Particle swarm optimization research base on quantum self learning behavior

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Abstract: - Particle swarm optimization algorithm is analyzed base on quantum-behaved self learning, particles searching action and local attract point are studied. To the different searching environment in searching progress, the searching actions are divided into four models. The proposed algorithm can self-learn the optimization problem, and utilize a suitable learning model, then the whole optimization performance is increased. The comparison and analysis of results with the proposed method and other improved QPSO are given based on CEC2005 benchmark function, the simulation results show the modified algorithm can greatly improve the QPSO performance.

Key-Words: - Particle Swarm Optimization(PSO), quantum behavior, self-learning, local attract point, searching model, evolutionary

1 Introduction

Particle swarm optimization (PSO) is a swarm intelligence algorithm, which was proposed by Kennedy and others for continuous search space problem [1]. PSO algorithm has been widely used for its programming is simple, its algorithm is fast convergence, but the algorithm itself is not a global convergence [2], many scholars have proposed many improved methods [3,4]. Sun studied forming mechanism of PSO algorithm, quantum theory are introduced in the algorithm, Quantum-behaved Particle Swarm Optimization(QPSO) was proposed, which has a global search capability of quantum behavior optimization algorithm[5], QPSO algorithm is computationally simple, and there are less control parameters and other characteristics, it attracted the attention of many scholars in related fields at home and abroad. For the algorithm only control parameters (systolic expansion factor), a number of improvements were proposed in the parameter selection method [6,7]. An improved method was proposed in terms of the particle population diversity. The probability distribution function was introduced QPSO algorithm, it was used to improve the performance of the algorithm. QPSO algorithm was combined with a number of other evolutionary methods, the overall performance of the algorithm was improved by combining the respective advantages of different algorithms [8-14].

In the above some QPSO improved algorithm, the group learning mode are the same, the complexity of the case does not distinguish between different treatment. For example, the search area of particles may be unimodal function type, or the type of multimodal function; or for a particle, the current peaks or troughs fall should have different learning patterns. In order to make the algorithm more in line with human thinking habits, with higher intelligence, an adaptive learning QPSO (Self-learning QPSO, SLQPSO) algorithm is presented in this study, it can improve the overall performance of the algorithm.

2 Materials and methods

2.1 Quantum Particle Swarm Optimization (QPSO)

Quantum Particle Swarm Optimization (QPSO) : The running track of PSO algorithm particles were analyzed by Clerc [14], he pointed out that in the PSO algorithm, if each particle can converge to its local attraction $P_i = (P_{i1}, P_{i2}, ..., P_{iD})$, then PSO algorithm may converge. Among them:

$$p_{id}(t) = \frac{c_1 r_{1d}(t) P_{id}(t) + c_2 r_{2d}(t) P_{gd}(t)}{c_1 r_{1d}(t) + c_2 r_{2d}(t)}, 1 \le d \le D \quad (1)$$

And set

$$\varphi_d(t) = \frac{c_1 r_{1d}(t)}{c_1 r_{1d}(t) + c_2 r_{2d}(t)}, 1 \le d \le D \quad (2)$$

In Formula (1) and (2), t is the current iteration number in algorithm, $r_{1d}(t)$ and r_{2d} (t) is a random number between [0,1], P_i is the current optimal position of the particle, P_g is global optimal position for groups.

In the PSO algorithm, the learning factor c_1, c_2 is usually set to the equal value, so the formula (2) can be rewritten as:

$$\varphi_d(t) = \frac{r_{1d}(t)}{r_{1d}(t) + r_{2d}(t)}, 1 \le d \le D \quad (3)$$

It can be seen that φ_d (t) is uniformly distributed random number between [0,1] , the formula (1) is rewritten as:

$$p'_{id}(t) = \varphi_d(t) P_{id}(t) + [1 - \varphi_d(t)] P_{gd}(t), \quad \varphi_d(t) \sim U[0,1] \quad (4)$$

As can be seen from the above equation, p'_i is located in super rectangle by point P_i and point P_g for vertices, it varies with the change of point Pi and Pg point. In fact, when the particles converge to their local attraction, their own optimum position, the local attraction point and global optimal position will converge to the same point, so it makes PSO algorithm converge. From the perspective of dynamics analysis, the particles in the search process has p' point attractor in the PSO algorithm, with the continuous reduction of the particle velocity, it is close to p' point, and it is finally dropped to p 'points. Thus, throughout the execution of the algorithm, there is some form of potential field to attract groups of particles at p' point, so that the population of particles maintained aggregation. However, in classical PSO system, the search process is implemented in the form of particles orbit, while the particle speed is limited, so in the search process, the search space of the particles is limited to a limited search space, which can not cover the entire feasible search space. General PSO algorithm is not guaranteed to converge to a global optimal solution in the probability, which is less than the PSO algorithm.

Assuming PSO system is a quantum system, in the quantum space, velocity and position of the particles can not be determined at the same time, the state of each particle is determined by the wave function ψ, $|\psi|^2$ is the probability density function of particle position. By analysis of PSO particle convergence system [14], it is assumed in the t-th iteration, particle i is in D-dimensional space of movement, the potential well of the particles in the j-th dimension is for p'_{ij} (t), the i particle wave function can be obtained at the t + 1 iteration:

$$\psi[x_{ij}(t+1)] = \frac{1}{\sqrt{L_{ij}(t)}} \exp[-|x_{ij}(t+1) - p_{ij}(t)| / L_{ij}(t)] \quad (5)$$

Thus, the probability density function Q can be obtained as:

$$Q[x_{ij}(t+1)] = |\psi[x_{ij}(t+1)]|^2 = \frac{1}{L_{ij}(t)} \exp[-2|x_{ij}(t+1) - p_{ij}(t)| / L_{ij}(t)] \quad (6)$$

Probability distribution function F is as:

$$F[x_{ij}(t+1)] = \exp[-2|x_{ij}(t+1) - p_{ij}(t)| / L_{ij}(t)] \quad (7)$$

By application of the Monte Carlo method, j-dimensional position of the particle i can be obtained in the t + 1 iteration:

$$x_{ij}(t+1) = p'_{i,j}(t) \pm \frac{L_{ij}(t)}{2} \ln[\frac{1}{u_{ij}(t)}], u_{ij}(t) \sim U[0,1] \quad (8)$$

Value L_{ij} (t) is determined by the following formula (9):

$$L_{ij}(t) = 2a|M_j(t) - x_{ij}(t)| \quad (9)$$

Wherein M is called the optimum average position, it is also referred to as mbest, which is the center of all particles themselves optimal position, it is obtained from the following equation (10):

$$M(t) = (m_1(t), m_2(t), \cdots, m_D(t)) = (\frac{1}{N}\sum_{i=1}^{N} P_{i1}(t), \frac{1}{N}\sum_{i=1}^{N} P_{i2}(t), \cdots, \frac{1}{N}\sum_{i=1}^{N} P_{iD}(t)) \quad (10)$$

Where, N is the size of population, P_i is the i-th particle best position itself. Thus, the update equation (11) can be obtained in the location of particles :

$$x_{ij}(t+1) = p'_{i,j}(t) \pm \alpha \cdot |M_{ij}(t) - x_{ij}(t)| \cdot \ln[\frac{1}{u_{ij}(t)}],$$
$$u_{ij}(t) \sim U[0,1] \quad (11)$$

Where, α is called compression expansion factor, it is used to adjust the convergence speed of the particles.

The current updated position P_i of the particle and the global best position in P_g updated manner are identical with the best updated way of the basic PSO algorithm, namely

$$P_i(t+1) = \begin{cases} P_i(t) & , f(P_i(t)) < f(X_i(t+1)) \\ X_i(t+1), & f(P_i) \geq f(X_i(t+1)) \end{cases} \quad (12)$$

$$P_g(t+1) = \arg\min_{P_i} f(P_i(t+1)), 1 \leq i \leq N \quad (13)$$

Here, the PSO algorithm in particle position update formula (11) is for quantum-behaved particle swarm optimization (Quantum-behaved Particle SwarmOptimization, QPSO).

2.2 An adaptive learning algorithm QPSO

In QPSO algorithm, it can be obtained by formula (4) that particle is attracted by a local attraction in the evolutionary process, the position of the local attraction point is located in super rectangle with vertices of point P_i and P_g , Figure 1 shows a particle state in two-dimensional space, in this case, a local attraction area is located in the area between P_i and P_g. In this region, the local points are calculated by the formula (4), but it does not guarantee that attract particles towards a better location area, perhaps it is even worse. Of course, through several evolved iterations, the particles can search for a better area, but it has delayed the search speed. In this case, the attracted point of the particles would make the particles be quickly toward to the target area for P_g . Therefore, it should be based on the position of the particle in the search process to determine the status of the local particle attractor.

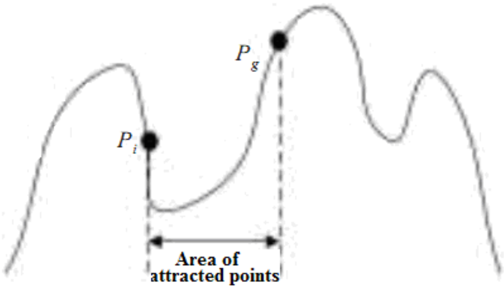


Figure 1 The calculating schematic of the local attractor in QPSO algorithm

2.3 Several learning strategies in adaptive learning QPSO algorithm

Based on the above analysis, It can be found when the particles are searching in different positions, different approach should be taked to learning. It is inspired by the human division of labor, the particles in the population can play different roles, such as convergence to the global optimum location, to search the individual best location, to explore the unknown feasible solution space, it is out of local optimal solution space. Accordingly, these particles also are correspond to the characters in the search process, the particles may be located. In unimodal function or peak far apart multimodal function, the particles can improve the effectiveness of search for the direct learning through the best individual information; for in the particle on the slopes, their optimal location information can be learned to make it easier search to better position; in case there are multiple local peaks, which are evenly distributed, it is more conducive to learning companion particle to explore new search space; for the local convergence of the particle, it can be a way out of local variation in the optimal solution. Thus, four learning strategies can handle different situations. For particle i, the corresponding learning equation can be modified as follows:

1) learn from global best position

$$x_{ij}(t+1) = p_{g,j}(t) \pm \alpha \cdot |M_{ij}(t) - x_{ij}(t)| \cdot \ln[\frac{1}{u_{ij}(t)}] \quad (14)$$

2) to study the best position itself

$$x_{ij}(t+1) = p_{i,j}(t) \pm \alpha \cdot |M_{ij}(t) - x_{ij}(t)| \cdot \ln[\frac{1}{u_{ij}(t)}] \quad (15)$$

3) study the companion particles

$$x_{ij}(t+1) = p_{rand,j}(t) \pm \alpha \cdot | M_{ij}(t) - x_{ij}(t) | \cdot \ln[\frac{1}{u_{ij}(t)}] \quad (16)$$

4) mutation operation

During the mutation operation, the each dimension values of the particles are initialized with a certain probability. Process is as follows:

Setting a mutation probability is γ

For each dimension d of Xi

If rand () <γ then

X_{id} = initialize (R_{lo}, R_{up})

Endif

Endfor

Which R_{lo} and R_{up} are particle initialization range and upper range limit.

2.4 Adaptive learning mechanism

The basis of adaptive learning mechanism is that the assumption made more successful in recent generations, particle evolution will be more successful in later generations of evolution. In the adaptive learning process, the particles depends on what sort of learning strategies, it depends on particle location, in the evolutionary process, a strategy for the optimal choice of the particles may change with the position of the particle changes. Four learning strategy are proposed in this study, they represent the four different topologies of particle population, population structure determines the manner in which the particles and information exchange for the population of other particles. Here, certain probability χ^i_j is selected for each learning strategy, i represents the i-th particle, j represents the number of learning strategies; at initialization, the χ^i_j initial value is 25%. With the evolution of the particles, these selected probabilities will also change with the selection probability, the specific value depends on the fitness value of the offspring, after the current success rate of the evolution is combined with the previously selected probability, the fitness is calculated, there is the learning strategy with a good performance, the selected probability increases.

First, the reward is calculated as follows:

$$R^i_j(t) = \omega \cdot \frac{r^i_j(t)}{\sum_{j=1}^{l} r^i_j(t)} + (1-\omega) \cdot \frac{n^i_j}{N^i_j} + \lambda^i_j \cdot \chi^i_j(t) \quad (17)$$

Wherein, R^i_j (t) is the bonus value of the current generation; l is 4, there are four kinds of learning strategies; ω is random number between (0, 1); λ^i_j is penalty factor.

The r^i_j (t) in formula (17) is the evolution of values, the calculation method is as follows for the minimization problem:

$$r^i_j(t) = \begin{cases} | f(x_i(t)) - f(x_i(t-1)) | \\ , f(x_i(t)) < f(x_i(t-1)) \\ 0, \quad f(x_i(t)) \geq f(x_i(t-1)) \end{cases} \quad (18)$$

n^i_j in Formula (17) is the number of evolutionary success after particle i uses learning strategies j since the latest update with the selectivity; N^i_j is the number of all particles i using evolutionary learning strategies j, it is for the selectivity since the most recent update.

According to experience, through multiple simulations, λ^i_j penalty factor was calculated as:

$$\lambda^i_j(t) = \begin{cases} 0.8, if\ n^i_j = 0 \quad and\ \chi^i_j(t) = \max{}^{l}_{j=1}(\chi^i_j(t)) \\ 0, \quad otherwise \end{cases} \quad (19)$$

Therefore, the calculation formula of selectivity can be obtained:

$$\chi^i_j(t+1) = (1-\phi) \cdot \frac{R^i_j(t)}{\sum_{j=1}^{l} R^i_j(t)} + \phi \quad (20)$$

Φ is selectivity with minimal learning strategies.

2.5 SLQPSO algorithm design

Step 1：nitialization parameter settings : which includethe number of groups, expansion contraction coefficient α ranges, algorithm iterations, mutation rate γ, minimum selection rate which is randomly selected, the initial solution x (i) which is randomly generated, and to set pbest(i) = x (i), and the global optimum gbest is calculated.

Step 2： An average optimum value mbest is calculated according to formula (8).

Step 3： Evolutionary values of particles are computed medianaccording to Formula (18).

Step 4： According to equation (19) , to determine the penalty factor.

Step 5： Learning reward strategy is calculated according to the formula (17) .

Step 6 ： According to equation (20), calculating the selected rate of four kinds of learning strategies.

Step 7： According to the selected rate, the appropriate model is selected in the four learning strategies, an iterative equation of algorithm is determined, the new position is updated for each particle. And a new population is generated.

Step 8： Each particle updates the position value in accordance with the formula (8), (9) and (10) , and it generates a new group.

Step 9： Return (2) until the end of the cycling conditions.

3 Simulation and performance analysis

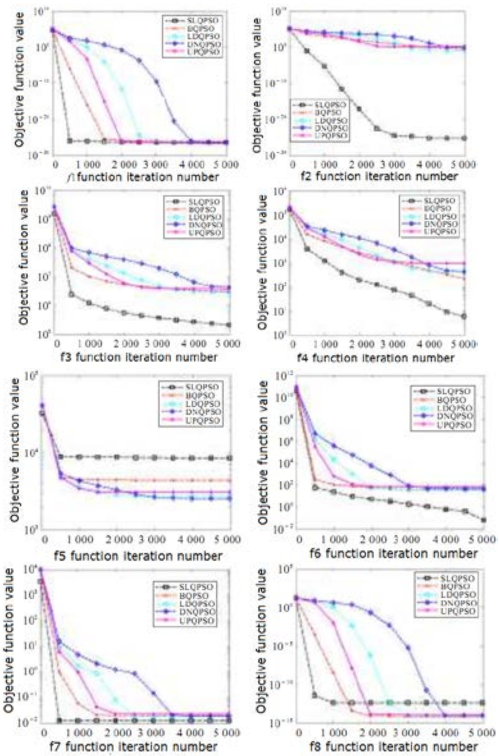
3.1 Simulation Setup

In QPSO algorithm, the search expansion factor α is the only performance-related control parameters of the algorithm. The parameter adjustment policies (LDQPSO) with a linear decrease and non-linear decreasing parameter adjustment policies (opening up UPQPSO were proposed [7], opening down DNQPSO), simulation results show that for different problems, these methods have their own advantages. In order to test and compare the performance of an adaptive learning algorithm QPSO, this study algorithm (SLQPSO) and fixed coefficient QPSO algorithm (BQPSO), LDQPSO, UPQPSO, DNQPSO were tested and compared, CEC2005 benchmark test functions are used in test function [15], where F1 to F6 are unimodal function , F7 to F12 are for multimodal function. Optimization problems set that α is 0.8 in BQPSO algorithm, α decreased from 1.0 to 0.5 in LDQPSO algorithm, UPQPSO algorithm, DNQPSO algorithm; in SLQPSO algorithm, the mutation probability γ is 0.05 in the mutation, the minimum selection rate φ is 0.01. In the maximum number of algorithm iterations and the settings of particle number, the effective set is needed for problem, the number of iterations and the number of particles in groups are too many, which would increase the overhead; if the number of iterations and the number of particles are too small, the optimization algorithm can not make or iteration algorithm stops before it has yet to find the optimal value. Therefore, for the testing function of this study, the problem dimension is 30, the maximum number of iterations in the algorithm is corresponding to 5000, the number of particles is 40; the solving algorithms are applied to each question, it randomly run independently 50 times.

3.2 Simulation results and discussion

The mean value and standard deviation of the objective function for five algorithms by the simulation results in 50 times, which are obtained under different number of particles, different dimensions and different iterations for each test functions.

Figure 2 shows the mean convergence curves of different algorithms at 40 particles, 30 dimensional case and 50 simulation.



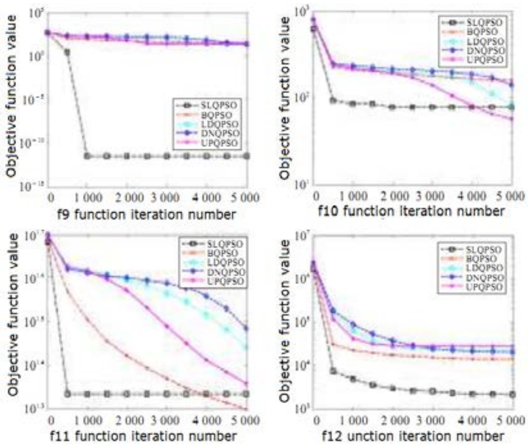


Figure 2 Optimization convergence curve of the objective function value comparison with five algorithms and with different standard test functions

From Table 1, the simulation results of various algorithms look that SLQPSO has achieved the best results in seven of 12 test functions, in BQPSO and UPQPSO, each made two times, LDQPSO made a sub-optimal value. In the 7 times case of sub-optimal in QLQPSO, significant advantage has been achieved for ShiftedSchwefel's Problem 1.2 function (F2), Shifted RotatedHigh Conditioned Elliptic Function (F3), Shifted Schwefel 's Problem 1.2 with Noise in Fitness (F4), Shifted Rosenbrock function (F6), Shifted Rastrigin 's function (F9), Schewefel' s Problem 2.13 function (F12) , especially for F2 and F9 function, the algorithm searches are close to the optimal value zero, it is far superior to several other algorithms; for Shifted Rotated Rastrigin 's (F10) and Shifted Rotated Weierstrass function (F11), the optimal value of SLQPSO algorithm is compared with several other algorithms, it has achieved suboptimal value. Further, it can be seen from the convergence curve of evolution in Figure 2 that after SLQPSO algorithm is used, 11 functions in 12 test functions are able to quickly move closer to the optimal solution (before step 500), even in the F5 function, it also made convergence speed with other similar algorithm; for F9 function tests, when SLQPSO algorithm is to iterative step 1000, it converges to the global optimal solution nearby. It is described above, Q learning algorithm is proposed, QPSO can adaptively determine the search position of the particle, and the appropriate learning strategies are used in a timely manner, learning efficiency is improved, thus, it is greatly improving the convergence rate, and SLQPSO algorithm has outstanding performance in the majority of test functions.

4 Conclusion

This study analyzes the evolution equation QPSO algorithm, and the algorithm local attraction learning mode is studied, local generation method is proposed for a single point of attraction, different methods of local attraction should be used for optimization problems. The particles search location in the optimization process is divided into four cases, and four local attraction learning modes were presented, adaptive learning algorithm process is given for a detailed four models. At last, the simulation is made and compared with test function CEC2005 between the improved algorithm and several different methods of other QPSO improvements. The simulation results show that the adaptive learning algorithm can achieve better results on most test functions, which is also able to achieve similar results with other methods for a number of other functions, and SLQPSO algorithm has faster convergence, and therefore the improved algorithm can greatly improve the overall performance of QPSO algorithm in degree largely.

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