Minimum distance of a triangle vertices for face classification

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Abstract: - This paper proposes a novel method for face classification based on principal component analysis (PCA) of a grayscale image. The proposed method classifies image by using the minimum distance between vertices of a triangle and the tested image. Vertices of a triangle are created from three distinct points obtained from the combination of a number of images per class. The recognition rate compares the proposed method with the nearest feature line (NFL), the shortest feature line segment (SFLS), the restricted nearest feature line with ellipse (RNFLE), and the shortest distance with the centroid of the triangle (SDC). The experimental results on the Grimace and faces94 databases show that the proposed method has recognition rate over 90%. For Grimace database, the proposed algorithm outperforms the other methods.

Key-Words: - face classification, triangle, vertices, minimum distance.

1 Introduction

Face recognition is a popular and interesting research. Moreover, many researches about face recognition have been increased and developed over the past few decades. Face Recognition is designed to compare the faces of individuals who are interested in the face database. The algorithms used in the template creation process and the comparison process may vary. A face recognition system can be divided into three steps such as face detection, feature extraction and face recognition. Many face recognition techniques have been proposed. One of the most popular techniques is PCA [1], an eigenvector method designed for model linear variation in high-dimensional data and performing dimensional reduction which makes a compact representation of each image. For face classification, there are many methods such as NN [2], NFL [3], SFLS [4], RNFLE [5], SDC [6], and the others [7-9].

The basic assumption is that at least two distinct feature points are available for each class [3]. Therefore, the feature line passes through the two points. Cover and Hart exhibited NN classifier. Since performance of NN is limited by the available point in each class, Li and Lu improved NN to be NFL. In 2011, Han, Han and Yang proposed SFLS using the length of the feature line segment satisfying given geometric relation with the tested image. Feng, Pan, and Yan proposed RNFLE using the ellipse to restrict the feature lines and considering the position of the tested image. In 2018, Klongdee and Ieosanurak presented SDC using the centroid of the triangle representation training images and finding the minimum distance between centroid points and the tested image.

In this paper, our goal is to introduce a new classification method which is classified by the minimum distance between vertices of a triangle and the tested image. In section 2 describes NFL, SFLS, RNFLE and SDC method. The proposed algorithm is described in Section 3. In Section 4 presents the experimental results and comparing between our method and the others. Finally, we provide some concluding in Section 5.

2 Literature Review

We propose several methods that relate face classification and the minimum distance.

2.1 Principal component analysis (PCA)

The principle component analysis approach was described by Turk and Pentland in 1991. PCA is one of the techniques to reduce the size of matrix of variables, or to find the correlation of the data that still contains most of the information in the large matrix. The training database contains M images which are represented to the same size of matrix. Each image matrix is normalized by converting to the equivalent image vector (column matrix) x_i. The training matrix X contains the image vectors as $X = [x_1\ x_2\ \dots\ x_M]$. Definition of eigenvector and eigenvalues are defined by:

Definition. An eigenvector of an $n \times n$ matrix A is a nonzero vector u such that $Au = \lambda u$ for some scalar λ are called an eigenvalue of A, if there is a nontrivial solution u of $Au = \lambda u$ such an u is called an eigenvector corresponding to λ.

The process of principal component analysis is described as follows:

Step 1. Set x_i is the image vector of i^{th} image and calculate the mean normalization of face images which is calculated by the row of matrices as follow:

$$\overline{x} = \frac{1}{M}\sum_{i=1}^{M} x_i. \quad (1)$$

Step 2. Reduce data from N-dimension to M-dimension. Compute the covariance matrix C by

$$C = \frac{1}{M} AA^T, \quad (2)$$

where $A = [\,(x_1 - \bar{x})\ \ (x_2 - \bar{x}) \ldots (x_M - \bar{x})\,]$.

Since the matrix C is high dimension, the eigenvectors of C are considered by the matrix $L = \frac{1}{M} A^T A$ of size $M \times M$.

Step 3. Calculate the eigenvectors and eigenvalues of the covariance matrix. Let V_i be the eigenvector of matrix L corresponding to the eigenvalue λ_i where $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_M$. Thus, $U_i = AV_i$ is eigenvector of C corresponding to the eigenvalue λ_i.

Step 4. Choose components and forming a feature vector. We choose the first two eigenvector with the highest eigenvalue and forming a matrix with these eigenvectors in the column. The eigenface is defined by

$$U = [U_1\ \ U_2\ \ U_3 \ldots U_M]. \quad (3)$$

Step 5. (Return to Vector Conversion) The weight of each eigenvector z_i represents the image in the eigenface space as given by

$$z_i = U^T(x_i - \overline{x}), \quad (4)$$

where U is the eigenface.

2.2 The nearest feature line (NFL)

NFL is the shortest feature line segment passing through two points. Let x_i^c and x_j^c be considered points and $\overline{x_i^c x_j^c}$ be the feature line segment of x_i^c and x_j^c, as shown in Fig. 1. Define x_p as the projection point of x which can be calculated by

$$x_p = x_i^c + t\left(x_j^c - x_i^c\right), \quad (1)$$

where $t = \frac{(x_p - x_i^c)^T (x_j^c - x_i^c)}{\left(x_j^c - x_i^c\right)^T \left(x_j^c - x_i^c\right)}$.

The distance between the tested image x and the feature line $\overline{x_i^c x_j^c}$ is defined as

$$d\left(x, \overline{x_i^c x_j^c}\right) = \left\|x - x_p\right\|_2, \quad (2)$$

where $\|\cdot\|_2$ is the Euclidean distance.

The classification decision called the nearest feature distance can be defined as follows:

$$C_{\text{NFL}} = \arg\min_c \left\{ \min_{1 \leq i < j \leq m} d\left(x, \overline{x_i^c x_j^c}\right) \right\} \quad (3)$$

for $c = 1,2,3, \ldots, N_c$ and C_{NFL} is the class of the tested image.

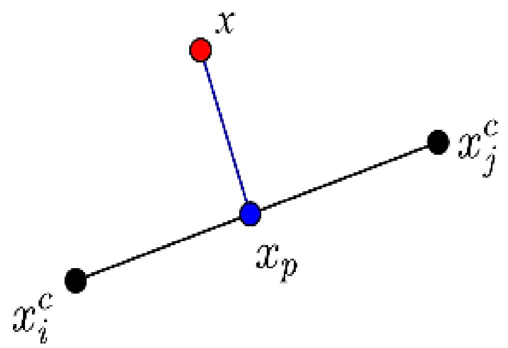

Fig. 1 the nearest feature line

2.3 The shortest feature line segment (SFLS)

Let x_i^c, x_j^c be two training images in the same class. Then the circle is constructed by $\overline{x_i^c x_j^c}$ as its diameter. After that classify the tested image by the shortest length diameter of circle which the tested image x inside the circle. In the worst case, if the tested image x is outside the circle. Then the classification uses the rule of NN as shown in Fig. 2. The distance of SFLS can be calculated by

$$d(x, x_i^c x_j^c) = ||x_i^c - x_j^c||, \quad (4)$$

The classification decision can be defined as follow:

$$C_{\mathrm{SFLS}} = \arg\min_c \left\{ \min_{i,j} d(x, x_i^c x_j^c) \right\} \quad (5)$$

for $c = 1,2,3, \ldots, N_c$. Where C_{SFLS} is the class of the tested image.

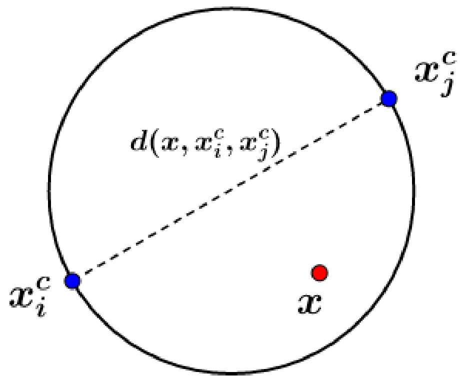

Fig. 2 the shortest feature line segment

2.4 The restricted nearest feature line with ellipse (RNFLE)

This method bases on nearest feature line which uses an ellipse to restrict the feature line. Let x_i^c, x_j^c be foci of any ellipse like Fig. 3, and set a_0 is threshold as the ratio between the length of ellipse long axis and the length of the centre to either focus.

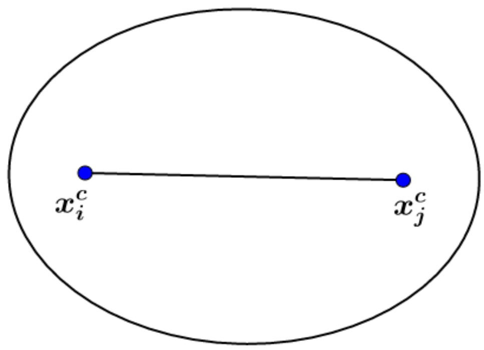

Fig. 3 x_i^C, x_j^C be the foci of ellipse

The classification decision can be defined as follows: if the tested image x is inside the ellipse which is shown in Fig. 4(a), the distance between x and $\overline{x_i^c x_j^c}$ is as follows:

$$\mathrm{d}(x, \overline{x_i^c x_j^c}) = \|x - x_p^c\|, \quad (6)$$

where x_p^c is the projection point of x on the feature line $\overline{x_i^c x_j^c}$. If not, shown in Fig. 4(b), the distance between the tested image and the feature line can be calculated by

$$\mathrm{d}(x, \overline{x_i^c x_j^c}) = \min\{\|x - x_i^c\|, \|x - x_j^c\|\}. \quad (7)$$

The procedure of decision is the nearest distance.

$$C_{\mathrm{RNFLE}} = \arg\min_{\mathrm{c}} \left\{ \min_{1 \le i < j \le m} \mathrm{d}(x, x_i^c x_j^c) \right\}, \quad (8)$$

where $\mathrm{c} = 1,2,3, \ldots, \mathrm{N_c}$.

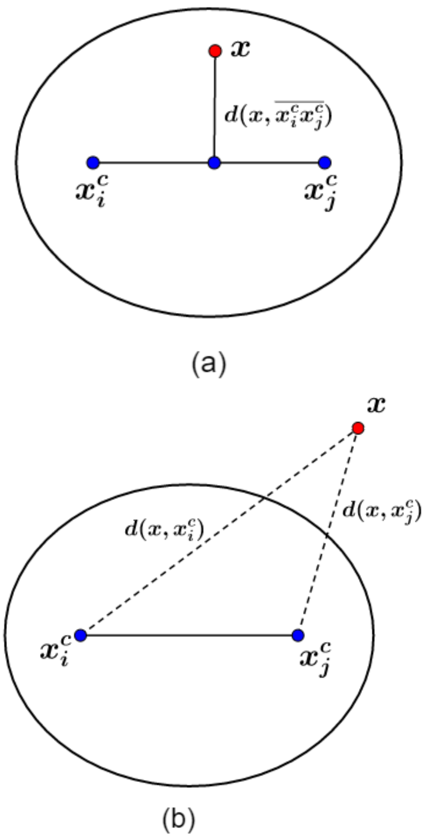

Fig. 4 the idea of RNFLE. (a) the tested image is inside the ellipse. (b) the tested image is outside the ellipse.

2.5 The shortest distance with the centroid of the triangle (SDC)

Let x_i^c, x_j^c and x_k^c be three training images in the same class. The triangle is created from these points and O_{ijk}^c is defined as the centroid of the triangle defined by

$$O_{ij\,k}^{c} = \frac{1}{3}\left(x_i^c + x_j^c + x_k^c\right). \tag{9}$$

The tested image is classified into class C_{SDC} as following form

$$C_{SDC} = \arg\min_{c}\left\{\min_{1 \le i \le j \le m} d\left(x, O_{ij\,k}^{c}\right)\right\}, \tag{10}$$

where $d\left(x, O_{ij\,k}^{c}\right)$ is distance between the tested image and the centroid of the triangle, m is the number of images per class.

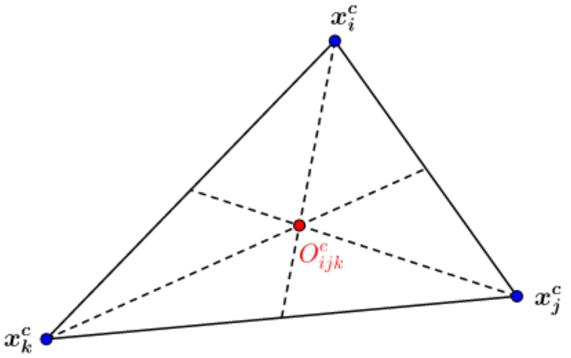

Fig 5 $O_{ij\,k}^{c}$ be the centroid of the triangle class c

3 The Proposed Algorithm

In this section, we proposed a new algorithm for face classification by using the minimum distance between vertices of a triangle and the tested image. First of all, we use PCA to find a subset of the principle component represent training images.

The proposed algorithm

Let x_i^c, x_j^c and x_k^c be the vertices of a triangle created by feature vectors from the same class C, where $1 \le i < j < k \le m, 1 \le c \le N_c$: N_c and m are the number of the class and the number of images per class, respectively.

The proposed algorithm can be shown as follows:

Step 1. Input the training image set and denoted by Y_i^c.

Step 2. Transform Y_i^c into a new column matrix as Z_i^c and use the PCA in order to get the feature vector as x_i^c.

Step 3. The triangle is produced by the three points, which are obtained from the combination of m distinct points taken of the same class, which means that the triangle is created from x_i^c, x_j^c and x_k^c.

Step 4. Input the tested image x and calculate the distance between each of a vertex of the triangle $d\left(x, x_{ij\,k}^{c}\right)$ as follows:

$$\begin{aligned} d\left(x, x_{ij\,k}^{c}\right) = \min_{1 \le i < j < k \le m} \{\|x - x_i^c\|_2, \|x \\ - x_j^c\|_2, \|x - x_k^c\|_2\}, \end{aligned} \tag{9}$$

where $\|\quad\|_2$ is the Euclidean distance.

Step 5. The tested image is classified into class C_{proposed} as the following form

$$C_{\text{proposed}} = \arg\min_{c}\left\{ d\left(x, x_{ij\,k}^{c}\right)\right\}, \tag{10}$$

for all $i, j, k = 1,2,3, \dots, m$.

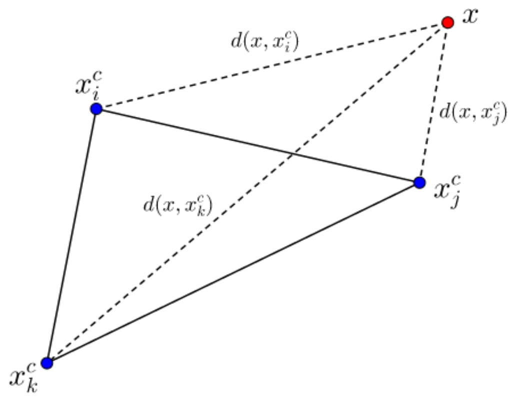

Fig. 6 show distance between the tested image and the vertices of the triangle.

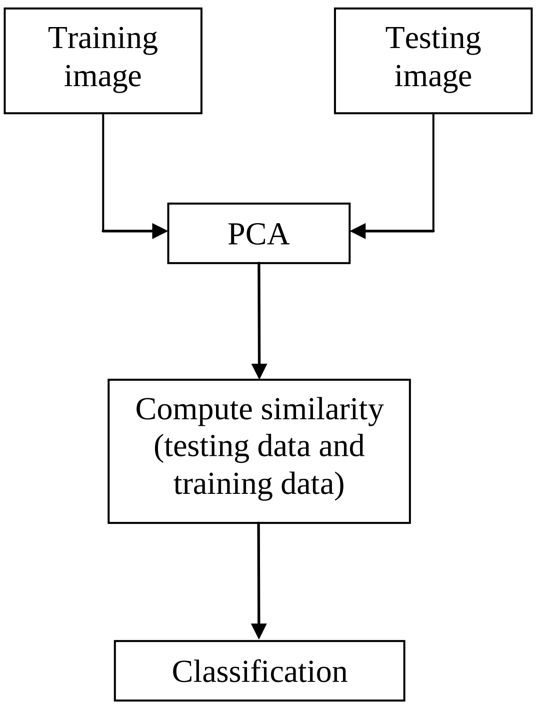

Fig 7 process of proposed approach

The recognition rate is calculated by

$$\text{The recognition rate} = \frac{N_{cr}}{N_{test}} \times 100\%, \quad (11)$$

where N_{cr} is the number of correct recognition of the tested face images and N_{test} is the total number of tested images.

4 Experimental Results

In the present study, a set of images is provided by the Grimace and Faces94 databases [25]. The Grimace database contains 18 distinct persons, each person having 20 different images, image resolution is $180x200$ pixels, the background is plain. During the sequence the subject moves his/her head One subject face images of Grimace database are shown in Fig 8.

Fig 8 one subject face images of Grimace database

The Faces94 database has two categories as female and male staff. The dataset consists of 19 and 20 classes and each class has 20 images. The subjects are asked to speak, whilst a sequence of images is taken. The speech is used to introduce facial expression variation. The background is plain green, position of face in image is minor changes, and facial details (glasses/no glasses). One subject face images of Faces94 database are shown in Fig 9 and Fig10.

Fig 9 some subject face image of Faces94 (female) database

Fig 10 some subject face image of Faces94 (male staff) database

We present the experimental results of NFL, SFLS, RNFLE, SDC, and our proposed method. Moreover, we transform the training images and the tested images as column vector and use PCA for feature extraction before classifying face images. For each database, the data is separated into two sets, the training set and the set of tested images. The training set is divided into 0.95, 0.90, 0.85, …, 0.60 proportion and the remaining is the set of tested images. Table 1, 2, and 3 show the recognition rate between our proposed algorithm and the other algorithms.

Table 1 shows the recognition rate of NFL, SFLS, RNFLE, SDC, and the proposed algorithm. The recognition rate of the proposed algorithm is better than the others except 0.90 proportion, SDC is better than the others.

As shown in Table 2, we see that SDC is better than the others. The recognition rate at 0.60 and 0.65 proportions, the proposed algorithm is better than the others.

Table 3 illustrates the recognition rate of NFL, SFLS, RNFLE, SDC and the proposed method. The recognition rate of SDC is better than the others except 0.60 proportion, the proposed algorithm is better than the others. For 0.70 proportion, SFLS and the proposed method are the best.

Table 1. The recognition rate of Grimace database with various algorithms

Grimace	**The recognition rate**				
Proportion	**NFL**	**SFLS**	**RNFLE**	**SDC**	**Proposed**
0.95	33.33	94.44	94.44	94.44	94.44
0.90	58.33	94.44	47.22	97.22	94.44
0.85	27.78	92.59	31.48	94.44	96.30
0.80	22.22	98.61	38.89	98.61	100
0.75	13.33	96.67	16.67	97.78	98.89
0.70	9.26	96.30	14.81	97.22	98.15
0.65	14.29	95.24	8.73	96.83	98.41
0.60	9.72	95.83	11.81	96.53	98.61

Table 2. The recognition rate of Faces94 (female) database with various algorithms

Faces94 (female)	**The recognition rate**				
Proportion	**NFL**	**SFLS**	**RNFLE**	**SDC**	**Proposed**
0.95	63.16	94.74	94.74	94.74	94.74
0.90	21.05	92.11	28.95	97.37	94.74
0.85	15.79	92.98	24.56	94.74	94.74
0.80	14.47	94.74	23.68	94.74	92.11
0.75	8.42	93.68	27.37	95.79	94.74
0.70	7.02	93.86	25.44	95.61	93.86
0.65	9.02	93.23	21.05	92.48	93.98
0.60	5.92	92.76	21.05	92.11	94.08

Table 3. The recognition rate of Faces94 (male staff) database with various algorithms

Faces94 (male staff)	**The recognition rate**				
Proportion	**NFL**	**SFLS**	**RNFLE**	**SDC**	**Proposed**
0.95	20.00	95.00	95.00	95.00	95.00
0.90	15.00	97.50	97.50	97.50	97.50
0.85	10.00	98.33	93.33	98.33	95.00
0.80	10.00	98.75	95.00	98.75	96.25
0.75	12.00	99.00	98.00	98.00	99.00
0.70	7.50	95.00	94.17	98.33	95.00
0.65	9.29	94.29	95.00	97.14	95.71
0.60	6.25	95.00	96.88	96.88	98.13

5 Conclusion

In this paper, we proposed a new algorithm for face classification by using the minimum distance between vertices of a triangle which is produced by three points based on PCA from the same class and the tested image. Mostly, the recognition rate of the proposed algorithm is better than the others, followed by SDC, SFLS, and RNFLE, respectively. However, experimental result of Grimace database has shown that the proposed algorithm is better than the others, except 0.90 proportion. Moreover, our method is suitable for little changes of position of face in image.

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