A multi-step method to calculate the equilibrium point of the Continuous Hopfield Networks: Application to the max-stable problem

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Abstract: - The Continuous Hopfield Networks (CHN) is a neural network tools which can be used to solve many problems like auto-memory and optimization problems. The dynamics of the CHN is described by differential equations system which is hard to solve analytically. That is why, the researchers use the Euler Cauchy method to calculate the CHN equilibrium point. Unfortunately, this method suffers from several problems, especially quality of the decision for a large step, sensibility to the slope function parameters and to the initial conditions. In this work, we use the well-known multi-step numerical method called Adams–Bashforth method, which is strong in terms of stability and performance, to calculate the equilibrium point of the CHN associated with the max stable problem. This method introduces an intermediary step to improve the Euler Cauchy method precision. The experimental results show that the (CHN+Adams-Bashforth) method produce a large max stable sets in comparison with the (CHN+Euler-Cauchy) method.

Key-Words: - Continuous Hopfield Networks, Euler Cauchy method, Adams–Bashforth method, max-stable problem

1 Introduction

Hopfield has shown its ability to handle a wide variety of optimization problems, such as the traveling salesman problem, and also has been used as an associative memory in the image processing field. In 1982, The Hopfield model of Neural Networks took the name of its inventor, John Hopfield that will strengthen later the neural network by a new model [1], [4].

In contrast to the other neural networks, in this Hopfield network, the information does not flow in a single direction but flows only from input to output. This network has shown that the use of a continuous transfer function ensures existence stable state if weight matrix is symmetric with zero diagonal. The energy function of this network is monotonically decreasing with time, which makes convergence to a local minimum can be guaranteed.

The dynamics of the Hopfield networks are formulated as analytically differential equation. This gives the numerical discretization neural network scheme in which each neuron state changes with iteration. Much of the work on this type of network has received remarkable importance in its application in different domains [5], [7],[24]. On the other hand, the discretization of a continuous system does not always benefited from mathematical tools to be applied. Many researchers have worked on the discretization Hopfield network by the method of Euler. However, this method suffers from several problems, especially quality of the decision for a step size. In order to overcome this problem, in this work, we discretize the Hopfield network with Adams-Bashforth method [21]; this method will show its power to calculate the equilibrium point for each iteration. In this work, we use the well-known multi-step numerical method called Adams–Bashforth method which is strong in terms of stability and performance. In fact, this method introduces an intermediary step to improve the Euler Cauchy method precision [19]. The discretization of the Hopfield neural network with the Adams–Bashforth method makes it more robust than the classical network. The proposed system is used in this work to solve the max-stable problem using the

CHN to evaluate the performance of this approach by comparing with the classical Hopfield.

This paper is organized as follows: In section 2, we present an introduction of the continuous Hopfield network. The equilibrium point of CHN and the local error of the second order Adams-Bashforth method are discussed in the section 3. In section 4, the maximum stable set problem is modelized as a 0 -1 quadratic program. Some numirecal results using both (CHN+Adams-Bashforth system) an d (CHN+Euler-Cauchy system) are presented in this section 5. Finally, section 6 provides a conclusion and future work.

2 The Continuous Hopfield Network (CHN)

The model proposed by John Hopfield in 1982 i s always presented as a powerful tool for solving many problems as the traveling salesman problem, linear programming problems, graph coloring problems, processing image, constraint satisfaction problems [7]. Due to the diversity of use of this model in various ways, this model has attracted many researchers attention to work with this model. The Hopfield networks are fully connected networks, the weight matrix is symmetric $T_{ij} = T_{ji}$ the strength of the connection from neuron j to neuron i. Each neuron i has an offset bias of i_i^b .The dynamics of the CHN is represented by the differential equation:

$$\frac{du}{dt} = -\frac{u}{\tau} + Tx + i^b$$

Where u, x and i_b will be the vectors of neuron states, outputs and biases. The output function $x_i = g(u_i)$ i s a hyperbolic tangent, which is bounded below by 0 a nd above by 1. $g(u_i) = \frac{1}{2}(1 + \tanh(\frac{u_i}{u_0})) \quad where \quad u_0 > 0$

Where u_0 is a parameter used to control the gain (or slope) of the activation function. If, for an input vector u_0, a point u^e exists such that $u(t) = u^e \quad \forall t > t_e$, for some $t_e \geq 0$, this point is called an equilibrium point of the system defined by the differential equation[16]. That point equilibrium point is also called the stable point system. The Hopfield model can be written as a Lyapunov function, so this model is stable and decreasing system over time. The evolution of each step gives a trajectory that converges to an equilibrium point, because the Lyapunov function provides the possibility of finding a local minimum. Hopfield showed that, if matrix T is symmetric, then the following Lyapunov function exists [6],[28]:

$$E(x) = -\frac{1}{2}x^t Tx - (i^b)^t x + \frac{1}{\tau}\sum_{i=1}^{n}\int g^{-1}(v)dv$$

The CHN can solve any combinatorial problems, which seeks to minimize an objective function:

$$E(x) = -\frac{1}{2}x^t Tx - (i^b)^t x$$

The main idea of this Lyapunov technique is in each step is stable and converges one of the local minima for any combinatorial problem. In this way, the output of the Hopfield network is seen as a solution for many combinatorial problems.

Consider the following quadratic assignment problem, with n variables and m linear constraints:

$$(P)\begin{cases} Min \quad \frac{1}{2}x^t Qx + q^t x \\ subject\ to \\ \qquad Ax = b \\ \qquad x_i \in \{0,1\} \quad i = 1,....,n \end{cases}$$

To solve the quadratic programming (P) using the Continuous Hopfield Networks, the following sets are needed:

H is a set of the Hamming hypercube: $H \equiv \{x \in [0,1]^n\}$

H_C is a set of the Hamming hypercube corners : $H_C \equiv \{x \in H : x_i \in \{0,1\}, \quad i = 1,....,n\}$

H_F is a set of feasible solutions: $H_F \equiv \{x \in H_C : Ax = b\}$

Thus the process for a given instance (n,m,Q, q, A,b), some conditions must be established on the problem so that its equilibrium points can be associated with local minima of the optimization problem, with m is the number of constraints.

An energy function must also be defined by: $E(x) = E^0(x) + E^R(x) \quad \forall x \in H$ Where:

$E^0(x)$ is directly proportional to the objective function of the problem.

$E^R(x)$ is a quadratic function that not only penalizes the violated constraints of the problem, but also guarantees the feasibility of the solution obtained by the CHN . This function must be constant $\forall x \in H_F$ and an appropriate selection of this function is crucial for correct mapping.

This energy function was introduced to overcome the problem observed with the energy functions

used by other authors, including Aiyer [8] and Brandt et al. [23].
In this paper, our goal is to solve the maximum stable set problem by a proposed new approach.
In this case, the next step is to discretize the Hopfield network with a new method called Adams-bashforth method. And in the second step we present a modelization of the maximum stable set problem as a quadratic 0-1 programming. From this model, implementation step becomes easy and general.

3 The equilibrium point of CHN and local error of the Adams-Bashforth method

Recently, continuous Hopfield networks (CHN) are used to solve very interested combinatorial problems like travelling salesman problem[26], graph coloring problems, placement of the electronic circuits problems, maximum stable set problem, constraint satisfaction problem and Optimization of the Kohonen Network Architectures Using the Continuous Hopfield Networks[14],[18],[23].
The dynamic of the CHN is characterized by the flowing differential equation that takes the general form:

$$\frac{du}{dt} = f(u).$$

Where $f(u) = -T \times \tanh(u) - I$
The discretization Hopfield network is often given by the numerical method of Euler method, which is defined by the following equation.

$$u_{n+1} = u_n + h \times f(u_n)$$

This method is highly sensitive to initial condition and step-size. In addition, the Euler method produces local solutions which are not enough good. To overcome this problem, we use in this work the second order Adams-Bashforth multi-step method. The idea of this method is to take different time steps for two components for to achieve the target accuracy, the components are integrated using larger step sizes. The large step sizes are at their turn integrated multiple of the small step sizes.
Euler method is part of an Adams–Bashforth family [21]. To derive Adams–Bashforth formulas, notice that:

$$u(t_{n+1}) = u(t_n) + \int_{t_n}^{t_{n+1}} f(t, u(t))dt$$

The approximation to the integral is obtained from polynomial interpolation at the points:

$$(t_n, f_n), \ldots, (t_{n-k+1}, f_{n-k+1})$$

For some integer k, and for k = 1, Adams-Bashforth order 1 is Euler's method: approximates the integral by the area of a rectangle whose base has length $(t_{n+1} - t_n)$ and whose height is $f(t_n, u_n)$:

$$u_{n+1} = u_n + \int_{t_n}^{t_{n+1}} f(t, u(t))dt \approx u_n + f(t_n, u_n)(t_{n+1} - t_n)$$

The figure 1, explains geometrically this approximation.

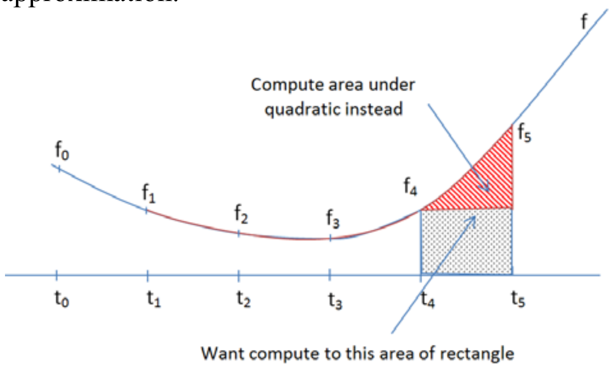

Figure 1: Adams-bashforth method with k=1.

For Adams–Bashforth with k = 2, the interpolation of the integral is done by a polynomial of degree 1, with $p(t_n) = f_n, \quad p(t_{n-1}) = f_{n-1,}$

The figure 2, explains geometrically this approximation.

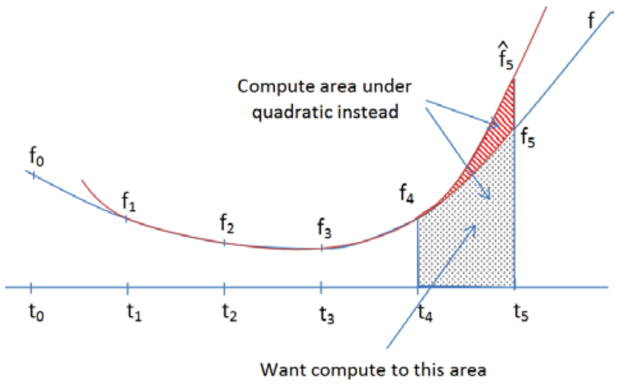

Figure 2: Adams-bashforth with k=2

So letting $h_n = t_{n+1} - t_n$ and $h_{n-1} = t_n - t_{n-1}$, we obtain:

$$u_{n+1} = u_n + \int_{t_n}^{t_{n+1}} f_{n-1} + \frac{f_n - f_{n-1}}{t_n - t_{n-1}}(t - t_{n-1})dt$$

$$= u_n + h_n f_{n-1} + \frac{f_n - f_{n-1}}{h_{n-1}} \frac{(h_n - h_{n-1})^2 - h_{n-1}^2}{2}$$

By setting $h_n = h_{n-1}$, we obtain the Adam-Bashforth scheme:

$$u_{n+1} = u_n + \frac{h}{2}(3f_n - f_{n-1})$$

To determine the error of each order of the Adams–Bashforth family, one can call for the Taylor series if u is smooth enough [20]; see the table 1.

Table 1: Order and local error by Adams-bashforth

Some Adams-Bashforth	Order	Local Error
$u_{n+1} = u_n + hf_n$	$k=1$	$\frac{h^2}{2}u^{(2)}(\eta)$
$u_{n+1} = u_n + \frac{h}{2}(3f_n - f_{n-1})$	$k=2$	$\frac{5h^3}{12}u^{(3)}(\eta)$

Here we have two parameters, step size h and order k, are used in control the size of the local error.
Practical methods for solving the differential equations use such estimates for the local error to determine whether the current choice of step size h is adequate.
The following example demonstrates the effectively of the method described above. In order to show the effectiveness of Adams-bashforth method, we compare the local error for each iteration. The example has been used for comparison:

a) u'=u+1, x_0 =0, u(x_0)=1;
b) u(x)=2e^x-1 is the exact solution

Figure 3, contains the results obtained by two methods for stepsize h=0.1. Again, a co mparison with the results of Adams-bashforth method and Euler method shows that Adams-bashforth gives comparable accuracy. The following figure shows the errors obtained for each of two methods:

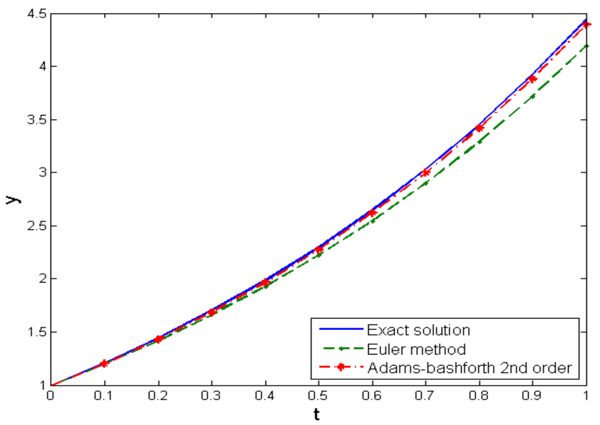

Figure 3: approximation by Adams-bashforth and Euler method

The results are shown in figure 3. There, the relative error ε_{rel}: and the number of iteration. Upon comparing the error of the second-order Adams-bashforth and Euler method, we notice that the local truncation error is smaller for the Adams-bashforth method; this reflects the effectiveness of the Adams-bashforth method. In front of the results were observed the importance of Adams-bashforth for calculating approximate solution of optimization problem. In this work, we use the Adams-Bashforth of order 2 to solve the Max-Stable problem and we compare the obtained results to those obtained with the Adams-Bashforth of order 1(Euler method) [22].

4 Continuous Hopfield network for the Max-Stable Problem (MSP)

In this part, we construct an adequate Continuous Hopfield Network for the max-stable problem. To this end, we express the (MSP) in terms of 0-1 a quadratic program. Basing on t his latter, we introduce our own energy function; then we use the hyper plan method to select adequate parameters to obtain a feasible stable set [12][13]. First, we define the MSP and we discuss the main proposed work to solve this problem.

4.1. The max-stable problem

Given an undirected graph G= (V, E) with $V = \{v_1, v_2, ..., v_n\}$. A stable set of a graph G is a set of nodes S with the property that the nodes of S are pairwise non adjacent. The Maximum Stable Set Problem (MSSP) consists of finding a stable set in graph G of maximum cardinality $\alpha(G)$. A side from its theoretical interest, the MSSP problem arises in applications in information retrieval, experimental design, signal transmission, and computer vision [25]. The stable set problem is NP-hard in the strong sense, and hard even to approximate. The MSSP problem can be solved using polynomial time algorithms for special classes of graphs such as perfect graphs and t-perfect graphs, circle graphs and their complements, claw-free graphs, and graphs with long odd cycles [27], [11] and [15]. But, the existence of a polynomial time algorithm for arbitrary graphs seems unlikely.
Different approaches have been discussed in the literature to solve the maximum stable set problem exactly. An implicit enumeration technique of Carrahan's and Pardalos's [13], computational results for different stable set linear programming relaxations have been reported by Gruber and Rendl [14], an effective evolution of the tabu search

approach is presented in the original work of Friden, Hertz and de Werra [15]. The MSSP problem can be solved via the Continuous Hopfield Network (CHN).

4.2. 0-1 a quadratic program for the max-stable problem

To solve the MSSP problem via the CHN, it must be expressed as an assignment problem with a quadratic constraint.

Let $S \subset V$ be a stable set of nodes. For each node v_i of the graph G, we introduce the binary variables x_i such that:

$$x_i = \begin{cases} 1 & if \quad v_i \in S \\ 0 & Otherwise \end{cases}$$

Two adjacent nodes v_i and v_j cannot be in the set S: $(v_i, v_j) \in E \Rightarrow x_i x_j = 0$ The constraints can be aggregated in a single one:

$$h(x) = \sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij} x_i x_j = 0$$

$$With \quad b_{ij} = \begin{cases} 1 & if \quad (v_i, v_j) \in E \\ 0 & Otherwise \end{cases}$$

The objective function of the mathematical programming model is:

$$f(x) = -\sum_{i=1}^{n} x_i .$$

Consequently, the MSSP problem can be expressed in the following algebraic form:

$$(QP)\begin{cases} Min \quad f(x) = -\sum_{i=1}^{n} x_i \\ subject\ to \\ \qquad h(x) = \sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij} x_i x_j = 0 \\ \qquad x \in \{0,1\}^n \end{cases}$$

4.3. The CHN for the Max-stable problem

The main objective of this section is to construct an adequate CHN to solve the maximum stable set problem (MSSP). Firstly, we begin by formulation of energy function associated with this MSSP problem. Then, we select a convenient parameters setting of this function [9]. The formulation of this energy function for maximum stable problem is done as follows:

$$E^0(x) = -\alpha\sum_{i=1}^{n} x_i + \frac{1}{2}\phi\sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij} x_i x_j + \gamma\sum_{i=1}^{n} x_i(1-x_i)$$

We determine the weights and thresholds as follows:

$$\begin{cases} T_{i,j} = -\phi b_{ij} + 2\delta_{ij}\gamma \\ i_i^b = \alpha - \gamma \end{cases} \qquad (1)$$

with $\delta_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$ is the Kronecker symbol.

The parameters ϕ, γ and α must be chosen so that the Hopfield network equilibrium point associated with the MSSP is realized. The parameter-setting procedure is obtained from the partial derivative of the energy function:

$$\frac{\partial E^0(x)}{\partial x_i} = E_i(x) = -\alpha + \phi\sum_{j=1}^{n} b_{ij} x_j + \gamma(1 - 2x_i)$$

The parameters-setting are determined by the hyper plane method [9]. Before treatment, some conditions are necessary to simplify the determination these parameters: $\phi > 0, \gamma > 0$.

To minimize the objective function, we impose the following constraint:

$\alpha > 0$.

The coming constraint is necessary for system stability and which is obtained by the following equation $x \in H - H_C$: $T_{i,i} = 2\gamma \geq 0$. Such as it was one constraint for the maximum stable set problem, we obtain: $H_C - H_F = \{x \in H_C / h(x) > 0\}$.

Let $x \in H_C - H_F$, in this case, two adjacent nodes v_i and v_j are in the stable set S , then $x_i = x_j = 1$ and therefore the value x_i will decrease if $E_i^0(x) \geq \varepsilon \quad where \quad \varepsilon > 0$. The following constraint is obtained: $-\alpha + \phi - \gamma \geq \varepsilon$.

All of these constraints will display the following result:

$$\begin{cases} \alpha > 0, \quad \phi > 0, \quad \gamma \geq 0 \\ -\alpha + \phi - \gamma = \varepsilon \end{cases}$$

A feasible solution could be the following:

$$\begin{cases} \alpha > 0, \quad \phi > 0, \quad \gamma \geq 0 \\ -\alpha + \phi - \gamma = \varepsilon \end{cases} \qquad (2)$$

Finally, the weights and thresholds Hopfield can be found based on the parameters of pre-treatment.

4.3. The proposed algorithm

Basing on the prosed Continuous Hopfield Network, for the max stable problem, the Adams-bashforth method and using the equations 1 and 2, we propose the following algorithm:

Algorithm : (Adams-bashforth discretization of the Hopfield network)
Input data
-The graph $G=(V, E)$;
-The weight matrix and bias vector are Calculated from the equation (1);
- The parameters α, ε *and* ϕ are positive real and γ is calculated from the system (2);
-The Adams-bashforth step h fixed to a small value.
-The stopping criterion is *MaxIter* or a small non negative real *eps.*
Out put
Vector of binary elements.
Step 0.
Initialize randomly the neurons states
Step 1.
While the stopping condition is false, do Steps 2-6.
Step 2. Perform Steps 3-5.
Step 3. Choose a unit at random.
Step 4. Change activity on selected unit:
$$u_{n+1} = u_n + h/2 \times (3 f_n - f_{n-1})$$
Step 5. Apply output function
$$v_n = 1/2 \times [1 + \tanh(u_n / u_0)]$$
Step 6. Check stopping condition.

As this algorithm converges rapidly to a local minima, we can turn it several time staring form several initial state; at the end we chose the best solution.

5 Simulation Results

In the present work, we showed the practicality of our approach in a series of experimentations to solve the max stable set problem. The evaluation instances are given to DIMACS Challenge [17]. These graphs were presented as test problems for aims solving the maximum clique problem. For these graphs, we tested each instance at the end of applying our approach to the maximum stable set problem. This implementation was done by using language Java and personal computer environment with an Intel CPU of Core i5 and 4 GB of RAM.

Calculate randomly generated initial states: $x_i = 0.55 + 10^{-5} t$
Where t is a random uniform variable in the interval [-0.5,0.5].
We choose the parameters:
$\alpha = 1.0250$, $\varepsilon = 10^{-6}$ and $\gamma = 0.7$; the parameter ϕ was computed from the equation $\phi = \alpha + \gamma + \varepsilon$
The results are supplied in table 2.

Table 2 : Computational results of the instances

graph	*V*	*|E|*	*α(G)*	*CHN Euler*	*CHN Adams*
				$\alpha_1(G)$	*$\alpha_2(G)$*
brock200_2	200	9876	12	11	11
brock200_4	200	13089	17	9	12
brock400_4	400	59765	33	6	9
brock800_2	800	208166	24	12	18
gen200_p0.9_44	200	17910	44	27	36
gen400_p0.9_55	400	71820	55	38	53
hamming8-4	256	20864	16	16	16
hamming10-4	1024	434176	40	40	40
keller4	171	9435	11	7	9
Keller5	776	225990	27	10	23
p_hat300_1	300	10933	8	8	8
p_hat300-3	300	33390	36	31	31
p_hat700-1	700	60999	11	11	11
p_hat700-2	700	121728	44	--	--
C125.9	125	6963	34	26	26
C250.9	250	27984	44	37	44
C1000.9	1000	450079	68	34	34
MANN_a27	378	70551	126	72	123

-- : the repetition of the unstable point.
$\alpha_1(G)$: the size of the stable set obtained by CHN combined with the Euler Cauchy method.
$\alpha_2(G)$: the size of the stable set obtained by CHN combined with the Adams-bashforth method.
In order to obtain these results the machine needed 200 steps with Adams-bashforth method and 200 steps with Euler method.
This table shows that the result is better when using the Hopfield network incorporated by the Adams-bashforth method. In fact, the (CHN+Adams-Bashforth) system produce a large max stable sets in comparison with the (CHN+Euler-Cauchy) system.
In addition, upon comparing the error of the second-order Adams-bashforth and Euler method, we notice that the local truncation error is smaller for the Adams-bashforth method; this reflects the effectiveness of the Adams-bashforth method. In front of the results were observed the importance of Adams-bashforth for calculating approximate solution of optimization problem.

6 Conclusion

In this work, we have used the well-known multi-step numerical method called Adams–Bashforth method to calculate the equilibrium point of the CHN associated with the max stable problem.
In order to confirm the practical effectiveness of this method, many simulations have been carried out, the graphs were taken from the 2nd D IMACS Challenge. The simulations results showed that the (CHN+Adams-Bashforth) method produce a large max stable sets in comparison with the (CHN+Euler-Cauchy) method. A such results are obtained thanks to the intermediary step, which permit to improve the Euler Cauchy method precision. In the future, we use the (CHN+Adams-Bashforth) method to solve some will known combinatorial problems such as the traveling salesman problem, linear programming problems, graph coloring problems, processing image and constraint satisfaction problems.

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