Constrained missile autopilot design based on model predictive control

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Abstract: - A new controller is proposed for a type of typical nonlinear missile autopilots using model predictive control method in the presence of constraints. Nonlinear model is first transformed into a linear structure, i.e. the form of state-dependent coefficient, which is used as the internal model for prediction. Then the constrained solution is obtained by solving an online quadratic programming problem at each sampling time, hence practical performances can be guaranteed. The resulting control law ensures nominal acceleration tracking for the missile. The closed-loop system has a good robustness against disturbances. Compared to the proportional integral controller, the proposed controller is more suitable to implement in practice. Simulation results confirm the effectiveness of the proposed control strategy.

Key-Words: - Nonlinear Systems, State-dependent Coefficient, Model Predictive Control, Missile Autopilot, Robustness

1 Introduction

For a missile autopilot design, fast response to commands and robustness against uncertainties are essential issues. In the considered flight envelope, the missile dynamics exhibits a highly nonlinear, rapid time-varying and uncertain behaviour, which leads to a great challenge for control [1]. Classically, based on linear time-invariant model obtained by linearization, the controllers for autopilots are designed by linear control techniques, such as linear quadratic regulator [2], proportional integral (PI) control [3], H_∞ design [4], and μ synthesis [5]. In order to achieve a better performance, many nonlinear control approaches have been proposed to treat the missile autopilot design, such as a power series expansion technique [6], nonlinear optimal control [7] and nonlinear H_∞ method [8]. Shamma [9] proposed the concept of linear parameter varying system, which is defined as a linear system whose dynamics depends on an exogenous variable, its values are unknown a priori but can be measured upon system operation. This is a breakthrough in methodology, and then many linear control theories can be utilized to design autopilot controllers [10, 11] based on LPV systems directly.

However, aforementioned methods can achieve satisfactory control performances in the absence of constraints. For an autopilot control design, considering hard constraints on the magnitude and rate of control surface deflection is a critical issue in practice. Otherwise, it may significantly degrade the control performance and even cause instability in the controlled systems. A feasible way to handle constraints is the anti-windup design. Kothare [12] presented a unified anti-windup framework for design: design a nominal controller by neglecting constraints and add on a compensation to reduce the windup effect on the performance. But, the compensation is sometimes not an easy task especially for nonlinear systems. Model predictive control (MPC), first proposed in the 1970s and referred to as a class of computer control algorithms that utilize a model to predict the future response of a plant based upon future input moves, has been used to treat constrained problems [13, 14]. The MPC is a systematic way to achieve the objective optimization and constraint treatment. Now, it has been applied to flight control systems [15]. Due to nonlinear dynamics, the nonlinear MPC techniques are also employed to design controllers for flight systems. Among them, series approximation is a main way for prediction [16, 17]. Nevertheless, the control performance is easily affected by the order of Taylor series expansion. At the same time, the control law was commonly obtained in the absence of constraints, and then a saturation function was used to handle limit on magnitude of the control. So, it is not convenient to deal with constraints, especially in the rate of change and magnitude of the output. In [18], a nonlinear model predictive controller in combination with recurrent neural network may be a solution to the aforementioned problems. The network treats nonlinear optimization without series approximation, and the resulting controller is capable of dealing with input and output constraints. However, the constraint on the rate of change of the control was not considered.

The purpose of this paper is to develop a new controller for a nonlinear missile autopilot based on a linear MPC. First of all, the missile dynamics is transformed into a linear structure, which is a form of state-dependent coefficient, and then the control law is obtained by solving a quadratic programming problem with constraints.

The rest of this paper is organized as follows. Section 2 states the missile dynamics. The missile autopilot design is formulated in Section 3. Section 4 is the simulation results and analysis. The final section 5 concludes the paper.

2 Missile Longitudinal Dynamics

The dynamics of a generic missile considered here is extracted from [3], which is representative of a missile flying at an altitude of 20 000 ft and at Mach number 3. However, it is not related to any particular missile airframe. Its nonlinear equations of motion are the following:

$$\dot{\alpha} = \cos(\alpha) K_\alpha M C_n(\alpha,\delta,M) + q \tag{1}$$

$$\dot{q} = K_q M^2 C_m(\alpha,\delta,M) \tag{2}$$

$$\eta = K_z M^2 C_n(\alpha,\delta,M)/g \tag{3}$$

where α, q, and η are angle of attack (rad), pitch rotational rate (rad / s) and nominal acceleration (g's), respectively. K_α, K_q, K_z, g are some constant coefficients. The stability derivatives $C_n(\alpha,\delta,M)$ and $C_m(\alpha,\delta,M)$ are given by

$$C_n(\alpha,\delta,M) = a_n\alpha^3 + b_n|\alpha|\alpha + c_n(2 - M/3)\alpha + d_n\delta \tag{4}$$

$$C_m(\alpha,\delta,M) = a_m\alpha^3 + b_m|\alpha|\alpha + c_m(-7 + 8M/3)\alpha + d_m\delta \tag{5}$$

where a_i, b_i, c_i, d_i $(i = m,n)$ are aerodynamic coefficients. Finally, the missile tailfin actuator can be modeled as a second-order system, described as

$$\ddot{\delta} = -\omega_a^2\delta - 2\xi_a\omega_a\dot{\delta} + \omega_a^2\delta_c \tag{6}$$

Remark 1. Generally, the ω_a is large enough that its response time is very short. Therefore, in many cases its dynamics is neglected in the design [9]. But this leads to a "proper" system that is not convenient to be dealt with using MPC.

Substituting the equations (4) and (5) into the equations (1) - (3) and in conjunction with the equation (6) yields

$$\begin{aligned}\dot{x}_1 = \cos(x_1) K_\alpha M[a_n x_1^3 + b_n|x_1|x_1 + \\ c_n(2 - M/3)x_1] + x_2 + \cos(x_1) K_\alpha M d_n x_3\end{aligned} \tag{7}$$

$$\begin{aligned}\dot{x}_2 = K_q M^2[a_m x_1^3 + b_m|x_1|x_1 + \\ c_m(-7 + 8M/3)x_1] + K_q M^2 d_m x_3\end{aligned} \tag{8}$$

$$\dot{x}_3 = x_4 \tag{9}$$

$$\dot{x}_4 = -\omega_a^2 x_3 - 2\xi_a\omega_a x_4 + \omega_a^2 u \tag{10}$$

$$y = K_z M^2 \left[a_n x_1^3 + b_n |x_1| x_1 + c_n(2 - M/3)x_1 \right] / g + K_z M^2 d_n x_3 / g \tag{11}$$

where $x = [\alpha, q, \delta, \dot{\delta}]^{\mathrm{T}}$, $u = \delta_c$, and $y = \eta$.

Obviously, the missile dynamics is highly nonlinear, which can not be handled using linear algorithms directly. Therefore, the dynamics should be described as a linear form. Based on a linear transformation, the system of (7) - (11) can be written in the form of state-dependent coefficient, which originates the state-dependent Riccati equation (SDRE) method, as

$$\dot{x} = \begin{bmatrix} \cos(x_1)K_\alpha M\Gamma & 1 & \cos(x_1)K_\alpha M d_n & 0 \\ \Pi & 0 & K_q M^2 d_m & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\omega_a^2 & -2\xi_a\omega_a \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \omega_a^2 \end{bmatrix} u \tag{12}$$

$$y = \left[K_z M^2 \Gamma / g \quad 0 \quad K_z M^2 d_n / g \quad 0 \right] x \tag{13}$$

where $\Gamma = a_n x_1^2 + b_n |x_1| + c_n(2 - M/3)$, and $\Pi = K_q M^2 [a_m x_1^2 + b_m |x_1| + c_m(-7 + 8M/3)]$.

The control objective is to make the output y track the desired acceleration y_c with a satisfactory performance by designing a controller in the presence of constraints on the actuator.

3 Missile Autopilot Design

The system of (12) and (13) can be described as in concise form of

$$\begin{aligned} \dot{x} &= A(x)x + B(x)u \\ y &= C(x)x \end{aligned} \tag{14}$$

where $A(x)$, $B(x)$, $C(x)$ are matrices of system, control and output, respectively. Similar to linear time-invariant systems, the system (14) can be converted into a discrete-time model of the following form:

$$\begin{aligned} x(k+1) &= A_k x(k) + B_k u(k) \\ y(k) &= C_k x(k) \end{aligned} \tag{15}$$

where A_k, B_k, C_k are matrices after discretizing. The system (15) is the internal model for prediction. In most predictive control design, the cost function penalizes the tracking error and the change of input u, i.e. Δu. Herein, the tracking error and the input like linear quadratic (LQ) problem are penalized in the cost function, which is more convenient to deal with constraints as seen in the sequel. Define the cost function as

$$J = \sum_{i=1}^{H_p} \| y(k+i\,|\,k) - r(k+i\,|\,k) \|_{Q(i)}^2 + \sum_{i=0}^{H_u-1} \| u(k+i\,|\,k) \|_{R(i)}^2 \tag{16}$$

where H_p, H_u are the prediction horizon and control horizon, respectively. r is the reference trajectory which can be generated by using the desired output trajectory, $Q(\cdot)$ and $R(\cdot)$ is the weighting matrices, $\|\cdot\|$ denotes Euclidean norm, $y(k+i\,|\,k)$, $r(k+i\,|\,k)$ and $u(k+i\,|\,k)$ indicate the prediction values at time $k+i$, which made at time k. In addition, it is assumed that $H_u \le H_p$, $Q(\cdot) \ge 0$, and $R(\cdot) \ge 0$. The cost function can be expressed in concise form as

$$J[U(k)] = \| Y(k) - \mathrm{T}(k) \|_Q^2 + \| U(k) \|_R^2 \tag{17}$$

where $Y(k) = [y(k+1\,|\,k), \ldots, y(k+H_p\,|\,k)]^{\mathrm{T}}$, $Q = \mathrm{diag}[Q(1), \ldots, Q(H_p)]$, $R = \mathrm{diag}[R(0), \ldots, R(H_u-1)]$, $U(k) = [u(k\,|\,k), \ldots, u(k+H_u-1\,|\,k)]^{\mathrm{T}}$, and $\mathrm{T}(k) = [r(k+1\,|\,k), \ldots, r(k+H_p\,|\,k)]^{\mathrm{T}}$.

The constraints imposed on the control are increments and magnitudes, which can be expressed by

$$E[\Delta u(k\,|\,k), \ldots, \Delta u(k+H_u-1\,|\,k), 1] \le 0 \tag{18}$$

$$F[u(k\,|\,k), \ldots, u(k+H_u-1\,|\,k), 1] \le 0 \tag{19}$$

here $\Delta u(k+i\,|\,k) = u(k+i\,|\,k) - u(k+i-1\,|\,k)$ is the control increment, E, F are matrices of suitable dimensions. Additionally, the right hand sides of above inequalities denote zero vectors of suitable dimensions. These constraints may represent actuator the slew rate, actuator range in practice.

Based on the equation (14), the state prediction can be obtained by recursion, expressed in matrix-vector form as

$$\begin{bmatrix} x(k+1|k) \\ \vdots \\ x(k+H_u|k) \\ x(k+H_u+1|k) \\ \vdots \\ x(k+H_p|k) \end{bmatrix} = \begin{bmatrix} A_k \\ \vdots \\ A_k^{H_u} \\ A_k^{H_u+1} \\ \vdots \\ A_k^{H_p} \end{bmatrix} x(k) + \begin{bmatrix} B_k & \cdots & 0 \\ \vdots & \cdots & \vdots \\ A_k^{H_u-1}B_k & \cdots & B_k \\ A_k^{H_u}B_k & \cdots & A_kB_k+B_k \\ \vdots & \cdots & \vdots \\ A_k^{H_p-1}B_k & \cdots & \sum_{i=0}^{H_p-H_u} A_k^i B_k \end{bmatrix} \begin{bmatrix} u(k|k) \\ \vdots \\ u(k+H_u-1|k) \end{bmatrix} \tag{20}$$

Then the output prediction can be given by

$$\begin{bmatrix} y(k+1|k) \\ \vdots \\ y(k+H_p|k) \end{bmatrix} = \begin{bmatrix} C_k & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & C_k \end{bmatrix} \begin{bmatrix} x(k+1|k) \\ \vdots \\ x(k+H_p|k) \end{bmatrix} \tag{21}$$

Furthermore, it can be expressed in concise form as

$$Y(k) = \Phi x(k) + \Theta U(k) \tag{22}$$

where Φ, Θ are products of matrices.

Define

$$\xi(k) = T(k) - \Phi x(k) \tag{23}$$

as known information, substituting the equations (22) and (23) into the equation (17) yields

$$J[U(k)] = \|\Theta U(k) - \xi(k)\|_Q^2 + \|U(k)\|_R^2 \tag{24}$$

i.e.

$$J[U(k)] = \xi^{\mathrm{T}}(k)Q\xi(k) - U^{\mathrm{T}}(k)L + U^{\mathrm{T}}(k)HU(k) \tag{25}$$

where $L = 2\Theta^{\mathrm{T}}Q\xi(k)$, and $H = \Theta^{\mathrm{T}}Q\Theta + R$.

In order to derive the control law, the constraints (18) and (19) should be expressed on $U(k)$. Suppose that E can be expressed in the form of

$$E = [E_1, E_2, ..., E_{H_u}, e] \tag{26}$$

then the inequality (18) becomes

$$\sum_{i=0}^{H_u-1} E_i[u(k+i|k) - u(k+i-1|k)] + e \le 0 \tag{27}$$

where $u(k-1|k) = u(k-1)$ denotes a known control effort at time $k-1$.

After arrangement, the inequality (27) can be written as

$$\tilde{E}U(k) \le \tilde{e} \tag{28}$$

where $\tilde{E} = [E_1 - E_2, E_2 - E_3, ..., E_{H_u-1} - E_{H_u}, E_{H_u}]$, and $\tilde{e} = -e + E_1 u(k-1)$.

Similarly, suppose that F has the form of

$$F = [F_1, F_2, ..., F_{H_u}, f] \tag{29}$$

then the inequality (19) can be written as

$$\tilde{F}U(k) \le \tilde{f} \tag{30}$$

where $\tilde{F} = [F_1, F_2, ..., F_{H_u}]$, and $\tilde{f} = -f$.

According to the equation (25), the constrained optimization problem is equivalent to minimize the following cost function:

$$\begin{aligned} J' &= U(k)^{\mathrm{T}} HU(k) - L^{\mathrm{T}}U(k) \\ &= \frac{1}{2}U(k)^{\mathrm{T}}(2H)U(k) + (-L^{\mathrm{T}})U(k) \end{aligned} \tag{31}$$

subject to the constraint

$$\begin{bmatrix} \tilde{E} \\ \tilde{F} \end{bmatrix} U(k) \le \begin{bmatrix} \tilde{e} \\ \tilde{f} \end{bmatrix} \tag{32}$$

Obviously, the optimization problem is a known quadratic programming counterpart, and standard algorithms are available for its solution, such as active set methods, interior point methods [19, 20].

After acquiring the control sequence $U(k)$, its first element is then applied to the plant.

4 Simulation Results and Analysis

The parameters of a missile are can be obtained in [3]. And, the prediction horizon and control horizon are chosen as $H_p = 10$, and $H_u = 2$. Weighting matrices Q, R are chosen as identity ones of $H_p \times H_p$ and $H_u \times H_u$ dimensions. In order to highlight advantages of the MPC, a comparative study is conducted in contrast to PI approach described in [3], herein, $k_0 = 1.017$, $k_1 = 0.2$, $k_2 = 5$, $k_3 = 0.5$.

Additionally, adopt the following reference trajectory of the form

$$r(k+i|k) = c(k+i) - e^{-iT_s/T_r}[c(k) - y(k)] \tag{33}$$

where c, T_r are the set-point and time constant, respectively. Note that e^{-T_s/T_r} should belong to interval $(0,1)$. Actually, the reference trajectory represents a suggested path by which the controlled variable should converg e on the set-point in a specified manner.

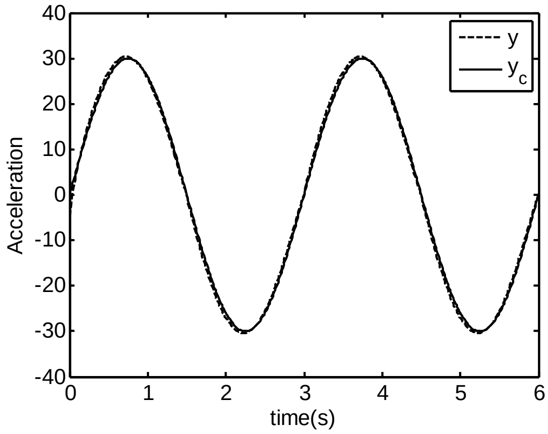

Fig. 1 Time response to sine command using MPC controller.

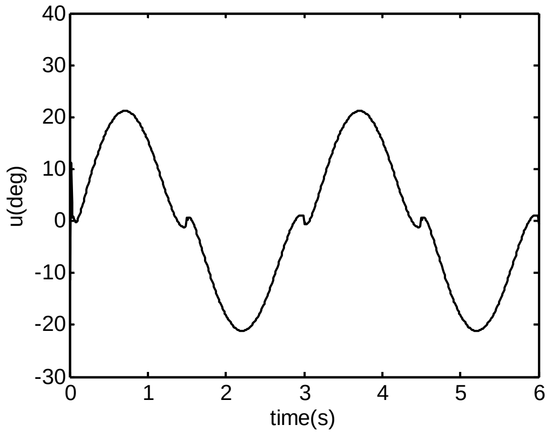

Fig. 2 Time history of control input using MPC controller.

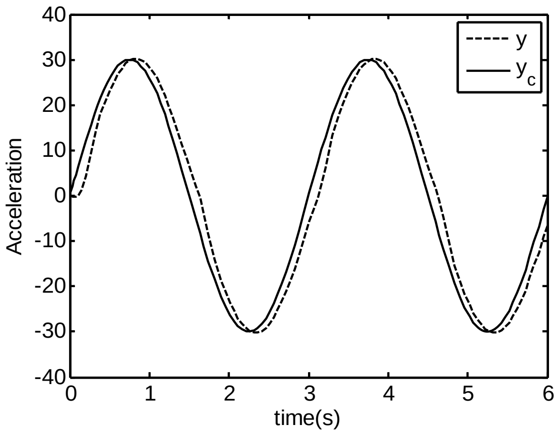

Fig. 3 Time response to sine command using PI controller.

Figure 1 - Figure 4 are th e simulation results of sine command tracking, which are under controls of MPC and PI for the no minal case and without considering constraints. It is o bserved that th e acceleration can track th e reference command in both cases. H owever, there are fewer time lags and control tailfin deflections under the control of MPC, so it is more suitable to implement in practice.

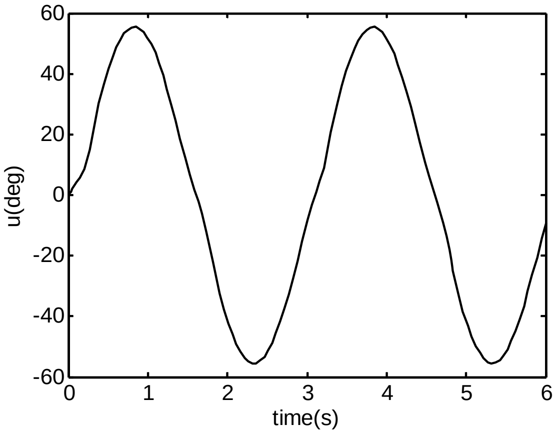

Fig. 4 Time history of control input using PI controller.

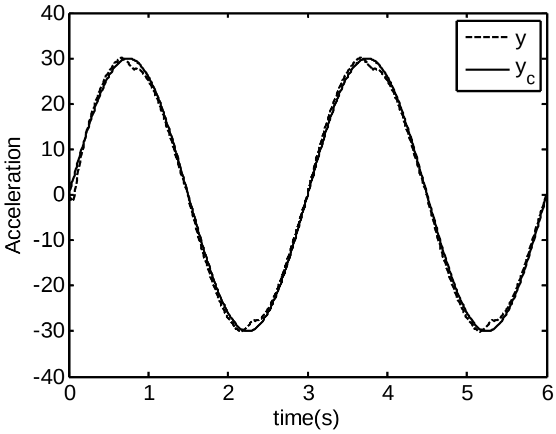

Fig. 5 Time response to sine command using MPC controller with constraints.

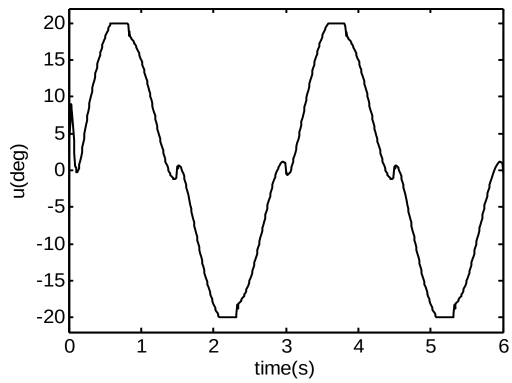

Fig. 6 Time history of control input using MPC controller with constraints.

Figure 5 and Figure 6 describe the sine tracking performance and the demanded control effort in the presence of control constraints. The constraints on the control are $u \in [-20°, 20°]$ and $\Delta u \in [-2°, 2°]$, that is, the rate of change was at . It is seen that the system output can track the expected output with a satisfactory performance except in the neighbourhoods of maximum and minimum due to the control limits. It shows that MPC has a powerful ability to handle the constrained systems.

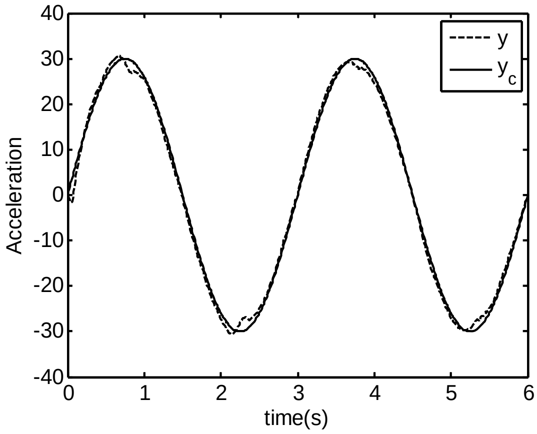

Fig. 7 Time response to sine command using MPC controller with uniform disturbances and constraints.

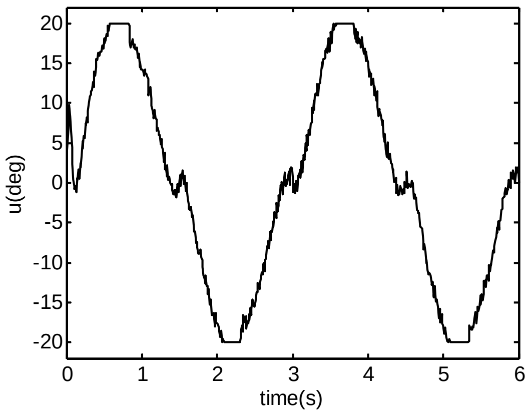

Fig. 8 Time history of control input using MPC controller with uniform disturbances and constraints.

Figure 7 and figure 8 are simulation results of sine command tracking in the presence of constraints and uniform disturbances. The disturbances are considered to inspect the robustness of the closed-loop system. The disturbances are uniformly distributed in the intervals $[-8000,8000]$ N and $[-800,800]$ Nm. It should be pointed out that these disturbances are active from beginning. It is concluded that the acceleration can well track the expected value except for the points around the peak and trough. At the same time, the control input is within the range of constraints. This shows the system is robust against the external disturbance under the control of MPC.

5 Conclusions

Model predictive control is known as a class of computer control algorithms that utilize an explicit process model to predict the future response of a plant, whose distinct advantage over traditional control approaches is capable of dealing with constraints. We combine MPC with state-dependent coefficient transformation to form a new powerful strategy, which greatly simplifies the design complexity for highly nonlinear systems. The simulations and analysis of a missile autopilot design show that the proposed approach is a meaningful attempt in the constrained nonlinear systems.

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