Fault Tolerant Control for Induction Motor Drive Using Descriptor Approach

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Abstract: - This paper presents an active fault tolerant control (FTC) strategy for induction motor drive that ensures trajectory tracking and offset the effect of the sensor faults despite the presence of load torque disturbance. The proposed approaches use a fuzzy descriptor observer to estimate simultaneously the system state and the sensor fault. The physical model of induction motor is approximated by the Takagi-Sugeno (T-S) fuzzy technique in the synchronous d-q frame rotating with field-oriented control strategy. The performances of the trajectory tracking are analyzed using the Lyapunov theory and L_2 *optimization. Finally the effectiveness of the proposed strategy has been illustrated in simulation results.*

Key-Words: - Induction motor, Fault tolerant control, Trajectory tracking, Takagi-Sugeno, LMI.

1 Introduction

It is well known that fault is inevitable in nonlinear complex system. It can degrade the control performances and in some case lead to the instability of the system. To overcome these drawbacks, fault detection and isolation (FDI) and fault tolerant control (FTC) has been integrated in control system scheme. Therefore, several researches have been developed around this subject [1, 2, 3, 4, 5, 6].

Fault-tolerant control is a control that possesses the ability to accommodate system failures automatically, and to maintain overall system stability and acceptable performance even in the faulty situation. Generally speaking, we find two approaches for the design of fault tolerant control: passive control and active control. In the first approach a priori information about the fault which may affect the system is required and considered as uncertainty or disturbances which are taken into account in the design of the control law [19]. In contrast, the second approach has the ability to compensate all possible faults on-line. It has the possibility to change structure according to the information provided by the FDI block [20, 21, 24].

Lately, Takagi-Sugeno (T-S) approach has been successfully used in nonlinear system modeling and control. It has the ability to approximate exactly complex system. The idea is to decompose the model of the nonlinear system into a series of linear models involving nonlinear weighting functions. The equivalent fuzzy model describes the dynamic of behavior of the system [23].

Recently, the problem of tracking control for T-S and a faulty model has been studied by few numbers of works. For example, in [10] the author describes an active fault tolerant tracking control based on the online estimation of actuator and sensor fault. In [11], a robust fault tolerant control for non linear systems subject to actuator fault is designed. This FTC strategy is based on the online estimation of the faults and allowing the systems state to track a desired reference corresponding to fault free situation.

In the last years, induction motor became very frequent in industrial processes. This due to their reliability, robustness and low cost. Regrettably, control of induction motor is well known to be difficult due to the fact that the dynamic is complex and always subject to various faults, such as stator short circuits and rotor failures including broken bars or rings [12]. The overall performance of induction motor drives with a feedback structure depends not only on the health of the motor itself but also on the performance of the driving circuits and sensors: the encoder, voltage sensors and current sensors. Therefore, FTC problem for induction motor has received considerable attention [13, 14, 15, 16, 17].

In this study we exploit the performances of the FTC for state feedback tracking control of induction motor. The goal is to guarantee the stability and the operating in safe despite the sensor fault. A fuzzy descriptor observer is designed to give simultaneous estimation of system state and sensor fault. This estimation will be exploited in an observer based FTC tracking control to guarantee the control performances of the induction motor with respect to load torque disturbance.

This paper is organized as follows: Section 2 introduces the physical model of the induction motor and an open-loop control strategy is designed. A fuzzy observer-based fault tolerant tracking control is considered in section 3. Finally, simulation result is provided to demonstrate the design effectiveness.

2 Open loop control

2.1 Physical model of induction motor

Under the assumptions of the linearity of the magnetic circuit, the electromagnetic dynamic model of the induction motor in the synchronous d-q reference frame can be described as

$$\dot{x}(t) = f(x(t)) + g(x(t))u(t) + w(t) \tag{1}$$

where

$$f(x(t)) = \begin{bmatrix} -\alpha i_{sd} + \omega_s i_{sq} + \dfrac{K_s}{T_r}\psi_{rd} + K_s n_p \omega_m \psi_{rq} \\ -\omega_s i_{sd} - \alpha i_{sq} - K_s n_p \omega_m \psi_{rd} + \dfrac{K_s}{T_r}\psi_{rq} \\ \dfrac{M}{T_r} i_{sd} - \dfrac{1}{T_r}\psi_{rd} + (\omega_s - n_p\omega_m)\psi_{rq} \\ \dfrac{M}{T_r} i_{sq} - (\omega_s - n_p\omega_m)\psi_{rd} - \dfrac{1}{T_r}\psi_{rq} \\ \dfrac{n_p M}{J L_r}(\psi_{rd} i_{sq} - \psi_{rq} i_{sd}) - \dfrac{f}{J}\omega_m \end{bmatrix}$$

$$g(x(t)) = \begin{bmatrix} \dfrac{1}{\sigma L_s} & 0 & 0 & 0 & 0 \\ 0 & \dfrac{1}{\sigma L_s} & 0 & 0 & 0 \end{bmatrix}^T,$$

$$w(t) = \begin{bmatrix} 0 & 0 & 0 & 0 & -\dfrac{C_r}{J} \end{bmatrix}^T$$

$$u(t) = \begin{bmatrix} u_{sq} & u_{sd} \end{bmatrix}^T, \alpha = \left(\frac{1}{\sigma\tau_s} + \frac{1-\sigma}{\sigma\tau_r}\right), \sigma = 1 - \frac{M^2}{L_s L_r},$$

$$\tau_s = \frac{L_s}{R_s}, \tau_r = \frac{L_r}{R_r}, K_s = \frac{M}{\sigma L_s L_r}$$

$$x(t) = \begin{bmatrix} i_{sd} & i_{sq} & \psi_{rd} & \psi_{rq} & \omega_m \end{bmatrix}^T$$

where ω_m is the rotor speed, ω_s is the electrical speed of stator, ψ_{rd}, ψ_{rq} are the rotor fluxes, i_{sd}, i_{sq} are the stator currents, and u_{sd}, u_{sq} are the stator voltages. The load torque C_r is considered as an unknown disturbance. The motor parameters are moment of inertia J, rotor and stator resistances R_s, R_r, rotor and stator inductances L_s, L_r mutual inductance M, friction coefficient f and number of pole pairs n_p.

2.2 Open loop control

In this section, the structure of the open loop control strategy is explored. If we replace the state variables of the motor $\begin{bmatrix} i_{sd} & i_{sq} & \psi_{rd} & \psi_{rq} & \omega_m \end{bmatrix}^T$ by the reference signal $\begin{bmatrix} i_{sdc} & i_{sqc} & \psi_{rdc} & 0 & \omega_{mc} \end{bmatrix}^T$ in (1) we obtain

$$\begin{cases} \dfrac{d}{dt} i_{sdc} = -\alpha i_{sdc} + \omega_s i_{sqc} + \dfrac{K_s}{\tau_r}\psi_{rdc} + \dfrac{1}{\sigma L_s} u_{sdc} \\ \dfrac{d}{dt} i_{sdc} = -\omega_s i_{sdc} - \alpha i_{sqc} - K_s n_p \omega_{mc}\psi_{rdc} + \dfrac{K_s}{\tau_r}\psi_{rqc} + \dfrac{1}{\sigma L_s} u_{sqc} \\ 0 = -(\omega_s - n_p w_{mc})\psi_{rdc} - \dfrac{M}{\tau r} i_{sqc} \\ \dfrac{d}{dt}\psi_{rdc} = \dfrac{M}{\tau r} i_{sqc} - \dfrac{1}{\tau_r}\psi_{rdc} \\ \dfrac{d}{dt}\omega_{mc} = \dfrac{n_p M}{J L_r}(\psi_{rdc} i_{sqc}) - \dfrac{f}{J}\omega_{mc} - \dfrac{1}{J} C_r \end{cases} \tag{2}$$

The open-loop reference stator current the electrical speed and the loop control can be written as follow:

$$\begin{cases} i_{sdc} = \dfrac{\psi_{rdc}}{M} + \dfrac{\tau_r}{M}\dfrac{d}{dt}\psi_{rdc} \\ i_{sqc} = \dfrac{J L_r}{n_p M \psi_{rdc}}\left(\dfrac{C_r}{J} + \dfrac{f}{J}\omega_{mc} + \dfrac{d}{dt}\omega_{mc}\right) \end{cases} \tag{3}$$

$$\omega_{sc} = n_p \omega_{mc} + \frac{M}{\tau_r \psi_{rdc}} i_{sqc} \tag{4}$$

$$\begin{cases} u_{sdc} = \sigma L_s \left(\dfrac{d}{dt} i_{sdc} + \alpha i_{sdc} - \omega_{sc} i_{sqc} - \dfrac{K_s}{\tau_r}\psi_{rdc}\right) \\ u_{sqc} = \sigma L_s \left(\dfrac{d}{dt} i_{sqc} + \alpha i_{sqc} + \omega_{sc} i_{sdc} + K_s n_p \omega_{mc}\psi_{rdc}\right) \end{cases} \tag{5}$$

2.3 Takagi Sugeno fuzzy model of induction motor

The nonlinear model of the induction motor can be written as

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + w(t) \\ y(t) = Cx(t) \end{cases} \tag{6}$$

where

$$B=\begin{bmatrix}\frac{1}{\sigma L_s} & 0\\ 0 & \frac{1}{\sigma L_s}\\ 0 & 0\\ 0 & 0\\ 0 & 0\end{bmatrix},C=\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&0&0&1\end{bmatrix}$$

$$A=\begin{bmatrix}-\alpha & \omega_s & \frac{K_s}{\tau_r} & K_s n_p \omega_m & 0\\ -\omega_s & -\alpha & -K_s n_p\omega_m & \frac{K_s}{\tau_r} & 0\\ \frac{M}{\tau_r} & 0 & -\frac{1}{\tau_r} & \frac{M}{\tau_r \psi_{rdc}} i_{sq} & 0\\ 0 & \frac{M}{\tau_r} & -\frac{M}{\tau_r\psi_{rdc}} i_{sq} & -\frac{1}{\tau_r} & 0\\ 0 & 0 & -\frac{n_p M}{JL_r} i_{sq} & -\frac{n_p M}{JL_r} i_{sd} & -\frac{f}{J}\end{bmatrix}$$

The fuzzy model can be constructing using the well-known sector nonlinearity technique. The system (6) is constituted by the following three nonlinearities:

$$\begin{cases} z_1(t)=i_{sd}(t)\\ z_2(t)=i_{sq}(t)\\ z_3(t)=\omega_m(t)\end{cases} \tag{7}$$

The local weighting functions are defined by:

$$\begin{cases} F_{1j}(t)=\dfrac{z_j(t)-z_{j\min}}{z_{j\max}-z_{j\min}}\\ F_{2j}(t)=\dfrac{z_{j\max}-z_j(t)}{z_{j\max}-z_{j\min}}\end{cases} \tag{8}$$

Thus we can transform the non linear terms under the following shape:

$$z_j(t)=F_{1j}(t)z_{j\max}+F_{2j}(t)z_{j\min} \tag{9}$$

Consequently, the global fuzzy model of the induction motor can be written in the following form:

$$\dot{x}(t)=\sum_{i=1}^{8} h_i(z(t))(A_i x(t))+Bu(t)+w(t) \tag{10}$$

where

$$\begin{cases} h_i(z(t))=\dfrac{\mu_i(z(t))}{\sum_{i=1}^{8}\mu_i(z(t))},\ \mu_i(z(t))=\prod_{k=1}^{3}F_{ik}(z_k(t))\\ h_i(z(t))>0,\ \ \sum_{i=1}^{8}h_i(z(t))=1\end{cases} \tag{11}$$

3 Observer based fault tolerant control

3.1 *Reference model*

As in [24], in order to specify the desired trajectory, we consider the following reference model

$$\dot{x}_r(t)=A_r x_r(t)+r(t) \tag{12}$$

where

$x_r(t)=\begin{bmatrix} i_{sdr} & i_{sqr} & \psi_{rdr} & \psi_{rqr} & \omega_{mr}\end{bmatrix}^T$ is the reference state of the closed loop system.

$$A_r=\begin{bmatrix}-\zeta_1 & \omega_{sr} & \zeta_2 & K_s n_p\omega_{mr} & 0\\ -\omega_{sr} & -\zeta_1 & -K_s n_p \omega_m & \frac{K_s}{\tau_r} & 0\\ \frac{M}{\tau_r} & 0 & -\frac{1}{\tau_r} & \frac{M}{\tau_r\psi_{rdr}} i_{sqr} & 0\\ 0 & \frac{M}{\tau_r} & -\frac{M}{\tau_r\psi_{rdr}} i_{sqr} & -\frac{1}{\tau_r} & 0\\ 0 & 0 & -\frac{n_p M}{JL_r} i_{sqr} & -\frac{n_p M}{JL_r} i_{sdr} & -\frac{f}{J}\end{bmatrix}$$

$\zeta_1=(\alpha+K_\Psi)\ ,\zeta_2=(\frac{K_s}{\tau_r}+\frac{MK_f}{L_s\tau_r})$,

$\omega_{sr}=n_p\omega_{mr}+\dfrac{M}{\tau_r\psi_{rdc}} i_{sqr}$ and K_f, K_Ψ are design positive constants introduced to improve the dynamic of the induction motor system. $r(t)$ is a bounded reference input given as

$r(t)=\begin{bmatrix} B & I\end{bmatrix}\begin{bmatrix} U_r(t)\\ w(t)\end{bmatrix}$ in which

$$U_r(t)=\begin{cases} U_{sdr}=\sigma L_s\left[\dfrac{d}{dt}i_{sdc}+(\alpha+K_i)i_{sdc}-\omega_{sc}i_{sqc}+\left(\dfrac{MK_\psi-L_sK_s}{\tau_R L_s}\right)\Psi_{rdc}\right]\\ U_{sqr}=\sigma L_s\left[\dfrac{d}{dt}i_{sqc}+(\alpha+K_f)i_{sqc}+\omega_{sc}i_{sdc}+K_s n_p\omega_{mc}\Psi_{rdc}\right]\end{cases}$$

Using the same technique presented in 2.3, the reference model can be written in the following form

$$\dot{x}_r(t)=\sum_{j=1}^{8} h_j(z_r(t))(A_{rj}x(t))+r(t) \tag{13}$$

To attenuate the external disturbances, we consider the H_∞ performances related to the tracking error $x_r(t)-x(t)$ as follow

$$\int_0^{t_f}\left(\left[x_r(t)-x(t)\right]^T Q\left[x_r(t)-x(t)\right]\right)\le\gamma^2\int_0^{t_f}\left(r(t)^T r(t)+w(t)^T w(t)dt\right) \quad (14)$$

3.1 *Fault tolerant control strategy*

The fault considered in this work is speed sensor fault. In order to point up the proposed approach additional fault are injected to the T-S model (10) representing the induction motor. The faulty system can be written in the following structure:

$$\begin{cases}\dot{x}(t)=\sum_{i=1}^{8}h_i(z(t))(A_i x(t)+Bu(t)+w(t))\\ y(t)=Cx(t)+Df(t)\end{cases} \quad (15)$$

An augmented system consisting of the system (15) and the sensor fault can be written in the following form using descriptor approach:

$$\begin{cases}\bar{E}\dot{\bar{x}}(t)=\sum_{i=1}^{r}h_i(z(t))\left(\bar{A}_i\bar{x}(t)+\bar{B}u(t)+\bar{H}w(t)+\bar{D}x_s(t)\right)\\ y(t)=\bar{C}\bar{x}(t)=C_0\bar{x}(t)+x_s(t)\end{cases} \quad (17)$$

where

$$x_s(t)=Df(t),\ \bar{x}(t)=\begin{bmatrix}x(t)\\ x_s(t)\end{bmatrix},\ \bar{E}=\begin{bmatrix}I_n & 0\\ 0 & 0\end{bmatrix}$$

$$\bar{A}_i=\begin{bmatrix}A_i & 0\\ 0 & -I_p\end{bmatrix},\bar{B}=\begin{bmatrix}B\\ 0\end{bmatrix},\bar{D}=\begin{bmatrix}0\\ I_p\end{bmatrix}$$

$$C_0=\begin{bmatrix}C & 0\end{bmatrix},\bar{C}=\begin{bmatrix}C & I_p\end{bmatrix},\bar{H}=\begin{bmatrix}I\\ 0\end{bmatrix}$$

and the vector $x_s(t)$ is considered as an auxiliary state of the augmented system (17).
The following fuzzy observer is designed in order to estimate the system state and the sensor fault:

$$\begin{cases}E\dot{z}(t)=\sum_{i=1}^{8}h_i(z(t))\left(N_i z(t)+\bar{B}u\right)\\ \hat{\bar{x}}(t)=z(t)+Ly(t)\end{cases} \quad (18)$$

where $z(t)\in\Re^{n+p}$ is the auxiliary state vector and $\hat{\bar{x}}(t)\in\Re^{n+p}$ is the state estimation vector. $E,N_i\in\Re^{(n+p)*(n+p)},L\in\Re^{(n+p)*p}$ are the gain matrices of the observer.
Let us define the observer error as follow

$$\bar{e}(t)=\bar{x}(t)-\hat{\bar{x}}(t)=\left[e_0^T(t)\quad e_s^T(t)\right]^T \quad (19)$$

From (17) and (18), we can obtain

$$\left(\bar{E}+EL\bar{C}\right)\bar{x}(t)-E\dot{\hat{\bar{x}}}(t)=\sum_{i=1}^{r}h_i(z(t))\left((\bar{A}_i+N_iLC_0)\bar{x}(t)-N_i\hat{\bar{x}}(t)+(\bar{D}+N_iL)x_s(t)+\bar{H}w(t)\right) \quad (20)$$

if we choose

$$\begin{aligned}N_i&=\bar{A}_i+N_iLC_0\\ E&=\bar{E}+EL\bar{C}\\ \bar{D}&=-N_iL\end{aligned} \quad (21)$$

the dynamic error can be written in the following form

$$E\dot{\bar{e}}(t)=\sum_{i=1}^{8}h_i(z(t))\left(N_i\bar{e}(t)+\bar{H}w(t)\right) \quad (22)$$

In order to guarantee the condition (21), the observer parameters E,N_i,L are chosen as follow

$$N_i=\begin{bmatrix}A_i & 0\\ -C & -I_p\end{bmatrix},L=\begin{bmatrix}0\\ I_p\end{bmatrix},E=\begin{bmatrix}I_n & 0\\ RC & R\end{bmatrix} \quad (23)$$

in which $R\in\Re^{p*p}$ is non singular matrix. Then the dynamic error can be written as follow

$$\dot{\bar{e}}(t)=\sum_{i=1}^{r}h_i(z(t))\left(S_i\bar{e}(t)+\tilde{H}w(t)\right) \quad (24)$$

where $S_i=E^{-1}N_i$ and $\tilde{H}=E^{-1}\bar{H}$.
Suppose the following fuzzy controller is employed to deal with the above control system design:

$$u(t)=\sum_{i=1}^{8}h_i(z(t))K_j(\hat{x}(t)-x_r(t)) \quad (25)$$

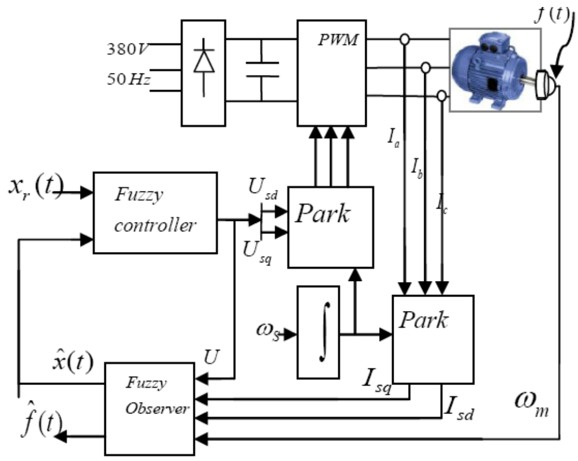

Fig.1: Fault Tolerant Control Strategy

The tracking error can be written as follow

$$e_t(t)=x(t)-x_r(t) \quad (26)$$

then we obtain:

$$\dot{e}_t(t)=\sum_{i=1}^{8}\sum_{j=1}^{8}h_j(z(t))h_i(z(t))\left((A_i+BK_j)e_r(t)-B\begin{bmatrix}K_j & 0\end{bmatrix}\bar{e}(t)+(A_i-A_r)x_r(t)+w(t)-r(t)\right) \quad (27)$$

Let us construct and augmented system containing the tracking error and the estimation error:

$$\dot{\tilde{x}}(t)=\sum_{i=1}^{8}\sum_{j=1}^{8}h_j(z(t))h_i(z_r(t))\left(\tilde{A}_i\tilde{x}(t)+\tilde{F}_{ij}\Phi(t)\right) \quad (28)$$

where

$$\tilde{e}(t)=\begin{bmatrix} e_t \\ e_0 \\ e_s \end{bmatrix}, \tilde{A}_i=\begin{bmatrix} A_i+BK_j & -BK_j & 0 \\ 0 & A_i & 0 \\ 0 & -CA_i-R^{-1}C & -R^{-1} \end{bmatrix}$$

$$\tilde{F}_{ij}=\begin{bmatrix} I & -I & A_i-A_{rj} \\ I & 0 & 0 \\ -C & 0 & 0 \end{bmatrix}, \Phi(t)=\begin{bmatrix} w(t) \\ r(t) \\ x_r(t) \end{bmatrix}$$

The H_∞ performances (14) related to the tracking error can be modified as follow

$$\int_0^{t_f} \tilde{e}^T \tilde{Q}\tilde{e}dt \le \rho^2 \int_0^{t_f} \tilde{\phi}(t)dt \tag{29}$$

where $\tilde{Q}=\begin{bmatrix} Q & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

Theorem 1

If there exists a symmetric and positive definite matrix $\tilde{P}=\tilde{P}^T>0$ and a prescribed positive constant ρ^2 such that

$$\tilde{A}_{ij}^T\tilde{P}+\tilde{P}\tilde{A}_{ij}+\frac{1}{\rho^2}\tilde{P}\tilde{F}_{ij}\tilde{F}_{ij}^T\tilde{P}+\tilde{Q}<0 \tag{30}$$

then the tracking control performance is guaranteed.

Proof.

Let us consider the following Lyapunov function:

$$V(\tilde{e}(t),t)=\tilde{e}^T(t)\tilde{P}\tilde{e}(t) \tag{31}$$

To guarantee the H_∞ tracking performance and the stability of the closed loop system, the following criterion will be holds:

$$\dot{V}(\tilde{e}(t),t)+\tilde{e}^T(t)\tilde{Q}\tilde{e}(t)-\rho^2\Phi^T(t)\Phi(t)<0 \tag{32}$$

Then we obtain

$$\sum_{i=1}^{8}\sum_{j=1}^{8} h_i(z(t))h_j(z_r(t))\Big(\tilde{e}^T(t)(\tilde{A}_{ij}^T\tilde{P}+\tilde{P}\tilde{A}_{ij})\tilde{e}(t) + \tilde{e}^T(t)\tilde{P}\tilde{F}_{ij}\Phi(t)+\Phi^T(t)\tilde{F}_{ij}^T\tilde{P}\tilde{e}(t) + \tilde{e}^T(t)\tilde{Q}\tilde{e}(t)-\rho^2\Phi^T(t)\Phi(t)\Big)<0 \tag{33}$$

Lemma 1: For any matrix X and Y with appropriate dimension, the following property holds

$$X^TY+Y^TX\le \rho^2X^TX+\frac{1}{\rho^2}Y^TY \tag{34}$$

Using Lemma 1, we can obtain

$$\tilde{e}^T(t)\tilde{P}\tilde{F}_{ij}\Phi(t)+\Phi^T(t)\tilde{F}_{ij}^T\tilde{P}\tilde{e}(t)\le \rho^2\Phi^T(t)\Phi(t)+\frac{1}{\rho^2}\tilde{e}^T(t)\tilde{P}\tilde{F}_{ij}\tilde{F}_{ij}^T\tilde{P}\tilde{e}(t) \tag{35}$$

Then we obtain the following condition

$$\tilde{e}^T(t)\left(\tilde{A}_{ij}^T\tilde{P}+\tilde{P}\tilde{A}_{ij}+\frac{1}{\rho^2}\tilde{P}\tilde{F}_{ij}\tilde{F}_{ij}^T\tilde{P}+\tilde{Q}\right)\tilde{e}(t)<0 \tag{36}$$

Consequently we obtain the condition in theorem 1. ■

Procedure resolution

We chose $\tilde{P}$ as follow

$$\tilde{P}=diag\begin{bmatrix} P_1 & P_2 & I \end{bmatrix} \tag{37}$$

By substituting (37) into (32) we obtain

$$\begin{bmatrix} \pi_{11} & \pi_{12} & \pi_{13} \\ \pi_{21} & \pi_{22} & \pi_{23} \\ \pi_{31} & \pi_{32} & \pi_{33} \end{bmatrix}<0 \tag{38}$$

where

$$\pi_{11}=P_1(A_i+BK_j)+(A_i+BK_j)^TP_1 + Q_1+\frac{1}{\rho^2}P_1\Big((A_i-A_{rj})(A_i-A_{rj})^T+2I\Big)P_1$$

$$\pi_{12}=-P_1BK_j+\frac{1}{\rho^2}P_1P_2$$

$$\pi_{13}=-\frac{1}{\rho^2}P_1C^T$$

$$\pi_{22}=P_2A_i+A_i^TP_2+\frac{1}{\rho^2}P_2P_1$$

$$\pi_{23}=\left(CA_i-R^{-1}C\right)^T-\frac{1}{\rho^2}P_2C^T$$

$$\pi_{33}=-R^{-1}-(-R^{-1})^T+\frac{1}{\rho^2}CC^T$$

Using the Schur complement we can obtain:

$$\begin{bmatrix} D_{11} & D_{12} & 0 & D_{14} \\ D_{12}^T & D_{22} & P_2 & 0 \\ 0 & P_2 & \rho^2 & D_{34} \\ D_{34}^T & 0 & D_{34}^T & D_{44} \end{bmatrix}<0 \tag{39}$$

in which

$$D_{11}=P_1(A_i+BK_j)+(A_i+BK_j)^TP_1 + Q+\frac{1}{\rho^2}P_1\Big((A_i-A_{rj})(A_i-A_{rj})^T+2I\Big)P_1$$

$$D_{12}=-P_1BK_j+\frac{1}{\rho^2}P_1P_2$$

$$D_{14}=-\frac{1}{\rho^2}P_1C^T$$

$$D_{34}=\left(CA_i-R^{-1}C\right)^T-\frac{1}{\rho^2}P_2C^T$$

$$D_{44}=-R^{-1}-(-R^{-1})^T+\frac{1}{\rho^2}CC^T$$

The inequality condition (39) contains the coupled variables of controller gains and observer gains.

There are no effective algorithms for solving them simultaneously. However, we can solve them in two-steps. First, we can find P_1 and K_j from the block diagonal D_{11} and then replace there's matrices in (39) to finds the variables P_2 and R.

In the first step after congruence (39) with $diag\begin{bmatrix} Z & I & I & I & I \end{bmatrix}$ and considering the change of variables $Z = P_1^{-1}, Y_i = K_j Z$, then using the Schur complement D_{11} can be written as:

$$\begin{bmatrix} A_i Z + ZA_i^T + BY_i + (BY_i)^T \\ + \frac{1}{\rho^2}\left((A_i - A_r)(A_i - A_r)^T + 2I\right) & Z \\ Z & -Q^{-1} \end{bmatrix} < 0 \quad (40)$$

The parameters $P_1 = Z^{-1}$ and $K_i = Y_i Z^{-1}$ are obtained by solving LMI (40).

In the second step by substituting P_1 and K_i into (39) we can easily find P_2 and R.

The sensor fault can be estimated by

$$\hat{f}(t) = (D^T D)^{-1} D^T \begin{bmatrix} 0 & I_n \end{bmatrix} \hat{\bar{x}}(t) \quad (41)$$

4 Simulation results

In this section numerical simulations have been performed to validate the developed control scheme. The induction motor is characterized by the following parameters:

TABLE I
Induction motor parameters

Pole pairs numbers	2
Stator resistance	10.5 Ω
Rotor resistance	4.3047 Ω
Stator inductance	0.4718 mH
Rotor inductance	0.4718 mH
Motor inertia	0.0293 $K_g.m^2$
Friction coefficient	0.001 $N.m.rad^{-1}$

A reference rotor speed of value 50rd/s is chosen at low speed operation which started at t=2sec and ends at t=3sec. whereas, at high speed, the reference rotor speed is fixed at 100rd/s between the instants 4sec and 10sec. A load torque of 5 N.m value is applied at t=1.5sec. An external additive sensor fault modeled as follow is injected in speed sensor:

$$f(t) = \begin{cases} 0 & t < 3 \\ 10 & 3 < t < 8 \\ 0 & 8 < t \end{cases}$$

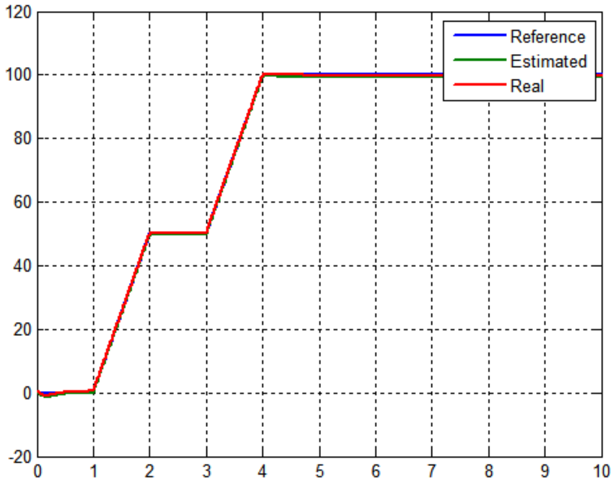

Fig. 2. Rotor speed

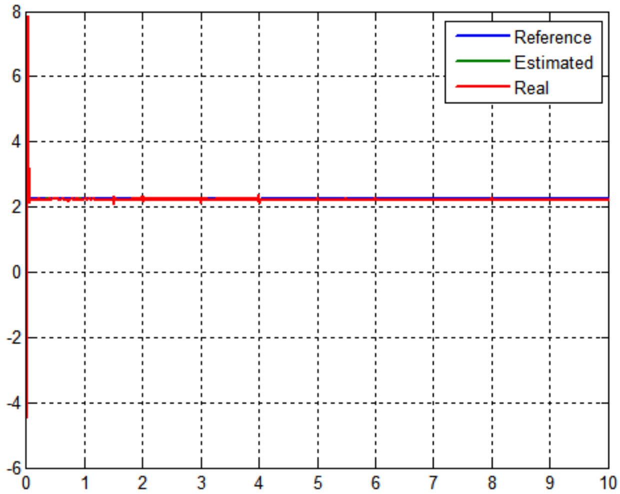

Fig. 3. d-axis stator current

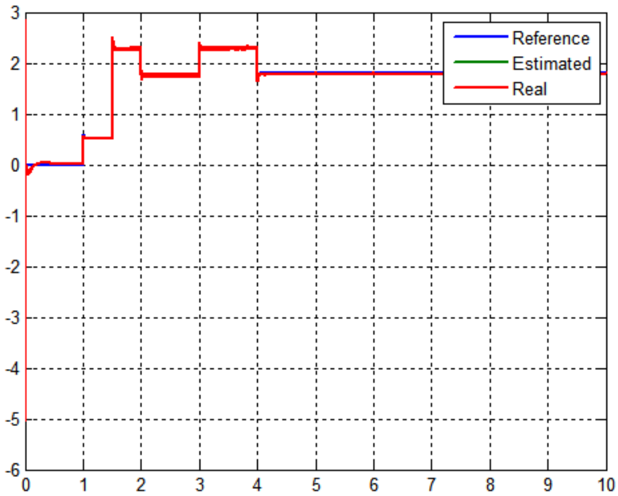

Fig. 4. q-axis stator current

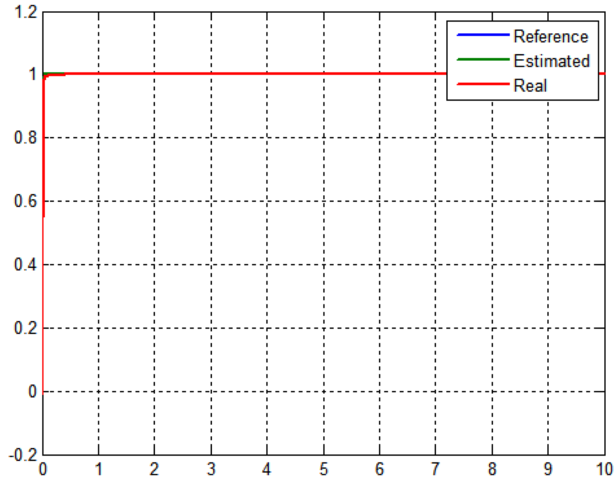

Fig. 5. d-axis rotor flux

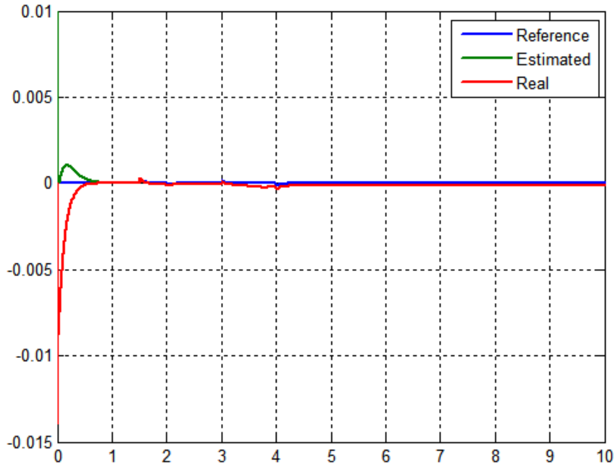

Fig. 6. q-axis rotor flux

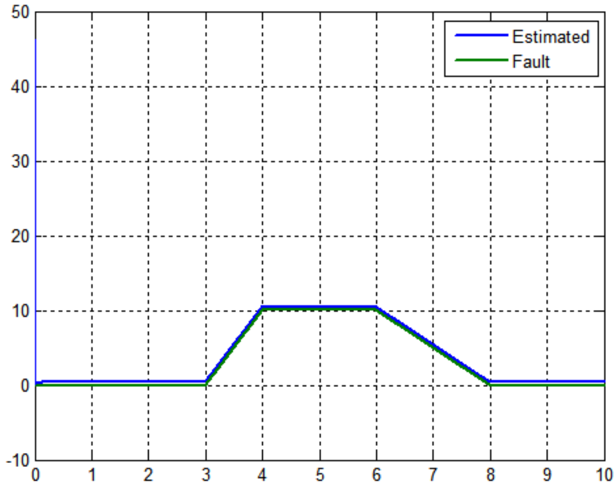

Fig.7. Fault and its estimated

The simulation results illustrated in fig. 2-7 show the trajectories of induction motor state together with the reference and the estimated states. The simulation results in Fig. 7 clearly demonstrate that the accurate estimates of the sensor fault signal are achieved via the descriptor observer.
In summary, it has been shown that the proposed scheme is able to estimate the sensor fault, through the descriptor technique. It can also compensate the unknown input load torque disturbance. It's clear that the proposed fuzzy FT controller forces the state variables to track the reference trajectory even in presence of sensors faults and then to achieve the decoupling control characteristic. This confirms that the performances of the FTC strategy are very satisfactory and allowing normal functioning of the system even in the occurrence of faults.

4 Conclusion

In this work, a fuzzy tracking control has been designed for the field oriented induction motor drive affected by external disturbance and sensor fault. The T-S fuzzy model is used to represent the induction motor in the synchronous d-q frame rotating. In order to guarantee the tracking performances, a fuzzy observer is used to estimate simultaneously the system state and the sensor fault. Finally, Simulation results showed the effectiveness of the proposed fuzzy controller.

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