An Improved Method for Simplification of the Large Scale Linear Time Invariant Control System

SHARAD KUMAR TIWARI, GAGANDEEP KAUR
Department of Electrical & Instrumentation Engineering
Thapar University, Patiala, INDIA
E-mail: sktiwari@thapar.edu
E-mail: gagandeep@thapaer.edu

Abstract:- In this paper, we introduce a new clustering algorithm based on Lehmer measure for simplifying the order of large scale dynamical system. This algorithm is applied with the combined advantages of retaining the dominant poles and the Pade approximation. The dominant pole algorithm adapted to select poles to obtain the cluster centre. The selection of poles for the cluster is based on from the viewpoint of important poles contribution to the system is preserved by dominant pole algorithm. Having obtained the denominator polynomial of the reduced model, the numerator terms are obtained using the Pade approximation method. The reduction algorithm is fully computer oriented. The reduced model is stable if the original model is stable. Moreover, this method gives a better approximation in both the transient and the steady-state responses of the original system.

Key- Words :- Order Reduction; Clustering; Lehmer measure; Dominant pole; Pade approximation; Integral Square Error.

1 Introduction

The model order reduction (MOR) is used to reduce high order system (HOS) into low order system (LOS) which retains the behavior of the original system which is important in many fields of engineering. The approximation of HOS with LOS is one of the important problems in system control theory. The use of a reduced order model makes it easier to implement analysis, simulations and control system designs. Therefore, it is necessary to obtain low order representation of original higher order models. The aim of the MOR is to reduce the order of a given linear system, such that the principal and important specifications of the HOS are retained in the reduced order model with minimum error.

A number of simplification methods exist in the frequency domain. Pade approximation [1], [2] uses the concept of dominant poles and matching the few initial time moments of the systems. It receives a drawback of deciding the dominant poles, which should retain in the reduced order model. The continued fraction expansion approximation method [3]–[5] has many useful properties such as computational simplicity, the fitting of the time moments, and the preservation of the steady-state responses.

Various methods have also been developed for obtaining a stable, low-order model if the original system is stable. [6] Uses the stability criterion of Routh for obtaining the reduced model. In their method, the low order models are derived by a suitable truncation of the original transfer function in the α-β expansion.[7], [8] suggested the equivalent but simpler procedures. The procedure in [8] eliminates the two reciprocal transformation used by Hutton and Friedland. This is achieved by modifying the alpha and beta tables used in [6]. [9] Using the Hurwitz polynomial approximants as characteristic polynomials, the numerator dynamics of reduced models are then determined by partial Pade approximation of a given large-order model. These two methods are correspondent [10] and called the Routh-Hurwitz method. It is also noted that if the dominant poles are not closest to the origin, the Routh-Hurwitz approximation fails to obtain a good lower order model. In fact the stability-equation method [11]–[13] also suffers from the same drawback because far-off poles and/or zeros of the stability equation of the characteristic polynomial are discarded. To overcome this problem, a generalization of the Routh method is developed by [14] to obtain several different reduced models. All stability based method is developed by

first approximating the characteristic polynomial followed by determining the coefficients of the numerator of the reduced model. Various constructions in consideration of stability problem are Chebyshev [15], [16] and Hermite [17] polynomial which deal with the stability problem with the original model. However, these methods may fail because of pure imaginary poles. [18] proposed stability-equation method using the Mihailov criterion and the Pade approximation technique. In clustering technique [19], zeroes and poles are collected to form clusters and these clusters are formulated by inverse distance measure (IDM) criterion to find cluster centers. [20] proposed a mixed method for finding stable reduced order models of large-scale systems using Pade approximation and clustering technique. [21], [22] modified the pole clustering by an iterative method. The difficulty with these methods is in selecting poles for the clusters. [23], suggested a method based on eigen spectrum analysis which consists of all poles of the high order system. The poles of the reduced model are uniformly spaced between the first and last poles.
In this paper, it is proposed that the denominator of reduced model is constructed from the new clustering method based on modified Lehmer measure [24]. The method uses the concept of dominant pole algorithm [25] for the selection poles for the clustering. Pade approximation method is used to find the parameters of the numerator polynomial of the reduced model. Examples of model reduction are included, which compare the proposed approach with earlier methods to illustrate the advantages of the method. This proposed method has the advantages that the reduced order model is always stable if the original one is stable. Due to the selection of the importance of the poles the accuracy of the reduced order model is improved, and also reduced model leads to good approximations in both transient and steady-state responses.

3 Problem formulation

Consider the linear, time-invariant (LTI) stable system defined by transfer function a

$$G(s)=\frac{a_m s^m + a_{m-1}s^{m-1}+.........+a_1 s \ + a_0}{b_n s^n + b_{n-1}s^{n-1}+..........+b_1 s \ + b_0} \tag{1}$$

In model reduction we are faced with the problem of finding a reduced-order LTI system,

$$G_r(s)=\frac{N_{r-1}(s)}{D_r(s)}=\frac{c_{r-1}s^{r-1}+c_{r-2}s^{r-2}+.........+c_1 s + c_0}{d_r s^r + d_{r-1}s^{r-1}+..........+d_1 s + d_0} \tag{2}$$

of order r, r ≪n, which approximates G(s). The aim of the model order reduction is to reduce the order of a given linear system, such that the principal and important specifications of the full order system are retained in the reduced order model with minimum error.

2 The proposed model reduction method

Step 1: A*nalysis for relative importance of poles*

Relative importance of all poles of original system is measured by the Dominant Pole Algorithm (DPA). We assume the transfer function H(s) can be expressed as

$$H(s)=\sum_{j=1}^{n}\frac{R_j}{s-\lambda_j} \tag{3}$$

Where, the sum is taken over all poles and R_j is called the residue. The weighted residue is defined as ρ_j

$$\rho_j = \frac{|R_j|}{|Re(\lambda_j)|} \tag{4}$$

A large weighted residue magnitude ρ_j implies dominance, i.e., good observability and controllability of the pole λ_j in the transfer function G(s). The peaks of the Bode magnitude plots occur at frequencies which are close to the imaginary parts of the dominant poles of G(s)[26]. For example, if ρ_3 has the largest value, then the term $\frac{R_3}{s-\lambda_3}$ is the most important term and – λ_3 is considered the most important pole. If ρ_2 has the next largest value, then the term $\frac{R_2}{s-\lambda_2}$ is the second most important term and – λ_2 is considered the next important pole of the original system. The poles are sorted following decreasing weighted residue, i.e.,

$$\rho_1 \geq \rho_2 \geq \rho_3 \geq \cdots \rho_n$$

We call $\lambda_1, \lambda_2 \ldots, \lambda_n$ are the corresponding n poles of the original system with their relative importance.

Step 2: *Determination of denominator polynomial $D_r(s)$*

Based on the distribution of the r important poles, r numbers of cluster centers are formed. The number of cluster centers to be calculated is equal to the order of the reduced system. The poles are distributed into the cluster center for the calculation such that none of the repeated poles present in the same cluster center. The minimum number of poles distributed per each cluster center is at least one.

Selection of poles for the clusters is based on relative importance of the poles computed using DPA. The first cluster includes first important pole and all others poles less than the second important poles of the original system and so on.

The cluster center of the reduced order model can be obtained by using the modified Lehmer measure criterion. The procedure described similar to the method proposed by [20] but the pole cluster calculated in the proposed scenario are based on the dominant pole in that particular cluster center.

Let k be the number of poles in a cluster are λ_1, λ_2, λ_3,…..,λ_k. Where λ_1 is the most important pole computed from step 1. And the λ_2-λ_k are the poles greater than the second important poles. Compute the pole cluster center using modified Lehmer measure criterion identifies the cluster center as

$$C_c = \left[\frac{\left(\frac{1}{|\lambda_1|}\right) + \sum_{i=2}^{k}\left(\frac{1}{|\lambda_i|}\right)^{k-1}}{\left(\frac{1}{|\lambda_1|}\right)^2 + \sum_{i=2}^{k}\left(\frac{1}{|\lambda_i|}\right)^{k}} \right] \tag{5}$$

Where C_c is the cluster center from r real poles of the original system

Let, m pair of complex conjugate poles in a cluster is $[(\alpha_1 \pm j\beta_1),(\alpha_2 \pm j\beta_2)..........(\alpha_m \pm j\beta_m)]$ then the modified Lehmer measure criterion is used to find the cluster center in the form of $(A_c \pm jB_c)$

$$A_c = \left[\frac{\left(\frac{1}{|\alpha_1|}\right) + \sum_{j=2}^{k}\left(\frac{1}{|\alpha_j|}\right)^{k-1}}{\left(\frac{1}{|\alpha_1|}\right)^2 + \sum_{j=2}^{k}\left(\frac{1}{|\alpha_j|}\right)^{k-1}} \right] \text{and}$$

$$B_c = \left[\frac{\left(\frac{1}{|\beta_1|}\right) + \sum_{j=2}^{k}\left(\frac{1}{|\beta_j|}\right)^{k-1}}{\left(\frac{1}{|\beta_1|}\right)^2 + \sum_{j=2}^{k}\left(\frac{1}{|\beta_j|}\right)^{k-1}} \right]$$

For obtaining the denominator polynomial for r^{th} order reduced system, following cases may occur as in [22].

Case-1: If all modified cluster centers are real, then the denominator polynomial of the k^{th} order reduced model can be obtained as

$$D_r(s) = (s-\lambda_{c1})(s-\lambda_{c2})...(s-\lambda_{cr}) \tag{6}$$

Where $\lambda_{c1}, \lambda_{c2}, ..., \lambda_{cr}$ are 1^{st} , 2^{nd} ,…..,r^{th} cluster center respectively.

Case2:- If all modified cluster centers are complex conjugate then the k^{th} order denominator polynomial is taken as:

$$D_r(s) = (s-\dot{\psi}_{c1})(s-\mathring{\psi}_{c1})........(s-\dot{\psi}_{cr/2})(s-\mathring{\psi}_{r/2}) \tag{7}$$

$\dot{\psi}$ and $\mathring{\psi}$ are complex conjugate cluster center.

$\dot{\psi}_{c1} = A_c + jB_c$ and $\mathring{\psi}_{c1} = A_c - jB_c$

Case-3: If some cluster centers are real and some are complex conjugate. For example, (k - 2) cluster centers are real and one pair of the cluster center is complex conjugate, then r^{th} order denominator can be obtained as:

$$D_r(s) = (s-p_{c1})(s-p_{c2})....(s-p_{c(r-2)})(s-\dot{\psi}_{c1})(s-\mathring{\psi}_{c2}) \tag{8}$$

Hence, the denominator polynomial $D_r(s)$ is obtained as:

$$D_r(s) = d_r s^r + d_{r-1}s^{r-1} + ...+ d_1 s + d_0 \tag{9}$$

Step 2: *Determination of the numerator polynomial using Pade approximation technique.*

Coefficients of numerator for the reduced model are obtained by Pade approximations. The original nth-order system can be expanded in power series about s = 0 as

$$G(s) = \frac{\sum_{i=0}^{n-1} a_i s^i}{\sum_{i=0}^{n} b_i s^i} = C_0 + C_1 s + C_2 s^2 + C_3 s^3 + \ldots.. \quad (10)$$

The constant coefficients of the power series expansion at s=0 are calculated as follows.

$$\begin{aligned} C_0 &= a_0 \\ C_i &= \frac{1}{b_0}\left[a_i - \sum_{i=1}^{i} b_i C_{i-1}\right] \\ a_i &= 0, i > n-1 \end{aligned} \quad (11)$$

Hence the reduced r^{th} order is given as

$$G_r(s) = \frac{N_r(s)}{D_r(s)} = \frac{\sum_{i=0}^{n-1} c_i s^i}{\sum_{i=0}^{n} d_i s^i} \quad (12)$$

Dr(s) is known through clustering method. For $G_r(s)$ equation is a Pade approximation of G(s).

Where,

$$\begin{aligned} c_0 &= b_0 C_0 \\ c_1 &= b_0 C_1 + b_1 C_0 \\ &\ldots\ldots\ldots\ldots\ldots\ldots \\ c_{r-1} &= b_0 C_{r-1} + b_1 C_{r-2} + \ldots + b_{r-1} C_1 + b_r C_0 \end{aligned} \quad (13)$$

where i=1,2,3,…….,r-1

The comparison is done by computing the error index known as integral square error (ISE) [6] in between the transient parts of the original and reduced order model, is computed to evaluate the quality, which is given by:

$$ISE = \int_0^{\infty} [y(t) - y_r(t)]^2 dt \quad (14)$$

Where y(t) and $y_r(t)$ are the unit step responses of original and reduced order systems respectively. This error index is computed for various reduced order models which are obtained by us and compared with the other order reduction methods available in the literature.

4 Numerical Experiments

To evaluate the efficiency of the proposed method, it has been applied to three test systems, where a step-by-step procedure is given for the first test system.

Test example 1: In this test case, consider the well-known 4^{th} order system with transfer function from the literature [23], [27]–[29].

$$G_4(s) = \frac{s^3 + 7s^2 + 24s + 24}{s^4 + 10s^3 + 35s^2 + 50s + 24}$$

To develop second order reduced model, two pole clusters centers are required. Based on step 1 the cluster one includes only one pole -1, and second cluster include pole (-2,-3 and -4). By applying the improved pole clustering method, the improved cluster poles are obtained as λ_{c1}=1and λ_{c2}=2.3844. The corresponding reduced order denominator polynomial is

$$D_r(s) = s^2 + 3.384s + 2.384$$

Furthermore, as specified in step 2, the coefficients of numerator polynomial of the required second order reduced model are obtained as.

$$G_2(s) = \frac{0.7523s + 2.384}{s^2 + 3.384s + 2.384}$$

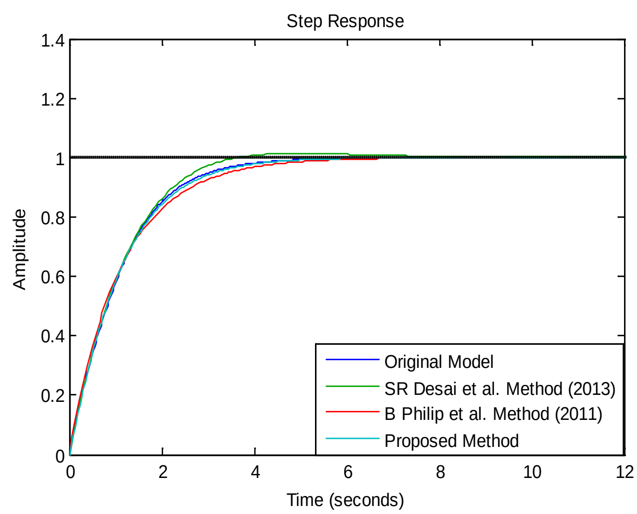

Figure 1: Comparison with step responses of original model and reduced model of example 1

The step response of the original and reduced order models is shown in figure 1 of test system 1. The quantitative comparison such as integral square error, steady state value, maximum overshoot, rise time, settling time between various 2^{nd} order models is presented in table 1. It is distinctly ascertained that the specification of the reduced order model achieved by the proposed method are close to the specification of the original system, and more serious than other methods.

Table 1. Quantitative Comparison between various reduced-orders for example 1

Method of order reduction	ISE	Steady state value	Overshoot (%)	Rise time (Sec.)	Settling Time (Sec.)
Original Model	-	1	0	2.26	3.93
Proposed method	2.6×10^{-4}	1	0	2.30	4.04
[29]	3.5×10^{-4}	1	1.24	2.13	3.15
[30]	2.78×10^{-3}	1	0	2.53	4.55
[23]	2.85×10^{-3}	0.99	1.3	2.19	3.22
[12]	4.296×10^{-3}	1	1.07	2.3	3.41
[31]	1.88×10^{-2}	1	2.69	2.2	5.57

Test example 2: Consider the well-known 6^{th} order system with transfer function from the literature [32],[33].

$$G_6(s)=\frac{2s^5+3s^4+16s^3+20s^2+8s+1}{2s^6+33.6s^5+155.94s^4+209.46s^3+102.42s^2+18.3s+1}$$

To develop second order reduced model, two pole clusters are required. Based on step 1 the cluster one includes poles (-0.1, -0.2, -0.5, -1.0), and second cluster include pole (-5 and -10). By applying the improved pole clustering method, the improved cluster poles are obtained as λ_{c1}=0.1066 and λ_{c2}=6. The reduced order denominator polynomial is obtained as

$$D_r(s)=s^2+6.107s+0.6393$$

Furthermore, as specified in step 2, the coefficients of numerator polynomial of the required second order reduced model are obtained as.

$$G_2(s)=\frac{-0.48088s+0.6396}{s^2+6.107s+0.6393}$$

The step response of the original and reduced order models is shown in figure 2 of test system 2. The quantitative comparison such as integral square error, steady state value, maximum overshoot, rise time, settling time between various 2^{nd} order models is presented in table 2. It is distinctly ascertained that the specification of the reduced order model achieved by the proposed method are close to the specification of the original system, and more serious than other methods.

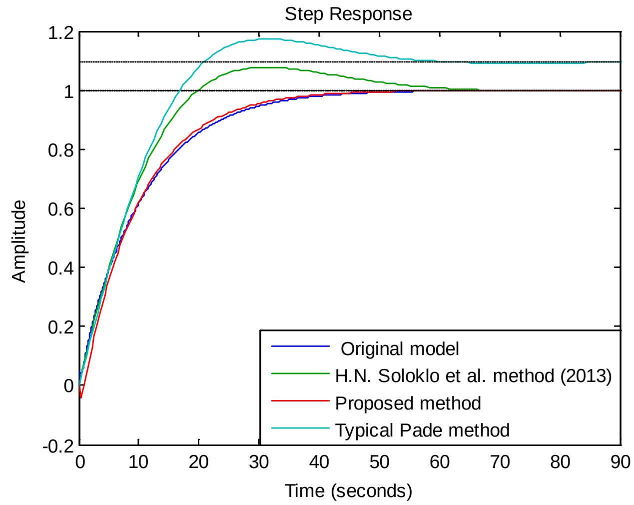

Figure 2: Comparison with step responses of original model and reduced model of example 2

Table 2.Quantitative Comparison between various reduced-orders for test system 2

Method of order reduction	ISE	Steady state value	Overshoot (%)	Rise time (Sec.)	Settling Time (Sec.)
Original Model	-	1	0	22.7	40
Proposed method	0.0227	1	0	20.6	37.2
[34]	0.5337	1	7.61	14.3	53
[33]	2.8887	1.1	7.46	15.1	50.4

Test example 3: In this final example, we will consider an 8^{th} order system transfer function that has been recently considered by different researchers[19], [35]. Consider 8^{th} order model given by

$$G_8(s)=\frac{19.82s^7+429.26156s^6+4843.8098s^5+45575.892s^4+241544.75s^3+905812.05s^2+1890443.1s+842597.95}{s^8+30.41s^7+358.4295s^6+2913.8638s^5+18110.567s^4+67556.983s^3+173383s^2+149172.19s+37752.826}$$

Furthermore, as specified in step 2, the coefficients of numerator polynomial of the required fourth order reduced model are obtained as

$$G_4(s)=\frac{-1.3501s^3+67.216s^2+647.8446s^2+357.0496}{s^4+1.91s^3+47.56s^2+56.35s+16}$$

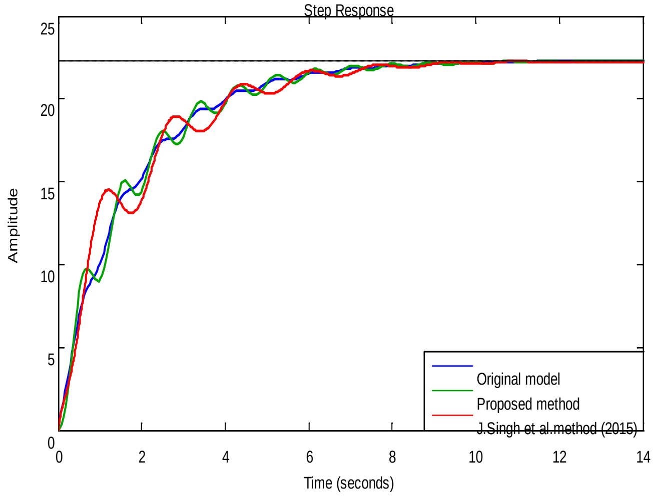

Figure 3: Comparison with step responses of original model and reduced model of example 3

Table 3.Quantitative Comparison between various reduced-orders for example 3

Method of order reduction	ISE	Steady state value	Over-shoot (%)	Rise time (Sec.)	Settling Time (Sec.)
Original Model	-	1	0	3.89	7.57
Proposed method	1.827	1	0	3.86	7.70
[35]	8.550	1	0	3.84	7.34

The step response of the original and reduced order models of test system 3 is shown in figure 3. The quantitative comparison such as integral square error, steady state value, maximum overshoot, rise time, settling time between various 2nd order models is presented in table 3. It is distinctly ascertained that the specification of the reduced order model achieved by the proposed method are close to the specification of the original system, and more serious than other methods.

4 Conclusion

In this method, new pole clustering method along with a simple mathematical process is suggested to obtain poles of the reduced order system. The selection of poles to the obtained cluster center is based on from the viewpoint of important poles contribution to the system is preserved by dominant pole algorithm. The coefficient of numerator polynomial is obtained using Pade approximation technique. The closeness between the original and approximated system is calculated by using ISE as quality parameters for the given step input. Ramp input and impulse input can also be examined. This method preserves stability in a reduced order model if the given higher order system is stable. This method has been extended for a discrete system as well with the combination of another method.

References

[1]. S. C. Chuang, “Homographic transformation for the simplification of continuous-time transfer functions by Padé approximation,” *Int. J. Control*, vol. 23, no. 6, pp. 821–826, Jun. 1976.

[2]. A. Bultheel and M. V. Barel, “Padé techniques for model reduction in linear system theory: a survey,” *J. Comput. Appl. Math.*, vol. 14, pp. 401–438, 1986.

[3]. C. F. Chen and L. S. Shieh, “An algebraic method for control systems design,” *Int. J. Control*, vol. 11, no. 5, pp. 717–739, May 1970.

[4]. R. Parthasarathy and S. John, “System reduction using Cauer continued fraction expansion about s= 0 and s=∞ alternately,” *Electron. Lett.*, vol. 18, no. 23, pp. 9–10, 1978.

[5]. R. Prasad and J. Pal, “Use of continued fraction expansion for stable reduction of linear multivariable systems,” *Inst. Eng. India Part Electr. Eng. Div.*, vol. 72, pp. 43–43, 1991.

[6]. M. Hutton and B. Friedland, “Routh approximations for reducing order of linear, time-invariant systems,” *IEEE Trans. Automat. Contr.*, vol. 20, no. 3, pp. 329–337, Jun. 1975.

[7]. V.Krishnamuthy and V. Seshadri, “Model Reduction Using the Routh Stability Criterion,” *IEEE Trans. Automat. Contr.*, vol. 23, pp. 729–731, 1978.

[8]. V. Krishnamurthi and V. Seshadri, “A simple and direct method of reducing order of linear systems using routh approximations in the frequency domain,” *Autom. Control. IEEE …*, vol. 21, no. 5, pp. 797–799, 1976.

[9]. R. K. Appiah, “Pade methods of Hurwitz polynomial approximation with application to linear system reduction,” *Int. J. Control*, vol. 29, no. 1, pp. 39–48, Jan. 1979.

[10]. Y. Shamash, “Failure of the Routh-Hurwitz method of reduction,” *IEEE Trans. Automat. Contr.*, vol. 25, no. 2, pp. 313–314, 1980.

[11]. T. C. Chen, C. Y. Chang, and K. W. Han,

"Reduction of transfer functions by the stability-equation method," *Journal of the Franklin Institute*, vol. 308, no. 4. pp. 389–404, 1979.

[12]. T. C. Chen, C. Y. Chang, and K. W. Han, "Model reduction using the stability-equation method and the Padé approximation method," *J. Franklin Inst.*, vol. 309, no. 6, pp. 473–490, 1980.

[13]. T. C. Chen, C. Y. Chang, and K. W. Han, "Model reduction using the stability-equation method and the continued-fraction method," *Int. J. Control*, vol. 32, no. 1, pp. 81–94, Jul. 1980.

[14]. Y. Shamash, "Stable biased reduced order models using the Routh method of reduction," *Int. J. Syst. Sci.*, vol. 11, no. 5, pp. 641–654, Jan. 1980.

[15]. G. Langhoz and Y. Bistrltz, "Model reduction of dynamic systems over a frequency interval†," *Int. J. Control*, vol. 31, no. 1, pp. 51–62, Jan. 1980.

[16]. Y. Bistritz and G. Langholz, "Model reduction by Chebyshev polynomial techniques," *IEEE Trans. Automat. Contr.*, vol. 24, no. 5, pp. 741–747, 1979.

[17]. H. N. Soloklo, O. Nali, and M. M. Farsangi, "Model Reduction by Hermite Polynomials and Genetic Algorithm," *J. Math. Comput. Sci.*, vol. 9, pp. 188–202, 2014.

[18]. B.W. Wan, "Linear model reduction using Mihailov criterion and Padé approximation technique," *Int. J. Control*, vol. 33, no. 6, pp. 1073–1089, Jun. 1981.

[19]. A. K. Sinha and J. Pal, "Simulation based reduced order modelling using a clustering technique," *Comput. Electr. Eng.*, vol. 16, no. 3, pp. 159–169, Jan. 1990.

[20]. C.B. Vishwakarma and R. Prasad, "Clustering Method for Reducing Order of Linear System using Pade Approximation," *IETE Journal of Research*, vol. 54, no. 5. p. 326, 2008.

[21]. R. Komarasamy, N. Albhonso, and G. Gurusamy, "Order reduction of linear systems with an improved pole clustering," *J. Vib. Control*, vol. 18, no. 12, pp. 1876–1885, Dec. 2011.

[22]. C. B. Vishwakarma and R. Prasad, "MIMO System Reduction Using Modified Pole Clustering and Genetic Algorithm," *Model. Simul. Eng.*, vol. 2009, pp. 1–5, 2009.

[23]. G. Parmar, S. Mukherjee, and R. Prasad, "System reduction using factor division algorithm and eigen spectrum analysis," *Appl. Math. Model.*, vol. 31, no. 11, pp. 2542–2552, Nov. 2007.

[24]. K. B. Stolarsky, "Holder Means, Lehmer Means, and," *J. Math. Anal. Appl.*, vol. 202, pp. 810–818, 1996.

[25]. M. Saadvandi, K. Meerbergen, and E. Jarlebring, "On dominant poles and model reduction of second order time-delay systems," *Appl. Numer. Math.*, vol. 62, no. 1, pp. 21–34, Jan. 2012.

[26]. N. Martins and P. E. M. Quintão, "Computing dominant poles of power system multlvariable transfer functions," *IEEE Trans. Power Syst.*, vol. 18, no. 1, pp. 152–159, 2003.

[27]. A. K. Mittal, R. Prasad, and S. P. Sharama, "Reduction of linear dynamic system using an error minimization technique," *J. Inst. Eng.*, vol. 84, pp. 201–204, 2004.

[28]. T. Lucas, "Factor division: a useful algorithm in model reduction," *Control Theory Appl. 130*, vol. 130, no. 6, pp. 1981–1983, 1983.

[29]. S. R. Desai and R. Prasad, "A new approach to order reduction using stability equation and big bang big crunch optimization," *Syst. Sci. Control Eng.*, vol. 1, no. 1, pp. 20–27, Dec. 2013.

[30]. B. Philip and J. Pal, "An evolutionary computation based approach for reduced order modelling of linear systems," *2010 IEEE Int. Conf. Comput. Intell. Comput. Res.*, pp. 1–8, Dec. 2010.

[31]. J. Pal, "Improved Pade approximants using stability equation method," *Electron. Lett.*, vol. 19, no. 11, pp. 3–4, 1983.

[32]. H. N. Soloklo and M. M. Farsangi, "Multiobjective weighted sum approach model reduction by Routh-Pade approximation using harmony search," *Turkish J. Electr. Eng. Comput. Sci.*, vol. 21, pp. 2283–2293, 2013.

[33]. M. Jamshidi, *Large-Scale Systems: Modeling, Control and F uzzy Logic*. New York, North-Holland, 1983.

[34]. H. N. Soloklo, R. Hajmohammadi, and M. M. Farsangi, "Model order reduction based on moment matching using Legendre wavelet and harmony search algorithm," *Iran. J. Sci. Technol. , Trans. Electr. Eng.*, vol. 39, no. E1, pp. 39–54, 2015.

[35]. J. Singh, C. B. Vishwakarma, and K. Chattterjee, "Biased Reduction Method by Combining Improved Modified Pole Clustering and Improved Pade Approximations," *Appl. Math. Model.*, 2015.