A Set of Speed Sensorless Observers For Estimation of the Rotor Flux and Load Torque For Asynchronous Machine

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Abstract: A state observer is proposed for asynchronous machine; with this observer it possible to observe rotor flux rotating speed and load torque. The gain of this observer involves a design function that has to satisfy some mild conditions which are given. Different expressions of such a function are proposed. Of particular interest, it is shown that high gain observers and sliding mode observers can be derived by considering particular expressions of the design function.

Key–Words: Nonlinear observer, High gain observer, Sliding mode observer, State transformation, LYAPUNOV equation candidate, Sensorless speed.

1 Introduction

The removal of the mechanical speed sensors offers an economic interest and may improve the reliability in the field of low power applications.

This article has as a principal objective to study the technique of determination mechanical speed and rotor flu of the asynchronous machine without velocity sensor.

The robustness, the low cost, the performances and the maintainability make the advantage of the asynchronous machine in many industrial applications or general public. Joint progress of the power electronics and numerical electronics makes it possible today to approach the controlling of axis at variable speed in applications low powers. Jointly with these technological projections, the scientifi community developed various approaches of order to control in real time the flu and the speed of the electric machines.

That it is the vectorial control, the scalar control or DTC control , to control the speed of the load it is necessary to measure this one by means of a mechanical sensor. For economic reasons and/or of safety of operation, certain applications force to be freed some. The information speed must then be rebuilt starting from the electric quantities. Multiple studies were undertaken, and without claim of exhaustiveness, we can distinguish several approaches (see [1], [2] and [3]).

The control speed sensorless must however have performances which do not deviate too much from those that we would have had with a mechanical sensor. It is thus significant during the development of an approach of velocity measurement without sensor to lay the stress on the static precise details and dynamics of this one according to the point of operation of the machine.

The article is organized in four sections:

1. Dynamic model of asynchronous machine;
2. Problem formulation;
3. Observers design;
4. Results and simulations.
5. Conclusion

2 Dynamic model of asynchronous machine

In this study, the model of the motor rests on the following hypothesis as [4]:

- The flu es and the currents are proportional by the intermediary of inductances and the mutual.
- The losses iron are neglected .
- The air-gap is constant (squirrel-cage rotor).
- The homopolar components are null.

It results from these assumptions that the various mutual between rotor and stator can be expressed like functions sinusoidal of the rotor position.

Its vector state is composed by the stator currents, rotor flu es and speed, as follows:

$$\left\{\begin{array}{rcl} \dot{i}_s &=& -\gamma i_s + KA(\Omega)\psi_r + \frac{1}{\sigma L_s}u_s \\ \dot{\psi}_r &=& \frac{M}{T_r}i_s - A(\Omega)\psi_r \\ \dot{\Omega} &=& \frac{p}{J}\frac{M}{L_r}i_s^T\mathcal{J}_2\psi_r - \frac{1}{J}T_L \\ \dot{T}_L &=& \varepsilon_{T_l} \end{array}\right. \quad (1)$$

The states variables accessible to measurement are the stator currents $i = \left[\begin{array}{cc} i_{s\alpha} & i_{s\beta}\end{array}\right]^T$ but to in no case rotor flu $\psi = \left[\begin{array}{cc}\psi_{s\alpha} & \psi_{s\beta}\end{array}\right]^T$ and possibly rotating speed. The source of energy $u = \left[\begin{array}{cc} u_{s\alpha} & u_{s\beta}\end{array}\right]^T$.

The parameters are define as follows:

$$\begin{array}{ll} T_r = \dfrac{L_r}{R_r} & ; \ \sigma = 1 - \dfrac{M^2}{L_sL_r} \\ K = \dfrac{M}{\sigma L_s L_r} & ; \ \gamma = \dfrac{R_s}{\sigma L_s} + \dfrac{R_rM^2}{\sigma L_sL_r} \\ \mathcal{I}_2 = \left[\begin{array}{cc} 1 & 0 \\ 0 & 1\end{array}\right] & ; \ \mathcal{J}_2 = \left[\begin{array}{cc} 0 & -1 \\ 1 & 0\end{array}\right] \\ A(\Omega) = \frac{1}{T_r}\mathcal{I}_2 - p\Omega\mathcal{J}_2 & . \end{array}$$

In these equations:

L_s	: Stator inductance cyclic ,
L_r	: Rotor inductance cyclic ,
M	: Cyclic mutual inductance between stator and rotor
R_s	: Stator resistance,
R_r	: Rotor resistance,
σ	: Scattering coefficient
T_r	: Time constant of the rotor dynamics,
J	: Rotor inertia,
T_l	: Resistive torque,
p	: Pole pair motor,
$\mathcal{I}_2$	: is the 2-dimensional identity matrix,
$\mathcal{J}_2$	: is a skew - symmetric matrix.

We need to transform system (1) to the triangular form. One will introduce the change of variable according to:

$$\left\{\begin{array}{l} z_1 = i_s \\ z_2 = \left(\frac{1}{T_r}\mathcal{I}_2 - p\Omega\mathcal{J}_2\right)\psi_r \\ z_3 = \left[\begin{array}{cc}\Omega & T_L\end{array}\right]^T = \left[\begin{array}{cc} z_{31} & z_{32}\end{array}\right]^T \end{array}\right. \quad (2)$$

Using this transformation and a time derivative of these states, we can rewrite from model (1), a following model :

$$\left\{\begin{array}{rcl} \dot{z}_1 &=& -\gamma z_1 + Kz_2 + \frac{1}{\sigma L_s}u_s \\ \dot{z}_2 &=& \frac{1}{T_r}\left(\frac{M}{T_r}z_1 - z_2\right) - pz_{31}\mathcal{J}_2\left(\frac{M}{T_r}z_1 - z_2\right) \\ && -pd\omega\mathcal{J}_2\psi_r + \frac{p}{J}z_{32}\mathcal{J}_2\psi_r \\ \dot{z}_3 &=& \left[\begin{array}{cc}\frac{p}{J}\frac{M}{L_r}z_1^T\mathcal{J}_2\psi_r + \varepsilon_\Omega & \varepsilon_{T_l}\end{array}\right]^T \\ y &=& z_1 \end{array}\right. \quad (3)$$

With $d\omega = \frac{p}{J}\frac{M}{L_r}z_1^T\mathcal{J}_2\psi_r$; and $\varepsilon_\Omega = -\frac{1}{J}T_L$.

3 Problem Formulation

Consider the nonlinear uniformly observable class of systems as the following form :

$$\left\{\begin{array}{rcl} \dot{z} &=& f(u,z) + \bar{\varepsilon}(t) \\ y &=& Cz = z_1 \end{array}\right. \quad (4)$$

with

$$z = \left[\begin{array}{c} z_1 \\ z_2 \\ z_3 \end{array}\right]; \quad f(z,u) = \left[\begin{array}{c} f_1(u,z_1,z_2) \\ f_2(u,z_1,z_2,z_3) \\ f_3(u,z)\end{array}\right]$$

$$= \left[\begin{array}{c} -\gamma z_1 + Kz_2 + \frac{1}{\sigma L_s}u_s \\ \left(\begin{array}{c}\frac{1}{T_r}\left(\frac{M}{T_r}z_1 - z_2\right) - pz_{31}\mathcal{J}_2\left(\frac{M}{T_r}z_1 - z_2\right) \\ -pd\omega\mathcal{J}_2\psi_r + \frac{p}{J}z_{32}\mathcal{J}_2\psi_r\end{array}\right) \\ \left[\begin{array}{cc}\frac{p}{J}\frac{M}{L_r}z_1^T\mathcal{J}_2\psi_r & 0\end{array}\right]^T \end{array}\right];$$

$$\bar{\varepsilon}(t) = \left[\begin{array}{c} 0 \\ 0 \\ \varepsilon_3(t)\end{array}\right]; \ \varepsilon_3(t) = \left[\begin{array}{c}\varepsilon_\Omega \\ \varepsilon_{T_l}\end{array}\right];$$

$$C = \left[\begin{array}{ccc}\mathcal{I}_2 & 0 & 0\end{array}\right]; \ u = u_s.$$

where the state $z \in \mathbb{R}^3$; the input $u \in \mathbb{U}^2$ the set of bounded absolutely continuous functions with bounded derivatives from $\mathbb{R}^+$ into $\mathbb{U}$ a compact subset of $\mathbb{R}^2$; $\bar{\varepsilon}(t) \in \mathbb{R}^3$. Our objective consists in designing state observers for system (4). We pose the following hypothesis (see. [7]).

$\mathcal{H}1$ **:** There are four positive constants α and β such as:

$$0 < \alpha^2 \leq \left(\frac{\partial f_1(u,z)}{\partial z_2}\right)^T\frac{\partial f_1(u,z)}{\partial z_2} \leq \beta^2 \quad (5)$$

$$0 < \alpha^2 \leq \left(\frac{\partial f_2(u,z)}{\partial z_3}\right)^T\frac{\partial f_2(u,z)}{\partial z_3} \leq \beta^2 \quad (6)$$

$\mathcal{H}2$ **:** The function $\varepsilon_3(t)$ is uniformly bounded by $\delta > 0$.

When $\varepsilon_3(t) = 0$, system (4) is identical to that considered in [5] and it characterizes a sub-class of locally $\mathbb{U}$-uniformly observable systems. In [6], the authors considered a sub-class of systems which involve the same uncertain term, $\bar{\varepsilon}(t)$, as (4). In the sequel, one shall use a strategy of observer design for asynchronous machine similar to that adopted in [5], [6] and [7].

4 Observers Design

One shall firstl introduce an appropriate state transformation allowing to easily design the proposed observers. Then, the equations of these observers will be derived in the new coordinates before being given in the original ones.

4.1 State Transformation

Consider the following change of states:

$\Phi : \mathbb{R}^3 \longmapsto \mathbb{R}^3$;

$$z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} \longmapsto x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \Phi(z) = \begin{bmatrix} \Phi(z)_1 \\ \Phi(z)_2 \\ \Phi(z)_3 \end{bmatrix}$$

where the $\Phi_j(z)$,$j = \overline{1,3}$ are define as follows:

$$\begin{cases} x_1 = \Phi(z)_1 = z_1 \\ x_2 = \Phi(z)_2 = K z_2 \\ x_3 = \Phi(z)_3 \\ = K \begin{bmatrix} -p\mathcal{J}_2\left(\frac{M}{T_r}z_1 - z_2\right) \\ \frac{p}{J}\mathcal{J}_2\psi_r \end{bmatrix}^T z_3 \end{cases} \Rightarrow \; x = \Phi(z) = \Lambda z \tag{7}$$

where Λ block diagonal matrix and Λ^{-1} is his left inverse:

$$\Lambda = diag\left(\mathcal{I}_2,\; K\mathcal{I}_2,\; -Kp\begin{bmatrix} \mathcal{J}_2\left(\frac{M}{T_r}z_1 - z_2\right) \\ -\mathcal{J}_2\frac{1}{J}\psi_r \end{bmatrix}^T\right)$$

Using this transformation and a time derivative of these states, we can rewrite from model (1), a following model :

$$\begin{cases} \dot{x}_1 &= x_2 + \varphi_1(u, x_1) \\ \dot{x}_2 &= x_3 + \varphi_2(u, x_1, x_2) \\ \dot{x}_3 &= \varphi_3(u, x) \\ y &= Cx = x_1 \end{cases} \tag{8}$$

Proceeding as in [5, 6], one can show that the transformation Φ puts system (1) under the following form:

$$\begin{cases} \dot{x} = Ax + \varphi(u, x) + \frac{\partial\Phi(z)}{\partial z}\bar{\varepsilon}(t) \\ y = Cx = x_1 \end{cases} \tag{9}$$

where $\varphi(u; x)$ has a triangular structure i.e.

$$A = \begin{bmatrix} 0 & \mathcal{I}_2 & 0 \\ 0 & 0 & \mathcal{I}_2 \\ 0 & 0 & 0 \end{bmatrix} \text{ and } \varphi(u, x) = \begin{bmatrix} \varphi_1(u, x_1) \\ \varphi_2(u, x_1, x_2) \\ \varphi_3(u, x) \end{bmatrix}$$

4.2 Observers synthesis

As in the works related to the high gain observers synthesis (see [6, 7, 8, 9]), one pose the hypothesis :

$\mathcal{H}3$ **:** The functions $\Phi(z)$ and $\varphi(u, x)$ are globally LIPSCHITZ with respect to x uniformly in u.

Before giving our candidate observers, one introduces the following notations.

1) Let Δ_θ is a block diagonal matrix define by:

$$\Delta_\theta = diag\left(\mathcal{I}_2, \frac{1}{\theta}\mathcal{I}_2, \frac{1}{\theta^2}\mathcal{I}_2\right);\; \theta > 0 \text{ is a real number.}$$

3) Let $S = S_{\theta=1}$ is a definit positive solution of the ALGEBRAIC LYAPUNOV EQUATION:

$$S + A^T S + SA - C^T C = 0 \tag{10}$$

Note that (10) is independent of the system and the solution can be expressed analytically. For a straightforward computation, its stationary solution is given by: $S_{(n,p)} = (-1)^{n+p} C_{n+p-2}^{n-1}$ where $C_n^p = \frac{n!}{p!(n-p)!}$ for $n \geq 1$ and $p \leq 3$; and then we can explicitly determinate the correction gain of (3) as follows:

$$\theta\Lambda^{-1}\Delta_\theta^{-1}S^{-1}C^T = \begin{bmatrix} 3\theta\mathcal{I}_2 \\ \dfrac{3\theta^2}{K}\mathcal{I}_2 \\ -\frac{\theta^3}{Kp}\left(\begin{bmatrix} \mathcal{J}_2\left(\frac{M}{T_r}z_1 - z_2\right) \\ -\frac{1}{J}\mathcal{J}_2\psi_r \end{bmatrix}^T\right)^{-1} \end{bmatrix} \tag{11}$$

4) $\forall\xi = \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix}$, set $\bar{\xi} = \Delta_\theta\xi$ and let $\Upsilon(\xi) = \begin{bmatrix} \Upsilon_1(\xi_1) \\ \Upsilon_2(\xi_2) \end{bmatrix}$ be a vector of smooth functions satisfying:

$$\forall\xi \in \mathbb{R}^2 :\; \bar{\xi}^T\Upsilon(\xi) \geq \frac{1}{2}\xi^T C^T C\xi \tag{12}$$

$$\exists \kappa > 0;\ \forall \xi \in \mathbb{R}^2 :\quad \|\Upsilon(\xi)\| \leq \kappa \|\xi\| \quad (13)$$

The system

$$\dot{\hat{x}} = A\hat{x} + \varphi(u,\hat{x}) - \theta\Delta_\theta^{-1}S^{-1}\Upsilon(\tilde{x}_1)$$
$$-\frac{\partial\Phi(z)}{\partial z}\left(\Lambda^{-1} - \left(\frac{\partial\Phi(z)}{\partial z}\right)^{-1}\right)\theta\Delta_\theta^{-1}S^{-1}\Upsilon(\tilde{x}_1) \quad (14)$$

is an observer for (9); Where $\tilde{x} = \hat{x} - x$ error in estimation; $\Upsilon(\tilde{x})$ satisfie conditions (12) and (13); u is the input of system (9) and $\theta > 0$ is a real number.

Finally one gives the following lemma (see[7]):

Lemma 1 *Assume that system (9) satisfies hypothesis $\mathcal{H}1$ to $\mathcal{H}3$. Then,*

$$\exists \theta_0 > 0;\ \forall \theta > \theta_0;\ \exists \lambda > 0;\ \exists \mu_\theta > 0;\ \exists M_\theta > 0;$$
$$\forall u \in \mathbb{U};\ \forall \hat{x}(0) \in \mathbb{R};\ \textit{one has}:$$
$$\|\tilde{x}\| \leq \lambda\theta^{-2}\exp^{-\mu_\theta t}\|\tilde{x}(0)\| + M_\theta\delta$$

where x is the unknown trajectory of (9) associated to the input u, $\hat{x}$ is any trajectory of system (14) associated to (u,y) and δ is the upper bound of $\varepsilon_3(t)$. Moreover, one has $\lim_{\theta\longrightarrow\infty}\{\mu_\theta\} = +\infty$ and $\lim_{\theta\longrightarrow\infty}\{M_\theta\} = 0$.

4.2.1 Analyse Stability

One has:

$$\begin{aligned}\dot{\tilde{x}} &= A\tilde{x} - \theta\Delta_\theta^{-1}S^{-1}\Upsilon(\tilde{x}_1) + \varphi(u,\hat{x}) - \varphi(u,x)\\ &\quad -\frac{\partial\Phi(\Phi^{-1}(x))}{\partial z}\varepsilon(t) - \Gamma(\hat{x})\theta\Delta_\theta^{-1}S^{-1}\Upsilon(\tilde{x}_1)\end{aligned}$$

where

$$\Gamma(\hat{x}) = \frac{\partial\Phi(\Phi^{-1}(\hat{x}))}{\partial z}\left(\Lambda^{-1} - \left(\frac{\partial\Phi(\Phi^{-1}(\hat{x}))}{\partial z}\right)^{-1}\right)$$

Notice that $\Gamma(\hat{x})$ is a lower triangular matrix with zeros on its main diagonal. Moreover, using hypothesis $\mathcal{H}1$ and $\mathcal{H}3$, one can easily deduce that $\Gamma(\hat{x})$ is bounded.

Now, one can easily check the following identities: $\theta\Delta_\theta^{-1}A\Delta_\theta = A$ and $C\Delta_\theta = C$. Set $\bar{x} = \Delta_\theta\tilde{x}$. One obtains :

$$\begin{aligned}\dot{\bar{x}} &= \theta A\bar{x} - \theta S^{-1}\Upsilon(\tilde{x}_1) + \Delta_\theta\left(\varphi(\hat{x}) - \varphi(u,x)\right)\\ &\quad -\Delta_\theta\frac{\partial\Phi(\Phi^{-1}(x))}{\partial z}\bar{\varepsilon}(t)\\ &\quad -\theta\Delta_\theta\Gamma(\hat{x})\Delta_\theta^{-1}S^{-1}\Upsilon(\tilde{x}_1) \qquad (15)\end{aligned}$$

To prove convergence, let us consider the following equation of LYAPUNOV $V(\bar{x}) = \bar{x}^TS\bar{x}$. By calculating the derivative of V along the $\tilde{x}$ trajectories, we obtains:

$$\begin{aligned}\dot{V} &= 2\bar{x}^TS\dot{\bar{x}}\\ &= 2\theta\bar{x}^TSA\bar{x} - 2\theta\bar{x}^T\Upsilon(\tilde{x}_1)\\ &\quad +2\bar{x}^TS\Delta_\theta\left(\varphi(u,\hat{x}) - \varphi(u,x)\right)\\ &\quad -2\bar{x}^TS\Delta_\theta\frac{\partial\Phi(\Phi^{-1}(x))}{\partial z}\bar{\varepsilon}(t)\\ &\quad -2\theta\bar{x}^TS\Delta_\theta\Gamma(\hat{x})\Delta_\theta^{-1}S^{-1}\Upsilon(\tilde{x}_1)\\ &= \theta\bar{x}^T(-S + C^TC)\bar{x} - 2\theta\bar{x}^T\Upsilon(\tilde{x}_1)\\ &\quad +2\bar{x}^TS\Delta_\theta\left(\varphi(u,\hat{x}) - \varphi(u,x)\right)\\ &\quad -2\bar{x}^TS\Delta_\theta\frac{\partial\Phi(\Phi^{-1}(x))}{\partial z}\bar{\varepsilon}(t)\\ &\quad -2\theta\bar{x}^TS\Delta_\theta\Gamma(\hat{x})\Delta_\theta^{-1}S^{-1}\Upsilon(\tilde{x}_1)\\ &= -\theta V + \theta\bar{x}^TC^TC\bar{x} - 2\theta\bar{x}^T\Upsilon(\tilde{x}_1)\\ &\quad +2\bar{x}^TS\Delta_\theta\left(\varphi(u,\hat{x}) - \varphi(u,x)\right)\\ &\quad -2\bar{x}^TS\Delta_\theta\frac{\partial\Phi(\Phi^{-1}(x))}{\partial z}\bar{\varepsilon}(t)\\ &\quad -2\theta\bar{x}^TS\Delta_\theta\Gamma(\hat{x})\Delta_\theta^{-1}S^{-1}\Upsilon(\tilde{x}_1)\end{aligned}$$

By taking account of the (10) and (12) the derivative of V becomes:

$$\begin{aligned}\dot{V} &= -\theta V + 2\theta\left(\frac{1}{2}\bar{x}^TC^TC\bar{x} - \bar{x}^T\Upsilon(x_1)\right)\\ &\quad +2\bar{x}^TS\Delta_\theta\left(\varphi(u,\hat{x}) - \varphi(u,x)\right)\\ &\quad -2\bar{x}^TS\Delta_\theta\frac{\partial\Phi(\Phi^{-1}(x))}{\partial z}\bar{\varepsilon}(t)\\ &\quad -2\theta\bar{x}^TS\Delta_\theta\Gamma(\hat{x})\Delta_\theta^{-1}S^{-1}\Upsilon(\tilde{x}_1)\\ &\leq -\theta V + 2\bar{x}^TS\Delta_\theta\left(\varphi(u,\hat{x}) - \varphi(u,x)\right)\\ &\quad -2\bar{x}^TS\Delta_\theta\frac{\partial\Phi(\Phi^{-1}(x))}{\partial z}\bar{\varepsilon}(t)\\ &\quad -2\theta\bar{x}^TS\Delta_\theta\Gamma(\hat{x})\Delta_\theta^{-1}S^{-1}\Upsilon(\tilde{x}_1) \qquad (16)\end{aligned}$$

Now, assume that $\theta \geq 1$, then, because of the triangular structure and the LIPSCHITZ assumption on φ, one can show that :

$$\|\Delta_\theta\left(\varphi(u,\hat{x}) - \varphi(u,x)\right)\| \leq \zeta\|\bar{x}\| \qquad (17)$$

where ζ is a constant of LIPSCHITZ. Similarly, according to hypothesis $\mathcal{H}1$ and to the LIPSCHITZ assumption on Φ (hypothesis $\mathcal{H}3$) , $\Gamma(\hat{x})$ is bounded. Moreover, and since $\Gamma(\hat{x})$ is lower triangular with zeros on the main diagonal, one has:

$$\left\|\theta\Delta_\theta\Gamma(\hat{x})\Delta_\theta^{-1}\right\| \leq \varrho \ \text{ for } \theta \geq 1 \qquad (18)$$

where $\varrho > 0$ is a constant that does not depend on θ. Finally, according to the structure of $\bar{\varepsilon}(t)$ and since $\frac{\partial\Phi(\Phi^{-1}(x))}{\partial z}$ is triangular, one can show that:

$$\left\| \Delta_\theta \frac{\partial\Phi(\Phi^{-1}(x))}{\partial z}\bar{\varepsilon}(t) \right\| \leq \frac{\beta^2}{\theta^2}\delta \tag{19}$$

where $\delta = \sup\{\bar{\varepsilon}(t)\}$ given in hypothesis $\mathcal{H}2$; β is given in hypothesis $\mathcal{H}1$. Using inequalities (17), (18), (13) and (19) inequality (16) becomes:

$$\begin{aligned} \dot{V} &\leq -\theta V + 2\lambda_{\max}(S)\|\bar{x}\|(\zeta\|\bar{x}\| + \varrho\eta(S)\|\tilde{x}_1\|) \\ &\quad +2\lambda_{\max}(S)\frac{\beta^2}{\theta^2}\delta\|\bar{x}\| \\ &\leq -(\theta - c_1)V + \frac{c_2}{\theta^2}\delta\sqrt{V} \end{aligned}$$

where $c_1 = 2\eta^2(S)(\zeta + \varrho\eta(S))$ and $c_2 = 2\beta^2\eta(S)\sqrt{\lambda_{\max}(S)}$ with $\lambda_{\min}(S)$ and $\lambda_{\max}(S)$ being respectively the smallest and the largest eigenvalues of S and $\eta(S) = \sqrt{\frac{\lambda_{\max}(S)}{\lambda_{\min}(S)}}$.

Now taking $\theta_0 = \max\{1, c_1\}$ and using the fact that for $\theta \geq 1$, $\|\bar{x}\| \leq \|\tilde{x}\| \leq \theta^2\|\bar{x}\|$, one can show that for $\theta > \theta_0$, one has :

$$\|\tilde{x}\| \leq \theta\eta(S)\exp\left[-\left(\frac{\theta - c_1}{2}\right)t\right]\|\tilde{x}(0)\| + 2\beta\frac{\eta^2(S)}{(\theta - c_1)}\delta$$

It is easy to see that λ, μ_θ and M_θ needed by the result 1 are: $\lambda = \eta(S)$, $\mu_\theta = \frac{\theta - c_1}{2}$ and $M_\theta = 2\beta^2\frac{\eta^2(S)}{(\theta - c_1)}$. This completes the proof.

4.3 Observers equations in the original coordinates

Proceeding as in [6], one can show that observer (14) can be written in the original coordinates z as follows:

$$\dot{\hat{z}} = f(u, \hat{z}) - \theta\Lambda^{-1}\Delta_\theta^{-1}S^{-1}\Upsilon(\hat{z}_1 - z_1) \tag{20}$$

Some expressions of $\Upsilon(\hat{z}_1 - z_1) = \Upsilon(\tilde{z}_1)$ that satisfying conditions (12) and (13) shall be given in this section and the so-obtained observers are discussed. These expressions will be given in the new coordinates x in order to easily check conditions (12) and (13) as well as in the original coordinates z in order to easily recognize the structure of the resulting observers.

4.4 High gain observer

Consider the following expression of $\Upsilon(\tilde{\xi})$:

$$\begin{aligned} \Upsilon_{HG}(\tilde{x}) &= C^T C\tilde{x} = C^T\tilde{x}_1 \\ &= C^T\tilde{z}_1 = C^T\bar{C}\tilde{z} \end{aligned} \tag{21}$$

One can easily check that expression (21) satisfie conditions (12) and (13). Replacing $\Upsilon(\tilde{x})$ by expression (21) in (20) gives rise to a high gain observer (see e.g. [9, 5, 6]):

$$\dot{\hat{z}} = f(u, \hat{z}) - \theta\Lambda^{-1}\Delta_\theta^{-1}S^{-1}C^T(\hat{z}_1 - z_1) \tag{22}$$

Or

$$\left\{\begin{array}{lcl} \dot{\hat{z}}_1 &=& -\gamma\hat{z}_1 + K\hat{z}_2 + \frac{1}{\sigma L_s}u_s - 3\theta(\hat{z}_1 - z_1) \\ \dot{\hat{z}}_2 &=& \frac{1}{T_r}\left(\frac{M}{T_r}\hat{z}_1 - \hat{z}_2\right) - p\hat{z}_{31}\mathcal{J}_2\left(\frac{M}{T_r}\hat{z}_1 - \hat{z}_2\right) \\ && -p\hat{d\omega}\mathcal{J}_2\hat{\psi}_r + \frac{p}{J}T_l\mathcal{J}_2\psi_r - \frac{3\theta^2}{K}(\hat{z}_1 - z_1) \\ \dot{\hat{z}}_3 &=& \begin{bmatrix} \frac{p}{J}\frac{M}{L_r}\hat{i}_s^T\mathcal{J}_2\hat{\psi}_r \\ 0 \end{bmatrix} + \frac{\theta^3}{Kp} \\ && \begin{bmatrix} \mathcal{J}_2\left(\frac{M}{T_r}\hat{z}_1 - \hat{z}_2\right) \\ -\frac{1}{J}\mathcal{J}_2\psi_r \end{bmatrix}^{-1}(\hat{z}_1 - z_1) \end{array}\right. \tag{23}$$

Referring to (7), the rotor flu is governed by the following equations:

$$\hat{\psi}_r = \left(\frac{1}{T_r}\mathcal{I}_2 - p\hat{\Omega}\mathcal{J}_2\right)^{-1}\hat{z}_2 \tag{24}$$

4.5 Sliding mode observers

At firs glance, the following vector seems to be a potential candidate for the expression of $\Upsilon(\tilde{x})$:

$$\begin{aligned} \Upsilon_{\text{sign}}(\tilde{x}) &= C^T C\,\text{sign}(\tilde{x}) = C^T\text{sign}(\tilde{x}_1) \\ &= C^T\text{sign}(\tilde{z}_1) = C^T\bar{C}\,\text{sign}(\tilde{z}) \end{aligned} \tag{25}$$

where sign is the usual signe function with $\text{sign}(\tilde{z}_1) = \begin{bmatrix} \text{sign}(\tilde{z}_{11}) \\ \text{sign}(\tilde{z}_{12}) \end{bmatrix}$; then:

$$\dot{\hat{z}} = f(u, \hat{z}) - \theta\Lambda^{-1}\Delta_\theta^{-1}S^{-1}C^T\text{sign}(\hat{z}_1 - z_1) \tag{26}$$

Or

$$\left\{\begin{array}{lcl} \dot{\hat{z}}_1 &=& -\gamma\hat{z}_1 + K\hat{z}_2 + \frac{1}{\sigma L_s}u_s - 3\theta\,\text{sign}(\hat{z}_1 - z_1) \\ \dot{\hat{z}}_2 &=& \frac{1}{T_r}\left(\frac{M}{T_r}\hat{z}_1 - \hat{z}_2\right) - p\hat{z}_{31}\mathcal{J}_2\left(\frac{M}{T_r}\hat{z}_1 - \hat{z}_2\right) \\ && -p\hat{d\omega}\mathcal{J}_2\hat{\psi}_r + \frac{p}{J}\mathcal{J}_2T_l\psi_r - \frac{3\theta^2}{K}\text{sign}(\hat{z}_1 - z_1) \\ \dot{\hat{z}}_3 &=& \begin{bmatrix} \frac{p}{J}\frac{M}{L_r}\hat{i}_s^T\mathcal{J}_2\hat{\psi}_r \\ 0 \end{bmatrix} + \frac{\theta^3}{Kp} \\ && \left(\begin{bmatrix} \mathcal{J}_2\left(\frac{M}{T_r}\hat{z}_1 - \hat{z}_2\right) \\ -\frac{1}{J}\mathcal{J}_2\psi_r \end{bmatrix}^T\right)^{-1}\text{sign}(\hat{z}_1 - z_1) \end{array}\right. \tag{27}$$

Indeed, condition (12) is trivially satisfie by (25). Similarly, for bounded input bounded output systems. However, expression (25) cannot be used due the discontinuity of sign function(see.[10]). Indeed, such discontinuity makes the stability problem not well posed since the LYAPUNOV method used throughout the proof is not valid. In order to overcome these difficulties one shall use continuous functions which have similar properties that those of the signfunction. This approach is widely used when implementing sliding mode observers. Indeed, consider the following function:

4.5.1 Tanh function:

$$\begin{aligned}\Upsilon_{\tanh}(\tilde{x}) &= C^T C \tanh(\tilde{x}) = C^T \tanh(\tilde{x}_1) \\ &= C^T \tanh(\tilde{z}_1) = C^T \bar{C} \tanh(\tilde{z})\end{aligned} \quad (28)$$

where $\tanh$ denotes the hyperbolic tangent function; then:

$$\dot{\hat{z}} = f(u,\hat{z}) - \theta\Lambda^{-1}\Delta_\theta^{-1}S^{-1}C^T\tanh(\hat{z}_1 - z_1) \quad (29)$$

Or

$$\left\{\begin{aligned}\dot{\hat{z}}_1 &= -\gamma\hat{z}_1 + K\hat{z}_2 + \frac{1}{\sigma L_s}u_s - 3\theta\tanh(\hat{z}_1 - z_1)\\ \dot{\hat{z}}_2 &= \frac{1}{T_r}\left(\frac{M}{T_r}\hat{z}_1 - \hat{z}_2\right) - p\hat{z}_{31}\mathcal{J}_2\left(\frac{M}{T_r}\hat{z}_1 - \hat{z}_2\right)\\ &\quad -p\hat{d\omega}\mathcal{J}_2\hat{\psi}_r + \frac{p}{J}T_l\mathcal{J}_2\psi_r - \frac{3\theta^2}{K}\tanh(\hat{z}_1 - z_1)\\ \dot{\hat{z}}_3 &= \begin{bmatrix}\frac{p}{J}\frac{M}{L_r}\hat{i}_s^T\mathcal{J}_2\hat{\psi}_r\\ 0\end{bmatrix} + \frac{\theta^3}{Kp}\\ &\quad \left(\begin{bmatrix}\mathcal{J}_2\left(\frac{M}{T_r}\hat{z}_1 - \hat{z}_2\right)\\ -\frac{1}{J}\mathcal{J}_2\psi_r\end{bmatrix}^T\right)^{-1}\tanh(\hat{z}_1 - z_1)\end{aligned}\right. \quad (30)$$

4.5.2 Arctan function:

$$\begin{aligned}\Upsilon_{\arctan}(\tilde{x}) &= C^T C \arctan(\tilde{x}) = C^T \arctan(\tilde{x}_1)\\ &= C^T \arctan(\tilde{z}_1) = C^T \bar{C}\arctan(\tilde{z})\end{aligned} \quad (31)$$

Similarly to the hyperbolic tangent function, one can easily check that the inverse tangent function:

$$\dot{\hat{z}} = f(u,\hat{z}) - \theta\Lambda^{-1}\Delta_\theta^{-1}S^{-1}C^T\arctan(\hat{z}_1 - z_1) \quad (32)$$

Or

$$\left\{\begin{aligned}\dot{\hat{z}}_1 &= -\gamma\hat{z}_1 + K\hat{z}_2 + \frac{1}{\sigma L_s}u_s - 3\theta\arctan(\hat{z}_1 - z_1)\\ \dot{\hat{z}}_2 &= \frac{1}{T_r}\left(\frac{M}{T_r}\hat{z}_1 - \hat{z}_2\right) - p\hat{z}_{31}\mathcal{J}_2\left(\frac{M}{T_r}\hat{z}_1 - \hat{z}_2\right)\\ &\quad -p\hat{d\omega}\mathcal{J}_2\hat{\psi}_r + \frac{p}{J}T_l\mathcal{J}_2\psi_r - \frac{3\theta^2}{K}\arctan(\hat{z}_1 - z_1)\\ \dot{\hat{z}}_3 &= \begin{bmatrix}\frac{p}{J}\frac{M}{L_r}\hat{i}_s^T\mathcal{J}_2\hat{\psi}_r & 0\end{bmatrix}^T + \frac{\theta^3}{Kp}\\ &\quad \left(\begin{bmatrix}\mathcal{J}_2\left(\frac{M}{T_r}\hat{z}_1 - \hat{z}_2\right)\\ -\frac{1}{J}\mathcal{J}_2\psi_r\end{bmatrix}^T\right)^{-1}\arctan(\hat{z}_1 - z_1)\end{aligned}\right. \quad (33)$$

5 Comparison of sensorless observers

To examine practical usefulness, the proposed observer has been simulated for a three-phase 1.5kw asynchronous machine(see [11]), whose parameters are depicted in Table 1.

Pole pair motor	p	2
Frequency	f	$50hz$
Stator inductance cyclic	L_s	$0.464H$
Rotor inductance cyclic	L_r	$0.464H$
Cyclic mutual inductance	M	$0.4417H$
Stator resistance	R_s	5.717Ω
Rotor resistance	R_r	3Ω
Rotor inertia	J	$0.00049Nm$

Table 1: Asynchronous machine parameters used in simulations

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In order to evaluate the observer behaviour in the realistic situation, the measurements of i_s issued from the model simulation have been corrupted by noise measurements with a zero mean value. The torque lead takes the shape of stair.

High gain observer : The adjustment parameter of the observer (23) is to chosen $\theta = 150$. The dynamic behaviour of the error of rotor flu is depicted in Figure 1 graph (a); when graph (b) shows the gaussian errors density and empirical errors histogram of rotor flu error. The means of error flu equal -3.10^{-3} with very small variance 4.9155×10^{-5} this is almost surety. The pace of speed error is given by the figur 2 graph (a) and the gaussian errors density and empirical errors histogram of rotor speed error are presented in graph (b)where means of error rotating speed equal 10.37×10^{-2} and variance equal 2.2929; the curve of load torque is illustrated on figur 3 graph (a).In graph (b) appear gaussian errors density and empirical errors histogram of load torque error where means of error load torque equal -5.88×10^{-2} and variance equal 62.72×10^{-2}

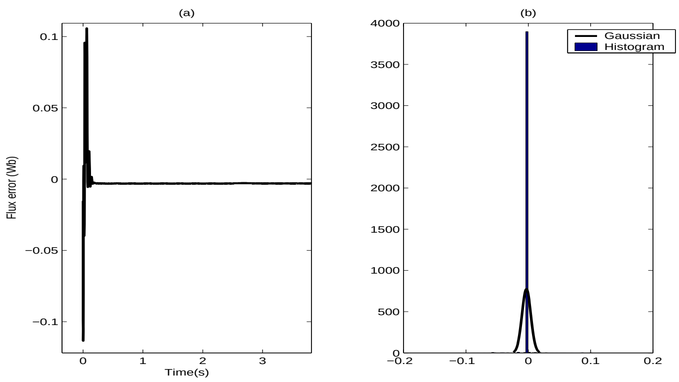

Figure 1: (a) Flux error. (b) Gaussian and histogram of error flux

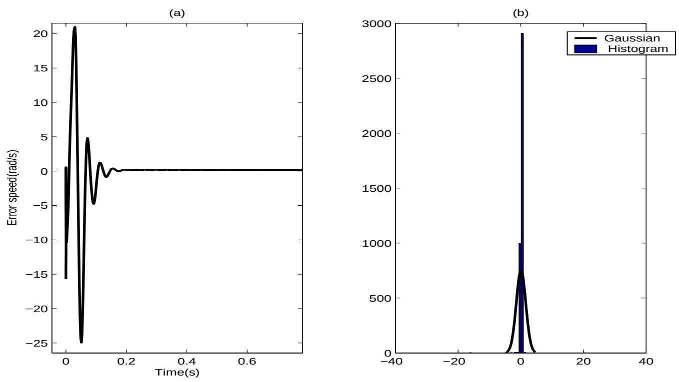

Figure 2: (a) Speed error. (b) Gaussian and histogram of error speed.

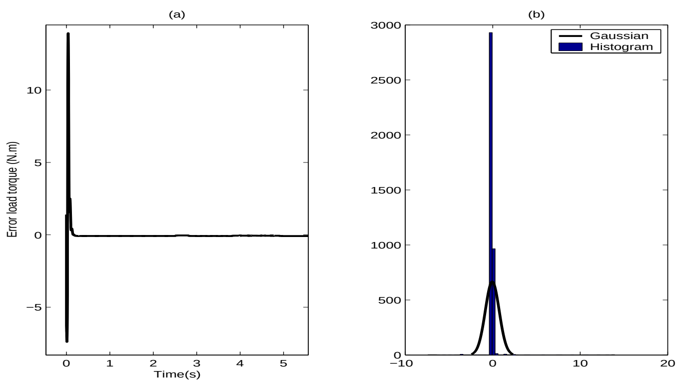

Figure 3: (a) Load torque error. (b) Gaussian and histogram of error load torque.

Sliding mode observer with $\tanh$: Estimation results of the proposed algorithm (30) with $\theta = 250$ is reported in Figure 4, 5 and 6. The behaviour of the error of rotor flu is depicted in Figure 4 graph (a); when graph (b) shows the gaussian errors density and empirical errors histogram of rotor flu error. The means of error flu equal -2.31×10^{-2} with very small variance 2.2×10^{-3} this is almost surety. The pace of speed error is given by the figur 5 graph (a) and the gaussian errors density and empirical errors histogram of rotor speed error are presented in graph (b)where means of error rotating speed equal -2.3373 and variance equal 33.8509; the curve of load torque is illustrated on figur 6 graph (a).In graph (b) appear gaussian errors density and empirical errors histogram of load torque error where means of error load torque equal -1.7535 and variance equal 14.4642

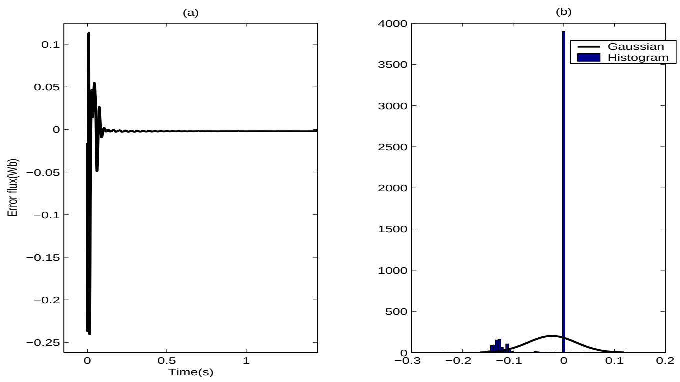

Figure 4: (a) Flux error. (b) Gaussian and histogram of error flux

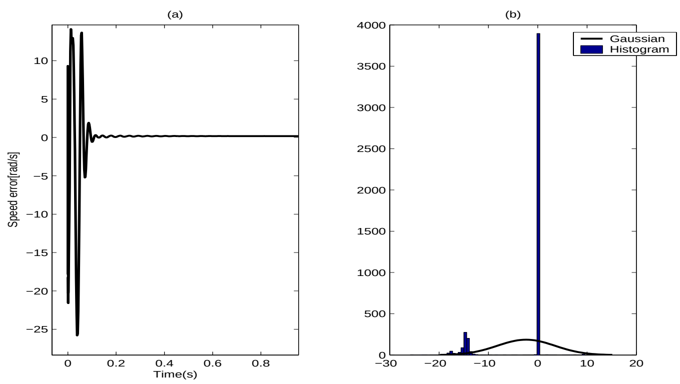

Figure 5: (a) Speed error. (b) Gaussian and histogram of error speed.

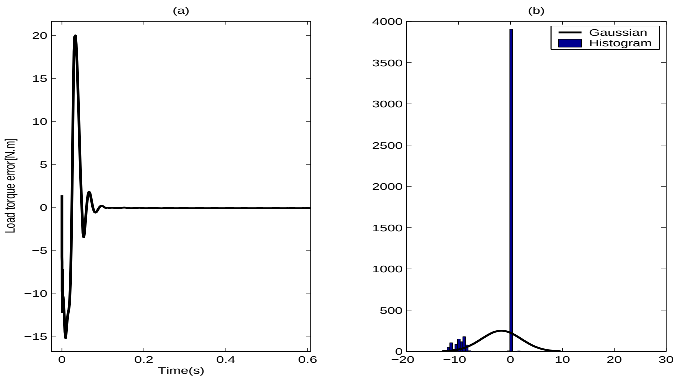

Figure 6: (a) Load torque error. (b) Gaussian and histogram of error load torque.

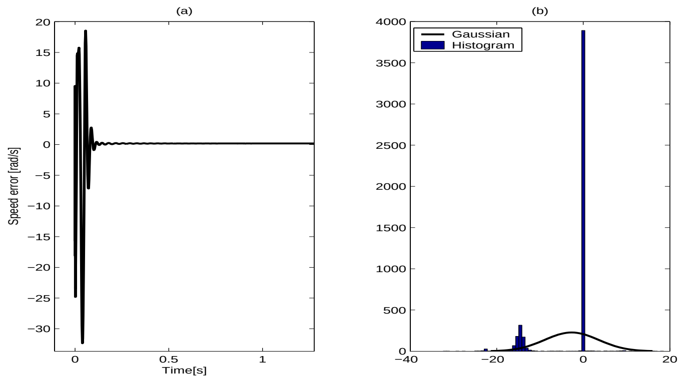

Figure 8: (a) Speed error. (b) Gaussian and histogram of error speed.

Sliding mode observer $\arctan$ **:** Under the same conditions with the function $\tanh$. One simulates for the function $\arctan$. The figur 7, 8 and 9 illustrates the pace of error flux error speed and error load torque in respectively. The behaviour of the error of rotor flu is depicted in Figure 7 graph (a); when graph (b) shows the gaussian errors density and empirical errors histogram of rotor flu error. The means of error flu equal -2.73×10^{-2} with very small variance 2.8×10^{-3} this is almost surety. The pace of speed error is given by the figur 8 graph (a) and the gaussian errors density and empirical errors histogram of rotor speed error are presented in graph (b)where means of error rotating speed equal -2.6758 and variance equal 38.7251; the curve of load torque is illustrated on figur 9 graph (a).In graph (b) appear gaussian errors density and empirical errors histogram of load torque error where means of error load torque equal -1.9414 and variance equal 15.9706

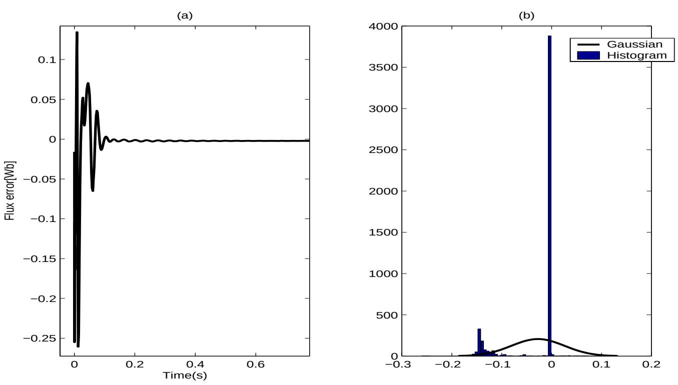

Figure 7: (a) Flux error. (b) Gaussian and histogram of error flux

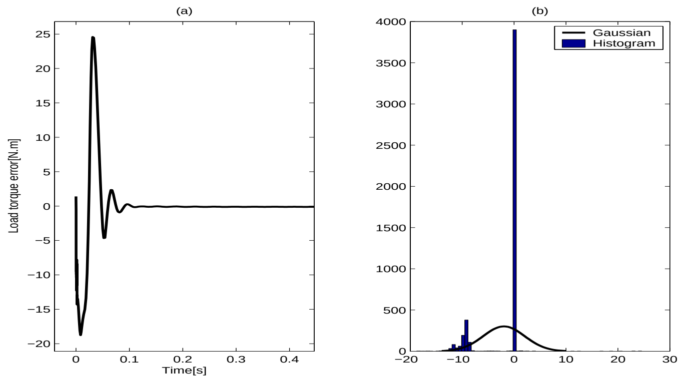

Figure 9: (a) Load torque error. (b) Gaussian and histogram of error load torque.

The preceding results, we notice the errors mean of observation with the high gain observer being very near to 0 with the very small variance (almost surely), then the high gain observer is the good in our case.

6 Conclusion

In this paper, high gain and alternative form for a sliding mode observers are presented. they is observer makes possible to observe, rotor flux rotor speed and load torque. An observer with high gain and three others with sliding mode which the functions sign, $\tanh$ and $\arctan$. Observer whose sign gives chattering. High gain observer is good for the observation of rotor flux rotating speed and load torque.

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