Braess' paradox and robustness of traffic network under dynamic equilibrium

CHUNXUE ZHAO
Anyang Normal University
School of Mathematics and Statistics
Xiange Street 436, Anyang
P.R.CHINA
zhaochunxue@aynu.edu.cn

Abstract: The Braess paradox is an important phenomenon of traffi networks, and the robustness is a characteristic that measures the network system performance under interference. Study of traffi network paradox and robustness is an important topic of traffi assignment. In this paper, we investigate the paradox and robustness of dynamic traffi network, in which, the influenc of all adjacent links on the link congestion is considered. In addition, we discuss the influenc of other links on whether the adding link works under dynamic system optimal, etc. The results show the shifty role of other links, which imply we should adjust the interaction between links rationally with traffi situations changing.

Key–Words: traffi network, dynamic user optimal, dynamic system optimal, Braess' paradox, robustness

1 Introduction

Dynamic traffi assignment (DTA) is one of the most important technologies of the intelligent transportation system (ITS)[1], which has received extensive attention of researchers and practitioners. During the process of investigating DTA, mathematical programming[2]–[5], optimal control[6]–[8], variational inequality[9]–[11] are labeled as three main analytical approaches. About the optimal control methods, dynamic user equilibrium (DUE), dynamic user optimal (DUO) and dynamic system optimal (DSO) are proposed; the variational inequality methods equivalent to DUO include the models based on the path congestion and link congestion.

The well-known Braess' paradox has been considerably investigated in the scientifi literatures since it was proposed by Braess[12]. For example, Yang and Bell[13] gave a capacity paradox design and studied how to avoid it. Pas and Principio[14] gained the specifi range that paradox occurred. In recent years, Hallefjord et al.[15] analyzed the traffi paradox when travel demand was elastic. Arnott et al.[16] discussed properties of dynamic traffi equilibrium including a paradox. Nagurney et al.[17] investigated the time-dependent Braess paradox using evolutionary variational inequalities. Zhao et al.[18] studied Braess paradox of traffi networks under user equilibrium. Zhao et al.[19] studied Braess paradox of traffi networks under stochastic user equilibrium. In addition, Zhao[20] discussed the paradox of traffi networks in which the link congestion is influence by the fl w on the link and adjacent links at the same path.

As we know, many distribution methods were proposed during the process of investigating the traffi assignment, such as the static methods including user equilibrium (UE), user optimal (UO) and system optimal (SO); the dynamic methods including DUE and DSO. The relationship between UE and SO has been investigated by a large number of researchers, from which[21], we know the solution of UE and SO is similar in the free fl w state and the difference becomes greater in the congested state.

Robustness is an important index in measuring the stability of a traffi network. It is often used to study the network under the partial degradation[34]–[24]. Sakakibara et.al.[25] used a topological index to quantify the road network dispersiveness. This approach can be used to evaluate the robustness of an urban highway network subject to catastrophic disaster. Scott et.al.[26] presented a method to identify the critical link and evaluated the network performance. Moreover, he compared with the traditional volume/capacity (V/C) ratio. Hoogendoorn et.al.[27] took into account the uncertainty in the predicted traffi condition and the system performance based on the controlled Markov process, the new control methodology showed how to control for the reliability in the condition of the generic control inputs and objectives. Tizghadam et.al.[28] presented a self-organizing management system of a network, in which, the requirement of the network was translated to a graph-theoretic metric, and the management system automatically evolved to a stable and robust control point by optimizing the metric. Mendes et.al.[29] used the electrical model and herding model to describe the emerging of the traffi jam up to the traffi gridlock and found the distributions of both the avalanche size and the flu follow a paw-law. Many different methods have been developed to study the system robustness and to identify the critical network components. Nagurney et.al.[30] proposed an index named relative total cost to measure the network efficien y drop under UE when a link capacity went down. Zhao et al.[19] studied the robustness of traffi networks under stochastic user equilibrium.

In the previous dynamic traffi models, the congestion on the link at time t was dependent on the link fl w, the infl w rate and outfl w rate on the link at time t, which increased the complexity when some traff c phenomena were discussed. Afterwards, some simplifie models were proposed, in which, the link congestion was related to the fl w on this link. But in reality the adjacent link fl w also has certain effect on link congestion, especially during peak hours. In order to simplify the traffi model and be more realistic, in this paper, we assume the link congestion at time t is related to the fl w on this link and all adjacent links to it at time t, and build dynamic traffi network model. On the basis, we investigate the paradox of dynamic traffi networks and the relationship between DUE and DSO. In addition, we use the relative total cost index to study the robustness of the network components under DUE when certain component is removed from the network.

The paper is organized as follows. We firs review the dynamic network equilibrium model and the equivalent variational inequality with it; then we discuss the paradox and whether the adding link makes sense under DSO, and we investigate the relationship of the total congestion between different assignments; in addition, we study the robustness of the network under DUE; at last, the conclusions about the results of the paper are given.

2 The DUE model and how the Braess paradox occurs

2.1 The DUE model

In this section, we firs review the DUE model, in the network $G = [N, L]$, where N, L denote the sets of nodes and links, respectively. Let W with n_W elements represent the set of origin/destination(O/D) and P_W represent the set of paths joining the O/D pair w; P with n_P elements denotes the set of all paths connecting all the O/D pairs in this network. Let $d_w(t)$ denote the demand at time t between O/D pair w, $f_a(t), x_r(t)$ stand for the fl w on link a and path r at time t, respectively. $[0, T]$ denotes the time interval under consideration. $c_a(t)$ is the congestion on link a at time t, $C_r(t)$ is the congestion on path r at time t

In this paper, we assume the congestion on the link is dependent on the fl w on this link and all adjacent links at time t, that is,

$$c_a(t) = c_a(f_a(t), f_1(t), f_2(t), \cdots, f_\Lambda(t)), \forall a \in L, \tag{1}$$

where $\{1, 2, \cdots, \Lambda\}$ is the set of the adjacent links to link a. The link fl ws and the route fl ws satisfy the following conservation of fl w equations:

$$f_a(t) = \sum_{r \in P} x_r(t)\delta_{ar}, \forall a \in L, \tag{2}$$

where $\delta_{ar} = 1$ if link a is contained in route r, and $\delta_{ar} = 0$, otherwise. Then we have

$$c_a(t) = c_a(x_1(t), x_2(t), \cdots, x_\Gamma(t)), \tag{3}$$

where $\{1, 2, \cdots, \Gamma\}$ is the set of paths containing link a or the adjacent links to link a. The path congestion and the link congestion satisfy the following equations:

$$C_r(t) = \sum_{a \in L} c_a(x_1(t), x_2(t), \cdots, x_\Gamma(t))\delta_{ar}, \forall r \in P. \tag{4}$$

The traffi demand at time t must satisfy the following conservation of fl w:

$$d_w(t) = \sum_{r \in P_w} x_r(t), \forall w \in W. \tag{5}$$

In addition, the model meets the following nonnegative constraint and boundary initial condition:

$$x_a(t) \geq 0, \tag{6}$$

$$x_a(0) = 0. \tag{7}$$

Then the definitio of dynamic network equilibrium satisfying Eq.(1)–(7) as follows[20]:

Definition 1 *A path flow pattern $x^*(t)$ is defined as a dynamic network equilibrium if, at each time t, only the minimum congestion routes are used for each O/D pair; which mathematical expression is given as follows:*

$$C_p(x^*(t)) \begin{cases} = \lambda_w(t), & if\ x_p^*(t) > 0; \\ \geq \lambda_w(t), & if\ x_p^*(t) = 0. \end{cases} \tag{8}$$

where $\lambda_w(t)$ is the minimal path congestion at time t, that is, $\lambda_w(t) = \min_{p \in P}\{C_p(t)\}$.

Theorem 2 *$x^*(t)$ is an equilibrium flow if and only if it satisfies the following variational inequality:*

$$\int_0^T < C(x^*(t)), x(t) - x^*(t) > dt \geq 0. \quad (9)$$

2.2 The dynamic network description and its equilibrium solution

In the following, we consider four-link Braess network in Fig.1. Let the total demand for travel from origin o to destination r be $d_w(t) = t$. Further, assume the problem is symmetric. Specificall , the link congestion functions of the four-link network in fig. are

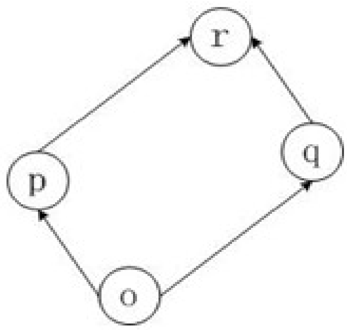

Fig.1 Four-link network

$$\begin{aligned} c_{op}(t) &= 3(\gamma f_{op}(t) + f_{pr}(t) + f_{oq}(t)) + 10, \\ c_{qr}(t) &= 3(\gamma f_{qr}(t) + f_{oq}(t) + f_{pr}(t)) + 10, \\ c_{oq}(t) &= (\gamma f_{oq}(t) + f_{qr}(t) + f_{op}(t)) + 20, \\ c_{pr}(t) &= (\gamma f_{pr}(t) + f_{op}(t) + f_{qr}(t)) + 20, \end{aligned}$$

where $c_{ij}(t)$ is the travel congestion on link ij at time t, $f_{ij}(t)$ is the fl w on link ij at time t, γ is the scaling parameter which differences the influenc between the given link and others. Generally, $\gamma \geq 1$. In the four-link network, there are two paths from the origin o to the destination r, and the path fl w satisfie the following relationship:

$$\begin{aligned} x_1(t) &= f_{op}(t) = f_{pr}(t), \\ x_2(t) &= f_{oq}(t) = f_{qr}(t), \end{aligned}$$

where x_k is the fl w from o to r on path k at time t, in addition, the costs on the paths are given as follows:

$$\begin{aligned} C_1(t) &= c_{op}(t) + c_{pr}(t) \\ &= 4(\gamma + 1)x_1(t) + 4x_2(t) + 30, \\ C_2(t) &= c_{oq}(t) + c_{qr}(t) \\ &= 4x_1(t) + 4(\gamma + 1)x_2(t) + 30, \end{aligned}$$

where $C_k(t)$ is the travel congestion from o to r on path k at time t; the total demand satisfie the following conservation of the fl w:

$$d_w(t) = x_1(t) + x_2(t).$$

The equilibrium solution of the four-link network is easily gets as follows according to Def. 1:

$$\begin{aligned} x_1^*(t) &= x_2^*(t) = \frac{t}{2}, \\ C_1(t) &= C_2(t) = 2(\gamma + 2)t + 30, \\ C^4(t) &= 2(\gamma + 2)t^2 + 30t, \end{aligned}$$

where $C^4(t)$ is the total system travel congestion under dynamic network equilibrium at time t for the four-link network.

Add the link pq based on Fig.1, there appears a new path $opqr$ from o to r in Fig.2.

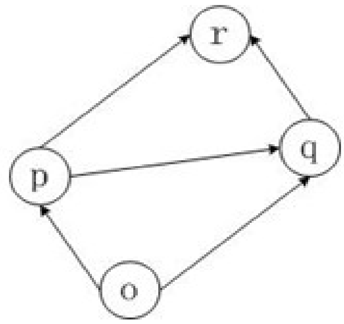

Fig.2 Five-link network

The link congestion functions are given as follows:

$$\begin{aligned} c_{op}(t) = &\ 3(\gamma f_{op}(t) + f_{pr}(t) + f_{pq}(t) + f_{oq}(t)) + 10, \\ c_{qr}(t) = &\ 3(\gamma f_{qr}(t) + f_{oq}(t) + f_{pq}(t) + f_{pr}(t)) + 10, \\ c_{oq}(t) = &\ (\gamma f_{oq}(t) + f_{qr}(t) + f_{pq}(t) + f_{op}(t)) + 20, \\ c_{pr}(t) = &\ (\gamma f_{pr}(t) + f_{op}(t) + f_{pq}(t) + f_{qr}(t)) + 20, \\ c_{pq}(t) = &\ 2(\gamma f_{pq}(t) + f_{op}(t) + f_{qr}(t) + f_{pr}(t) + f_{oq}(t)) \\ &+5. \end{aligned}$$

The traffi fl w on each link is as follows:

$$\begin{aligned} f_{op}(t) &= x_1(t) + x_3(t), \\ f_{pr}(t) &= x_1(t), \\ f_{oq}(t) &= x_2(t), \\ f_{qr}(t) &= x_2(t) + x_3(t), \\ f_{pq}(t) &= x_3(t). \end{aligned}$$

The costs on the paths of f ve-link network are given as follows:

$$\begin{aligned} C_1(t) &= 4(\gamma + 1)x_1(t) + 4x_2(t) + (3\gamma + 6)x_3(t) + 30, \\ C_2(t) &= 4x_1(t) + 4(\gamma + 1)x_2(t) + (3\gamma + 6)x_3(t) + 30, \\ C_3(t) &= (3\gamma + 10)(x_1(t) + x_2(t)) + 2(4\gamma + 5)x_3(t) + 25. \end{aligned}$$

The total demand satisfie the conservation of the fl w as follows:

$$d_w(t) = x_1(t) + x_2(t) + x_3(t).$$

According to theorem 2, the variational inequality of the f ve-link network over $t \in [0, T]$ is given as follows:

$$\int_0^T (4(\gamma+1)x_1^*(t) + 4x_2^*(t) + (3\gamma+6)x_3^*(t) + 30)$$
$$(x_1(t) - x_1^*(t)) + (4x_1^*(t) + 4(\gamma+1)x_2^*(t)$$
$$+ (3\gamma+6)x_3^*(t) + 30)(x_2(t) - x_2^*(t))$$
$$+ ((3\gamma+10)(x_1^*(t) + x_2^*(t)) + 2(4\gamma+5)x_3^*(t)$$
$$+ 25)(x_3(t) - x_3^*(t))dt \geq 0.$$

Because $d_w(t) = x_1(t) + x_2(t) + x_3(t)$, $d_w(t) = x_1^*(t) + x_2^*(t) + x_3^*(t)$, $x_1^*(t) = x_2^*(t)$, the above variational inequality implies that:

$$\int_0^T (4(2\gamma-1)x_1^*(t) - (5\gamma+4)t + 5)(x_1(t) + x_2(t)$$
$$- 2x_1^*(t))dt \geq 0,$$

we consider the term:

$$(4(2\gamma-1)x_1^*(t) - (5\gamma+4)t + 5)(x_1(t) + x_2(t) - 2x_1^*(t)),$$

for the fi ed t and analyze when its value is greater than or equal to zero. It implies, if $x_1^*(t) = 0$, we have $-(5\gamma+4)t + 5 \geq 0$, then $t \leq \frac{5}{5\gamma+4}$, that is, when $t \in [0, \frac{5}{5\gamma+4}]$,

$$x_1^*(t) = x_2^*(t) = 0, x_3^*(t) = t.$$

If $x_3^*(t) = 0$, we have $x_1^*(t) = \frac{t}{2}$, then $t > \frac{5}{\gamma+6}$, that is, when $t \in (\frac{5}{\gamma+6}, +\infty)$,

$$x_1^*(t) = x_2^*(t) = \frac{t}{2}, x_3^*(t) = 0.$$

when $t \in (\frac{5}{5\gamma+4}, \frac{5}{\gamma+6}]$,

$$x_1^*(t) = x_2^*(t) = \frac{(5\gamma+4)t - 5}{4(2\gamma-1)},$$

$$x_3^*(t) = \frac{-(\gamma+6)t + 5}{4\gamma-2}.$$

Assume $\gamma = 2$, the equilibrium fl w of the f ve-link network is pictured in Fig.3($time(h)\ vs\ flow(veh)$), in range I and II: $[0, \frac{5}{14}]$, only the new path is used; in range III: $(\frac{5}{14}, \frac{5}{8})$, all three paths are used; in range IV: $[\frac{5}{8}, +\infty)$, only the firs two paths are used, i.e., the third path is never used when $t > \frac{5}{8}$. Corresponding to different ranges, the total travel congestion of f ve-link network is given as follows:

$$C^5(t) = 2(4\gamma+5)t^2 + 25t, \ if\ t \leq \frac{5}{5\gamma+4},$$
$$= t[\frac{(7\gamma^2+4\gamma-20)t + 5(\gamma+2)}{4\gamma-2} + 30],$$
$$if\ \frac{5}{5\gamma+4} < t < \frac{5}{\gamma+6},$$
$$= 2(\gamma+2)t^2 + 30t, \ if\ t \geq \frac{5}{\gamma+6}.$$

where $C^5(t)$ is the total congestion for the f ve-link network.

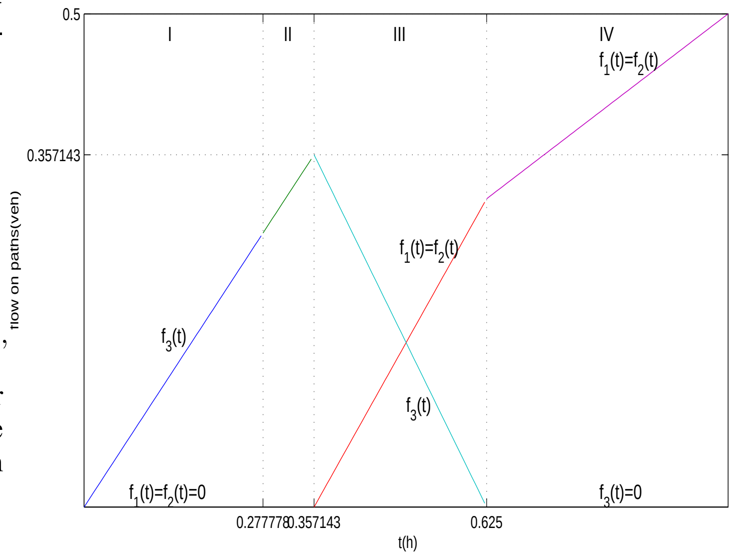

Fig.3 Equilibrium fl w of the dynamic network($t(h)\ vs\ flow(veh)$)

2.3 The paradox under dynamic network equilibrium

Let $C^5 > C^4$, we get $t \in (\frac{5}{6(\gamma+1)}, \frac{5}{\gamma+6})$, i.e., the paradox occurs in the range. Let $\gamma = 2$, we can fin the paradox occurs in range II and III in Fig.3.

In order to capture the trend of the paradox when γ changes, we give the definitio of interval length and use it to represent the appearance of the paradox. The interval length of $(\frac{5}{6(\gamma+1)}, \frac{5}{\gamma+6})$ is $L = \frac{5}{\gamma+6} - \frac{5}{6(\gamma+1)}$. It is well known that probability of paradox occurrence and interval length is in scale. In Fig.4, we give the variation tendency of L with γ changing, and fin the probability of paradox occurrence firstl increases then decreases with γ increasing. When $\gamma = \sqrt{6}$, the probability reaches the maximum, which explains the interaction of links influ ences the occurrence of paradox. Thus we may take some appropriate measures to affect the interaction of the links in order to control the occurrence of the paradox.

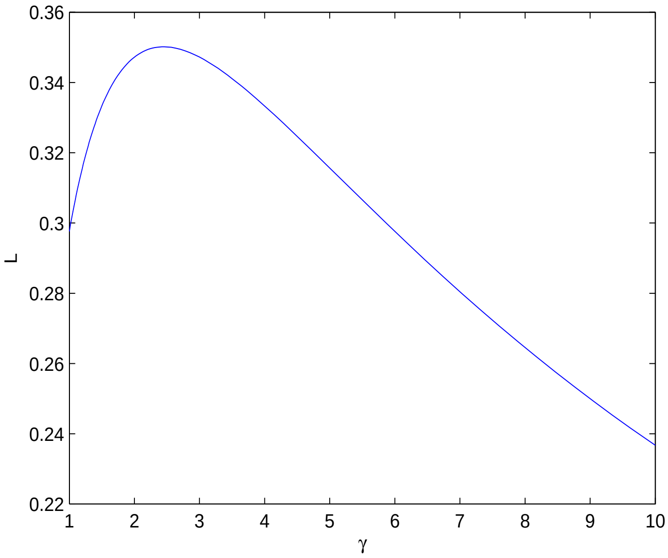

Fig.4 Occurrence trend of the paradox

2.4 Does the adding link make sense under DSO?

We have known, under the static traffi assignment, adding a new link does not reduce the total system travel time even under system optimal[14]. Subsequently, we discuss the phenomenon under DSO. As we have known, DSO is obtained by charging users the marginal cost of traveling, for the link congestion in this work, the marginal link congestion functions are given as follows:

$$\begin{aligned}
c'_{op}(t) = &\ 3(2\gamma f_{op}(t) + f_{pr}(t) + f_{pq}(t) + f_{oq}(t)) \\ &+10, \\
c'_{qr}(t) = &\ 3(2\gamma f_{qr}(t) + f_{oq}(t) + f_{pq}(t) + f_{pr}(t)) \\ &+10, \\
c'_{oq}(t) = &\ (2\gamma f_{oq}(t) + f_{qr}(t) + f_{pq}(t) + f_{op}(t)) \\ &+20, \\
c'_{pr}(t) = &\ (2\gamma f_{pr}(t) + f_{op}(t) + f_{pq}(t) + f_{qr}(t)) \\ &+20, \\
c'_{pq}(t) = &\ 2(2\gamma f_{pq}(t) + f_{op}(t) + f_{pr}(t) + f_{oq}(t) \\ &+f_{qr}(t)) + 5.
\end{aligned}$$

The corresponding path marginal cost equations are

$$\begin{aligned}
C'_1(t) = &\ 4(2\gamma+1)x_1(t) + 4x_2(t) + 6(\gamma+1)x_3(t) \\ &+30, \\
C'_2(t) = &\ 4x_1(t) + 4(2\gamma+1)x_2(t) + 6(\gamma+1)x_3(t) \\ &+30, \\
C'_3(t) = &\ (6\gamma+10)(x_1(t)+x_2(t)) + 2(8\gamma+5)x_3(t) \\ &+25.
\end{aligned}$$

Then the variational inequality of the f ve-link dynamic traffi network over $t \in [0, T]$ under DSO is given as follows:

$$\begin{aligned}
\int_0^T &(4(2\gamma+1)x_1^*(t) + 4x_2^*(t) + 6(\gamma+1)x_3^*(t) + 30) \\
&(x_1(t) - x_1^*(t)) + (4x_1^*(t) + 4(2\gamma+1)x_2^*(t) \\
&+ 6(\gamma+1)x_3^*(t) + 30)(x_2(t) - x_2^*(t)) + ((6\gamma \\
&+ 10)(x_1^*(t) + x_2^*(t)) + 2(8\gamma+5)x_3^*(t) \\
&+ 25)(x_3(t) - x_3^*(t))dt \geq 0.
\end{aligned}$$

Because $d_w(t) = x_1(t) + x_2(t) + x_3(t)$, $d_w(t) = x_1^*(t) + x_2^*(t) + x_3^*(t)$, $x_1^*(t) = x_2^*(t)$, we have

$$\begin{aligned}
\int_0^T &(4(4\gamma-1)x_1^*(t) - 2(5\gamma+2)t + 5)(x_1(t) + x_2(t) \\
&- 2x_1^*(t))dt \geq 0.
\end{aligned}$$

Let $x_3^*(t) = 0$, then $x_1^*(t) = x_2^*(t) = \frac{t}{2}$, if the value of $\int_0^T (4(4\gamma-1)x_1^*(t) - 2(5\gamma+2)t + 5)(x_1(t) + x_2(t) - 2x_1^*(t))dt$ is greater than or equal to zero, we must have

$$4(4\gamma-1)\frac{t}{2} - 2(5\gamma+2)t + 5 \geq 0,$$

then we obtain $t \leq \frac{5}{2\gamma+6}$, it implies when $t \geq \frac{5}{2\gamma+6}$, $x_3 = 0$, i.e., the adding link is not used. Thus when $t \in (0, \frac{5}{2\gamma+6})$, the adding link makes sense under DSO. In the following, we give the trend of the upper bound under which the adding link works under DSO with the parameter γ changing in Fig.5 and fin the bound becomes smaller as γ increasing. It explains that the less the influenc of the other links is, the less the possibility that the adding link works under DSO, which warns us of improving the influenc between the links appropriately if we want to make the adding link work under DSO.

2.5 Relationship between the cost under DUE and DSO

About the relationship between total congestion under different kinds of distributions, researchers have done

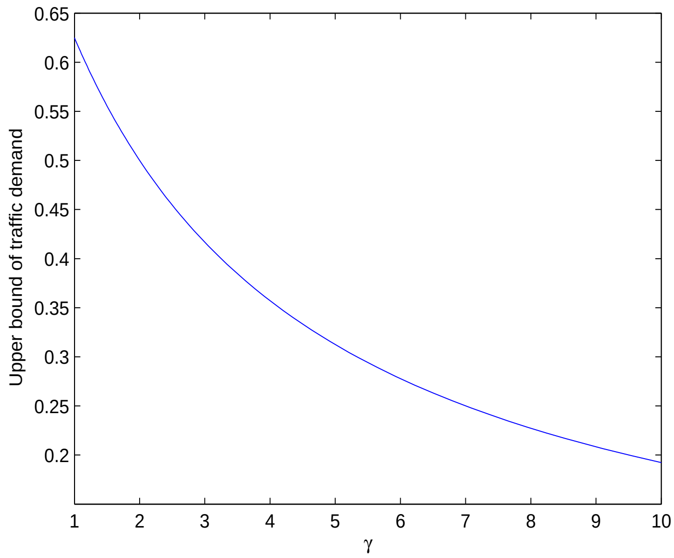

Fig.5 The bound above which an adding link can not make the total costs increase under DSO

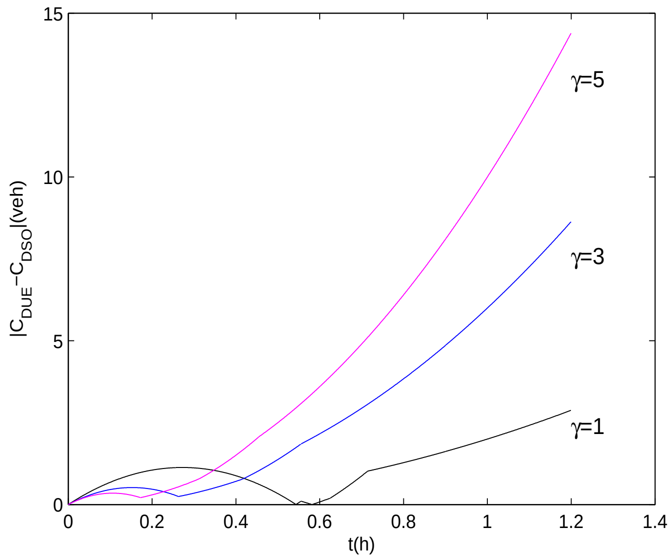

Fig.6 The distance between T_{DUE} and T_{DSO}

a large number of study[31], where we know, under static traffi assignment, the solution between UE and SO is approximative in the free fl w state; as the traffi becomes more congested, the difference between the solution under UE and that under SO becomes greater. In this work, it shows the influenc of other links to the congestion relationship between DUE and DSO with a plot Fig. 6, in which, the distance between total congestion i.e.,$|C_{DUE} - C_{DSO}|(Veh)$ vs $t(h)$. In Fig.6, let γ be $1, 3, 5$, respectively, and find the distance between different assignments under congested state is greater than that under free fl w state, which is same as the situation under the static assignments; in addition, $|C_{DUE} - C_{DSO}|$ becomes larger with γ increasing under congested state, which is explained as follows: the influenc of other links on the congestion on the given link decreases with γ increasing, that is, the influenc of other links is ignored, which is contrary to the choice principle of DSO.

3 Importance identification of network components

As an important branch of the traffi network robustness, the importance identificatio of network components has been widely investigated[19][32]–[34]. In this section, we use the relative total cost index[30] to study the importance identificatio of the network components under DUE when certain component is removed. First, we briefl review a few concepts. The total congestion on link a is denoted as follows:

$$\hat{t_a} = t_a f_a,$$

where $\hat{t_a}$ is the total congestion on link a, t_a is the unit congestion on link a, f_a is the traffi fl w on link a. The link fl w f is nonnegative and and satisfie the fl w conservation. The total congestion TC of the network is given by:

$$TC = \sum_{a \in L} \hat{t_a},$$

where L is the set of links. Then the relative total congestion index of the link l can be define as follows:

$$\Gamma^l = \frac{TC(G-l) - TC}{TC}, \tag{10}$$

Where Γ^l is define as the importance indicator of link l under DUE, $TC(G-l)$ denotes the total congestion when link l is removed from the network. Similarly, the importance indicator of node M is denoted as follows:

$$\Gamma^M = \frac{TC(G-M) - TC}{TC}. \tag{11}$$

It is stated in Sect.2 that the total congestion is different in different ranges. In this section, the values of the link congestion given in Sect.2 are used to investigate the importance indicators and its contributing factors. Let $\gamma = 2$, before the removal of the network components, the total congestion of the f ve-link

Table 1: Total congestion after removing different links in different ranges.

	I	II	III	IV
$TC(G-op)$	$12t^2+30t$	$12t^2+30t$	$12t^2+30t$	$12t^2+30t$
$TC(G-oq)$	$26t^2+25t$	$26t^2+25t$	$12t^2+30t$	$12t^2+30t$
$TC(G-pr)$	$26t^2+25t$	$26t^2+25t$	$12t^2+30t$	$12t^2+30t$
$TC(G-qr)$	$12t^2+30t$	$12t^2+30t$	$12t^2+30t$	$12t^2+30t$
$TC(G-pq)$	$8t^2+30t$	$8t^2+30t$	$8t^2+30t$	$8t^2+30t$

Table 2: Total congestion after removing different nodes.

$TC(G-o)$	$+\infty$
$TC(G-p)$	$12t^2+30t$
$TC(G-q)$	$12t^2+30t$
$TC(G-r)$	$+\infty$

Table 3: Importance rankings of links in different ranges.

	I	II	III	IV
op	1	1	1	1
oq	3	2	1	1
pr	3	2	1	1
qr	1	1	1	1
pq	2	3	2	2

Table 4: Importance rankings of nodes in different ranges.

	I	II	III	IV
o	1	1	1	1
p	2	2	2	2
q	2	2	2	2
r	1	1	1	1

network is as follows:

$$I: TC = 26t^2 + 25t$$
$$II: TC = 26t^2 + 25t$$
$$III: TC = \frac{8t^2 + 100t}{3}$$
$$IV: TC = 8t^2 + 30t$$

After link op oq pr qr and pq are respectively removed, the explicit formulas of the total congestion of the network can be showed in table 1.

When node o p q or r is respectively removed, the total congestion of the network can be showed in table 2.

Where $+\infty$ denotes that the traffi process cannot be achieved if the node is missing, so the node cannot be deleted.

After the components are removed, the explicit formulas of the importance indicators are easily obtained according to formula (10)–(11), they will not be listed here to save space. Only the importance rankings of links and nodes in different ranges is listed in Table 3–4.

From Table 3–4, it can be seen that the robustness of different components is different. If traffi situation change, that is, the traffi network is paralytic, we should firs repair the work of important links or nodes as soon as possible according to importance rankings of links and nodes based on the decomposable nature of origin-destination fl w of the traffi network. In addition, when the traffi demand is in different ranges, the importance rankings of the corresponding components are also different. At this point, we should further analyze traffi network path congestion and get the optimal path of traffi network, at the same time, partial balance paths are also taken into account to obtain the actual traffi network structure. Therefore, the discussion about the robustness is very important to plan the link indicators appropriately so that the system can adapt to the random change of traffi situation in real life.

4 Conclusions

In this work, we assume the link congestion is related to the fl w on the link and all adjacent links to this link, and investigate the paradox phenomenon and robustness of the traffi network by methods of the variational inequality. Using the Braess network, we fin that the possibility that paradox occurs firstl increases then decreases with the influenc of other links decreasing; but the possibility that the adding link makes no sense decreases all the time under DSO, which reminds us of adjusting the interaction between the links correctly according to the different purposes in the traffi assignment; in addition, we fin the difference between the total congestion under DUE and that under DSO increases with γ increasing, which further explains the essential difference between DUE and DSO. At last, we fin the robustness of different components is different and the robustness changes with traffi demand changing. The mechanisms for the dynamic traffi network which are closer to the reality need further study.

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