Synthesis of Astatic MPC-Regulator for Magnetic Levitation Plant

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Abstract: - This paper is devoted to linear astatic regulator design using model predictive control (MPC) with the application to magnetic levitation plant. The control objective is the stabilization of a metal ball position at the given point in the air by means of electromagnet. The considered mathematical model is nonlinear with input and output constraints and external disturbances. This allows to state that MPC strategy is a quite suitable approach to be used here. In this paper the algorithm for linear astatic MPC regulator design is proposed. This algorithm is based on predictive model representation in the form of augmentations. The effectiveness of the approach is demonstrated by real-time experiments for a particular magnetic levitation plant.

Key-Words: - model predictive control, astatic regulator, magnetic levitation, constraints, optimization

1 Introduction

There are a lot of control processes with essentially nonlinear dynamics, constraints and uncertainties, where traditional linear systems control synthesis can not be used. In such cases it is necessary to use more sophisticated control design approaches in order to provide high-quality performance subject to all constraints and disturbances, which influence system dynamic. Nowadays available computational resources allow us to implement quite complicated control algorithms in real-time.

This paper is focused on Model Predictive Control (MPC) and its application for magnetic levitation plant (Maglev) control. Maglev plant consists of a ball, levitating in the air in a vertical direction, and an electromagnet, which produce control influence. It is important to note that Maglev system control has an essential practical meaning. For instance, this problem is similar to plasma vertical stabilization and serves as a prototype for plasma control in modern tokamaks [1], [2].

Maglev system has essentially nonlinear dynamic with input and output constraints and external disturbances. At the same time, computations in control loop must be very fast due to vertical instability and very small time constant of the considered Maglev plant.

Mathematical model of a Maglev plant is known inexactly. The main difficulties here arise with the description of a magnetic field, which varies in dependence of a distance from electromagnet. It is very difficult to obtain exact mathematical equations for magnetic forces as were discussed in [3–5]. In further discussion we will suppose that unknown part of magnetic forces is incorporated in external disturbances and in the vicinity of some operating point this disturbances has approximately constant value.

Mentioned reasons makes MPC approach very suitable to be used here. It allows us to take into account constraints imposed on controlled and manipulated variables while provide optimal performance with respect to a given cost functional. In order to provide zero steady-state error (astatic property) it is necessary also to predict external disturbances impact on future evolution of the control process. In this paper we propose an astatic linear MPC control algorithm, which guarantee offset-free performance in the case of slowly-varying external disturbances.

It is well-known that the MPC algorithms are time-consuming since they require the repeated on-line solution of the optimization problems at each sampling instant. This drawback prevents the wide expansion of MPC algorithms in practical applications, especially for systems with fast dynamic. But if linear predictive model is used, MPC algorithms can be successfully implemented in real-time even for processes with fast dynamic.

The paper is organized in the following way. Firstly, the mathematical model of magnetic levitation plant is described. Secondly, the optimal control problem is formulated taking into account constraints and slowly-varying disturbances. Thirdly, brief description of the MPC control scheme is given and its main features are discussed. Fourthly, the procedure of linear MPC astatic regulator synthesis is proposed. In the last section the results obtained in the simulation study and in real-time implementation are presented.

2 Problem Formulation

In this section mathematical model of magnetic levitation system and control problem statement are considered.

2.1 Mathematical Model of Maglev system

The scheme of magnetic levitation system is shown in Fig. 1. The main components of system – an electromagnet above, a pedestal below, where the ball rests initially, and a steel ball. The control objective is the stabilization of the ball position in the particular point between the electromagnet and the pedestal by means of the controlled voltage which is applied to the electromagnet. The ball dynamic in the air is influenced by two forces: gravitational force F_g and electromagnetic field force F_m. The system parameters are as follows: I – current of the electromagnet, x_b – ball position, V – voltage applied to electromagnet, R and L are resistance and inductance of the electromagnet loop correspondently. The origin of the coordinate system Oxy is displaced on the electromagnet surface so that axe Ox is directed downward. Control input is a voltage applied to electromagnet.

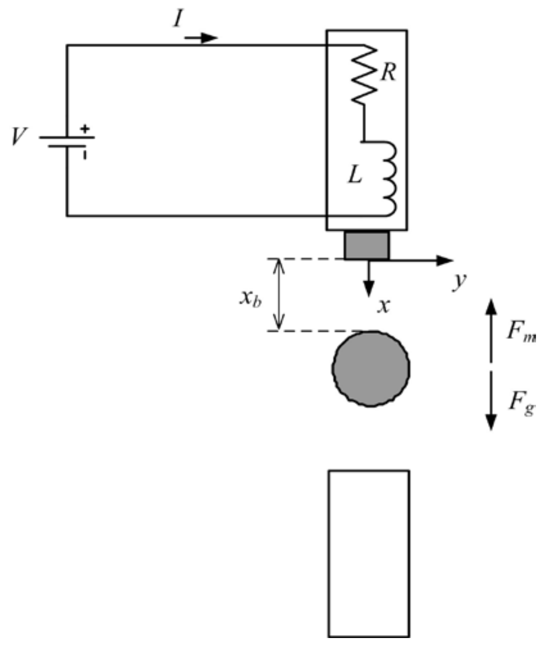

Fig. 1. Scheme of the magnetic levitation system.

Magnetic levitation system has the current sensor and optical sensor, which measure the current in the electromagnet and the distance between ball position and the electromagnet surface correspondently. The optical sensor is very sensitive to the changes in temperature and light conditions and its measurements are very noisy.

It is important to notice that the magnetic field created by the electromagnet, is non-uniform, especially nearby the magnet surface and consequently it is difficult enough to describe it mathematically. Thereby the corresponding mathematical model of the control process is essentially nonlinear. One more important feature of the system is the ball vertical position instability.

Let consider mathematical model of the Maglev plant, which is represented by the system of nonlinear differential equations:

$$\begin{aligned} \dot{\bar{x}}_1 &= \bar{x}_2, \\ \dot{\bar{x}}_2 &= g - \frac{1}{2} \cdot \frac{K_m \bar{x}_3^2}{M \bar{x}_1^2}, \\ \dot{\bar{x}}_3 &= -\frac{R}{L}\bar{x}_3 + \frac{1}{L}\bar{u}. \end{aligned} \tag{1}$$

Here $\bar{x}_1 = x_b$, $\bar{x}_2 = \dot{x}_b$, $\bar{x}_3 = I$ are state vector components, $\bar{u} = V$ is the control input signal; M – mass of the ball, K_m – magnetic field constant, g – gravitational constant. Equations (1) are easily obtained using Newton's second law and the laws of electrical circuits. Two components of the state vector are measured:

$$\begin{aligned} \bar{y}_1 &= \bar{x}_1, \\ \bar{y}_2 &= \bar{x}_3. \end{aligned} \tag{2}$$

In order to stabilize ball position we will use linearized model, representing ball dynamics in the vicinity of operating point $(\bar{x}_{10}, 0, \bar{x}_{30}) = (x_{b0}, 0, I_0)$, which corresponds to nominal voltage value $\bar{u}_0$. Let introduce following variables

$$x_1 = \bar{x}_1 - \bar{x}_{10},\ x_2 = \bar{x}_2,\ x_3 = \bar{x}_3 - \bar{x}_{30},\ u = \bar{u} - \bar{u}_0$$

representing system deviation from the operating point. Then linear model is given by the equations

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= \frac{2g}{x_{b0}} x_1 - \frac{2g}{I_0} x_3, \\ \dot{x}_3 &= -\frac{R}{L} x_3 + \frac{1}{L} u. \end{aligned} \tag{3}$$

The measurement equations are

$$y_1 = x_1,\ y_2 = x_2, \tag{4}$$

where $y_1 = \bar{y}_1 - \bar{y}_{10}$, $y_2 = \bar{y}_2 - \bar{y}_{20}$. It can be noted that zero equilibrium position of the system (3) is not stable due to ball vertical position instability. In addition, the coefficients values are only rough and can be refined using identification procedure.

2.2 Control problem statement

Let accept a control processes performance index, which is represented by the quadratic cost functional of the form

$$J = J(\bar{u}) = \int_0^{\infty} \left((x_b - x_{b0})^2 + \lambda \bar{u}^2 \right) dt \qquad (5)$$

where x_{b0} – desired ball position, $\lambda > 0$ – weight multiplier. Let us also introduce following constraints, which should be satisfied during the transient process:

$$|\bar{u}(t)| \le u_0, \ |\dot{\bar{u}}(t)| \le \dot{u}_0, \\ \rho(t) = \frac{|x_b - x_{b0}|}{x_{b0}} \le \rho_0, \ \forall t \in [0, t^*], \qquad (6)$$

where $\bar{u}_0, \Delta\bar{u}_0, \rho_0$ are given real positive numbers, t^* is a transient process settling time.

The goal is to design a feedback regulator on the base of MPC approach in order to provide the desirable position x_{b0} of a ball while minimizing cost functional (5) subject to mathematical model (1) – (4) and constraints (6). The application of MPC control strategy is quite suitable here, because the nominal linear mathematical model (4) is inexact and thus can be considered as if the ball is subject to slowly-varying external disturbances due to difficulties with magnetic field description. Moreover, the input and output constraints (6) are imposed and, in contrary to other control synthesis approaches, MPC provides optimal solution for the given problem in the sense of a cost functional (5).

3 MPC Basic Ideas

Let consider the mathematical model of the control process represented by the system of nonlinear equations in the discrete form [6], [7]:

$$\bar{\mathbf{x}}[k+1] = \bar{\mathbf{f}}(\bar{\mathbf{x}}[k], \bar{\mathbf{u}}[k], \mathbf{w}[k]), \\ \bar{\mathbf{y}}[k] = \mathbf{C}\bar{\mathbf{x}}[k] + \mathbf{v}[k], \qquad (7)$$

where $\bar{\mathbf{x}}[k] \in \mathbf{E}^n$, $\bar{\mathbf{u}}[k] \in \mathbf{E}^m$, $\bar{\mathbf{y}}[k] \in \mathbf{E}^l$, $\mathbf{w}[k] \in \mathbf{E}^{n_e}$ – state vector, control input, vector of output variables and external disturbances at sample instant k correspondently, $\mathbf{v}[k]$ – measurement noise. On the base of the model (7) the following predictive model could be formed:

$$\mathbf{x}[i+1] = \mathbf{f}(\mathbf{x}[i], \mathbf{u}[i]), \quad i = k, k+1, ..., \\ \mathbf{y}[i] = \mathbf{C}\mathbf{x}[i], \qquad \mathbf{x}[k] = \tilde{\mathbf{x}}[k], \qquad (8)$$

where $\mathbf{x}[k] \in \mathbf{E}^n$, $\mathbf{u}[k] \in \mathbf{E}^m$, $\mathbf{y}[k] \in \mathbf{E}^l$ are the state, input and output vectors as previously. But, unlike the ordinary mathematical model (7), predictive model is used to predict future outputs of the process given the programmed input control sequence over some time interval named prediction horizon. As it can be seen from (8), the initial condition for predictive model is defined by the actual state $\tilde{\mathbf{x}}[k]$ of the plant.

The prediction is performed in the following way. Let us consider that the programmed control over a prediction horizon is represented by the sequence $\{\mathbf{u}[k], \mathbf{u}[k+1], ..., \mathbf{u}[k+P-1]\}$, where P is a prediction horizon, i.e. the number of steps over which the prognosis is calculated. Then the sequence of vectors $\{\mathbf{y}[k+1], \mathbf{y}[k+2], ..., \mathbf{y}[k+P]\}$, which is obtained using equations (8), represents prediction of future plant behavior. The scheme of the prediction is shown in Fig. 2. Here C is a control horizon that is a number of steps where control input can vary, and for the remaining steps it must be constant.

It can be noted that the function $\mathbf{f}$ in (8) can differ from the function $\bar{\mathbf{f}}$. This is due to the fact that the predictive model must be integrated very fast for possibility of real-time implementation, so predictive model can be simplified in relation to the initial model (7). Besides that, the predictive model can include additional components of the state vector which are used to model the external disturbances.

The idea of MPC approach is to chose programmed control $\{\mathbf{u}[k], \mathbf{u}[k+1], ..., \mathbf{u}[k+P-1]\}$ that minimize following quadratic cost functional over the prediction horizon

$$J_k = \sum_{j=1}^{P} \Big\{ \left(\mathbf{y}[k+j] - \mathbf{r}^y_{k+j}\right)^T \mathbf{R}_{k+j} \left(\mathbf{y}[k+j] - \mathbf{r}^y_{k+j}\right) + \\ + \left(\mathbf{u}[k+j-1] - \mathbf{r}^u_{k+j-1}\right)^T \mathbf{Q}_{k+j} \left(\mathbf{u}[k+j-1] - \mathbf{r}^u_{k+j-1}\right) \Big\}, \qquad (9)$$

where $\mathbf{R}_{k+j}$ and $\mathbf{Q}_{k+j}$ are the positive definite weight matrices, $\mathbf{r}^y_i$ and $\mathbf{r}^u_i$ are the output and input reference signals and

$\overline{\mathbf{u}} = (\mathbf{u}[k]\ \mathbf{u}[k+1] \dots \mathbf{u}[k+P-1])^{\mathrm{T}} \in \mathbf{E}^{mP}$,

$\overline{\mathbf{y}} = (\mathbf{y}[k+1], \mathbf{y}[k+2], \dots, \mathbf{y}[k+P])^{\mathrm{T}} \in \mathbf{E}^{lP}$

are the auxiliary vectors. In addition, the programmed control sequence $\overline{\mathbf{u}}$ should satisfy all of the constraints imposed on the state and control variables. Therefore, the programmed control $\overline{\mathbf{u}}$ over a prediction horizon is chosen in order to provide minimum of the following optimization problem

$$J_k = J_k(\overline{\mathbf{x}}(\overline{\mathbf{u}}), \overline{\mathbf{u}}) = J_k(\overline{\mathbf{u}}) \to \min_{\overline{\mathbf{u}} \in \Omega \subseteq \mathbf{E}^{mP}}, \qquad (10)$$

where

$$\Omega = \left\{ \overline{\mathbf{u}} \in \mathbf{E}^{mP} : \mathbf{u}[k+j-1] \in \mathbf{U}, \mathbf{x}[k+j] \in \mathbf{X},\ j = \overline{1, P} \right\}$$

is the admissible set. Here $\mathbf{U} \subseteq \mathbf{E}^m$ is the set of feasible input values and $\mathbf{X} \subseteq \mathbf{E}^n$ is the set of feasible state values. The functional J_k in the considered situation is a function of mP variables. Generally, this function is nonlinear and Ω is a non-convex set. Therefore, the optimization task (10) is a nonlinear programming problem.

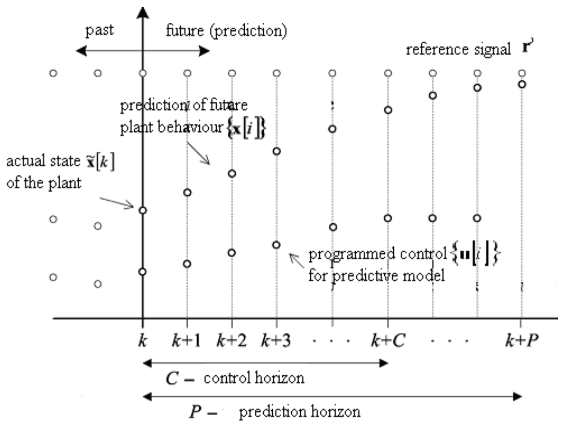

Fig. 2. Prediction of the future plant behaviour.

Denote by $\overline{\mathbf{u}}^*$ the solution of the optimization problem (10). According to the basic MPC idea, the obtained optimal programmed control $\overline{\mathbf{u}}^*$ is used as the input only on the current sample instant k, i.e. only first component of $\overline{\mathbf{u}}^*$ is implemented. At the next time instant the whole procedure – prediction and optimization – is repeated again to find new optimal programmed control over time interval $[k+1, k+P]$. Summarizing, real-time MPC-algorithm works as follows:

- obtain the state estimation $\hat{\mathbf{x}}[k]$ based on measurements $\overline{\mathbf{y}}[k]$ using the observer;
- solve the nonlinear programming problem (10) subject to prediction model (8) with initial conditions $\mathbf{x}[k] = \hat{\mathbf{x}}[k]$ and cost functional (9). It should be noted that the value of the J_k is obtained by numerically integrating the prediction model (8) and then substituting the prediction behaviour $\overline{\mathbf{y}}$ in to the cost functional (9) given the programmed control $\overline{\mathbf{u}}$ over the prediction horizon and initial conditions $\hat{\mathbf{x}}[k]$;
- let $\overline{\mathbf{u}}^* = \left(\mathbf{u}^*[k], \mathbf{u}^*[k+1], \dots, \mathbf{u}^*[k+P-1]\right)^T$ be the solution of the problem (10). Implement only first component $\mathbf{u}^*[k]$ of the obtained optimal sequence at the current step k;
- repeat the whole procedure 1–3 at next time instant $k+1$.

It is obvious that the scheme of MPC approach, presented above, realizes a feedback control loop, which has both significant advantages and certain drawbacks [8].

One of the main positive features is that MPC is an adaptive control algorithm, because the control input is adjusted to the changing conditions at each sample of discrete time. On the other hand, one of the essential disadvantages consists of that there is no guarantee of the closed-loop motion stability in general case.

Nevertheless, in order to avoid drawbacks of this approach, usually the following practical techniques are used. The time consumptions are reduced by decreasing the optimization problem order, for example, by means of control horizon. The stability property is provided by choosing enough long prediction horizon P.

4 Astatic Linear MPC-Regulator

Linear MPC scheme is based on the linear prediction model. The corresponding algorithms are computationally efficient that is especially important from the real-time implementation point of view. Generally, linear prediction model is presented by

$$\begin{aligned} &\mathbf{x}[i+1] = \mathbf{A}\mathbf{x}[i] + \mathbf{B}\mathbf{u}[i], \quad i = k, k+1, \dots, \\ &\mathbf{y}[i] = \mathbf{C}\mathbf{x}[i], \quad \mathbf{x}[k] = \tilde{\mathbf{x}}[k]. \end{aligned} \qquad (11)$$

The predictive model of the form (11) can not be used to provide offset-free performance if the plant is influenced by external disturbances. The traditional approach to overcome this problem is the estimation of the disturbances using asymptotic observer and after that to predict object dynamic evolution using model (11) with fixed disturbances values over prediction horizon. Let us propose another approach to astatic MPC regulator design. This approach provides offset-free performance in the case of slowly-varying disturbances and don't require asymptotic observer using for disturbances estimation.

Let suppose that the linear mathematical model of the control object is represented by the system

$$\bar{\mathbf{x}}[k+1] = \mathbf{A}\bar{\mathbf{x}}[k] + \mathbf{B}\bar{\mathbf{u}}[k] + \mathbf{H}\mathbf{w}[k], \quad \bar{\mathbf{y}}[k] = \mathbf{C}\bar{\mathbf{x}}[k], \tag{12}$$

where $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{H}$ are constant matrices, obtained by means of nonlinear system (1) linearization in the vicinity of equilibrium position. Vector $\mathbf{w}$ represents slowly-varying external disturbances and additional unmodeled dynamics. Let notice that the disturbance $\mathbf{w}$ time constant is much greater than prediction horizon. So, we can assume that the disturbances is constant over prediction horizon P.

Now let us derive linear predictive model on the base of equations (12) in the form of augmentations. To this end, let define predictive model expanded state vector $\mathbf{p}[i] = (\Delta\mathbf{x}[i] \;\; \mathbf{y}[i])^T$, where $\Delta\mathbf{x}[i] = \mathbf{x}[i] - \mathbf{x}[i-1]$ is a state augmentation at time instant i. In accordance with the model (12) subject to slowly-varying disturbances, we obtain following linear predictive model:

$$\mathbf{p}[i+1] = \overline{\mathbf{A}}\mathbf{p}[i] + \overline{\mathbf{B}}\Delta\mathbf{u}[i], \quad i = k, k+1, ... \quad \mathbf{z}[i] = \overline{\mathbf{C}}\mathbf{p}[i], \quad \mathbf{p}[k] = (\Delta\tilde{\mathbf{x}}[k] \;\; \tilde{\mathbf{y}}[k])^T. \tag{13}$$

Here matrices $\overline{\mathbf{A}}, \overline{\mathbf{B}}, \overline{\mathbf{C}}$ are defined as

$$\overline{\mathbf{A}} = \begin{bmatrix} \mathbf{A} & \mathbf{0}_{n\times l} \\ \mathbf{CA} & \mathbf{I}_{l\times l} \end{bmatrix}, \quad \overline{\mathbf{B}} = \begin{bmatrix} \mathbf{B} \\ \mathbf{CB} \end{bmatrix}, \quad \overline{\mathbf{C}} = (\mathbf{0}_{l\times n} \quad \mathbf{I}_{l\times l}),$$

vector $\Delta\mathbf{u}[i] = \mathbf{u}[i] - \mathbf{u}[i-1]$ is an input augmentation at time instant i, $\mathbf{z}[i] = \mathbf{y}[i]$ is an output vector, $\mathbf{p}[k]$ represents initial conditions calculating using measurements and asymptotic observer estimates.

Let us suppose that

$$\Delta\bar{\mathbf{u}} = (\Delta\mathbf{u}[k] \quad \Delta\mathbf{u}[k+1] \quad \ldots \quad \Delta\mathbf{u}[k+P-1])^T$$

is the programmed control over the prediction horizon. Then, solving (13) we obtain future outputs of the plant in the form

$$\bar{\mathbf{z}} = \mathbf{L}\mathbf{p}[k] + \mathbf{M}\Delta\bar{\mathbf{u}}, \tag{14}$$

where $\mathbf{L}$ and $\mathbf{M}$ are constant matrices, dependent on the matrices of predictive model (13). Now, let introduce the quadratic cost functional in the following form

$$J_k(\Delta\bar{\mathbf{u}}) = \sum_{i=1}^{P} \Big\{ (\mathbf{z}[k+i] - \mathbf{r})^T \mathbf{R} (\mathbf{z}[k+i] - \mathbf{r}) + \Delta\mathbf{u}[k+i-1]^T \mathbf{Q} \Delta\mathbf{u}[k+i-1] \Big\}. \tag{15}$$

Here $\mathbf{R}$ and $\mathbf{Q}$ are positive definite weighted matrices, $\mathbf{r}$ – reference signal, which remains constant over prediction horizon. The cost function (15) is slightly different from the general form (13) presented early, but it provides in the unconstrained case optimal solution in the form

$$\Delta\bar{\mathbf{u}} = \mathbf{H}(\bar{\mathbf{y}} - \bar{\mathbf{r}}),$$

which guarantees zero steady-state error. Taking into account auxiliary vectors $\bar{\mathbf{y}}$ and $\Delta\bar{\mathbf{u}}$ introduced above, we can represent cost (15) as a quadratic function

$$J_k = J_k(\Delta\bar{\mathbf{u}}) = \Delta\bar{\mathbf{u}}^T \mathbf{H} \Delta\bar{\mathbf{u}} + 2\mathbf{f}^T \Delta\bar{\mathbf{u}} + g. \tag{16}$$

The matrix $\mathbf{H}$ and vector $\mathbf{f}$ in (16) are as follows

$$\mathbf{H} = \mathbf{M}^T \overline{\mathbf{R}} \mathbf{M} + \overline{\mathbf{Q}}, \quad \mathbf{f} = \mathbf{M}^T \overline{\mathbf{R}} (\mathbf{L}\mathbf{p}[k] - \bar{\mathbf{r}}),$$

where $\overline{\mathbf{R}}$ and $\overline{\mathbf{Q}}$ are block-diagonal matrices with $\mathbf{R}$ and $\mathbf{Q}$ on the diagonal correspondently.

The control input over prediction horizon is computed subject to imposed constraints on input and output variables. Let define the corresponding admissible set $\mathbf{\Omega}$ of the programmed control sequences. This set comprises next constraints:

$$|\mathbf{u}_j[i]| \le \mathbf{u}_j^0, \; |\Delta\mathbf{u}_j[i]| \le \Delta\mathbf{u}_j^0, \quad |\mathbf{x}_j[i]| \le \mathbf{x}_j^0, \; i = k, ..., k+P-1, \tag{17}$$

where index j denotes components of the control vector $\mathbf{u}$ and state vector $\mathbf{x}$, $\mathbf{u}^0, \Delta\mathbf{u}^0, \mathbf{x}^0$ are given constant vectors. The admissible set $\mathbf{\Omega}$ is determined by linear inequalities (17) and therefore can be represented as

$$\mathbf{\Omega} = \left\{ \Delta\bar{\mathbf{u}} \in E^{mP} \mid \mathbf{A}_c \Delta\bar{\mathbf{u}} \le \mathbf{b}_c \right\}, \tag{18}$$

taking in use notations introduced above. Let notice that the right part in inequalities (18) is not stationary.

In accordance with MPC basic idea, the programmed control sequence $\Delta\bar{\mathbf{u}}$ over the prediction horizon is chosen as a solution of the optimization problem. This optimization has the following form

$$J_k = J_k(\Delta\bar{\mathbf{u}}) \to \min_{\Delta\bar{\mathbf{u}} \in \mathbf{\Omega} \subset E^{mP}}, \tag{19}$$

where $\mathbf{\Omega}$ is the admissible set (18). It is easy to show that in the case of linear predictive model (13) optimization task (19) reduces to quadratic programming problem of the form

$$J_k(\Delta\bar{\mathbf{u}}) = \Delta\bar{\mathbf{u}}^T \mathbf{H}\Delta\bar{\mathbf{u}} + 2\mathbf{f}^T\Delta\bar{\mathbf{u}} + g \to \min_{\Delta\bar{\mathbf{u}}\in\mathbf{\Omega}\subset E^{mP}}, \quad (20)$$

where $\mathbf{\Omega}$ is the admissible set (18), matrix $\mathbf{H}$ and vector $\mathbf{f}$ are determined by the formulas presented above.

Proposed linear astatic MPC based scheme provides an optimal control with respect to a given cost functional (16), taking into account linear predictive model of the process (13) and imposed constraints (18). Generally, the resulting control law is nonlinear.

The advantage of the proposed approach relative to other methods is that it does not require an estimation of the external constant disturbances and a prediction of its impact for future process dynamics.

The disadvantage of the approach is that it uses nominal inexact mathematical model of a plant. The further development of the proposed approach can be related with the ensuring robust properties of the control algorithms.

5 Astatic MPC-Regulator for Maglev Plant

The experiments were carried out for a particular Maglev plant named Quanser Maglev [9]. This system has following specific parameters: ball position can vary in the range from 0 to 0.014 m. If position $x_b = 0.014$ then the ball is resting on the post. Physical parameters are as follows: $L = 0.41\,\text{H}$, $R = 11\,\text{Om}$, $Km = 6.5308e-5 N\cdot m/A^2$, $M = 0.068\,\text{kg}$. Assume that the reference ball position is $x_{b0} = 0.009$ m. At the beginning of the process ball is resting on the post.

Let consider practical application of the proposed approach in real-time. The first challenge is the identification of the parameters of linear model, which describes the process dynamics nearby the operating point $x_{b0} = 0.009\,\text{m}$. Such a linear model is varied in dependence of the distance from the electromagnet. Parameters estimation of the linear model can be performed with the help of the prediction error method, which are implemented, for example, by the function **pem** in the System Identification Toolbox of MATLAB package. As a result of the identification the estimations of the parameters and the corresponding linear model are obtained.

The obtained nominal linear model is used to derive predictive model in the form of augmentations. The experimental data with astatic linear MPC regulator are presented in the fig. 3. Notice that the periodical reference input signal x_{b0} is used here. This signal provides ball position variations in the range $\pm 0.001\,\text{m}$ in the vicinity of nominal value $x_{b0} = 0.009$ m. As it can be seen from the picture, astatic MPC-regulator provides zero steady-state error with insignificant overshoot and all of the constraints imposed are hold.

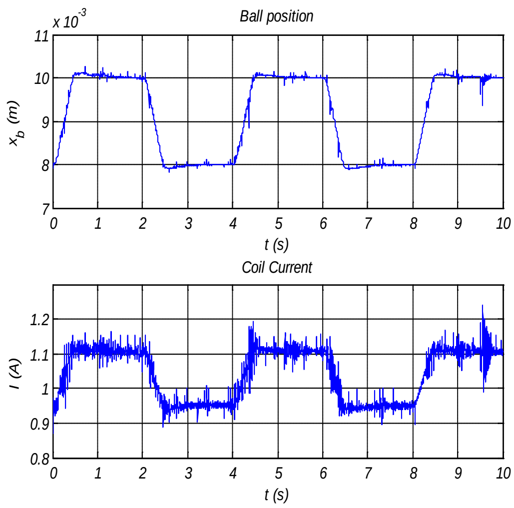

Fig. 3. Experimental data with linear astatic MPC.

6 Conclusion

In this article the linear astatic MPC control design algorithm is proposed. This algorithm operates with augmentations of state and input variables. The application of astatic MPC algorithm to magnetic levitation system control is considered. It was shown that this algorithm provide zero steady state error in the presence of slowly-varying disturbances. In the case of Maglev system this disturbances appear due to inexact mathematical model of the magnetic field. As the experimental results show, linear astatic MPC algorithm can be implemented in real-time taking into account imposed constraints. This allows us to use linear MPC algorithms in real-time control even for fast systems.

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