Adaptive Integral Sliding Mode Control Method for Synchronization of Supply Chain System

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Abstract: In this paper, synchronization problem of supply chain chaotic system is carried out with active and adaptive integral sliding mode controlling method. Active integral sliding mode synchronization is performed for two identical supply chain systems by assuming that all parameters of the systems are known. When the internal and external distortion parameters of the system are considered unknown, an appropriate feedback controller is developed based on the adaptive integral sliding mode control mechanism to synchronize two identical supply chain chaotic systems and to estimate the unknown parameters of the system. The stability evaluation of the synchronization methods are performed by the Lyapunov stability theorem. In addition, the performance evaluation of the designed controllers and the theoretical analysis are verified by some illustrative numerical simulations. Simulation results indicate excellent convergence from both speed and accuracy points of view.

Key–Words: Supply chain system, Integral sliding mode control , Active control, Adaptive control

1 Introduction

Chaos synchronization is an extension of the concept of chaos control. In the past to decades, the chaos synchronization has got considerable attentions by the researchers because of its unpredictable complex behaviour. The ultimate goal of the synchronization is to design a feedback controller such that the follower system tracks the trajectories of the leader chaotic system after a short initialization time interval. However, the challenges occur when the chaotic systems are exposed to some uncertainty, different initial values and unknown system parameters. So, some actions have to be taken in order to stabilize and to improve the synchronization task.

Chaos phenomenon usually appears in nonlinear dynamical systems. A nonlinear dynamical system has chaotic behaviour if it is sensitive to the initial values of its state variables. Since Lorenz [1] in 1963 has found his 3D chaotic system, many chaotic systems have discovered and studied by the researchers, such as: Chen system [2], Lü system [3], Liu system [4], Genesio system [5], BhalekarGejji system[6], supply chain system [7], and many other chaotic systems [8].

Nowadays, supply chain system, has got considerable attention in analysis, modelling and planning because of its so many economical applications[9, 10]. The purpose of this paper is to investigate the synchronization and the control of chaos in the supply chain system.

1.1 Literature review

In the literature, the first method on control of the chaotic system has published by Ott et al in [11] as name OGY method and the first identical synchronization method has developed by Pecora et al in [12]. Since then, different methods have been proposed for the synchronization and the anti-synchronization of the chaotic systems either identical or non-identical ones. Active method [13, 14], impulsive method [15], projective method [16, 17], lag method [18], sliding mode method [19, 20] and backstepping control method [21] are some of the investigated synchronization methods. Nevertheless, more often the parameters of a chaotic system are fully or partially uncertain or unknown, so these methods are hardly applicable. To overcome this problem, many researchers concentrate on Adaptive method [22, 23, 24, 25], which are extensions of adaptive control theory.

However there exist many adaptive related approaches in the literature, but there are minor works carried out on adaptive integral sliding mode synchronization method. Integral sliding mode method for controlling a pneumatic servo system in [26], adaptive integral sliding Control of single electromagnetic guiding system in [27], synchronization of unified chaotic systems via sliding mode in [28], adaptive sliding mode synchronization of two chaotic systems in [29], genetic algorithm based integral sliding mode method in [30] and synchronization of the

GenesioTesi chaotic system via adaptive sliding mode method in [31] are some of the researches based on the adaptive sliding mode method.

Recently, supply chain system has got a lot of attention by the researchers [32, 33, 34, 10, 35, 36]. Supply chain system wants to afford of the customers demands accurately on time with minimum possible cost. Supply chain systems have usually some unknown/uncertain coefficient in their dynamical systems. The behaviour of the supply chain system would be chaotic in some situations as the behaviour results of the customers or purchasing decisions. The deficiency of supply shortages, order batching, price fluctuations and lead times may result a phenomenon namely, bullwhip effect. A number of studies are devoted to find the bullwhip effect resources to reduce the uncertainty[7, 37, 38].

Chaotic behaviour of the supply chain system at the production or inventory levels is not pleasant. So the control of a supply chain system may eliminate its nonlinear factors of the system. And also the synchronization task of the supply chain systems can equilibrium the demand and resource planning of the system.

Anne et al in [39], have proposed an adaptive method for synchronization of the supply chain system with unknown internal or external disturbances. In order to improve their competitiveness every enterprises have to use supply chain management system. Goksu et al in [40], have designed a linear feedback controller to control and to synchronize the supply chain system. Chaos synchronization of supply chain system is carried out by using radial basis function in [7] to counteracted the bullwhip effect. In [37], the bullwhip effect is challenged by the linear control theory. So far, there is not any published article on adaptive-integral sliding mode synchronization of the supply chain chaotic systems, which is the novelity of this paper.

1.2 approach and contribution

The sliding mode control method is often used because of its inherent advantages of easy realization, fast response and good transient performance, as well as its insensitivity to parameter uncertainties and external disturbances.

In the following, the supply chain system and its chaotic behaviour is described. Then an active integral sliding mode scheme is developed for synchronization of the leader-follower supply chain systems. After that adaptive integral sliding mode synchronization of suppy chain system is performed with unknown internal and external distortions; and also an appropriate designed feedback controller is proposed to track the trajectories of the leader supply chain system by the corresponding follower system. Chaos synchronization of the leader-follower systems are proved by the Lyapunov stability theorem. At the end, the validity of the proposed method is assessed by some provided numerical simulations.

The structure of the consequent sections of this paper are outlined as follows: In Section 2, some preliminaries and theoretical information about chaotic supply chain system is provided. The proposed identical active integral sliding mode control of the supply chain system is investigated in Section 3. Adaptive integral sliding mode synchronization is given in Section 4, where the internal and the external distortions of the supply chain system are considered unknown. Some numerical simulation results related to the represented approaches are carried out in Section 5to study the effectiveness of the proposed synchronization schemes. Finally, in Section 6, some conclusion remarks are provided.

2 Preliminaries and mathematical modelling

The supply chain system can be represented based on the three main components: producers, distributors and final customers. In [39], the dynamic behaviour of the supply chain system is given by a three dimensional equations as:

$$\begin{aligned} \dot{x}_1 &= (m+\delta_m)x_2 - (n+1+\delta_n)x_1 + d_1 \\ \dot{x}_2 &= (r+\delta_r)x_1 - x_2 - x_1x_3 + d_2 \\ \dot{x}_3 &= x_1x_2 + (k-1-\delta_k)x_3 + d_3 \end{aligned} \tag{1}$$

Where $\dot{X} = (\dot{x_1}, \dot{x_2}, \dot{x_3})$ is the time derivative of the state variables vector $X = (x_1, x_2, x_3)$. Linear disorders $\delta_m, \delta_n, \delta_r$ and δ_k are the amount of purturbance of the constant parameters m, n, r and k, respectively. d_1, d_2 and d_3 are the three nonlinear external distortions related to the states x_1, x_2 and x_3, which are corresponding to the three quantities as demand, inventory, and produced, respectively. The component m indicate the distributors delivery efficiency; the constant parameter n denote the customer demand rate; the constant parameter r implies the distortion coefficient and k is the safety stock coefficient.

Chaotic behaviour of the supply chain system is obtained with distributors values as: $m = 10, \delta_m = 0.1, n = 9, \delta_n = 0.1, r = 28, \delta_r = 0.2, k = -5/3, \delta_k = 0.3$ and external perturbation values as $d_1 = 0.2\sin(t), d_2 = 0.1\cos(5t)$ and $d_3 = 0.3\sin(t)$. The initial state values are considered as $x_1 = 0, x_2 = -0.11$ and $x_3 = 9$ along this paper. Time series of the supply chain system are given in Figure (1). In addition, the 3D phase plane chaotic behaviour of the system are shown in the Figure (2).

The 3D chaotic supply chain system presented in (1) can be re-written as follows:

$$\dot{X} = (A_P + A_\Delta)X + x_1 \cdot BX + D \tag{2}$$

Where $D = (d_1, d_2, d_3)^T$ is the nonlinear distortion vector. $P = (m, n, r, k)$ denotes the constant distribution parameter of the leader system (1) and $\Delta = (\delta_m, \delta_n, \delta_r, \delta_k)$ is the distortion vector of the leader system (1). The coefficient matrix of $A_P, A_\Delta, B \in R^{3\times3}$ are given as:

$$A_P = \begin{bmatrix} -n-1 & m & 0 \\ r & -1 & 0 \\ 0 & 0 & k-1 \end{bmatrix},$$
$$A_\Delta = \begin{bmatrix} -\delta_n & \delta_m & 0 \\ \delta_r & 0 & 0 \\ 0 & 0 & -\delta_k \end{bmatrix}, \; B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \tag{3}$$

The matrixes $(A_P + A_\Delta)$ as the coefficient of vector X in Equation (2) has three eigenvalues as: $\lambda_1 = -23.0292, \lambda_2 = 11.9292, \lambda_3 = -2.9667$. Since eigenvalue λ_2 is positive, it can conclude from the Lyapunov stability theory[41] that the supply chain attractor presented in (2) is not stable at its origin equilibrium point $E_0 = (0, 0, 0)$.

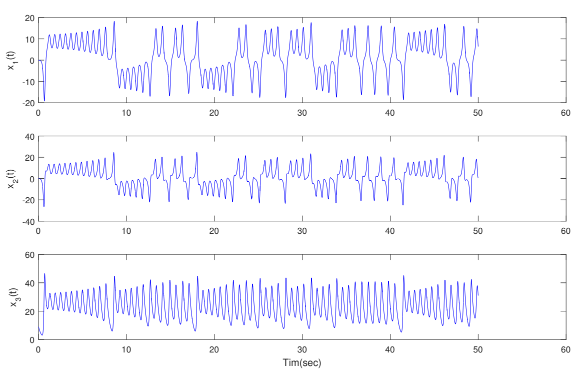

Figure 1: Time series phase portrait of the chaotic supply chain system for state values $x_1(t), x_2(t)$ and $x_3(t)$, respectively.

3 Active integral sliding mode synchronization

In this section, an active integral sliding mode for synchronization of two identical supply chain systems is designed. Let the supply chain chaotic attractor represented in matrix form (2) as the leader one. Then the follower supply chain attractor can be presented in matrix form as follows:

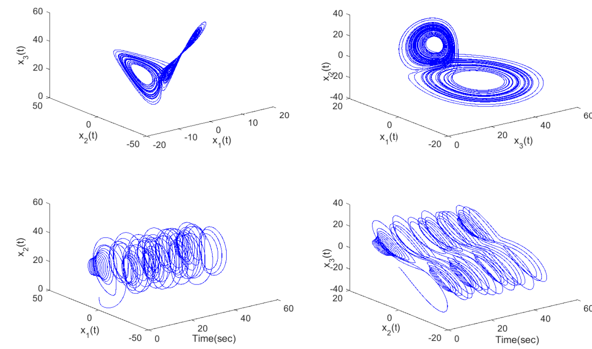

Figure 2: Three-dimensional phase portrait of the chaotic supply chain system

$$\dot{Y} = (A_P + A_\Delta)Y + y_1 \cdot BY + D + U \tag{4}$$

Where $Y = (y_1, y_2, y_3) \in \mathbf{R}$ denotes the follower state variables vector and $U = (u_1, u_2, u_3)$ indicates the feedback controller to be designed. The matrix coefficients $A_P, A_\Delta, B \in \mathbf{R}^{3\times3}$ are defined in (3) and also the external distortion $D \in \mathbf{R}^{3\times1} = (d_1, d_2, d_3)^T$ is considered constant vector related to state variables.

The error vector between the states of two identical leader chaotic attractor (2) and the follower attractor (4) can be given as:

$$E_s = Y - X \tag{5}$$

Then, the error dynamics between two chaotic attractors can be obtained as follows:

$$\dot{E}_s = \dot{Y} - \dot{X} \tag{6}$$

The ultimate goal of synchronization is to design an appropriate feedback controller to force the motion trajectories of the follower chaotic attractor to track the leader one. To this end, an active integral sliding mode controller is designed in this section that is capable to synchronize the leader and the follower attractors; and to force the synchronization error in (5) to converges to zero. Given the dynamic synchronization errors in (6), then the sliding surface of the active integral sliding mode control $S = (s_1, s_2, s_3)$ can be defined as follows:

$$s_i = \left(\frac{d}{dt} + \lambda_i\right)\left(\int_0^t e_i(\tau)d\tau\right) = e_i + \lambda_i \int_0^t e_i(\tau)d\tau \quad , \quad \forall i = 1, 2, 3 \tag{7}$$

Where $\lambda = (\lambda_1, \lambda_2, \lambda_3)$ is the constant coefficient in the sliding surface function. Then the time derivative of the sliding surface can be given in matrix form as follows:

$$\dot{S} = \dot{E}_s + \boldsymbol{\lambda} E_s \tag{8}$$

Where S=(s_1, s_2, s_3) is the sliding surface vector.

According to the exponential reaching law presented in [41], the derivative of the sliding surface can be given as follow:

$$\dot{S} = -\boldsymbol{\xi}\ sgn(S) - KS \tag{9}$$

Where $\boldsymbol{\xi} = (\xi_1, \xi_2, \xi_3)$ and $\boldsymbol{K} = (k_1, k_2, k_3)$ are the positive real constant vectors and $sgn(.)$ denotes the signum function.

Considering the time derivative of the sliding surface in (8) and the exponential reaching law presented by Equation (9), one can obtain:

$$\dot{E}_s + \boldsymbol{\lambda} E_s = -\boldsymbol{\xi}\ \text{sgn}(S) - KS \tag{10}$$

This equation can be simplified by substituting the synchronization error E_s and its dynamical representative $\dot{E}_s$ from the equations (5) and (6) as follows:

$$\dot{Y} - \dot{X} + \boldsymbol{\lambda} Y - \boldsymbol{\lambda} X = -\boldsymbol{\xi}\ \text{sgn}(S) - KS \tag{11}$$

Substituting the state variable vectors X and Y and their dynamic representatives $\dot{X}$ and $\dot{Y}$ from the equations (2) and (4), gives:

$$\begin{aligned}&(A_P + A_\Delta)Y + y_1 \cdot BY + D + U\\ &- (A_P + A_\Delta)X - x_1 \cdot BX - D\\ &+ \boldsymbol{\lambda} Y - \boldsymbol{\lambda} X = -\boldsymbol{\xi}\ \text{sgn}(S) - KS\end{aligned} \tag{12}$$

Then the following feedback controller can be resulted:

$$\begin{aligned}U = &- (A_P + A_\Delta)E_s - y_1 \cdot BY\\ &+ x_1 \cdot BX - \boldsymbol{\lambda} E_s - \boldsymbol{\xi}\ \text{sgn}(S) - KS\end{aligned} \tag{13}$$

Theorem 1 *The motion trajectories of the follower chaotic attractor in Equation (4) with the initial state values $Y(0) \in \mathbf{R}^3$, using the feedback controller presented in (13) with coefficient vectors $\boldsymbol{\lambda}, \boldsymbol{\xi}, \boldsymbol{K} > 0$, will track the trajectories of the leader attractor in Equation (2). Furthermore, the synchronization error vector E_s in Equation (5) asymptotically converges to zero.*

Proof: Let the Lyapunov candidate function as follows:

$$V = \frac{1}{2} S^2 \tag{14}$$

Clearly V is positive definite. The derivative of Lyapunov function V with respect to the time is:

$$\begin{aligned}\dot{V} &= S\dot{S} = S(\dot{E}_s + \boldsymbol{\lambda} E_s)\\ &= S\Big[(A_P + A_\Delta)Y + y_1 \cdot BY + D + U\\ &\quad - (A_P + A_\Delta)X - x_1 \cdot BX - D + \boldsymbol{\lambda} Y - \boldsymbol{\lambda} X\Big]\end{aligned} \tag{15}$$

With substituting the designed feedback controller U in (13), the derivative of Lyapunov function in (15) can be simplified as:

$$\dot{V} = -\boldsymbol{\xi}\ S\ \text{sgn}(S) - KS^2 \tag{16}$$

Hence $\dot{V}$ is negative definite when the coefficient vectors $\boldsymbol{\xi}$ and K are positive. Consequently, according to the Lyapunov stability theorem, the leader supply chain attractor in (2) and the follower attractor in (4) will be asymptotically synchronized with the control input vector of Equation (13). So the proof is complete.

4 Adaptive integral sliding mode synchronization

This section is devoted to adaptive integral sliding mode synchronization of the supply chain chaotic attractor. Along this section, the internal $\Delta = (\delta_m, \delta_n, \delta_r, \delta_k)$ and the external disturbance $D = (d_1, d_2, d_3)$ parameter vectors of the supply chain attractor are considered unknown. So the designed feedback controller and the parameter estimation strategy are performed simultaneously.

Consider the supply chain chaotic attractor presented in matrix form (2), as the leader system. Then the follower system can be given as:

$$\dot{Y} = (A_P + A_{\hat{\Delta}})Y + y_1 \cdot BY + \hat{D} + U \tag{17}$$

Where $Y = (y_1, y_2, y_3)$ is the state vector of the follower supply chain attractor and $U = (u_1, u_2, u_3)$ indicates the feedback control law of the closed-loop control system, which have to be designed in such way that aligns the behaviour of the follower attractor to track the trajectories of the leader attractor, which means two identical chaotic attractor (2) and (17) synchronize with different initial leader and follower state values. The constant matrix A_P and $B \in R^{3\times 3}$ can be defined as follow:

$$A_P = \begin{bmatrix} -n-1 & m & 0 \\ r & -1 & 0 \\ 0 & 0 & k-1 \end{bmatrix},\ B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \tag{18}$$

Where $P = (m, n, r, k)$ represents the constant distribution vector and the estimated vector $\hat{D} = (\hat{d}_1, \hat{d}_2, \hat{d}_3)$ implicates the estimation of the external nonlinear distortion parameter vector $D = (d_1, d_2, d_3)$; and also matrix $A_{\hat{\Delta}}$ can be set as:

$$A_{\hat{\Delta}} = \begin{bmatrix} -\hat{\delta_n} & \hat{\delta}_m & 0 \\ \hat{\delta}_r & 0 & 0 \\ 0 & 0 & -\hat{\delta}_k \end{bmatrix} \quad (19)$$

Where $\hat{\Delta} = (\hat{\delta}_m, \hat{\delta}_n, \hat{\delta}_r, \hat{\delta}_k)$ denotes the estimation of the internal linear distortion parameter vector $\Delta = (\delta_m, \delta_n, \delta_r, \delta_k)$ of the leader system (2). In this section, an adaptive integral sliding mode control law will be designed to estimate the unknown parameters and to synchronize the leader and the follower supply chain attractors, simultaneously.

The synchronization error vectors between two identical leader and follower attractors (2) and (17) can be described as:

$$E_s = Y - X \quad , \quad E_\Delta = \hat{\Delta} - \Delta \ , \ E_D = \hat{D} - D \quad (20)$$

Then, the dynamics of synchronization errors can be obtained as follows:

$$\dot{E}_s = \dot{Y} - \dot{X} \ , \ \dot{E}_\Delta = \dot{\hat{\Delta}} = (\dot{\delta}_m, \dot{\delta}_n, \dot{\delta}_r, \dot{\delta}_k) \ , \ \dot{E}_D = \dot{\hat{D}} = (\dot{d}_1, \dot{d}_2, \dot{d}_3) \quad (21)$$

The objective of adaptive synchronization is to design an appropriate feedback controller to force the motion trajectories of the follower chaotic attractor to track the leader attractor when the parameters of the leader attractor are unknown or uncertain. In this section, an adaptive integral sliding mode controller is designed that is capable to synchronize the leader and follower attractors and to force the synchronization errors in (20) to converge to zero.

Given the dynamic synchronization errors in (21), then the sliding surface of the adaptive integral sliding mode control $S = (s_1, s_2, s_3)$ and its time derivative of sliding surface can be identified in the same way as (7) and (8), respectively. And also the derivative of the sliding surface obtained by exponential reaching law presented in [41] can be defined as Equation (9). Considering the time derivative of the sliding surface in (8) and the exponential reaching law presented by Equation (9), one can obtain:

$$\dot{E}_s + \boldsymbol{\lambda} E_s = -\boldsymbol{\xi} \ \mathrm{sgn}(S) - KS \quad (22)$$

By substituting the synchronization error E_s from equation (20), its dynamical representative $\dot{E}_s$ from the equation (21), and the state variable dynamics $\dot{X}$ and $\dot{Y}$ from equation (2) and (17), respectively, gives:

$$\begin{aligned} &(A_P + A_{\hat{\Delta}})Y + y_1 \cdot BY + \hat{D} + U \\ &- (A_P + A_\Delta)X - x_1 \cdot BX - D \\ &+ \boldsymbol{\lambda} Y - \boldsymbol{\lambda} X = -\boldsymbol{\xi} \ \mathrm{sgn}(S) - KS \end{aligned} \quad (23)$$

Then the adaptive feedback controller can be given as:

$$\begin{aligned} U = &- A_P E_s - \boldsymbol{\lambda} E_s - A_{\hat{\Delta}} E_s - y_1 \cdot BY \\ &+ x_1 \cdot BX - \boldsymbol{\xi} \ \mathrm{sgn}(S) - KS \end{aligned} \quad (24)$$

Furtheremore, the parameter estimations can be obtained as follows:

$$\begin{aligned} \dot{\hat{\delta}}_m &= +s_1 x_1 - k_{\Delta 1}(\hat{\delta}_m - \delta_m) \\ \dot{\hat{\delta}}_n &= -s_1 x_2 - k_{\Delta 2}(\hat{\delta}_n - \delta_n) \\ \dot{\hat{\delta}}_r &= -s_2 x_1 - k_{\Delta 3}(\hat{\delta}_r - \delta_r) \\ \dot{\hat{\delta}}_k &= +s_3 x_3 - k_{\Delta 4}(\hat{\delta}_k - \delta_k) \end{aligned} \quad (25)$$

and,

$$\dot{\hat{D}} = -S - \boldsymbol{K_D} E_D \quad (26)$$

Where $\boldsymbol{K_\Delta} = (k_{\Delta 1}, k_{\Delta 2}, k_{\Delta 3}, k_{\Delta 4})$ and $\boldsymbol{K_D} = (k_{d_1}, k_{d_2}, k_{d_3})$ are constant positive coefficients.

Theorem.2 The leader chaotic system represented in (2) with unknown parameter vectors Δ and D and the follower chaotic system in (17) as the representatives of the supply chain attractors are globally and asymptotically synchronized with any initial state values $X(0)$ and $Y(0) \in \mathbf{R}^3$, by the feedback control law given in (24) with coefficient vectors $\boldsymbol{\lambda}, \boldsymbol{\xi}, \boldsymbol{K} > 0$, and the estimation parameters presented in (25). Furthermore, the vectors of synchronization errors E_s, E_Δ and E_D in Equation (20) asymptotically converges to zero.

Proof. Let the Lyapunov candidate function as follows:

$$V = \frac{1}{2}\left(S^2 + E_\Delta^2 + E_D^2\right) \quad (27)$$

It is obvious that V is positive definite. The derivative of Lyapunov function V with respect to the time is:

$$\begin{aligned} \dot{V} &= S\dot{S} + E_\Delta \dot{E_\Delta} + E_D \dot{E_D} \\ &= S(\dot{E}_s + \boldsymbol{\lambda} E_s) + E_\Delta \dot{\hat{\Delta}} + E_D \dot{\hat{D}} \\ &= S\Big[(A_P + A_{\hat{\Delta}})Y + y_1 \cdot BY + \hat{D} + U \\ &- (A_P + A_\Delta)X - x_1 \cdot BX - D \\ &+ \boldsymbol{\lambda} Y - \boldsymbol{\lambda} X\Big] + E_\Delta \dot{\hat{\Delta}} + E_D \dot{\hat{D}} \end{aligned} \quad (28)$$

Substituting the control law U in (24), the derivative of parameter estimations $\dot{\hat{\Delta}}$ and $\dot{\hat{D}}$ in (21), gives

$$\begin{aligned}\dot{V} &= S\Big[\hat{D} - (A_{\hat{\Delta}} - A_{\Delta})X - D - \boldsymbol{\xi}\,\mathrm{sgn}(S) - KS\Big] \\ &+ E_{\Delta}\dot{\hat{\Delta}} + E_D\dot{\hat{D}} \\ &= -\boldsymbol{\xi}\;S\,\mathrm{sgn}(S) - KS^2 - K_{\Delta}\|E_{\Delta}\|_2^2 \\ &- K_D\|E_D\|_2^2 \qquad (29)\end{aligned}$$

Where $\|.\|_2$ denotes the euclidean norm. Then, the derivative of V is negative definite as the coefficient vectors $\boldsymbol{\xi}, K, K_{\Delta}$ and K_D are all positive. Consequently, according to the Lyapunov stability theorem, the leader supply chain system in (2) and the follower system in (17) will be asymptotically synchronized with the control input vector of Equation (24) and the estimation parameters defined in (25). So the proof is complete.

5 Numerical simulations

The objective of numerical simulations is to validate the effectiveness and feasibility of the proposed approach for synchronization of two chaotic systems and also identification of unknown distributions. In this section, some numerical results related to the synchronization of the two identical supply chain systems are given. Numerical simulations have been carried out using Matlab Simulink. The implementation program is written based on Forth-order Runge-Kutta iterative method with a fixed time-step size and a tolerance of 10^{-7}.

For simulation purposes, the supply chain system presented in (1) is considered as the leader system. Then the synchronization between the leader and the corresponding follower systems are done based on the designed feedback controller and parameter estimation strategy.

For chaotic behaviour of the supply chain system (1), parameters are selected as: $m = 10, \delta_m = 0.1, n = 9, \delta_n = 0.1, r = 28, \delta_r = 0.2, k = -5/3, \delta_k = 0.3$ and external perturbation values as $d_1 = 0.2\sin(t), d_2 = 0.1\cos(5t), d_3 = 0.3\sin(t)$.

5.1 Active integral sliding mode synchronization

Now, some results related to the synchronization of the two identical supply chain chaotic systems via active integral sliding mode presented in Section 3 is given. The initial state values of the leader and the follower chaotic systems are assumed typically as $X(0) = (0, -0.11, 9)^T$ and $Y(0) = (7, 8, 2)^T$, respectively.

The behaviour of the leader and follower systems are given in Fig. (3). It is clearly evident from Figures. (3)that the expected synchronization between the leader and the follower systems are obtained; and the synchronization errors converge to zero as time tends to infinity.

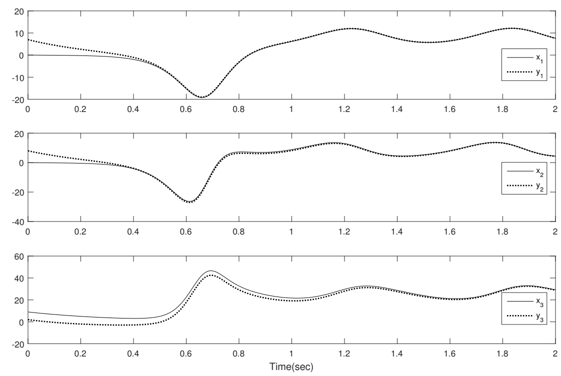

Figure 3: Motion trajectories of the leader and follower chaotic supply chain systems state variables obtained by active integral sliding mode synchronization method.

5.2 Adaptive integral sliding mode synchronization

Some simulation results obtained from the synchronization of the two identical supply chain system described in Section 4 is represented here via designing an adaptive integral sliding mode controller. Let the initial state values for the leader and the follower chaotic systems as: $X(0) = (0, -0.11, 9)^T$ and $Y(0) = (7, 8, 2)^T$, respectively. The initial values for the internal and the external distortions are considered as: $\hat{\Delta} = (\hat{\delta}_m, \hat{\delta}_n, \hat{\delta}_r, \hat{\delta}_k) = (0.7, 0.8, 0.3, 0.4)$ and $D = (\hat{d}_1, \hat{d}_2, \hat{d}_3) = (7, 8, 2)$, respectively.

The time response of the leader and the follower chaotic systems state variables obtained by synchronization process are shown in Fig. (4). The internal and external parameter estimation errors are given in Fig. (5) and (6), respectively. It is clearly apparent from Fig. (4) to Fig (6) that the expected synchronization between leader and follower systems is obtained.

6 Conclusion

Nonlinear behaviour and internal/external distortions are not desirable factors in a supply chain system.

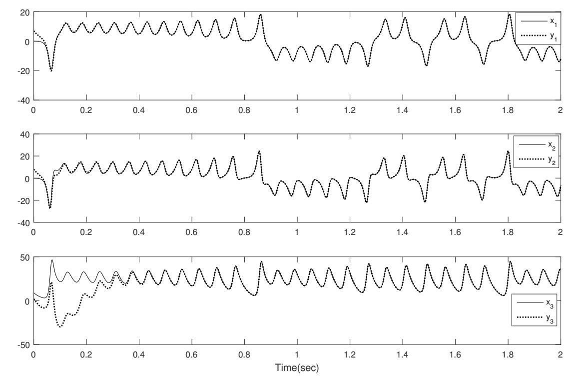

Figure 4: Motion trajectories of the leader and follower chaotic supply chain systems state variables obtained by adaptive integral sliding mode synchronization method.

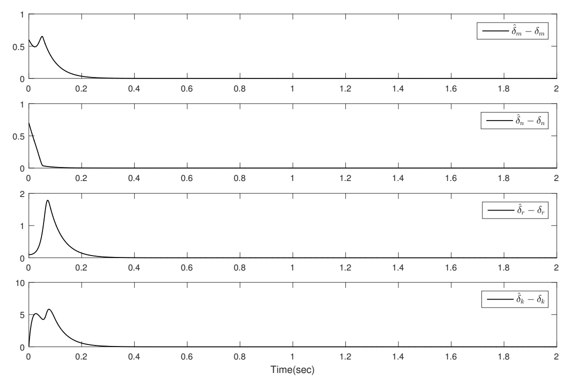

Figure 5: Estimation errors of unknown internal distortion parameters $\delta_m, \delta_n, \delta_r$ and δ_k.

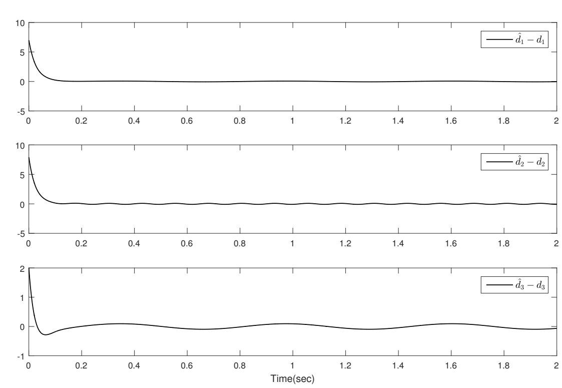

Figure 6: Estimation errors of unknown external distortion parameters d_1, d_2 and d_3.

The undesirable behaviour of the system may caused by a phenomenon is named as bullwhip effect which causes chaos in the supply chain system. Bullwhip effect can be detected sometimes under certain circumstances. But the bullwhip effect can be reduced by enterprise resource planning system with retrieving correct demand information of the customers. Hence, an appropriate synchronization method can reduce the nonlinear behaviours and bullwhip effect of the supply chain system.

Chaos synchronization of the supply chain system via integral sliding mode approach is addressed in this paper. The sliding mode control method synchronization is often used because of its inherent advantages of easy realization, fast response and good transient performance, as well as its insensitivity to parameter uncertainties and external disturbances. So chaos synchronization of the supply chain systems are carried out by designing an appropriate feedback controller via active/adaptive integral sliding mode control schemes. Suitable sufficient conditions for active or adaptive synchronization of the leader and the follower supply chain systems are derived. The performance of the proposed feedback controller and developed synchronization method is proved by Lyapunov stability theorem. Furthermore, as we can see from the simulation results, the excellent synchronization of the two identical supply chain systems are obtained from both speed and accuracy points of view; and also synchronization errors for either internal or external unknown parameters converge asymptotically to zero.

References:

[1] E. N. Lorenz, Deterministic nonperiodic flow, Journal of the atmospheric sciences 20 (2) (1963) 130–141.

[2] G. Chen, T. Ueta, Yet another chaotic attractor, International Journal of Bifurcation and chaos 9 (07) (1999) 1465–1466.

[3] J. Lü, G. Chen, S. Zhang, Dynamical analysis of a new chaotic attractor, International Journal of Bifurcation and Chaos 12 (05) (2002) 1001–1015.

[4] C. Liu, T. Liu, L. Liu, K. Liu, A new chaotic attractor, Chaos, Solitons & Fractals 22 (5) (2004) 1031–1038.

[5] R. Genesio, A. Tesi, Harmonic balance methods for the analysis of chaotic dynamics in nonlinear systems, Automatica 28 (3) (1992) 531–548.

[6] S. B. Bhalekar, Forming mechanizm of bhalekar-gejji chaotic dynamical system, American Journal of Computational and Applied Mathematics 2 (6) (2012) 257–259.

[7] Z. Lei, Y.-j. Li, Y.-q. Xu, Chaos synchronization of bullwhip effect in a supply chain, in: 2006 International Conference on Management Science and Engineering, IEEE, 2006, pp. 557–560.

[8] R. Martínez-guerra, C. A. Pérez-pinacho, G. C. Gómez-cortés, A. D. Algebraic, Synchronization of Integral and Fractional Order Chaotic Systems, 2015.

[9] G. R. Nasiri, R. Zolfaghari, H. Davoudpour, An integrated supply chain production–distribution planning with stochastic demands, Computers & Industrial Engineering 77 (2014) 35–45.

[10] E. Claypool, B. A. Norman, K. L. Needy, Modeling risk in a design for supply chain problem, Computers & Industrial Engineering 78 (2014) 44–54.

[11] E. Ott, C. Grebogi, J. A. Yorke, Controlling chaos, Physical review letters 64 (11) (1990) 1196.

[12] L. M. Pecora, T. L. Carroll, Synchronization in chaotic systems, Physical review letters 64 (8) (1990) 821.

[13] H. S. Nik, J. Saberi-Nadjafi, S. Effati, R. A. Van Gorder, Hybrid projective synchronization and control of the baier–sahle hyperchaotic flow in arbitrary dimensions with unknown parameters, Applied Mathematics and Computation 248 (2014) 55–69.

[14] H. Richter, Controlling chaotic systems with multiple strange attractors, Physics Letters A 300 (2) (2002) 182–188.

[15] C. Lia, X. Liao, X. Zhang, Impulsive synchronization of chaotic systems, Chaos 15 (2005) 023104.

[16] L. Chun-Lai, Z. Mei, Z. Feng, Y. Xuan-Bing, Projective synchronization for a fractional-order chaotic system via single sinusoidal coupling, Optik-International Journal for Light and Electron Optics 127 (5) (2016) 2830–2836.

[17] H. Xi, Y. Li, X. Huang, Adaptive function projective combination synchronization of three different fractional-order chaotic systems, Optik-International Journal for Light and Electron Optics 126 (24) (2015) 5346–5349.

[18] M. G. Rosenblum, A. S. Pikovsky, J. Kurths, From phase to lag synchronization in coupled chaotic oscillators, Physical Review Letters 78 (22) (1997) 4193.

[19] J. Sun, Y. Shen, X. Wang, J. Chen, Finite-time combination-combination synchronization of four different chaotic systems with unknown parameters via sliding mode control, Nonlinear Dynamics 76 (1) (2014) 383–397.

[20] M. P. Aghababa, A. Heydari, Chaos synchronization between two different chaotic systems with uncertainties, external disturbances, unknown parameters and input nonlinearities, Applied Mathematical Modelling 36 (4) (2012) 1639–1652.

[21] J. Yu, B. Chen, H. Yu, J. Gao, Adaptive fuzzy tracking control for the chaotic permanent magnet synchronous motor drive system via backstepping, Nonlinear Analysis: Real World Applications 12 (1) (2011) 671–681.

[22] H. Adloo, M. Roopaei, Review article on adaptive synchronization of chaotic systems with unknown parameters, Nonlinear Dynamics 65 (1-2) (2011) 141–159.

[23] X. Zhang, H. Zhu, H. Yao, Analysis and adaptive synchronization for a new chaotic system, Journal of dynamical and control systems 18 (4) (2012) 467–477.

[24] T. Ma, J. Zhang, Y. Zhou, H. Wang, Adaptive hybrid projective synchronization of two coupled fractional-order complex networks with different sizes, Neurocomputing 164 (2015) 182–189.

[25] K.-S. Hong, et al., Adaptive synchronization of two coupled chaotic hindmarsh–rose neurons by controlling the membrane potential of a slave neuron, Applied Mathematical Modelling 37 (4) (2013) 2460–2468.

[26] H.-M. Chen, Z.-Y. Chen, M.-C. Chung, Implementation of an integral sliding mode controller for a pneumatic cylinder position servo control system, in: Innovative Computing, Information and Control (ICICIC), 2009 Fourth International Conference on, IEEE, 2009, pp. 552–555.

[27] Q. Hu, H. B. Du, D. M. Yu, Adaptive integral sliding mode control of single electromagnetic guiding system suspension altitude in linear elevator, in: Applied Mechanics and Materials, Vol. 321, Trans Tech Publ, 2013, pp. 1704–1707.

[28] J.-J. Yan, Y.-S. Yang, T.-Y. Chiang, C.-Y. Chen, Robust synchronization of unified chaotic systems via sliding mode control, Chaos, Solitons & Fractals 34 (3) (2007) 947–954.

[29] M. Pourmahmood, S. Khanmohammadi, G. Alizadeh, Synchronization of two different uncertain chaotic systems with unknown parameters using a robust adaptive sliding mode controller, Communications in Nonlinear Science and Numerical Simulation 16 (7) (2011) 2853–2868.

[30] S. Dereje, M. K. Pattanshetti, A. Jain, R. Mitra, Genetic algorithm based integral sliding surface design and its application to stewart platform manipulator control, International Journal of Systems applications, Engineering and Devolopment 5 (2011) 518–528.

[31] M. Ghamati, S. Balochian, Design of adaptive sliding mode control for synchronization genesio–tesi chaotic system, Chaos, Solitons & Fractals 75 (2015) 111–117.

[32] S. E. Fawcett, M. A. Waller, Making sense out of chaos: Why theory is relevant to supply chain research, Journal of Business Logistics 32 (1) (2011) 1–5.

[33] A. Göksu, U. E. Kocamaz, Y. Uyaroğlu, Synchronization and control of chaos in supply chain management, Computers & Industrial Engineering 86 (2015) 107–115.

[34] G.-Q. Yang, Y.-K. Liu, K. Yang, Multi-objective biogeography-based optimization for supply chain network design under uncertainty, Computers & Industrial Engineering 85 (2015) 145–156.

[35] J. Hou, A. Z. Zeng, L. Zhao, Achieving better coordination through revenue sharing and bargaining in a two-stage supply chain, Computers & Industrial Engineering 57 (1) (2009) 383–394.

[36] S. K. Kumar, M. Tiwari, Supply chain system design integrated with risk pooling, Computers & Industrial Engineering 64 (2) (2013) 580–588.

[37] W. Hui, C. Yan, F. Ce, L. Yan-wei, H. You-ming, The linear control theory for counteracting the bullwhip effect, in: 2006 International Conference on Management Science and Engineering, IEEE, 2006, pp. 434–438.

[38] H. L. Lee, V. Padmanabhan, S. Whang, Information distortion in a supply chain: The bullwhip effect, Management science 43 (4) (1997) 546–558.

[39] K. Anne, J. Chedjou, K. Kyamakya, Bifurcation analysis and synchronisation issues in a three-echelon supply chain, International Journal of Logistics: Research and Applications 12 (5) (2009) 347–362.

[40] A. Göksu, U. E. Kocamaz, Y. Uyaroğlu, Synchronization and control of chaos in supply chain management, Computers & Industrial Engineering 86 (2015) 107–115.

[41] J.-J. E. Slotine, W. Li, et al., Applied nonlinear control, Vol. 199, prentice-Hall Englewood Cliffs, NJ, 1991.