in the input signal will be modelled in this case (6th, 7th, 8th. 9th and 10th). For case 15, the Kalman filter will model only the fundemental signal and it will not model any of the input signal harmonics (i.e 2nd, 3rd, 4th and 5th), where the Kalman filter will model (6th, 7th, 8th. 9th , 10th and 11th) and none of them are existed in the input signal. In the first four cases (case1 to case4) of Table-1, only one frequency of the input signal is un modelled, in the next 6 cases (case5 to case10) two frequencies of the input signal are not modelled, for case11 to case14 three frequencies of the input signal are not modelled, and in the last case only the fundemental signal is modelled. And in all the cases, the Kalman filter models 8 harmonic signals, which means that the Kalman filter models extra harmonics' signals that are not existed in the input signal.

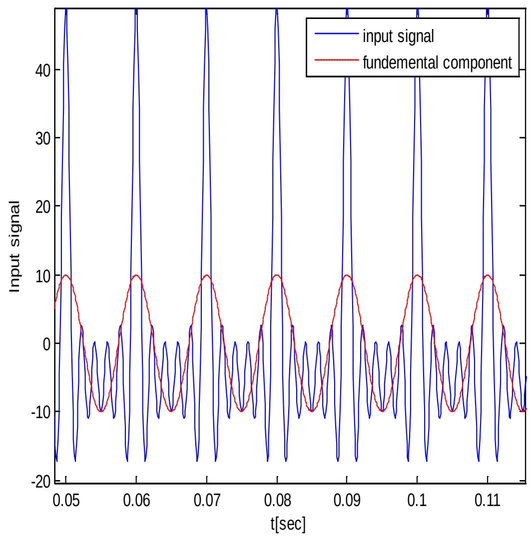


Fig.1. First Input signal

Table.1. harmonics that will be modelled in Kalman filter in each case.

Case#	Measured signal harmonics that are included in the Kalman model	Extra harmonic's signal are not existed in the input signal
1	1st, 3rd, 4th , 5th	6th, 7th, 8th, 9th
2	1st, 2nd, 4th , 5th	6th, 7th, 8th, 9th
3	1st, 2nd, 3rd , 5th	6th, 7th, 8th, 9th
4	1st, 2nd, 3rd , 4th	6th, 7th, 8th, 9th
5	1st, 2nd , 3rd	6th, 7th, 8th, 9th, 10th
6	1st, 2nd, 4th	6th, 7th, 8th, 9th, 10th
7	1st, 2nd, 5th	6th, 7th, 8th, 9th, 10th
8	1st, 3rd, 4th	6th, 7th, 8th, 9th, 10th
9	1st, 3rd, 5th	6th, 7th, 8th, 9th, 10th
10	1st, 4th, 5th	6th, 7th, 8th, 9th, 10th
11	1st, 2nd	6th, 7th, 8th, 9th, 10th, 11th
12	1st, 3rd	6th, 7th, 8th, 9th, 10th, 11th
13	1st, 4th	6th, 7th, 8th, 9th, 10th, 11th
14	1st, 5th	6th, 7th, 8th, 9th, 10th, 11th
15	1st	6th, 7th, 8th, 9th, 10th, 11th, 12th

Since the fundamental component is known in the input signal, two error signals will be studied; the **error signal** which is defined as the difference between the input signal and the estimated output signal, and the **fundamental error signal** which is defined as the difference between the fundamental signal and the estimated fundamental signal, both of these error signals are shown in Fig.2 for case1. Both of them are not equal zero, since the 2nd harmonic signal isn't modelled. Now FFT will be used to determine the frequencies of all the estimated harmonics signals, the result of case 1 is shown in details then the results of all the other cases are listed in Table 2 to Table-4. Fig.3 and Fig.4 show the frequencies in each of the estimated signals for case1, for the estimated fundamental signal, there are two frequencies; one of them is the fundamental frequency and the other is the 2nd harmonic frequency, where the amplitude of the fundamental component in the estimated fundamental signal is equal to the amplitude of the fundamental component in the input signal, which means that, Kalman filter can accurately estimate the amplitude of the fundamental signal, and the fundamental error signal is due to the second harmonic component that appears in the estimated fundamental signal. The estimated 3rd order harmonic is also contained two frequencies one of them is the 3rd harmonic frequency and the other is the 2nd harmonic frequency, and again the amplitude of the third order harmonic in the estimated signal is equal to the 3rd order harmonic amplitude in the input signal. The same results are obtained for the estimated 4th and 5th harmonics signals. For the estimated signals that are not existed in the input signal such as 6th ,7th , 8th and 9th harmonics, there is one harmonic frequency on each one of them which is the 2nd harmonic frequency, so again the Kalman filter is estimated accurately the amplitude of these harmonics in the input signal, which is equal to zero.

Two important results can be noticed from these figures, the first one is that the 2nd harmonic frequency is appeared in all the estimated signals but the amplitude of it is not equal in all the estimated components, where the biggest amplitude of it appeared in the estimated fundamental and estimated 3rd harmonics signals, the second result that the kalman filter estimates the amplitude of the harmonics in the input signal accurately.

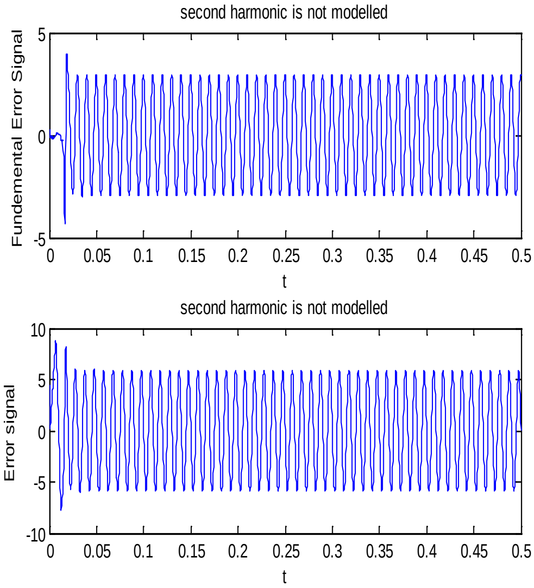


Fig.2. Error and fundamental error signal of case1

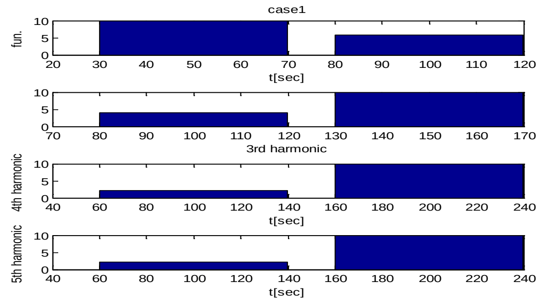


Fig.3. Amplitude of the harmonics components in the estimated harmonics signal by Kalman filter for case 1.

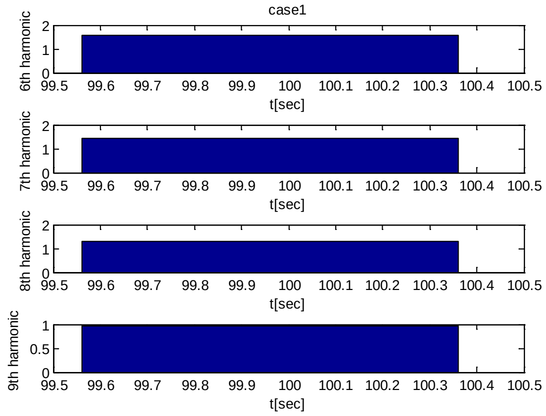


Fig.4. Amplitude of the harmonics components in the estimated harmonics signal by Kalman filter for case 1.

Table2, summarized the results obtained from the FFT of the estimated fundemental signal for all the cases listed in Table-1, in all the cases, the estimated fundemental signal contains a fundemental signal component with amplitude equals to the amplitude of the fundemental input signal, and when only one of the input signal frequency is not modelled (Case1-Case4), the estimated fundemental signal has two frequencies; one of them is the fundemental frequency and the other is the un modelled harmonic frequency, the amplitude of the un modelled frequency is decreased as the order of the un modelled harmonic is increased. when two of the input signals are not modelled (Case5-Case10), both of the un modelled harmonic frequencies are appeared in the estimated fundemental signal, their amplitude are not equal, where the amplitude of them depends on how much their frequencies are close to the fundemental frequency, for example, when the 2nd , 3rd harmonic signals are not modelled , the amplitude of the 2nd harmonic component is greater than the amplitude of the 3rd harmonic component since the second harmonic frequency is closer to the fundemental frequency more than the 3rd harmonic frequency, the same result can be obtained when more than two of input signal harmonics are not modelled (Case11-Case15) , where all the un modelled frequencies are appeared in the estimated fundemental signal with different amplitudes and their amplitudes depend on their orders, when the order of the un modelled harmonic is close to the estimated fundemental frequency, its amplitude is large.

In Table-3 and Table-4, two estimated harmonic signals are analysed using FFT, one of them is for the 2nd harmonic signal, which is an example of harmonic signal that is existed in the input signal, and the other Table is for the estimated 5th harmonic signal which is not existed in the input signal.

Table-3 shows the results obtained from FFT of the estimated 2nd harmonic signal for all the cases that are listed in Table-1. For Case1, Case8-Case10, and Case12-Case15, the 2nd harmonic signal is not modelled in the Kalman filter. in all the remaining cases , the estimated 2nd harmonic signal always has 2nd harmonic frequency with amplitude equals to the amplitude of the 2nd harmonic in the input signal, and there are extra components of frequencies equal to the un modelled harmonics frequencies of the input signal, their amplitudes are depends on their orders, when the un modelled frequency is closer to the 2nd harmonic signal. its amplitude will be larger.

In Table-4, the estimated 5th harmonic signal is analysed for all the cases that are listed in Table-1, where it emphasizes the results that are obtained from Table-2 and Table-3.

Two important facts can be noticed from Table2-Table4, the first one is, the un modelled harmonics of the input signal are appeared in all the estimated harmonics by Kalman filter, the second fact, the amplitude of the un modelled harmonics in the estimated harmonics are varied, where the highest amplitude of them appear in the nearest estimated harmonic order.

In previous input signal, all harmonics components have the same amplitude, now the effect of the amplitude will be investigated, the result here will focus on the estimated fundamental, fundamental error and the error signals. let the second input signal to be as follows:

$$\begin{aligned} y_2(t) = 10\cos(wt) + \beta_2 \cos(2wt) + \beta_3 \cos(3wt) \\ + \beta_4 \cos(4wt) + \beta_5 \cos(5wt) \end{aligned} \quad (6)$$

First, the second harmonic will not be modelled in the Kalman filter, the amplitude of β_2 will change from 0.5 to 20, the amplitude of β_3, β_4 and β_5 will set to 10, the percent of the amplitude of the 2nd harmonic in the estimated fundamental, fundamental error and error signals are shown in Fig.5, the percent of the 2nd harmonic is almost constant and reaches approximately to 60% of its amplitude in both of the estimated fundamental and fundamental error signals, while it reaches about 30% in the error signal.

The values of β_3, β_4 and β_5 are also changed individually from 0.5 to 20 , the results are shown in Fig.6 to Fig.8 respectively, the results show that the percent of the amplitude of them in the estimated fundamental signal are almost constant, the percent of the un modelled harmonic in the estimated fundamental signal is decreased when the order of the un modelled harmonic is increased as it is shown in Fig.9. The fundamental error and the estimated fundamental signals have exactly the same results and this emphasis the result obtained before, that the estimated fundamental signal has a fundamental component with the same amplitude of the fundamental component in the input signal.

For the error signal, the percent of the un modelled harmonic signals is constant regardless of their order.

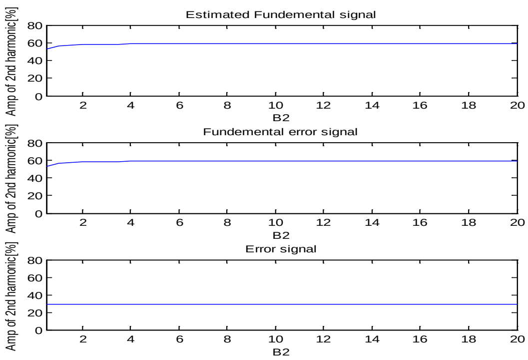


Fig.5. Percent of 2nd harmonic amplitude in the estimated fundamental, fundamental error and error signal when the 2nd harmonics is not modelled and its amplitude is changed from 0.5 to 20.

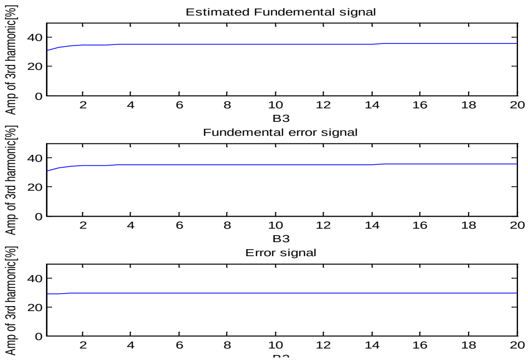


Fig.6. Percent of 3rd harmonic amplitude in the estimated fundamental, fundamental error and error signal when the 3rd harmonics is not modelled and its amplitude is changed from 0.5 to 20.

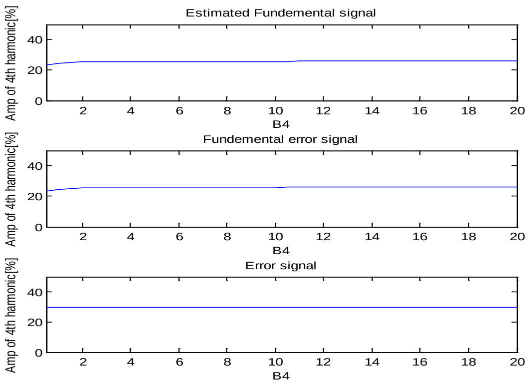
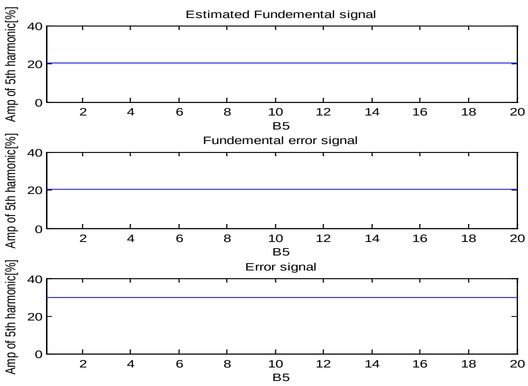


Fig.7. Percent of 4th harmonic amplitude in the estimated fundamental, fundamental error and error signal when the 4th harmonics is not modelled and its amplitude is changed from 0.5 to 20.



.8. Percent of 5th harmonic amplitude in the estimated fundamental, fundamental error and error signal when the 5th harmonics is not modelled and its amplitude is changed from 0.5 to 20.

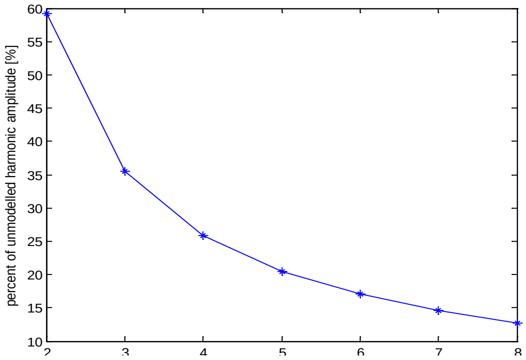


Fig.9. Percent of un modelled harmonics in the estimated fundamental signal

4 Suggested Algorithm

. If Kalman filter can model all the input signal harmonics, this will lead to zero error signal, but this will be difficult since the harmonic's orders can't be known prior the measurement process, and may the harmonic orders increase or change due to some nonlinear devices in power system.

Based on the frequency analysis, a slight modification in Kalman filter calculation is proposed in this section to reduce the error in the estimated fundamental signal.

Let us start with a Kalman filter can model only 9 order of harmonics, our choice will be modelling the first 9 harmonic order from (2nd to 9th), since these harmonics are the most effective on the estimated fundamental signal, this step will ensure that the error in the estimated fundamental signal will have frequency order greater than or equal to 10, and the percent of amplitude of the un modelled harmonics will not exceed 15% in the estimated fundamental signal (based on the result obtained from Fig.9), let the input signal to be as follows:

$$\begin{aligned} y_3(t) = {} & 10\cos(wt) + 10\cos(2wt) + 10\cos(3wt) \\ & + 10\cos(4wt) + 10\cos(5wt) + 10\cos(6wt) \\ & + 10\cos(7wt) + 10\cos(8wt) + 10\cos(9wt) \\ & + 10\cos(10wt) \end{aligned} \tag{7}$$

The input signal has harmonic's orders from 2nd to 10th , the Kalman filter in this case will model all the harmonics except the 10^{th} harmonic, and the error in the estimated fundamental signal will have frequency equals to the 10^{th} harmonic frequency, the error and the fundamental error signals are shown in Fig. 10.

Now, if the calculation of the Kalman filter is changed slightly, where it will be divided into four stages instead of two stages as it is shown in Fig.11, the size of the Kalman matrices will be also modified, where, the size of the original Kalman filter was 18, the size of the matrices in the new stages will be the half of the size of the original Kalman matrices. In the first two stages, The fundamental and the harmonic's orders from (2^{nd} to 5^{th}) will be modelled, while in second two stages, the Kalman filter will model also the fundamental and the harmonics orders from (6^{th} to 9^{th}), note here the modelled harmonics are the same in both of the original and the proposed Kalman filter.

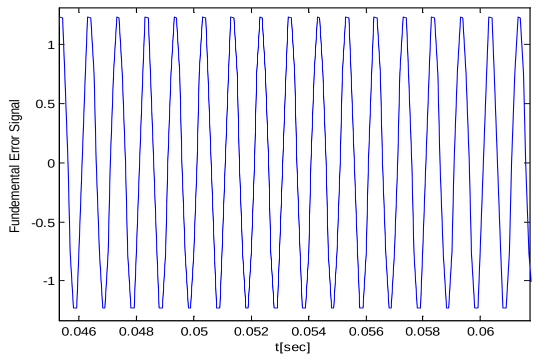


Fig.10. Fundemental error signal for the input signal 3.

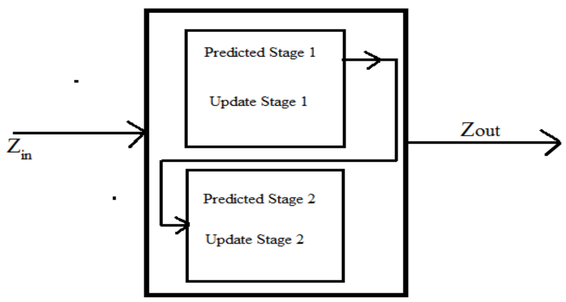


Fig.11.Proposed modification of Kalman filter calculations.

The input of the first two stages is the measured signal, while the input of the second two stages is the estimated fundamental signal from the first two stages, this will not cause a delay on the output signal, since it depends on the current measured signal, but it is required to have a processor compatible with the calculation time, and the size of the matrices in both of stages is half of the original Kalman filter and this will reduce the calculation time.

The advantage of this modification, is that the output of the first two stage will be the fundemental signal beside a percent of amplitude of the harmonics components exceeds 6^{th} harmonic order, then when this signal is considered as the input of the second two stages, the output signal will be the fundamental signal and a percent of harmonic that their order greater than 9, so the un modelled harmonics is multiply by a percent doesn't exceeds 15% **twice**, so the amplitude of the un modelled harmonic in the estimated fundemental signal will not exceed 2.25% (i.e 15% multiply by 15%), the results of the proposed Kalman filter is shown in Fig.12.

Another suggestion can be proposed here, is to combine the original Kalman filter with a Low Pass Filter (LPF), where the estimated fundamental signal from the Kalman filter will have harmonic orders equal or greater than 10 , and their amplitudes are reduced to 15% of their amplitude in the input signal, so using a simple LPF can easily get rid of these harmonics since the Kalman creates a wide frequency band between the passing frequency (fundamental frequency) and the stopping frequencies (equal or greater than 10^{th} order frequencies).

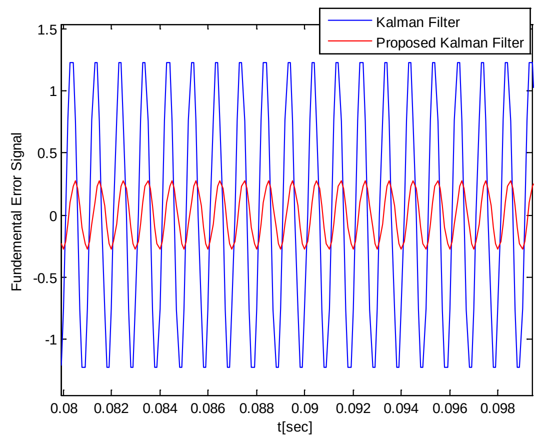


Fig.12. Fundemental Error Signal for the original Kalman filter and the proposed Kalman filter.

4 Conclusions

A frequency analysis of the estimated signals of Kalman filter was done in this paper using FFT, where it is found that the Kalman filter has the capability of estimating the exact amplitude of all the harmonics that are modelled in the kalman filter, and the error in the estimated signal is due to the un modelled frequencies in the input signal, where the un modelled harmonic frequencies are appeared in all the estimated signals, and it is also noticed that, the un modelled harmonics affected mostly the nearest estimated frequencies, and the percent of its amplitude in the estimated signal is almost constant. Based on the frequency analysis of the Kalman filter, different techniques can be proposed to enhance the Kalman filter performance, where one of them is proposed in this paper, in the proposed Kalman filter the number of the modelled harmonics are not changed but the calculations in the Kalman filter is divided into two cascaded stages, to multiply the amplitude of the error twice by the percent error. In future work the number of stages will be studies carefully and the calculation time will be computed.

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Table-2. Amplitude of the harmonics components in the estimated Fundemental signal for the cases listed in Table-1

	Amplitude														
Freq. order	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9	Case 10	Case 11	Case 12	Case 13	Case 14	Case 15
1st	9.918	9.983	10.01	10.03	10.09	10.00	10.03	9.936	9.923	9.908	10.12	9.968	9.926	9.918	9.93
2nd	5.901	0	0	0	0	0	0	5.943	5.999	6.491	0	6.047	6.520	6.566	6.59
3rd	0	3.541	0	0	0	3.659	3.985	0	0	3.487	3.973	0	3.576	4.143	4.34
4th	0	0	2.580	0	2.854	0	2.832	0	2.327	0	3.537	2.708	0	2.423	2.95
5th	0	0	0	2.046	2.328	2.045	0	1.829	0	0	2.216	2.021	1.680	0	1.86
6th	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7th	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8th	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9th	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10th	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11th	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12th	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table-3. Amplitude of the harmonics components in the estimated 2^{nd} harmonic signal for the cases listed in Table-1

	Amplitude														
Freq. order	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9	Case 10	Case 11	Case 12	Case 13	Case 14	Case 15
1^{st}	-	0	0	0	0	0	0	-	-	-	0	-	-	-	-
2^{nd}	-	9.932	9.987	10.01	10.06	9.953	9.906	-	-	-	9.824	-	-	-	-
3^{rd}	-	5.292	0	0	0	5.380	5.868	-	-	-	6.013	-	-	-	-
4^{th}	-	0	3.060	0	3.467	0	2.891	-	-	-	3.391	-	-	-	-
5^{th}	-	0	0	2.240	2.379	1.914	0	-	-	-	1.918	-	-	-	-
6^{th}	-	0	0	0	0	0	0	-	-	-	0	-	-	-	-
7^{th}	-	0	0	0	0	0	0	-	-	-	0	-	-	-	-
8^{th}	-	0	0	0	0	0	0	-	-	-	0	-	-	-	-
9^{th}	-	0	0	0	0	0	0	-	-	-	0	-	-	-	-
10^{th}	-	0	0	0	0	0	0	-	-	-	0	-	-	-	-
11^{th}	-	0	0	0	0	0	0	-	-	-	0	-	-	-	-
12^{th}	-	0	0	0	0	0	0	-	-	-	0	-	-	-	-

Table-4. Amplitude of the harmonics components in the estimated 6^{th} harmonic signal for the cases listed in Table-1

	Amplitude														
Freq. order	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9	Case 10	Case 11	Case 12	Case 13	Case 14	Case 15
1^{st}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2^{nd}	1.561	0	0	0	0	0	0	0.916	1.183	1.406	0	0.579	0.655	0.787	0.985
3^{rd}	0	1.894	0	0	0	1.335	1.688	0	0	1.784	1.267	0	1.448	1.847	2.000
4^{th}	0	0	2.641	0	2.343	0	2.690	0	2.531	0	2.912	2.371	0	2.377	3.533
5^{th}	0	0	0	4.972	5.448	4.914	0	4.834	0	0	5.665	5.549	5.006	0	5.810
6^{th}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7^{th}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8^{th}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9^{th}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10^{th}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11^{th}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12^{th}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0