

# Multilinked position-path control for autonomous vehicle

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*Abstract:* - In this paper control forces and torques automatic distribution algorithms for a vehicle actuators are considered. Base control algorithm is designed by position and path control method for vehicles [1] – [3]. This control algorithm is based on kinematics and dynamics equations of vehicle. Distribution of control forces and torques between actuators is solved by applying of the pseudoinverse matrix. Another approach is mathematical programming problem solution. These two approaches does not separate control channel as it is in conventional control systems [4]. The considered methods are applied in the control system of unmanned airship. Modeling results and estimation of algorithms accuracy and performance are presented.

*Key-Words:* - Actuators, control, multilinked systems, vehicles.

## 1 Introduction

Modern control systems of vehicles are based on movement separation and control of actuator as single input – single output system [5], [6]. In other word separate component of vehicle motion is controlled by separate actuator. For example, an aircraft pitch is controlled by an elevator [7]. This approach limits the abilities of vehicle control systems and modern control design methods.

In this paper control system consists of two levels. The high level is designed by position and path control design method for vehicle. The result of the system high level operation is control forces and torques. These forces and torques are inputs of the low level of the control system. But the output of the system low level is actuator's thrusts and angles. Such approach is valid if performance of actuators is high. This approach is used in different vehicles control systems [3], [8] – [10], [19] – [21]. In this case control system of actuator is local SISO system. Therefore the required thrusts and angles of engines are calculated as solution of the algebraic equations system. For instance, in [11] – [13] the algebraic equations system with rectangular matrix is solved by minimization of the required thrusts. In [12] – [13] the number of the algebraic system solutions is 96. Therefore the optimal solution is finding by exhaustive search in real time. In common case it is necessary to apply high performance algorithms.

## 2 Task Statement

Consider mathematical model. Let us consider a vehicle in a  $n$ -dimensional space. Control forces and

torques are included in  $n \times 1$  vector  $F_u$ . The number of a vehicle actuators is  $m$ . Every actuator has three components  $[P_{ix} P_{iy} P_{iz}]^T$ ,  $i = \overline{1, m}$ . Coordinates of actuator is given by the vector  $[x_i y_i z_i]^T$ ,  $i = \overline{1, m}$ , in associated coordinate system. In this case components of actuator's forces and components of the control vector satisfy the next matrix equation:

$$F_u = UP \quad (1)$$

where  $P = [P_{1x} P_{1y} P_{1z} P_{2x} P_{2y} P_{2z} \dots P_{mx} P_{my} P_{mz}]^T$ ;

$$U = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & \dots & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & \dots & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & \dots & 0 & 0 & 1 \\ 0 & z_1 & -y_1 & 0 & z_2 & -y_2 & \dots & 0 & z_m & -y_m \\ z_1 & 0 & -x_1 & z_2 & 0 & -x_2 & \dots & z_m & 0 & -x_m \\ -y_1 & x_1 & 0 & -y_2 & x_2 & 0 & \dots & -y_m & x_m & 0 \end{bmatrix} \quad (2)$$

It is necessary to find vector  $P$  as the most accurate solution of system (1).

## 3 Problem solution on base of pseudoinverse matrix

If  $U$  (2) is rectangular matrix, then system (1) has infinite set of solutions [14]. But the single solution determined by linear superposition of the rows and columns of conjugate matrix  $U^*$ . This solution is called pseudoinverse matrix  $U^+$  [14]. It is known that the pseudoinverse matrix  $U^+$  determines the best solution of system (1) in term of criterion of a minimum of least squares.

Thus vector  $P$  is:

$$P = U^+ F_u \tag{3}$$

Thrusts and rotation angles of actuators are:

$$p_i = \sqrt{P_{ix}^2 + P_{iy}^2 + P_{iz}^2} \tag{4}$$

$$\alpha_i = \arctan \frac{P_{iy}}{P_{ix}} \tag{5}$$

$$\beta_i = \arctan \frac{P_{iz}}{P_{ix}} \tag{6}$$

where  $p_i$  is the thrust of  $i$ -th actuator;  $\alpha_i$  is the rotation angle of  $i$ -th actuator in the vertical plane of associated coordinate system;  $\beta_i$  is the rotation angle of  $i$ -th actuator in the horizontal plane of associated coordinate system. Consider example of vehicle control system based on position and path method and algorithms (1) – (6). Consider control system of the prototype of stratospheric airship, similar to Lockheed-Martin P-791. It is shown in Fig. 1.

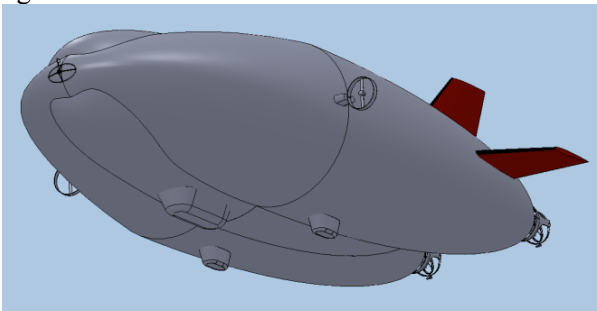


Fig. 1 Hybrid airship

Main parameters of the airship: length 38 m, width 17 m, height 10 m, envelope volume 4 100 m<sup>3</sup>, weight (with empty ballonets) 3 300 kg, one ballonet volume 900 m<sup>3</sup>. Coordinates of gravity center in reference to volume center (0 m, -1.5 m, 0 m). Main propulsion engines generate thrust of 5 000 N each. Engines are rotated in vertical plane in range from -180° up to +180°. Coordinates of main engines gravity centers are (0 m, 0 m, ±9 m). Tail steering motors generates up to 500 N each. They rotate in range from -90° up to +90° both in horizontal and vertical planes. They are located in tail part of airship and have coordinates (-20 m; 0 m; ±3.5 m).

Thus vector  $P$  for the given airship is:

$$P = [P_{1x} P_{1y} P_{2x} P_{2y} P_{3x} P_{3y} P_{3z} P_{4x} P_{4y} P_{4z}]^T \tag{7}$$

The power of steering motors is poor to control lateral motion of the airship. Therefore position and path control system calculates  $5 \times 1$  vector of the control forces and torques:

$$F_u = [F_{ux} F_{uy} N_{ux} N_{uy} N_{uz}]^T \tag{8}$$

Algorithms of calculation of vector (5) are presented in [2], [3], [15], [16].

In this case matrix (2) is:

$$U = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & z_1 & 0 & z_2 & 0 & z_3 & -y_3 & 0 & z_4 & -y_4 \\ z_1 & 0 & z_2 & 0 & z_3 & 0 & -x_3 & z_4 & 0 & -x_4 \\ -y_1 & x_1 & -y_2 & x_2 & -y_3 & x_3 & 0 & -y_4 & x_4 & 0 \end{bmatrix} \tag{9}$$

Coordinates of the airship propellers are:  $x_1=0$ ;  $x_2=0$ ;  $x_3=-21.63$ ;  $x_4=-21.63$ ;  $y_1=0$ ;  $y_2=0$ ;  $y_3=0$ ;  $y_4=0$ ;  $z_1=-10.7$ ;  $z_2=10.7$ ;  $z_3=-4.1$ ;  $z_4=4.1$ .

Modeling results of the airship closed-loop system are presented in fig. 2 – 4. Linear coordinates of the airship are presented in fig. 2. The airship motors thrusts and rotation angles are presented in fig. 3 and fig. 4.

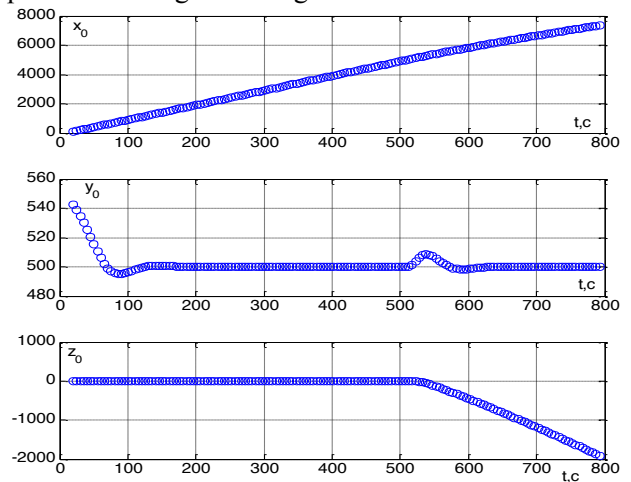


Fig. 2 The airship linear coordinates

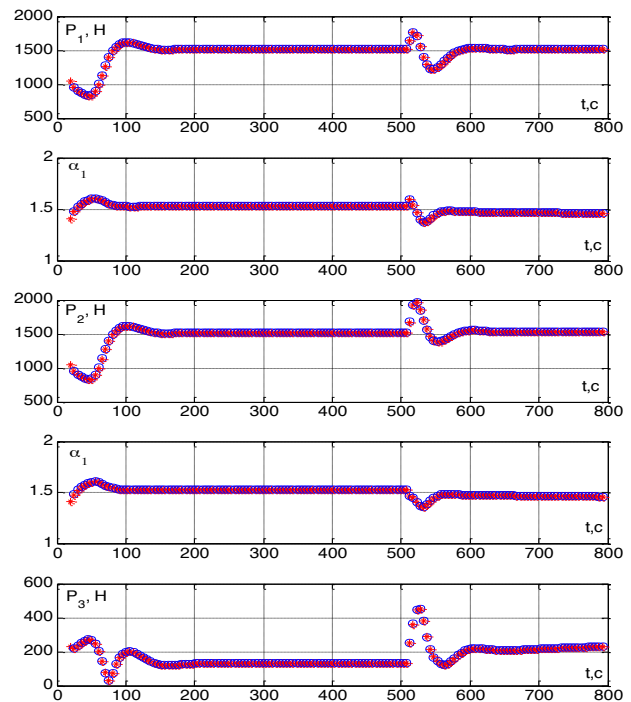


Fig. 3 The airship main motors thrusts and rotation angles

Notations in fig. 2 are:  $x_0$  and  $z_0$  are the airship

coordinates in horizontal plane;  $y_0$  is altitude of the airship.

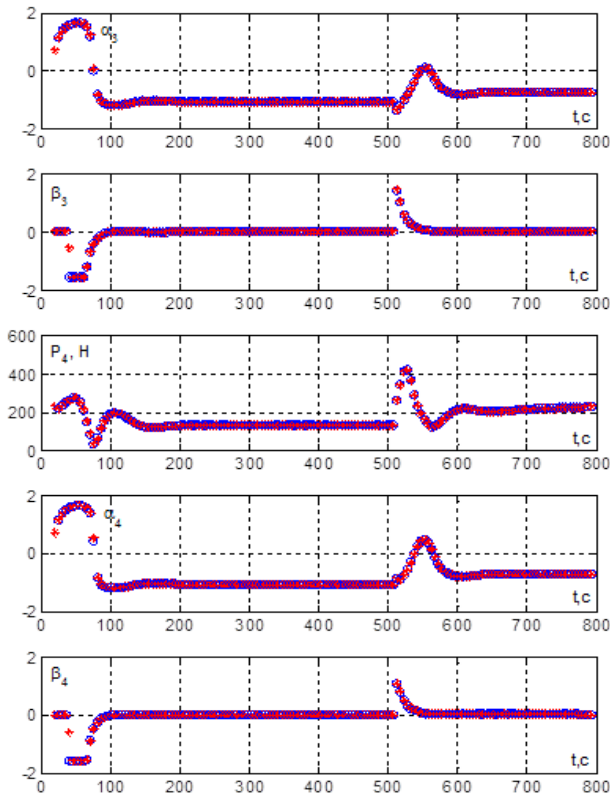


Fig. 4 The airship steering motors thrusts and rotation angles

On the first stage the airship is moving from point (0; 550; 0) to point (5000; 500; 0). After the airship is moving to point (7000; 500; 6000). The airship ground speed is 10 m/s. Wind speed is 5 m/s.

The main advantage of this method is automatic distribution of the control forced and torques between actuators. Searching operations and iterations are not used. Disadvantage of the method is miss of the motors thrusts and rotation angles limitations. But the limitations can be accounted after performing (1) – (6).

It is necessary to note that algorithms (1) – (6) allow synchronize vehicle actuators in the steady-state modes. In conventional vehicles control systems the synchronization of actuators is performing by designers for every movement mode. From fig. 3 and fig. 4 it is clear that thrusts and rotation angles of the left and right motors are equal to same values. In the transients thrusts and rotation angles of the left and right motors are different.

#### 4 Problem solution on base of the method of mathematical programming

The problem described in section II can be formulated as the problem of mathematical

programming:

$$P^* = \min(\text{norm}(F_u - UP)) \tag{10}$$

$$P_{1x}^2 + P_{1y}^2 < P_1^{\max}, P_{2x}^2 + P_{2y}^2 < P_2^{\max}; \tag{11}$$

$$P_{3x}^2 + P_{3y}^2 + P_{3z}^2 < P_3^{\max}, P_{4x}^2 + P_{4y}^2 + P_{4z}^2 < P_4^{\max}$$

where  $P_1^{\max}, P_2^{\max}, P_3^{\max}, P_4^{\max}$  are maximal values of the airship thrusts.

The problem (10), (11) is solved by Matlab function *fmincon*. This function is based on the trust-region-reflective algorithm [17]. Modeling results of the closed-loop control system of the airship are presented in fig. 5 and fig. 6.

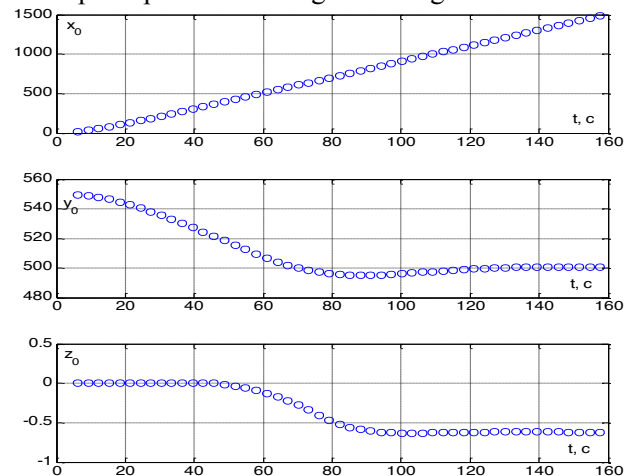


Fig. 5 The airship linear coordinates with problem (10), (11)

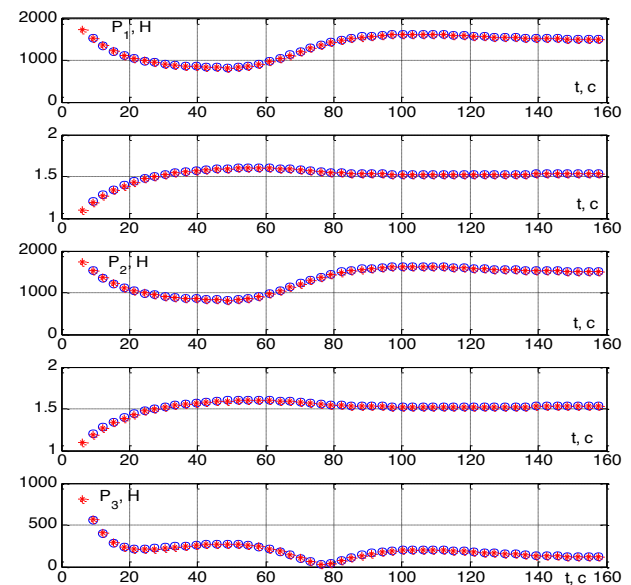


Fig. 6 The airship main motors thrusts and rotation angles with problem (10), (11)

The method of nonlinear mathematical programming allows to account nonlinear convex limitations in the searching area of (1). But the method of nonlinear mathematical programming requires high performance of the computer and depends from initial point of solution.

### 5 Position and path control

Design of motion control algorithm is performed on the base of position-trajectory control for mobile objects [2, 3, 15, 16]. Main cruising engines have time constant of 1 s, when steering – 0.5 s, that is in order of magnitude less than airship time constants. It allows not to include engines and motors equation in main control loop and control system is developed with equations of dynamics and kinematics presented in [8, 18]. In accordance with position-trajectory control method we introduce trajectory error

$$\psi_{tr} = A_1 Y_m Y + A_2 Y + A_3 \tag{12}$$

Herein  $Y = [x_0 \ y_0 \ z_0 \ \psi \ \vartheta \ \gamma]^T$  is position vector of airship in ground coordinate system;  $Y_m = \text{diag}([x_0 \ y_0 \ z_0 \ \psi \ \vartheta \ \gamma]^T)$  is diagonal matrix;  $A_1, A_2, A_3$  are matrixes and vector, which coefficients define motion trajectory.

We require variable (12) to be:

$$\dot{\psi}_{tr} + T_1 \psi_{tr} = 0 \tag{13}$$

Herein  $T_1$  is the tune coefficient of controller.

From (12), (13) we obtain:

$$(2A_1 Y_m + A_2)RX + T_1 \psi_{tr} = 0 \tag{14}$$

Herein  $R$  is a matrix of the airship kinematics.

We make requirements for airship velocity:

$$\psi_{vel} = A_4 X + A_5 \tag{15}$$

Herein  $X = [V_x \ V_y \ V_z \ \omega_x \ \omega_y \ \omega_z]^T$  is vector of airship velocities and angular rates in body coordinate systems;  $A_4, A_5$  are matrix and vector, defining required velocity of airship for motion along trajectory.

Total error is:

$$\psi_{\Sigma} = (2A_1 Y_m + A_2)RX + T_1 \psi_{tr} + A_4 X + A_5 \tag{16}$$

We require variable (16) to be

$$\dot{\psi}_{vel} + T_2 \psi_{vel} = 0 \tag{17}$$

Here in  $T_2$  is the tune coefficient of controller.

Hence from (12) – (17) we define control:

$$F_{con} = -F_d - M \left( (2A_1 Y_m + A_2)R + A_4 \right)^{-1} \times \left( 2A_1 \dot{Y}_m RX + (2A_1 Y_m + A_2) \dot{R}X + T_1 \dot{\psi}_{tr} + T_2 + T_1 \dot{\psi}_{vel} \right) \tag{18}$$

Here in  $F_d = [\bar{F}_{dyn} + \bar{F}_{ext}, \bar{N}_{dyn} + \bar{N}_{ext}]^T$  is vector of dynamic and external forces and moments, acting on airship;  $\dot{Y}_m = \text{diag}(RX)$  is diagonal matrix;  $\dot{R}$  is matrix of time derivatives of  $R$  matrix elements,  $M$  is matrix of inertial parameters of the airship.

Let straightforward motion of an airship with 10

m/s velocity along linear trajectory described with following equations has to be provided

$$y_0 = 2000 + \frac{x_0}{12}, \ z_0 = 0, \ \psi = 0, \ \vartheta = \frac{1}{12}, \ \gamma = 0,$$

Than matrixes and vectors  $A_1, A_2, A_3, A_4, A_5$  are:

$$A_1 = \text{zeros}(6,6), \ A_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{12} & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 0 \\ -2000 \\ 0 \\ 0 \\ -\frac{1}{12} \\ 0 \end{bmatrix}, \ A_4 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \ A_5 = \begin{bmatrix} -10 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

### 6 Conclusion

In this paper two methods of the control forces and torques distribution between vehicle actuators are studied. The problem is solution of the linear algebraic equations system with rectangular matrix. The first method is solution based on the pseudoinverse matrix. The second method is minimization of the solution error norm by the method of mathematical programming. These two methods are applied in the airship control system. The first method ensures for the given control system accuracy about  $10^{-14}$  N. Time of the problem solution is about 1 nanoseconds. The second method ensures accuracy about  $10^{-3}$  N and time of the problem solution about 1,5 microseconds. The studied methods are characterized by the absence of decomposition procedure "control channel - controlled variable". The problem is solved, if the time constants of actuators are much less than the time constants of a vehicle. If the inertia of the actuators is comparable with the inertia of a vehicle, the system (1) becomes the differential one. In this case it is possible to apply control algorithms described in [15], [16]. Described methods are able to compensate failure of actuators. Consider failure of the airship tail motors. In this case matrix (2) is

$$U = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & z_1 & 0 & z_2 \\ z_1 & 0 & z_2 & 0 \\ -y_1 & x_1 & -y_2 & x_2 \end{bmatrix} \tag{12}$$

Simulation results of the airship flight with only

main propulsion motors in fig. 7. Wind speed is 5 m/s. Coordinates of the main propulsion engines are:  $x_1=5$  m;  $x_2=5$  m;  $y_1=-2$  m;  $y_2=-2$  m;  $z_1=-10.7$  m;  $z_2=10.7$  m. From fig. 7,8 it is clear that error of control system is about 70 m. In addition in steady state mode we can see oscillations of the airship altitude.

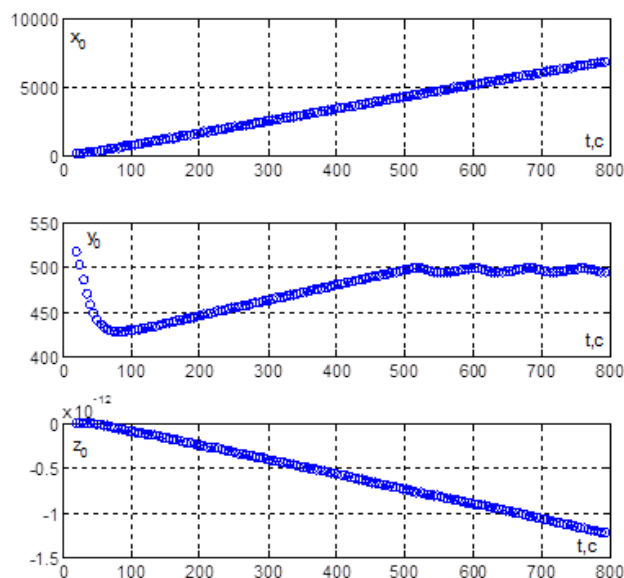


Fig. 7 The airship linear coordinates with motors failure

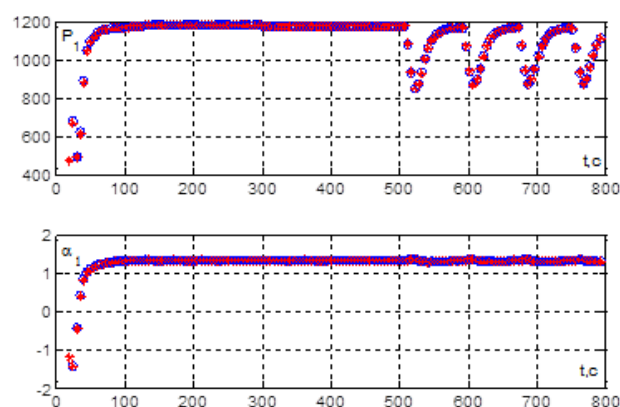


Fig. 8 The airship motors thrusts and rotation angles with failure

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