Determination and Calculation the Critical Levels in Development of Complex Systems of Different Nature with Shifted Arguments for their Investigation and Optimal Control

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Abstract: - Mathematical modeling and analysis is presented for revealing and investigation of the critical phenomena in a development of complex systems for various natures, the living and the technical ones associated with diverse complicated factors, in particular with a presence of shifted arguments of the system. Intensive research in this direction and developed techniques may optimize the management of the complex systems in financial-economic, natural and other fields. Construction of adequate physical and mathematical models for development of complex systems with shifted arguments, critical modes and their effective control are important tasks for a wide range of contemporary issues. Critical levels in the development of economic, banking, industry, technical, political and other systems necessary to determine and anticipate, to manage their system requirements stable development of the system, without being hit in a critical situation, leading to growing oscillations of system settings.

Key-Words: - Critical Levels, Optimal Control, Economic, Banking, Development, Stability, Modeling

1 Introduction

Critical processes and regimes in the development of complex systems of various nature (the living and the technical, as well as the other ones) [1-6] associated with various complex factors, in particular with the presence of shifted arguments of the system are interested in the contemporary area of research. Intensive research in this direction started in the last few decades and developed techniques to optimize the control of those systems, for example financial-economic, natural and other ones.

The effective research method for such systems has been developed in [1], where many examples from different industries, of different origin and nature have been done too.

Development and investigation of physical and mathematical models for development of complex systems with shifted arguments and critical modes, their effective control are critical tasks for a wide range of modern problems, which are of paramount interest. Critical levels in the development of economic, banking, industry, technical, political and other systems, which may cause instability of the system and destroy a stable development of the system due to growing oscillations in a system. Therefore the problem stated is vital for study, mathematical modelling and simulation.

2 History of the problem and the statement for investigation

The theory of nonlinear dynamical systems with shifted (deviated) arguments provides a powerful mathematical tool for the study of complex systems and the determination of critical levels in their development. Properties of many real objects essentially depend on the after-effects due to which their behaviour in the next moment of time depends on the previous history of development, and not only on the current state of the object.

The simplest cases of such systems are studied in the theory of functional differential equations with shifted arguments (delay and forecast) [7-21]. Real objects are more complicated and the mathematical models describing them, even with simplification described by systems of differential equations, contain the arguments depending on many deviating arguments [2-6], which, moreover, can themselves be time-dependent and to be linked.

Over several decades the fundamental results in the theory of dynamical systems with delayed and forecasting arguments formed the theory of differential equations with the shifted arguments, which for the last 20 years were applied to modelling various complex systems from a wide range of diverse fields of science, technology, wildlife, economics and banking, and the like. The development of numerical algorithms of problems solution and their application devoted a lot of effort, for example, [8-11, 14-20], where the equations with shifted arguments almost no attention was given. Only in [9, 10] provides a classification task with a forecasting arguments.

In relation to nonlinear dynamical systems with delayed and forecasting arguments, it should be noted that they were considered [2-6, 10, 21] for modelling of potentially hazardous objects of nuclear energy [2-6, 21], dynamics of populations crashing in biological systems and transmission of electrical signals in electrical systems, high voltage power lines. Interestingly, in the theory of motion control with delay the application of necessary optimality conditions in the form of Pontryagin's maximum principle leads to the conjugate system of the equations with forecasting arguments.

2.1 Solution of the differential equations with shifted arguments

For solution of differential equations with delayed and forecasting arguments there are effective numerical methods and methods of averaging of differential and of integral and integro-differential operators allowing performing mathematical simulation of a wide range of complex processes and systems. In particular, modelling the dynamics of behaviour of potentially hazardous industries, based on statistical information about the objects.

The aggregate model for the development of nuclear energy facilities is constructed and studied in [2-6]. It allows computer experiments to identify the interesting features in development of nuclear energy industry or an individual nuclear power plant, the optimal strategy, identification the critical and dangerous situations, etc. To some extent this can contribute to improving the management of the relevant objects and reduce their negative impact on the environment.

Since such complex objects in most cases do not allow constructing the precise deterministic mathematical model because of the large number of influential parameters and often unknown links between them, then the aggregate model, built on statistics about the object, can be useful for studying the nature and behaviour of the objects.

2.2 Statement of the problem

This paper focuses on the modelling and analysis of the behaviour of different systems based on the latest achievements of the theory, in particular, the method developed in [1]. The equations for development of the systems with delayed and forecasting arguments are considered. It is shown which must be the intensities of development and other parameters so that the system is kept in a state of stable development and not subject to the modes of oscillations growing in time by amplitude, which rapidly destroy it.

The focus of study is on the peculiarities of the behaviour of systems in a methodological and mathematical aspects, the task of constructing and using such models of applied nature. Also the features performances of a number of outstanding tasks are discussed as well.

3 Development of the mathematical models for complex systems

Development of the physical and mathematical models for evolution of complex systems in a rather general setting requires consideration of the delayed and forecasting arguments, because development of the system is really accompanied by some delays compared to the planned indicators due to various reasons and orientation for leading indicators what is known as the term "foreseen adaptation" [1].

3.1 The simplest equation of development

Such phenomena have actually been observed in a number of different processes and systems as a wildlife and technical origin. First the equation for system development was considered already in the 18-19 centuries in the following form

$$\frac{dy}{dt} = ky, \qquad (1)$$

where y,t respectively are the function describing evolution of some magnitude, and time, k is the growth coefficient of the system, which generally can be function of time.

For the first time the solution of equation (1) for the simplest case with a constant growth rate, was received by the priest Malthus who made a conclusion about exponential growth of the population over time, if it is described by the equation (1). A law of population growth was seen as threatening to society and spawned the philosophical course of Malthusianism, which justified the war as a necessary mechanism of regulation of population growth. It was a big mistake based on a simplified growth equation of the system.

In fact, later on it was shown that the system can start development of the law (1), and further the coefficient k depends on time, e.g., decreases, following the hyperbolic law (algometric process) and, therefore, the function increases with time slowed, not exponential law. Since that time, there were many attempts to use this as a simple equation of development, and many others, more accurately given the characteristics of the systems under consideration. Thus, the observation of the number of systems has shown that development processes are better described in several other equations: over time, in some cases, coefficient *k* is being gradually or abruptly falling down, and this leads to solutions, in which there is no exponential growth function.

This behaviour corresponds better to the realities of different systems of diverse nature: first there is an intensive development process, which is adjusted according to the results of development and analysis of development needs.

3.2 The peculiarities of the real systems

In the real systems some delays and forecasting in time always are met (the first ones are caused by delays in control mechanisms, and the second ones are available in the result of planning and orientation the control system to the desired future performance rather than current real state).

3.2.1 Nonlinear effects

In many systems the linearity of development is broken by more complex behaviour [2-6], which in the example of equation (1) can be demonstrated as follows:

$$\frac{dy}{dt} = ky(A - y), \qquad (2)$$

where *A* is the maximum possible value of *y*.

Limit values are defined by natural or artificial means under the system. For example, it may be a limit to the number of population under the existing conditions with respect to existing power supplies, the number of workers for the industry, if the industry is considered, and the limiting possible value is known, a limited amount of financial provision of development banking institutions and the like.

In such cases, the fact that the closer the functionality of the growth of the system to its limit, the growth rate of the system will naturally decrease, takes into account in equation (2) and upon reaching the border, the further growth stops at that level $y=A$.

3.2.2 Nonlinearity due to time delay effects

Another kind of the system (1) nonlinearity may arise due to dependence of the coefficient k against time and the function *y* against deviating arguments [1-6], therefore it yields:

$$\frac{dy}{dt} = ky(t - \tau), \qquad (3)$$

where τ - delay of the system in time. To solve equation (3) is necessary to specify initial conditions on the time interval τ that precedes the starting point in time or to examine the mechanism of Delay (lag) only after a period of time τ. This is a significant feature of equations with delay, which greatly complicates their solution (the numerical solution of differential equations with automatic selection of time step must approximate point values with a shift in time, which may not be available during automatic partitioning of the time interval).

Equation (3) has a much broader range of applications to modelling the development of complex systems because it takes into account the possibility of delays in development of the system (process) relative to the current state of the system.

3.2.3 Evolutionary and destroying regimes in the equations with shifted arguments

If the growth coefficient k in equation (3) is positive, increasing functions u, if it is negative – attenuation system (decrease of the function *y*, that is, the drop development, extinction of populations, reducing the Bank's funding, etc. - depending on the nature of the complex system that is modelled).

The equations with shifted arguments have more complex modes and features, in particular, critical stages of development and the possible instabilities that rapidly lead to the destruction of the system [1]. Such regimes are particularly important and must be investigated thoroughly. In complex real systems the equations of the type (3) can be many different and each of the plurality of interrelated system parameters may have its own delay, which significantly complicates the mathematical model of the system [2-6].

4 Solution of the equations with shifted arguments

4.1 Development equation with time delay

The solution of equation (3) with the constant time delay and can a coefficient of growth can be sought in the form similar to the solution of equation (1):

$$y = y_0 e^{zt} \qquad (4)$$

where y_0 is the initial value y at $t = 0$, $z = u + iv$ are the Eigen values of the differential operator, u, v respectively, the real and imaginary part of the Eigen values, $i = \sqrt{-1}$ is the imaginary unit.

By substitution of the requested solution in the form (4) into the equation (3) the equation for calculation of the Eigen values is got (after deletion by e^{zt}):

$$z = ke^{-z\tau}. \quad (5)$$

Applying the Euler's formula for exponents of imaginary numbers to the equation (5) for real and imaginary parts of the (5) yields the following:

$$u = ke^{-u\tau}\cos v\tau, \quad v = -ke^{-u\tau}\sin v\tau, \quad (6)$$

which implies that at $v=0$ will be two cases: exponentially increasing ($u>0$) and exponentially decreasing ($u<0$) in time processes.

When $v\neq 0$ are, respectively, the oscillation modes of development of the system with exponentially decreasing ($u<0$) and exponentially increasing ($u>0$) amplitudes become available. In the first case, the process of vibration decays over time and is sustainable (can lead to the degeneration of the system, the cessation of its operation), whereas in the second case, growing over time, the vibrations will quickly destroy the system.

For example, in the case of financial systems this means that the rocking of growth and decrease will lead to a total collapse. So you need to find the conditions under which it is possible to prevent mode oscillations for the increasing amplitude and to control the system, so that it has to evolve, growing smoothly, without oscillations.

From these general speculations one can, for example, come to conclusion that the financing of the project should be closely monitored for features of the development, which are modelled by appropriate equations and the possible scheduled delays in time. If the time delay exceeds the upper limit for the system, there may be fluctuations in the system parameters growing over time. The growing amplitude of oscillations may destroy the system. Thus, with the time delays in the development up to an extreme level nothing dangerous happens. But when the limits are exceeded, the whole system quickly collapses due to growing oscillations.

4.2 Critical levels of the system and its control in the smooth pre-crisis regime

The study of equations (6) when $v\neq 0$ concerning an occurrence of values $u>0$ leads to the following conditions:

$$v\tau = (\tfrac{3}{2} + 2n)\pi, \quad k\tau = v\tau = (\tfrac{3}{2} + 2n)\pi, \quad (7)$$

where n= 0, 1, 2,... Analysis of (7) shows that the first critical value, which causes fluctuations of the system with growing amplitude will be $k\tau=3\pi/2$. Therefore, from the expression (6) we get the following: $u\tau = \frac{3}{2}\pi e^{-u\tau}$, and this leads to the numerical solution of $u\tau = 1.293$.

Thus, a stable development of the system is possible only to certain limits, and it will be a quick breakdown of the system due to the growing fluctuations. To prevent this, it is necessary to manage growing system (process) to it exponential or other nature growth continued no further as time delay value $\tau = 1.293/u$ is reached. And this means that for a given level of growth u the constant increase in the time delay of the system development is permitted only up to a critical value of delay $\tau = 1.293/u$, followed by rapid destruction of the system due to oscillations of increasing amplitude.

Based on the above mentioned, the strategy of sustainable development of the system with time delay requires to manage the system so that, starting from any value of the growth rate of the system $u = u_1$, once it attains a critical value of delay $\tau = \tau_1 = 1.293/u_1$, the system control must reduce the time delay or reduce the rate of the system's development. These features of system development can be clearly traced in Fig. 1:

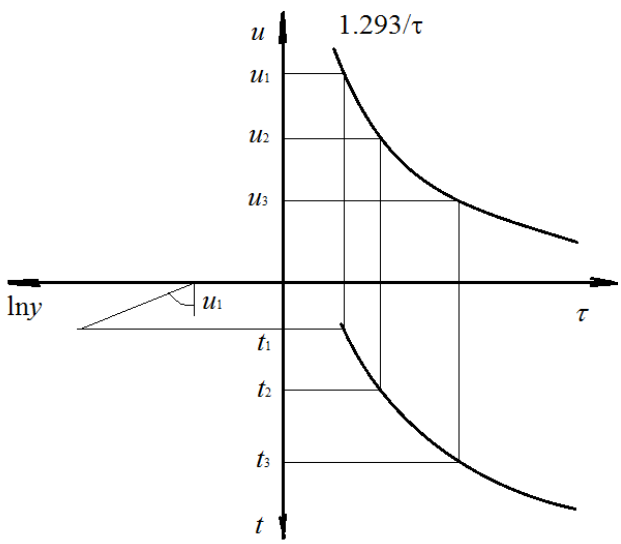

Fig. 1 Critical levels of dependence of the growth rate of the system versus the time delay and corresponding values y and time delay against time

Here the critical levels of dependence of the growth rate of the system and the associated delay in the first quadrant show the critical dependence of the rate of time delay for the system's development. And in the fourth quadrant the dependence of the lag (time delay) is shown, while in the third – lny versus time.

4.2.1 The regimes of stable development

Regimes of stable development of the system according to Fig. 1 are below the curve $u = 1.293/\tau$ shown in the first quadrant (the border region for sustainable development). Therefore, the development can predict optimally in the following way. If the growth of the system starts with the given desired rate of u_1 at the beginning of such system's grow rate, it may be maintained only until the time $t = t_1$, when the increasing delay of the system at the time $\tau_1(t_1)$ becomes critical (line $u = u_1$ crosses a critical curve $= \frac{1.293}{\tau}$) at the point $\tau_1(t_1)$. Then further growth of the system with the specified rate is impossible, provided the further growth with the same time delay.

It is necessary to reduce the rate of growth of the system, for example, up to some value $u = u_2$, then there is an additional system resource regarding the increase in the delay to the intersection of the critical curve at a lower level of growth.

If possible, for sustainable development it is necessary to reduce the growth of time delay for the system, which in many cases is poorly controlled or not controlled at all. Under the case of uncontrolled time delay one can improve the situation and control the system reducing the growth rate every moment when the critical values of time delay are got.

4.2.2 Optimal control of system's development

In reality, the shift points of the control system according to the above described scenario can be realized in the following way. For example, the Bank finances the project, which for the equation of development has been identified in a specified form, whose solution was found.

The function $y(t)$ for sustainable development is known. If one needs to take into account some possible time delays in the system and manage it for optimal development and lack of critical modes, which destroy the system, it is necessary to determine a delay by comparing the characteristics of the system in real-time calculated, as shown in Fig. 2. The forecasted system parameters $y_p(t)$ obtained without taking into account the possible time delay in comparison with such parameters $y(t)$ obtained for a given delay are compared to identical values, resulting in delays expressed by the following expressions: $\tau_1 = t_1 - t_{p1}$, $\tau_2 = t_2 - t_{p2}$, $\tau_3 = t_3 - t_{p3}$, and so on.

Thus, it is possible to determine the actual delay, if the results of the behavior of a real system are known. This allows determining the optimal modes of development of the system and preventing appearance of the critical (catastrophic) situations.

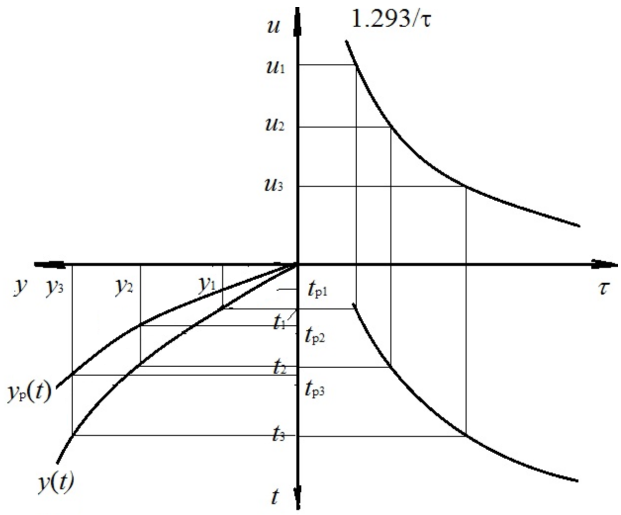

Fig. 2 Comparative characteristics of the system's development of the real system with time delay and forecasting system without delays

4.2.3 Important features of system's development

In the monograph [1], in addition to theoretical issues a large number of examples of applications were considered. Some amazing features of the development of bright diverse fields' systems with time delays were revealed and analyzed in detail. In particular, the question concerning development strategy in the region below the critical one (below the critical curve, shown in Fig. 1) to what extent should reduce time delay or the growing rate to remain sustainable exponential development was stated. How to rebuild the system so that it continued to grow?

It is a very important issue for many natural and technical systems. So, practice has shown that most financial and economic systems do not receive such control and after a period of intensive development get into a state of slow development or stagnation or total destruction. This is because the laws of development for the systems with time delays and forecasting, with the manifestation of significant nonlinearities are too complex for comprehending without solid mathematical modelling and optimization of strategy for development, without taking into account the most essential features of the behaviour of the system, the switching mechanisms of the adjustment systems for the future desirable characteristics.

Any control of complex systems has not only to consider the time lag, but it is also based on the forecast, and therefore the mechanism of adjustment of the managed system, its sustainability must be proactive. Therefore, in many cases the development management of the systems requires simultaneously and consistently take into account both the time delay and the switching mechanisms of adjustment (adaptation) of the system under the future features.

4.3 Features of the equations with forecasting terms by time

To study characteristic mechanisms of timing and their impact on sustainable development of systems it can be considered, for example, the following mathematical model:

$$\frac{dy}{dt} = ky(t+\tau), \qquad (8)$$

where τ is the time forecasting term of the system.

Equation (8) means that the rate of development of the system described by such a mathematical model, that focuses not on current performance, as in equation (1) and not on previous performance, as in equation (3), but on a future performance.

To solve equation (8) it is necessary to specify initial conditions on the time interval from zero to τ. For example, funded the project and its execution at each moment of time has a rate that does not match the current level of funding, and one that meets the future, a higher level, which is the head. This is a case of development in advance. Modelling the development of complex systems with forecasting terms has interesting features that are worth exploring and are used for optimal control of systems' development (processes).

4.3.1 Solution of the equation with forecasting

Considering solution of the equation (8) in the form (4) and substituting the solution sought $y = y_0 e^{zt}$ into equation (8), we can obtain the equation to determine the Eigen values (after reduction for e^{zt}):

$$z\tau = k\tau e^{z\tau}. \qquad (9)$$

After researching possible solutions to equation (9) it follows that the highest value of $k\tau$ at which there exists a solution of equation (9) satisfies the conditions:

$$u\tau = 1, \quad k\tau = 1/e, \qquad (10)$$

where it is clear that as similarly to equation with time delay, when *v=0* there are two cases: exponentially increasing (*u>0*) and exponentially decreasing (*u<0*) in time. When *v≠0* are, respectively, the oscillating modes of development in a system with exponentially decreasing (*u<0*) and exponentially increasing (*u>0*) amplitudes take place. In the first case, the process of oscillation i damped over time and is sustainable (can lead to the degeneration of the system, the cessation of its operation). But in the second case, growing over time, the oscillations quickly destroy the system. Only here the conditions (10), in contrast to the case of delay in the system, define the lower boundary of the steady growth of the system.

The rates of development of the system, taking into account forecasting in time that are below $u = 1/\tau$, will lead to increasing time-oscillations of the system, leading to its destruction.

4.3.2 Strategies of the correlated time delay and time forecasting in system development

The above considered forces to conclude that for sustainable development of the system's development both processes of the time delays and time forecasting must be correlated in a system development [1]. That can be considered as the global law of optimal control for systems.

Thus, from expressions (10) and earlier obtained similar expressions for critical levels of delay systems, it follows that the ratio of the rate of growth for the delayed type and forecasting type systems' development is approximately equal to 1,293. By results of researches [1], it is possible to recommend the following method for determining the development of the system and its critical points, illustrating the process according to Fig. 3, where in the first quadrant are two critical curves, respectively, for strategies with a delay (top) and forecasting (bottom).

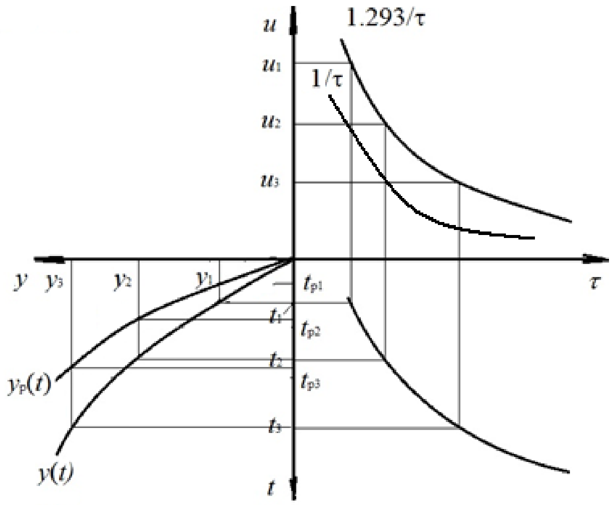

Fig. 3 The scheme for stable development with correlated time delay and forecasting strategies

In any case, a stable development is possible only between these two strategies and the actual behaviour of the system can be only inside the region between them.

4.3 Example of building the control strategy for object with prevention of critical regimes

Example of building the control strategy for system with prevention of critical regimes in steady development is considered below. Given the peculiarities of the behaviour of systems development with shifted arguments, we can construct a control strategy in the following way.

Let at the initial moment of time (*t = 0*) the intensity of development equal to u_0, delay is absent (*τ=0*), so the system according to (4) develops according to the law

$$y = y_0 e^{u_0 t} \quad (11)$$

at time $t = t_1$, where the delay is equal to $\tau_1(t_1) = 1{,}293/u_0$, then it must be the switching rate of development of the system to the border region leading arguments $u_1 = 1/\tau_1 = u_0/1{,}293$.

Next, the system continues stable growth law (11) with the new rate u_1 point-in-time $\tau_2(t_2) = 1.293/u_1$when the delay reaches a critical level of development for a strategy with delay (see Figs 1-3). Then again, you can go to the lower critical curve, which corresponds to the development strategy with forecasting, that is $u_2 = 1/\tau_2 = u_1/1.293$. And so on to provide the desired level of system development.

It is clear that over time development of the system with growing time delay becomes ineffective so that it's best to stop and then continue the development, starting it again with zero delay.

4.3.1 Control of the time delay

For the control of the time delay it is useful to have function $\tau(t)$, which can be obtained for example as a second order polynomial satisfying the following conditions: $t = 0,\ \tau = 0;\ t = t_1, \tau = \tau_1;\ t = t_2, \tau = \tau_2$. Such function has a form

$$\tau = (a_1 + a_2 t)t, \quad (12)$$

where $a_1 = \frac{\tau_1 t_2^2 - \tau_2 t_1^2}{t_1 t_2 (t_2 - t_1)}$, $a_2 = \frac{\tau_2 t_1 - \tau_1 t_2}{t_1 t_2 (t_2 - t_1)}$.

Hence, considering the above formula (12) implies the following expression for delay as a function of time:

$$\tau = \frac{1.293t}{t_1 t_2 (t_2 - t_1)} \left[\frac{t_1}{u_1}(t - t_1) + \frac{t_2}{u_0}(t_2 - t) \right]. \quad (13)$$

4.3.2 Stable development of income system

Thus, the stable development of the system is according to formula (11) to the next critical point in time where the tempo switches to the next and so every critical moment. And the shift points of the rate of development are calculated by formula (13).

For example, $y_0 = 1$ billion USD, the initial growth rate of the financial plan adopted is $u_0 = 1.293/year$, so for the year according to the formula (11) needs to obtain $y = y_0 e^{1.293} \approx 3.63 y_0$, that is, to 3.63 billion USD of income. But the time delay is $\tau_1 = 1$ year. So for next year one needs to change the growth rate to $u_1 = 1/year$.

Over the next year the revenue should be about 2.71 billion USD. Having the delay equal to $\tau_2 = 1.293/year$, so for the time of 1.293 years income will be 3.63 billion USD. And then one needs to change the rate of development to the level of $u_2 = 1/1{,}293 \approx 0{,}77$. The latest rate gives an income of about 2.15 billion USD for the year. And so on.

5 Nonlinear systems

Equation (1) is linear; equation (2) is nonlinear. The nonlinearity in the system by the mathematical model (2) is relatively simple. But the equation (3) with delayed arguments seems linear, but it contains the worst type of nonlinearity (2).

5.1 Monotonous systems with time delay satisfying the Elsholtz theorem

If in equation (3) replace the function of delayed argument *y(t-τ)* according to theorem of Elsholtz [8], which for monotone functions in the Taylor series relative to the point *t* by τ with accuracy to the linear terms, since the linear approximation will be the most precise in this case according to the theorem: $y(t - \tau) \approx y(t) - \tau \frac{dy}{dt}$, then the equation (3) takes the following approximate form:

$$(1 + k\tau) \frac{dy}{dt} = ky(t), \quad (14)$$

Equation (14) differs relatively little from (1) and the delay affects only the time deformation. But when the function is non-monotonic, and such solutions can be oscillatory with increasing amplitude, the theorem of Elsholtz does not take place and instead the left side in (14) there will be a complete Taylor series to the derivative of the function *y* and the powers of delay *τ*. The last case is exactly critical, examples of which have just been considered.

5.2 Strong nonlinear effect

Significantly stronger effect of the time delay in the case of the nonlinear equation (2) is:

$$\frac{dy}{dt} = ky(t-\tau)(A - y(t-\tau)), \qquad (15)$$

which even for the case of monotonous growing, when Elsholtz theorem satisfies, leads to the strong nonlinear equation of the form

$$(1 + Ak\tau - 2k\tau y)\frac{dy}{dt} + \left(\tau\frac{dy}{dt}\right)^2 = ky(A - y), \quad (16)$$

containing the nonlinear terms of different type.

The nonlinearity of the systems and processes always cause unpredictable properties and characteristics of their behaviour including the existence of various special and critical parameters and system modes.

Possible points of bifurcation in a system correspond to the situations, where the system abruptly jumps from one mode to another (usually completely different from the previous one). There are available also strange attractors (sets of trajectories in the phase space of the system motion in which all other trajectories approach under any initial conditions), and the other features.

The study of nonlinear behaviour of systems (15), (16) in many cases is crucial for understanding their behaviour, which can be highly unpredictable. So, knowing the basic critical modes and the corresponding parameters and laws of management, one can try to optimize the system and to prevent the ingress of an object into a critical state, as shown in some relatively simple examples above.

5.3 Complex systems with a number of parameters and shifted arguments

Complex systems with a large number of governing parameters and individual values of possible delays and forecasting terms are difficult to investigate in the above-mentioned way. But the general patterns are similar on a qualitative level and must be taken into account.

For example, the aggregated mathematical model of a potentially hazardous object in nuclear energy (like the mathematical model can be developed also for other objects) has the following form [2-6]:

$$\frac{dz_4}{dt} = [b_{40} + b_{43}z_3(t-\tau_{43}) + b_{44}z_4(t-\tau_{44}) + b_{45}z_5(t-\tau_{45}) + b_{46}z_6(t-\tau_{46})]z_4(t-\tau_{40}),$$

$$\frac{dz_1}{dt} = [b_{10} + b_{11}z_1(t-\tau_{11}) + b_{12}z_2(t-\tau_{12}) + b_{13}z_3(t-\tau_{13})]z_1(t-\tau_{10}),$$

$$\frac{dz_2}{dt} = [b_{20} + b_{21}z_1(t-\tau_{21}) + b_{22}z_2(t-\tau_{22}) + b_{23}z_3(t-\tau_{23})]z_2(t-\tau_{20}),$$

$$\frac{dz_3}{dt} = [b_{30} + b_{31}z_1(t-\tau_{31}) + b_{32}z_2(t-\tau_{32}) + b_{33}z_3(t-\tau_{33}) + b_{34}z_4(t-\tau_{34}) + b_{35}z_5(t-\tau_{35}) + b_{36}z_6(t-\tau_{36})]z_3(t-\tau_{30}),$$

$$\frac{dz_5}{dt} = [b_{50} + b_{53}z_3(t-\tau_{53}) + b_{54}z_4(t-\tau_{54}) + b_{55}z_5(t-\tau_{55}) + b_{56}z_6(t-\tau_{56})]z_5(t-\tau_{50}),$$

$$\frac{dz_6}{dt} = [b_{60} + b_{61}z_1(t-\tau_{61}) + b_{62}z_2(t-\tau_{62}) + b_{63}z_3(t-\tau_{63}))]z_6(t-\tau_{60}),$$

(17)

where τ_{ij} is the time delay for the corresponding parameters. Such indexes in a more general case also, in turn, depend on the time (and possibly each other). Here z_i - system parameters, b_{ij} - coefficients of the mathematical model, which are determined for each model based on the results of its functioning using the methods of identification.

The system of differential equations (17) discussed above is much more difficult equations of the type (15), (16) and allows the analysis of critical levels by described methods. It was investigated numerically. From Figs 4-6 it is seen that modes can be critical and catastrophic, and detect parametric dependencies and characteristics is much more complicated than the above, where the concept of main features is clearly understandable.

5.4 Examples of computer simulations for potentially hazardous objects

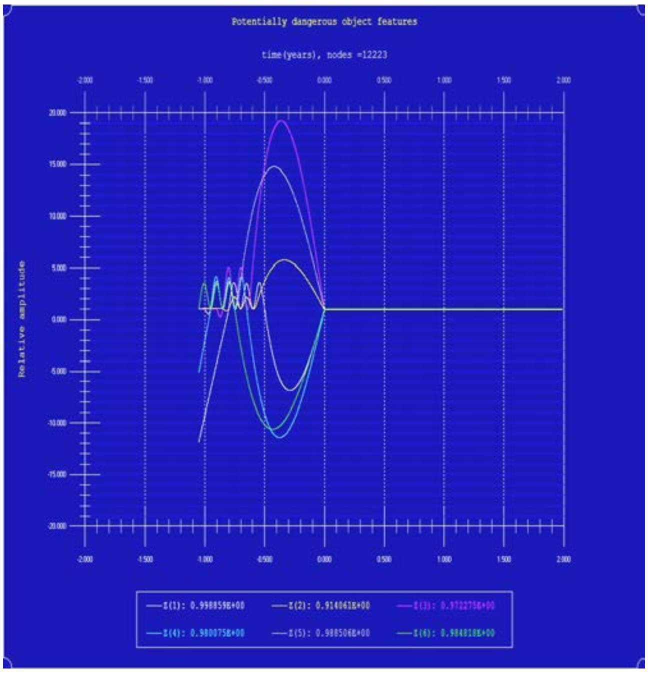

Fig. 4 Oscillation processes in potentially hazardous object's development

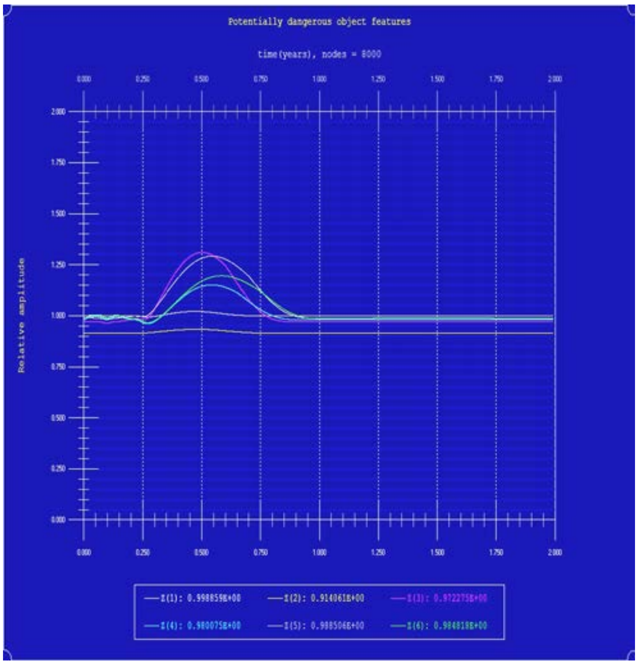

Fig. 5 Oscillation processes in the development of aggregate nuclear power energy object

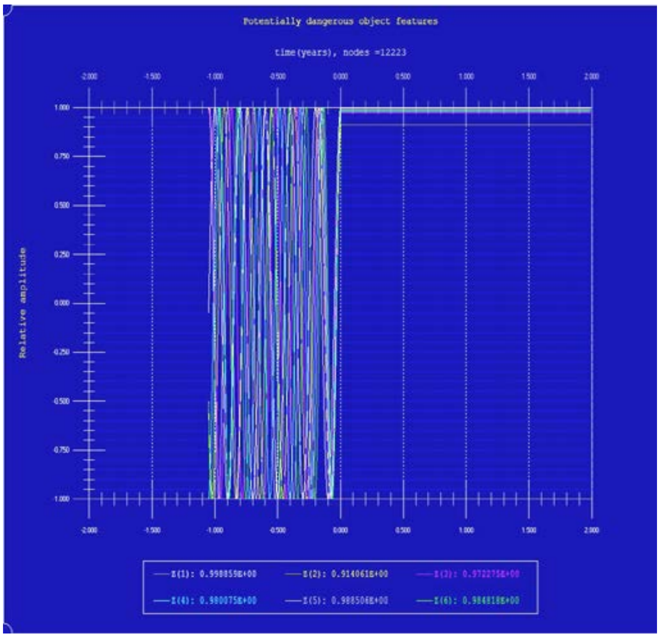

Fig. 6 Functions z_i (for $t>0$) and interpolations for interval till the initial moment ($t<0$)

Developed aggregated mathematical models of complex objects for a variety of applications, taking into account the effect of deviating arguments and nonlinear effects, allowed investigating some features of objects by computer simulation. So, it is possible to set some negative impact on the environment and people, possible ways of weakening the negative actions and eliminate them, to determine the features of development dynamics of the object, change the safety culture on it and possibilities to improve it. Construction and research of aggregated dynamic models on a computer can be useful also for the study of tactical and strategic management of facilities at various levels (enterprise, industry or simply complex technical devices, etc.).

The results of computer situational simulations in a wide range of variable parameters given the shift of the arguments enable to identify and explore some of the most important features and phenomena as an optimizing plan, and dangerous for the development and operation of the facility. The identified critical conditions and catastrophic situations and scenarios for which the system can get to them, should be studied carefully with the purpose of their conditions in a real object, which is investigated by simulation.

6 Conclusion

The article considers the approaches for the development of the systems with shifted arguments and their implementation. The result presented in this paper, modeling methodology and analysis of the obtained data, modeling of complex systems with shifted arguments showed how knowledge about the dynamics of development allows systematizing the data about the critical situation and the mutual influence of different parameters on each other, in which a variety of undesirable or even catastrophic events become possible.

Example development of financial system showed a variation of the management policies between the delays and forecasting based on piecewise continuous matching, the beginning when development starts from the selected tempo, which is maintained until achieving a critical level with increasing delay over time, and then switches to mode critical curve of the trajectory, which corresponds to strategy of development ahead. And so it's constantly switching between strategies with delay and forecast.

References:

[1] Zhirmunsky A.V., Kuzmin V.I. Critical Levels of Natural Systems Development.- L.: Nauka, 1990. 222 p.

[2] Kazachkov I.V., Chesnokov Ye.V. and Kazachkova O.M. *Modelling of Potentially Hazardous Objects with Time Shifts*// WSEAS

Trans. on Business & Economics. 2004, Issue3, №1, p. 37-43.

[3] Jamshid Gharakhanlou, Ivan V. Kazachkov, Oleksandr V. Konoval. Development and Investigation of the Mathematical Models for Potentially Hazardous Nuclear Power Objects with Deviated Arguments// WSEAS Trans. on Applied and Theoretical Mechanics.- 2013.- Vol. 8.- Issue 4.- P. 241-257.

[4] Jamshid Garakhanlou, I.V. Kazachkov. Mathematical modeling of potentially hazardous nuclear power objects with shifted arguments// J. Nuclear and Radiation Safety.- 3 (55) .- 2012.- P. 21-26.

[5] Jamshid Garakhanlou, I.V. Kazachkov. Development and Investigation of Aggregate Models for Nuclear Objects with Time Shifts// J. Nuclear and Radiation Safety.- 2 (54) .- 2012.- P. 36-41.

[6] Jamshid Gharakhanlou, Oleksandr V. Konoval, Ivan V. Kazachkov. About development of the aggregate mathematical models for complex non-linear systems with deviated arguments// Recent advances in mathematical methods, mathematical models and simulation in science and engineering - Interlaken, Switzerland 2014.– P. 42-47.

[7] L.E. Elsholz, S.B.Norkin. Introduction to the theory of differential equations with delay.- Moscow, Nauka, 1971.- 296p.

[8] Allen P.M. Evolution, Population Dynamics and Stability/ Proceedings of the National Academy of Sciences of the USA.- 1976.- March.- Vol.73.- No.3.- P.665-668.

[9] Baker C.T.H., Paul C.A.H., Wille D.R. Issues in the numerical solution of evolutionary delay differential equations// Advances in Comput. Math.- 1995.- V. 3.- P. 171-196.

[10] Yan J. Oscillation of first-order impulsive differential equations with advanced argument// Computers and Mathematics with Applications.- 2001.- V. 42.- N 6.- P. 1353-1363.

[11] Hansheng Wu. Adaptive robust control of uncertain nonlinear systems with nonlinear delayed state perturbations// Automatica.- 2009.- V. 45.- P. 1979-1984.

[12] Yanlai Liang, Lijie Li, Lansun Chen. Almost periodic solutions for Lotka–Volterra systems with delays// Commun Nonlinear Sci Numer Simulat.- 2009.- V. 14.- P. 3660–3669.

[13] Svante Björklund, Lennart Ljung. An improved phase method for time-delay estimation// Automatica.- 2009.- V. 45.- P. 2467-2470.

[14] Guirong Jiang, Qigui Yang. Complex dynamics in a linear impulsive system// Chaos, Solitons and Fractals.- 2009.- V. 41.- P. 2341–2353.

[15] Yuanliang Zhang, Jae Byung Park, Kil To Chong. Controller design for nonlinear systems with time delay using model algorithm control (MAC)// Simulation Modelling Practice and Theory.- 2009.- V. 17.- P. 1723–1733.

[16] Jian-Qiao Sun, Bo Song. Control studies of time-delayed dynamical systems with the method of continuous time approximation// Commun Nonlinear Sci Numer Simulat.- 2009.- V. 14.- P. 3933–3944.

[17] S.M. Lee, Ju H.Park. Delay-dependent criteria for absolute stability of uncertain time-delayed Lur'e dynamical systems// Journal of the Franklin Institute.- 2010.- N 347.- P. 146–153.

[18] Liping Wen, Wansheng Wang, Yuexin Yu. Dissipativity and asymptotic stability of nonlinear neutral delay integro-differential equations// Nonlinear Analysis.- 2010.- V. 72.- P. 1746-1754.

[19] P. Pue-on, S.V. Meleshko. Group classification of second-order delay ordinary differential equations// Commun Nonlinear Sci Numer Simulat.- 2010.- V. 15.- P. 1444–1453.

[20] Rajeev K. Azad, J. Subba Rao, Ramakrishna Ramaswamy. Information-entropic analysis of chaotic time series: determination of time-delays and dynamical coupling// Chaos, Solitons and Fractals.- 2002.- V. 14.- P. 633–641.

[21] Begun V.V., Begun S.V., Kazachkov I.V., et al. *Safety culture on nuclear power plants of Ukraine*.- Kyiv.- NTUU "KPI".- 2009.- 386 pp. (In Russian).