Design of Robust Fractional Order Proportional Integral Controller Based on Vector Method

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Abstract: This paper detailedly discusses the geometry idea, design steps and algorithm implementation in designing a FOPI controller based on a vector method. It takes first order controlled plant, first order plus time delay controlled plant and fractional order plant as examples, using traditional method and vector method to solve the parameters of the FOPI controller. The results show that the vector method has the same dynamic response performance as the traditional method, which demonstrates the reasonability of the vector method. Compared with traditional method, the FOPI controller design method based on the geometry idea (vector method) is simpler and more convenient. Besides, the proposed method overcomes the traditional phenomenon of multiple solutions of controller parameters.

Key–Words: Fractional order PI controller; Vector method; Tuning parameters; Robust; Optimization; Unique solution

1 Introduction

Fractional calculus is a more than 300-years-old topic. Because it can be a powerful and widely used tool for better modeling and control of processes, in recent years, an increasing number of studies can be found related to the application of fractional calculus in many areas of science and engineering [1–3], such as continuous stirred tank reactor in chemical process, the path tracking control of tractors, unmanned aerial vehicle system, and hybrid adaptive cruise control [4–7].

Design of a fractional order proportional integral (FOPI) controller which aims at enhancing the system control performance is an important issue in studying fractional calculus control area. Monje et al. [8–10] designed the $PI^{\lambda}D^{\mu}$ controller by three constraints, phase margin constraint, amplitude margin constraint and the constraint of sensitivity function and compensation function. Barbosa et al. [11] designed the $PI^{\lambda}D^{\mu}$ controller based on ideal Bode transfer function. Besides, other design methods of the fractional order controller based on particle swarm [12, 13] and neural network [14] have been proposed. Chen added the constraint of robustness to open-loop gain variation on the basis of the constraints of phase and amplitude margins, and proposed a new design method of robust PI^{λ} controller. That demands the phase is flat around the crossover frequency ω_c in phase-frequency characteristic [15–22]. In our previous work, Chen's method has been successfully applied to different plants [19, 21, 22] and some experiments of real systems have been done to verify the effectiveness of the method [19].

References [15–19, 21, 22] give the traditional design processes of a robust FOPI controller. However, it exists enormous calculated amount and complex parameter tuning processes in designing a FOPI controller because of the extra real parameters λ. Besides, sometimes it would appear a new phenomenon that solution parameters of controller are not unique. Multiple solutions will make engineers hard distinguish which parameters they should adopt to design a FOPI controller in engineering practice. Compared with traditional method, the proposed method can settle the problems.

This paper proposes an optimization algorithm based on the vector method used for simplifying the calculated amount and parameter solution processes of a robust FOPI controller. The idea of the simplified algorithm is that a vector model of fractional order controller is built in advance in complex plane, and then geometrical relationship in complex plane is used to solve the controller parameters. This paper takes robust FOPI controller (a FOPI controller to achieve the robustness to the loop-gain variations) as an example and discusses the FOPI controller parameter solution process in detail. The results show that not only does the proposed algorithm decrease the calculated amount, but also the FOPI controller parameters which are solved out by the method are unique and effective.

The paper is organized as follows: In Section II, vector model of an integer order PID controller and vector model of a fractional order PID controller are built in complex plane. In Section III, on the basis of traditional robust fractional order PID controller design method, tuning parameter rules for robust FOPID controller based on vector model are established. Section IV is the key content. It illustrates how the vector model of fractional calculus is applied to solution of FOPI controller parameters. Section V concludes the parameter solution process of robust FOPI controller based on vector method. Section VI proves the uniqueness of controller parameter solution based on the vector method and overcomes the traditional phenomenon of multiple solutions of controller parameters. In Section VII, we take first order controlled plant, first order plus time delay controlled plant and fractional order plant as examples to design a FOPI controller, and give the simulation diagram and analysis. Finally, the paper is concluded in section VIII.

2 Vector Model of Controller

We can use mapping approach to map a controller into complex plane and build the vector model of controller.

2.1 Vector Model of an Integer Order PID (IOPID) Controller

The transfer function of IOPID controller can be expressed as,

$$C(s) = K_p + \frac{K_i}{s} + K_d s \tag{1}$$

The frequency response can be got,

$$C(j\omega) = K_p + \frac{K_i}{j\omega} + K_d j\omega \tag{2}$$

So the vector model of IOPID controller can be shown in Fig.1.

From Fig.1 we can see that the IOPID controller is made up of three parts, which respectively are proportional vector, integral vector and differential vector, where the proportional vector is located in positive real axis, the integral vector is located in negative imaginary axis and the differential vector is located in positive imaginary axis. The IOPID controller vector which is synthesized by the three vectors can only appear in right half complex plane. The module value of IOPID controller vector and its phase angle change along with the variation of controller parameters K_p, K_i and K_d.

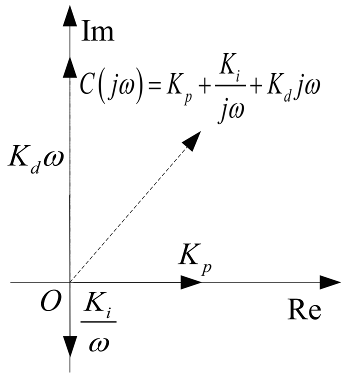

Fig. 1. Vector model of IOPID controller

2.2 Vector Model of a Fractional Order PID (FOPID) Controller

The transfer function of FOPID controller can be expressed as,

$$C(s) = K_p + \frac{K_i}{s^\lambda} + K_d s^\mu \tag{3}$$

The frequency response can be got,

$$C(s) = K_p + \frac{K_i}{(j\omega)^\lambda} + K_d (j\omega)^\mu \tag{4}$$

So the vector model of FOPID controller can be shown in Fig.2.

From Fig.2 we can see that the proportional vector of the FOPID controller vector is still located in positive real axis. However, because of existing non-integer order integral operator $\lambda \in (0, 2)$ and non-integer order differential operator $\mu \in (0, 2)$, it will make integral vector and differential vector distribute in any position of complex plane with the variations of λ and μ. So the synthetic FOPID controller vector can appear in any position of complex plane.

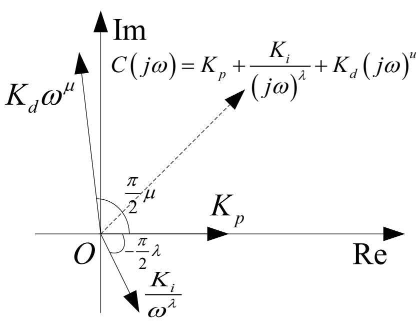

Fig. 2. Vector model of FOPID controller

3 Tuning Parameter Rules for Robust FOPID Controller Based on Vector Model

According to the known plant $P(s)$, unknown controller $C(s)$ and open-loop transfer function $G(s) = C(s)P(s)$, references [15–22] give the tuning parameter rules for robust FOPID controller. On the basis of traditional three constraints, This paper deduces new tuning equations based on vector model.

(i) The amplitude of system's open-loop transfer function at crossover frequency ω_c should satisfy the following expression,

$$|G(j\omega_c)| = |C(j\omega_c)P(j\omega_c)| = 1 \tag{5}$$

(ii) The phase of system's open-loop transfer function at crossover frequency ω_c should satisfy the following expression,

$$Arg[G(j\omega_c)] = Arg[C(j\omega_c)P(j\omega_c)] = -\pi + \phi_m \tag{6}$$

where ϕ_m is the phase margin value.

(iii) In order to insure the robustness to open-loop gain variation, it demands that the phase of system's open-loop transfer function around cut-off frequency ω_c should remain unchanged. It should satisfy the following expression,

$$\frac{d(Arg[G(j\omega)])}{d\omega}\Big|_{\omega=\omega_c} = 0 \tag{7}$$

From expressions (5) and (6), we can get the vector of system's open-loop transfer function $G(j\omega)$ at crossover frequency ω_c,

$$G(j\omega_c) = 1\angle(\phi_m - 180^\circ) \tag{8}$$

The frequency response of the plant is known as $P(j\omega)$, we can obtain the vector of plant at crossover frequency ω_c,

$$P(j\omega_c) = |P(j\omega_c)|\angle P(j\omega_c) \tag{9}$$

According to the expression $G(j\omega) = C(j\omega)P(j\omega)$, we can get the vector of FOPID controller at crossover frequency ω_c,

$$C(j\omega_c) = \frac{1}{|P(j\omega_c)|}\angle(\phi_m - \angle P(j\omega_c) - 180^\circ) = A\angle\theta \tag{10}$$

where

$$A = 1/|P(j\omega_c)|$$

$$\theta = \phi_m - Arg[P(j\omega_c)] - 180^\circ$$

It can be got from expression (7),

$$\frac{d(Arg[P(j\omega)])}{d\omega}\Big|_{\omega=\omega_c} + \frac{d(Arg[C(j\omega)])}{d\omega}\Big|_{\omega=\omega_c} = 0 \tag{11}$$

Assume that

$$\frac{d(Arg[P(j\omega)])}{d\omega}\Big|_{\omega=\omega_c} = \varphi_p \tag{12}$$

$$\frac{d(Arg[C(j\omega)])}{d\omega}\Big|_{\omega=\omega_c} = \varphi_c \tag{13}$$

An equation can be obtained,

$$\varphi_c = -\varphi_p \tag{14}$$

where φ_c and φ_p are respectively phase angle gradient of the controller and phase angle gradient of the plant at the crossover frequency ω_c. So based on the new vector method, the robust FOPID controller design should satisfy the formula(10) and formula(14).

4 Parameter Solution of FOPI Controller Based on Vector Method

The transfer function of FOPI controller can be expressed as,

$$C(s) = K_p + \frac{K_i}{s^\lambda}, (0 < \lambda < 2) \tag{15}$$

The frequency response can be obtained,

$$C(j\omega) = K_p + \frac{K_i}{(j\omega)^\lambda}, (0 < \lambda < 2) \tag{16}$$

So the vector model of FOPI controller can be shown in Fig.3.

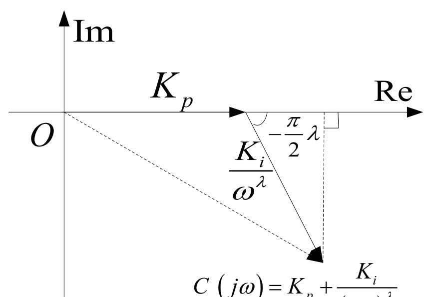

Fig. 3. Vector model of FOPI controller

From Fig.3, we can get the phase angle of FOPI controller,

$$Arg[C(j\omega)] = \arctan\frac{\frac{K_i}{\omega^\lambda}\sin(-\frac{\pi}{2}\lambda)}{K_p + \frac{K_i}{\omega^\lambda}\cos(-\frac{\pi}{2}\lambda)} \tag{17}$$

So the phase angle gradient of FOPI controller can be obtained,

$$\frac{d(Arg[C(j\omega)])}{d\omega}\Big|_{\omega=\omega_c} = \frac{\lambda K_p \frac{K_i}{\omega_c^{\lambda+1}}\sin(\lambda\pi/2)}{\Big(K_p+\frac{K_i}{\omega_c^\lambda}\cos(\lambda\pi/2)\Big)^2+\Big(\frac{K_i}{\omega_c^\lambda}\sin(\lambda\pi/2)\Big)^2} = \varphi_c \tag{18}$$

According to formula (14), we can get,

$$\frac{-\lambda K_p \frac{K_i}{\omega_c^{\lambda+1}}\sin(\lambda\pi/2)}{\Big(K_p+\frac{K_i}{\omega_c^\lambda}\cos(\lambda\pi/2)\Big)^2+\Big(\frac{K_i}{\omega_c^\lambda}\sin(\lambda\pi/2)\Big)^2} = \varphi_p \tag{19}$$

Fig.4 shows the vector model of FOPI controller which satisfies the formula(10),

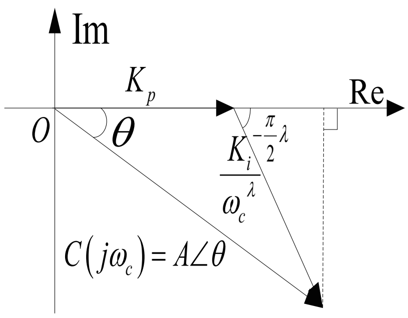

Fig. 4. Specified condition of FOPI controller vector

From Fig.4 and the cosine law, we can get,

$$\frac{K_i}{\omega_c^\lambda} = \sqrt{K_p^2+A^2-2K_pA\cos\theta} \tag{20}$$

According to (20) and laws of right triangle, we can get

$$\cos(-\frac{\pi}{2}\lambda) = \frac{A\cos\theta-K_p}{\sqrt{K_p^2+A^2-2K_pA\cos\theta}} \tag{21}$$

We can obtain other relationships from Fig.4,

$$\Big(K_p+\frac{K_i}{\omega_c^\lambda}\cos(\lambda\pi/2)\Big)^2+\Big(\frac{K_i}{\omega_c^\lambda}\sin(\lambda\pi/2)\Big)^2 = A^2 \tag{22}$$

$$\frac{K_i}{\omega_c^\lambda}\sin(-\lambda\pi/2) = A\sin\theta \tag{23}$$

From equations (19) ,(22) and (23), we can get,

$$\frac{\lambda K_p \sin\theta}{A\omega_c} = \varphi_p \tag{24}$$

So the tuning parameter equations of FOPI controller based on vector model can be concluded as follows,

$$\begin{cases} \frac{K_i}{\omega_c^\lambda} = \sqrt{K_p^2+A^2-2K_pA\cos\theta} & (25)\\ \cos(\frac{\pi}{2}\lambda) = \frac{A\cos\theta-K_p}{\sqrt{K_p^2+A^2-2K_pA\cos\theta}} & (26)\\ \frac{\lambda K_p\sin\theta}{A\omega_c} = \varphi_p & (27)\end{cases}$$

It can be seen that compared with traditional method of solving controller parameters in references [15–19, 21, 22], using the vector method to solve the controller parameters has smaller calculated amount and simpler derivation process.

5 Parameter Solution Process of Robust Fractional Order Controller Based on Vector Model

(i) According to the known plant $P(s)$, solve the amplitude-frequency characteristic $|P(j\omega)|$ and phase-frequency characteristic $\angle P(j\omega)$.
(ii) Given crossover frequency ω_c and phase margin ϕ_m.
(iii) Obtain module value of the plant $|P(j\omega)|$, phase angle of the plant $\angle P(j\omega)$ and phase angle gradient of the plant φ_p in the condition of $\omega = \omega_c$.
(iv) Obtain module value, phase angle and phase angle gradient of the controller. Those are
$A = 1/|P(j\omega_c)|$, $\theta = \phi_m - Arg[P(j\omega_c)] - 180°$ and $\varphi_c = -\varphi_p$.
(v) Solve equations (25), (26) and (27), and get the controller parameters.

6 Uniqueness of FOPI Controller Parameter Solution Based on Vector Method

The uniqueness of FOPI controller parameter solution will be discussed in this section.

(i) From equation (26), one can obtain,

$$\cos\big(\frac{\pi}{2}\lambda\big) = \frac{A\cos\theta-K_p}{\sqrt{(K_p-A\cos\theta)^2+A^2\sin^2\theta}} \tag{28}$$

It is not difficult to see that λ is the continuous function of K_p.

When $0 < K_p < A\cos\theta$, we can get from expression (28)

$$\cos\big(\frac{\pi}{2}\lambda\big) = \frac{1}{\sqrt{1+\frac{A^2\sin^2\theta}{(A\cos\theta-K_p)^2}}} \tag{29}$$

The FOPI controller satisfies the condition of $0 < \lambda < 2$, so the left of equation (29) is the monotone decreasing function of λ and the right of equation (29) is the monotone decreasing function of K_p. So λ is the monotone increasing function of K_p.

When $K_p > A\cos\theta$, the expression (28) can be expressed as,

$$\cos\left(\frac{\pi}{2}\lambda\right) = -\frac{1}{\sqrt{1+\frac{A^2\sin^2\theta}{(K_p-A\cos\theta)^2}}} \quad (30)$$

The left of equation (30) is the monotone decreasing function of λ and the right of equation (30) is the monotone decreasing function of K_p. So λ is the monotone increasing function of K_p.

When $K_p = A\cos\theta$, from (28) we can get $\lambda = 1$. In conclusion, λ is the monotone increasing function of K_p in expression (26).

We define

$$\lambda = g_1(K_p) = \frac{2}{\pi}\arccos\frac{A\cos\theta - K_p}{\sqrt{K_p^2 + A^2 - 2K_pA\cos\theta}} \quad (31)$$

where λ is the monotone increasing function of K_p. That is to say $g_1^{'}(K_p) > 0$.

(ii) From equation (27), we can get λ is the function of K_p and we define

$$\lambda = g_2(K_p) = \frac{A\varphi_p\omega_c}{K_p\sin\theta} \quad (32)$$

According to $0 < \lambda < 2$ and $K_p > 0$, we can get from (32)

$$\frac{A\varphi_p\omega_c}{\sin\theta} > 0 \quad (33)$$

So the derivative of $g_2(K_p)$ is

$$g_2^{'}(K_p) = -\frac{A\varphi_p\omega_c}{K_p^2\sin\theta} < 0 \quad (34)$$

That is to say, λ is the monotone decreasing function of K_p.

(iii) We define

$$h(K_p) = g_1(K_p) - g_2(K_p) \quad (35)$$

We can get the derivative function $h^{'}(K_p)$

$$h^{'}(K_p) = g_1^{'}(K_p) - g_2^{'}(K_p) \quad (36)$$

According to the conclusions from (i) and (ii), we can know that the derived function $h^{'}(K_p)$ is greater than zero. So $h(K_p)$ is the monotone increasing function. So equations (26) and (27), if any, have only one solution. That means the parameters λ and K_p are unique.

Put the parameters λ and K_p into equation (25), the parameter K_i is also unique.

Taken (i), (ii) and (iii) together, the uniqueness of FOPI controller parameter solution is proved.

7 Simulation and Verification

In this section, robust FOPI controllers for first order plant, first order plus time delay controlled plant and fractional order plant are respectively designed based on vector model to verify the effectiveness and correctness of the proposed vector method.

It gives the unit step response when open-loop gain are respectively 0.9, 1 and 1.1(the variation of open-loop gain is $\pm 10\%$). Fig.5 shows the structure chart of FOPI control system. We can change K or $P(s)$ and get simulation results to verify the effectiveness.

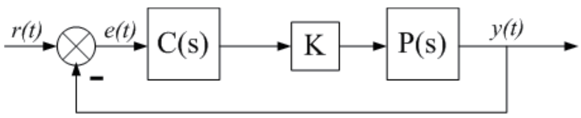

Fig. 5. Structure chart of FOPI control system

• **Case-1**: In reference [21], given the plant $P(s) = 1/(Ts+1)$, where $T = 0.4s$, the gain crossover frequency $\omega_c = 10rad/s$ and the phase margin $\phi_m = 70°$.

According to the known conditions, we can obtain module value of the plant $|P(j\omega_c)|$, phase angle of the plant $\angle P(j\omega_c)$ and phase angle gradient of the plant φ_p in the condition of $\omega_c = 10rad/s$. Those respectively are

$$|P(j\omega_c)| = -12.3dB, \angle P(j\omega_c) = -76°$$
$$\varphi_p = \frac{d(Arg[P(j\omega)])}{d\omega}\Big|_{\omega=10rad/s} = -0.0235$$

And then, we can get module value of fractional order controller $A = 12.3dB$, phase angle of fractional order controller $\theta = -34° < 0$ and phase angle gradient $\varphi_c = 0.0235$. So the fractional order controller parameters are respectively $K_p = 2.376$, $\lambda = 0.7299$ and $K_i = 13.5761$. The transfer function of fractional order controller can be obtained as follows,

$$C_1(s) = 2.376 + 13.5761s^{-0.7299} \quad (37)$$

The Bode plot of open-loop transfer function is shown in Fig.6(a) and the unit step responses with open-loop gains changing from 0.9 to 1.1 are shown in Fig.6(b).

Compared with reference [21], we can see that the designed robust FOPI controller based on the proposed vector method is basically consistent with the robust FOPI controller

$$C(s) = 2.3702(1+\frac{5.747}{s^{0.73}}) = 2.3702 + \frac{13.6215}{s^{0.73}} \quad (38)$$

designed by traditional method and the simulation results also comply with the design requirement.

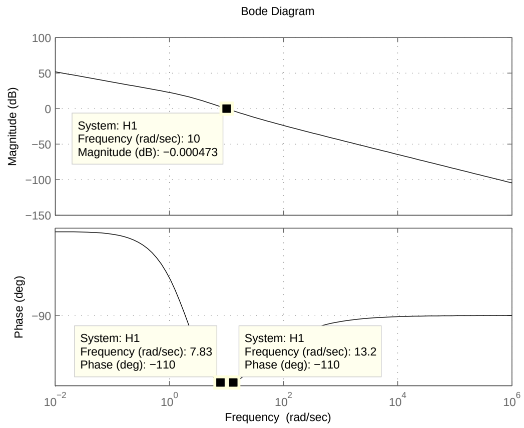

Fig. 6(a). Bode plot of open-loop transfer function

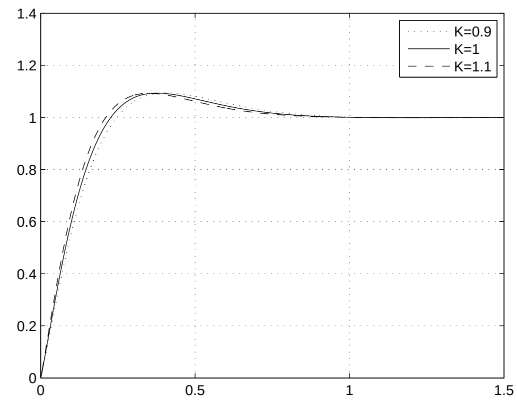

Fig. 6(b). Unit step responses with open-loop gain variations

Fig. 6. Bode plot of open-loop transfer function and unit step responses in case-1

• **Case-2**: In reference [22], given the plant $P(s) = e^{-\tau s}/(Ts+1)$, where $T = 0.4s, \tau = 0.01$, the gain crossover frequency $\omega_c = 10rad/s$ and the phase margin $\phi_m = 50°$.

According to the known conditions, we can obtain module value of the plant $|P(j\omega_c)|$, phase angle of the plant $\angle P(j\omega_c)$ and phase angle gradient of the plant φ_p in the condition of $\omega_c = 10rad/s$. Those respectively are

$$|P(j\omega_c)| = -12.3dB, \angle P(j\omega_c) = -81.8°$$

$$\varphi_p = \frac{d(Arg[P(j\omega)])}{d\omega}\Big|_{\omega=10rad/s} = -0.0335$$

And then, we can get module value of fractional order controller $A = 12.3dB$, phase angle of fractional order controller $\theta = -48.2° < 0$ and phase angle gradient $\varphi_c = 0.0335$. So the fractional order controller parameters are respectively $K_p = 2.124$, $\lambda = 0.8727$ and $K_i = 23.3818$. The transfer function of fractional order controller can be obtained as follows,

$$C_2(s) = 2.124 + \frac{23.3818}{s^{0.8727}} \tag{39}$$

The Bode plot of open-loop transfer function is shown in Fig.7(a) and the unit step responses with open-loop gains changing from 0.9 to 1.1 are shown in Fig.7(b).

Compared with reference [22], we can see that the designed robust FOPI controller based on the proposed vector method is basically consistent with the robust FOPI controller

$$C(s) = 2.1204(1 + \frac{11.06}{s^{0.8731}}) = 2.1204 + \frac{23.4516}{s^{0.8731}} \tag{40}$$

designed by traditional method and the simulation results also comply with the design requirement.

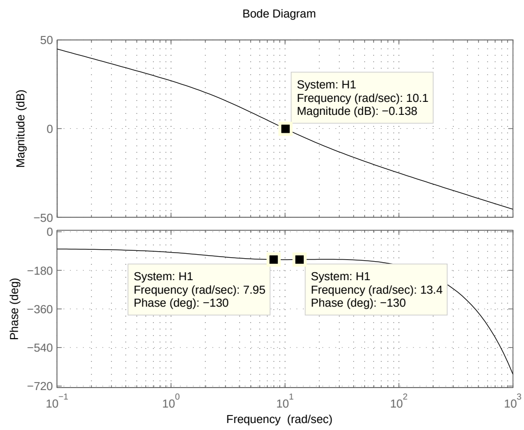

Fig. 7(a). Bode diagram of open-loop transfer function in case-2

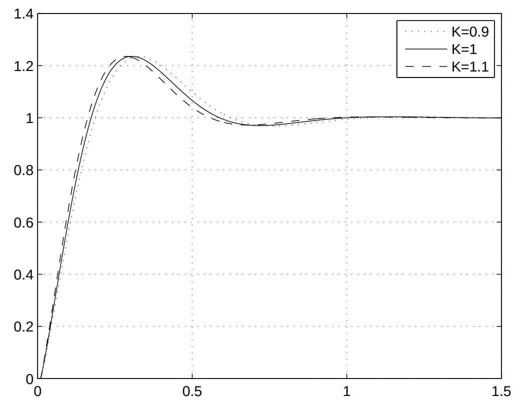

Fig. 7(b). Unit step responses of closed-loop system with open-loop gain variations in case-2

Fig. 7. Bode diagram of open-loop transfer function and unit step responses in case-2

• **Case-3**: In reference [19], given the plant $P(s) = 1/(Ts^\beta + 1)$, where $T = 0.4s, \beta = 0.5$, the gain crossover frequency $\omega_c = 10rad/s$ and the phase margin $\phi_m = 50°$.

According to the known conditions, we can obtain module value of the plant $|P(j\omega_c)|$, phase angle of the plant $\angle P(j\omega_c)$ and phase angle gradient of the plant φ_p in the condition of $\omega_c = 10rad/s$. Those respectively are

$$|P(j\omega_c)| = -6.43dB, \angle P(j\omega_c) = -25.3°$$

$$\varphi_p = \frac{d(Arg[P(j\omega)])}{d\omega}\Big|_{\omega=10rad/s} = -0.0102$$

And then, we can get module value of fractional order controller $A = 6.43dB$, phase angle of fractional order controller $\theta = -84.7° < 0$ and phase angle gradient $\varphi_c = 0.0102$. So the fractional order controller parameters are respectively $K_p = 0.1815$, $\lambda = 1.215$ and $K_i = 35.2293$. The transfer function of fractional order controller can be obtained as follows,

$$C_3(s) = 0.1815 + 35.2293s^{-1.215} \quad (41)$$

The Bode plot of open-loop transfer function is shown in Fig.8(a) and the unit step responses of closed-loop system with open-loop gains changing from 0.9 to 1.1 are shown in Fig.8(b).

Compared with reference [19], we can see that the designed robust FOPI controller based on the proposed vector method is basically consistent with the robust FOPI controller

$$C(s) = 0.1817(1 + \frac{194.4}{s^{1.216}}) = 0.1817 + 35.32s^{-1.216} \quad (42)$$

designed by traditional method and the simulation results also comply with the design requirement.

From Fig.6(a),7(a) and Fig.8(a), we can know that they all meet requirements of the given gain crossover frequency, the phase margin and flat phase around cross-over frequency. From Fig.6(b), 7(b) and Fig.8(b), it can be seen that the overshoots of step responses are basically the same when the open-loop gain K has variation of $\pm 10\%$, so it meets the requirement of robustness to open-loop gain variations. So the proposed vector method for designing a robust FOPI controller is correct.

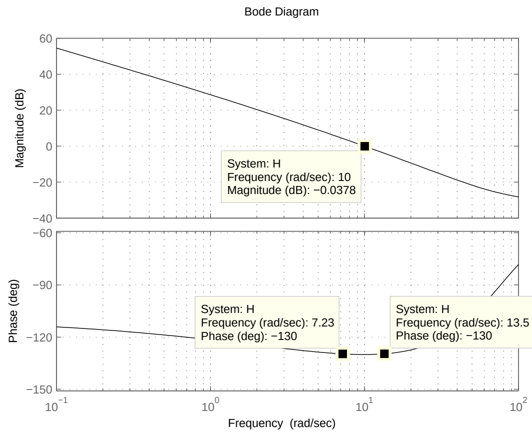

Fig. 8(a). Bode plot of open-loop transfer function

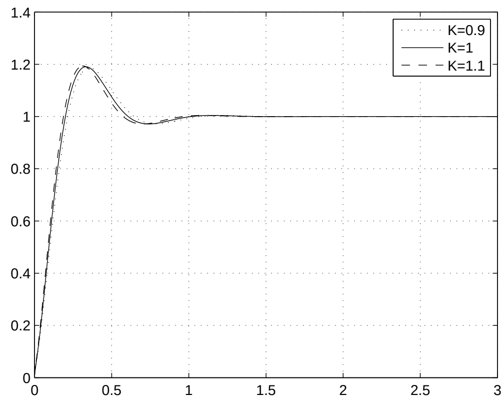

Fig. 8(b). Unit step responses with open-loop gain variations

Fig. 8. Bode plot of open-loop transfer function and unit step responses in case-3

8 Conclusion

On the basis of traditional design method of robust fractional order PI controller, this paper proposes and illustrates an optimization method for designing robust fractional order controller based on vector model, and proves the uniqueness of FOPI parameter solution. Compared with references [15–19, 21, 22], the proposed method which is used to design a robust FOPI controller has smaller calculated amount and simpler derivation process. The results show that the FOPI controller which is designed by vector method not only can meet the performance index, but also be unique.

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