The Deduction and Application of Climate Index Based on the L-M BP Neural Network Algorithm in Chinese Real Estate Market

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Abstract: The paper introduces the intelligent neural network to make forecast for the climate index in the real estate market. At the same time, deeply mine the practical use of intelligent system of neural network, thus, abundant chaos phenomenons are obtained in the hidden nodes by numerical simulation. The bifurcation diagram and chaotic attractors are drawn from two dimensional chaotic neurons to seven dimensional neurons, and the sensitivity of initial weights for neuronal cell is verified. By comparing the traditional BP neural network with the improved L-M algorithm, we obtain the conclusion that the parameters of the nonlinear climate index system can affect the output of climate index system because certain ranges of parameters of hidden nodes may cause chaos. However, the improved L-M algorithm can choose the best parameters for the system, so that the complexity of the climate index system is reduced and chaos disappears, in addition it can shorten the training time, getting the higher precision and the faster convergence speed compared with conventional BP neural network in the system of climate index, which is better for the evaluation of climate index in the real estate market.

Key–Words: The traditional BP neural network, Improved L-M algorithm, Climate index, Highdimensional hidden nodes, Bifurcation diagram, Chaotic attractors.

1 Introduction

The real estate industry is an important industry in the national economy. Its development not only affects the macro economy, but also be related to the living of persons closely. The climate index of the real estate development can reflect the current situation and trend of the real estate industry comprehensively. Thus, climate index is the barometer of the real estate market.

In order to reflect the trend overall and the fluctuation of the real estate market accurately, we construct the climate index system scientifically in the real estate market, which can work with the regulation and control measures of government, the investment decision of developers and the purchase strategy of consumers. Predict the future state of the real estate boom fluctuation structure by the climate index of the real estate market by analyzing the pilot index, and provide the accurate market volatility trend for investors, developers, operators, consumers and government regulation in the real estate market, which helps to prevent the deterioration of market fluctuations and the emergence of the real estate bubble.

It has been in continuous fluctuations for the real estate market in the city since the development. Fluctuation is not a mechanical movement of up and down around a point or line, but make the complex nonlinear wave around a trend value. In the real estate economy, sometimes the fluctuation can be expressed as wave appearing alternately between absolute expansion and absolute shrinkage, however, the performance of movement appears fluctuating around a nonlinear trend more times. It is irregular not only in the length of time of fluctuation, but also in the amplitude, waveform, wave height, wave trough depth. If we understand the real estate market as mechanical wave action, we cannot describe the domestic real estate market situation objectively, and it is not conducive to monitoring and early warning in the real estate market, and the regulation and standard development of the real estate market in China. Therefore, there are obvious nonlinear characteristics for the climate index in the real estate market, and analyze how the dynamic nonlinear characteristics affect the output of the climate index. It have been proved that both fuzzy system (FS) and artificial neural networks are popular techniques for modeling systems. We can find there are universal approximatory in many research literatures. In this paper, we use the fuzzy system (FS) and artificial neural networks to analyze the nonlinear characters for the climate index in the real estate market.

An understanding of dynamical nonlinear characters of neural networks is important significance to the analysis of functions of the brain in applications and information processing. Wang et al.[1] have discussed the global exponential stability in the Lagrange sense for a non-autonomous Cohen- Gross berg neural network (CGNN) with time-varying and distributed delays, and obtained the results which can be applied to analyze mono-stable as well as multi-stable biology neural networks due to making no assumptions on the number of equilibria, this is more challenging than the existing references. Jian and Zhao [2] have analyzed new estimations for ultimate boundary and synchronization control for a disk dynamo system, and some new sufficient algebraic criteria for the globally exponential synchronization of two 4D disk dynamo systems are obtained analytically. Barak et al.[3] have studied and compared three models of delayed discrimination ranging from a highly organized line attractor model to a randomly connected network with chaotic activity, with data recorded during this task from fixed points to chaos, and obtained the results that prefrontal networks may begin in a random state relative to the task and initially rely on modified readout for task performance. With further training, more tuned neurons with less time varying responses should emerge as the networks become more structured. Zhu et al.[4] have studied the amoeba-based computing for Traveling Salesman Problem: Long-term Correlations between Spatially separated individual cells of Physarum polycephalum, and found that a single-celled, multi-nucleated amoeboid organism, a plasmodium of the true slime mold Physarum polycephalum, can perform sophisticated computing by exhibiting complex spatiotemporal oscillatory dynamics while deforming its amorphous body. Li et al.[5]have analyzed an integrated module partition approach for complex products and systems using weighted complex networks. Xu et al.[6]have constructed mathematical model and algorithm of toolpath optimisation on aircraft structural parts Bachlaus et al.[7]have used a chaos-embedded simulated annealing algorithm to analyze sequencing of parts on single-stage multifunctional machining systems. Thus, more and more nonlinear systems use neural network to solve the problems in practice.

Cheng and Cheng[8] have discussed an asymmetric image cryptosystem based on the adaptive synchronization of an uncertain unified chaotic system and a cellular neural network, and obtained useful conclusions. Yang et al.[9] have proposed a novel self-constructing least-Wilcoxon generalized radial basis function neural-fuzzy system and its applications to non-linear function approximation and chaos time sequence prediction, and studied a novel self-constructing radial basis function neural-fuzzy system, getting some conclusions. Yang et al.[10] have proposed a parallel chaos search based incremental extreme learning machine (PC-ELM) with additional steps to obtain a more compact network architecture. At each learning step, optimal parameters of hidden node that are selected by parallel chaos optimization algorithm will be added to exist network in order to minimize the residual error between target function and network output. The optimization method is proposed parallel chaos optimization method. Kundu et al.[11] have dealt with the complex dynamics of a four neuron network model having a pair of short-cut connections with multiple delays, and hopf bifurcations with respect to two other delays which are associated with short-cut connection are also obtained. Kozma and Puljic [12] have analyzed the learning effects in coupled arrays of cellular neural oscillators, and the interconnected oscillators can produce a wide range of dynamics, including quasi-periodic limit cycles, chaotic waveforms, and intermittent chaotic oscillations. Liu et al.[13] have studied hyper chaotic behavior in arbitrary-dimensional fractional-order quantum cellular neural network model, and more complex and abundant fractional-order hyper chaotic behaviors can be observed by these two examples. Ma et al.[14-16]have analyzed the complex dynamics in the games. Feng et al.[17] have made a study on hyper chaotic system implemented by SETMOS, and Saito hysteresis chaos generator(SHCG)is constructed using a cellular neural network(CNN). Ge and Yang[18] have studied hyper chaos of four state autonomous system with three positive Lyapunov exponents. Karimi-Nasab et al.[19]have made developing approximate algorithms for EPQ problem with process compressibility and random error in production. Tsao et al.[20]have constructed a piecewise nonlinear model for a production system under maintenance, trade credit and limited warehouse space. Egilmez [21]have studied the impact of risk on the integrated cellular design and control. Benyoucef et al.[21]have constructed supply chain network design with unreliable suppliers: a Lagrangian relaxation-based approach. Thus, we have some valuable conclusions in many research literatures.

Yang and Huang[23] have presented chaos and hyperchaos in a class of simple cellular neural networks modeled by ODE, and the chaoticity of these neural networks is indicated by positive Lyapunov exponents calculated by a computer Chen and Li[24] have analyzed hyperchaos in RTD-based cellular neural networks, and presented the Lyapunov exponents spectrum. Some typical Lyapunov exponents are calculated in a range of parameters. Huang and Huang[25] have studied chaos, bifurcation and robustness of a class of Hopfield Neural Networks.

There are various types of game systems in the real estate market between the main behaviors. Chaotic behaviors are in certain ranges of parameters in the process of dynamic games. Thus, we construct a nonlinear intelligent evaluation system on the boom by simulation of artificial neural networks. By comparing the traditional BP neural network and improved LM algorithm, we found there are differences between them because of the different internal construction, and deeply mine the reason of the difference. At the same time, we obtain that obtained the internal nonlinear hidden neurons can produce chaotic characteristics in the range of some parameters, thus, we draw the corresponding bifurcation diagram and the strange attractors.Thus, we establish the artificial intelligence system, which uses BP neural network and improved LM algorithm. The test company is an effective conclusion.

There are characteristics of the BP neural network, thus, the input data and output data are prone to produce chaos. Thus, the BP neural network with chaotic dynamics is studied by numerical simulation, and randomly select from two dimensional nodes to seven dimensional nodes to draw the bifurcation and the chaotic attractor which reflect the characteristics of dynamic system for climate index in the real estate market. Finally, we obtain some conclusions by judging the improved BP neural network LM algorithm with the conventional BP neural network. The innovations of this paper are: 1. the intelligent method is applied to the climate index of real estate market; 2. Recent study show that some hidden nodes may play a minion role in the network output and thus eventually increase the complexity of the network. we draw the bifurcation diagram, chaotic attractors and limit cycle which represent the dynamic characteristic of the system. The rest of the paper is organized as follows: in section 2, the climate index system is established in the real estate market, the index is quantified by the AHP and fuzzy mathematics method; the use of BP neural network is trained on fifty enterprises; and compare the difference of the two methods in the system of climate index, and found that the reason is that the choose of parameters of the weights is very important. In some ranges of parameters, the phenomenon of chaos appears, which has a great influence on the output of the climate index, but the improved LM algorithm can choose the best parameters in the process of training. Thus, the choose of the parameters is the most important in the nonlinear system of climate index. Besides this, we analyze the complexity of the hidden nodes, drawing the bifurcation diagram, attractors and limit cycles on behalf of chaos dynamic system; in section 3: the simulation using the improved BP neural network is trained by the LM algorithm, the BP neural network and the consistent conclusion, and the speed is quick, high precision. In section 4: Some useful conclusions are obtained.

2 The Establishment of Traditional BP Neural Network and Improved LM Algorithm for the Climate Index Analysis in the Real Estate Market

We randomly select fifty Real Estate Company from the real estate market. Such as Vanke, Evergrande, Country Gardon, Lu jin, Longhu et al.

2.1 The Built of Dynamic Equation

We take the index by the method of AHP from the experts and the main behaviors in the real estate market. Thus, we obtain five main factors which affect the climate index of the real estate market. 1 on the interest rate; 2 on the consumer price index, what is city residents consumer price index; 3 on the financing, what is the financing of real estate development enterprises; 4 on the completion, what is the completion of investment in real estate development; 5 on the commercial housing, what is commercial housing sales; 6 on the score, what is the climate index, prosperity index or boom index.

The fuzzy comprehensive evaluation results of evaluation and weight for the behaviors are:

The results of comprehensive forecast for behaviors are: 56%think ”extreme boom”, 26% think ”high boom”, 13%think ”the tiny boom”, 5% think ”no boom”.

The fuzzy comprehensive forecast results and weight for the experts are:

The results of comprehensive evaluation for experts are: 75% think ”extreme boom”, 11%think ”high boom”, 9% think ”the tiny boom”, 5% think ”no boom”.

It is different for the behaviors and experts, because there is difference in professional level. Behaviors are more likely to considering the overall benefit from the macro level, such as profits, market value, sustain ability; while the experts are inclined to focus on the individual factors of climate index .

Thus,

$$B = \begin{pmatrix} 0.56 & 0.26 & 0.13 & 0.05 \\ 0.75 & 0.11 & 0.09 & 0.05 \end{pmatrix} \quad (1)$$

$$A = \begin{pmatrix} 0.70 & 0.30 \end{pmatrix} \quad (2)$$

the results of fuzzy comprehensive evaluation for both behaviors and experts are: 61% think ”extreme boom”, 22% think ”high boom”, 12% think ”tiny boom”, 5% think ”no boom”. According to the principle of fuzzy comprehensive forecast of the maximum membership degree. The test value is 61%, so the result is ”extreme boom”.

According to the same method, we obtain:

$$RA_t = \begin{pmatrix} 0.55 & 0.24 & 0.15 & 0.06 \\ 0.70 & 0.19 & 0.10 & 0.08 \\ 0.51 & 0.21 & 0.17 & 0.14 \\ 0.63 & 0.27 & 0.06 & 0.03 \\ 0.46 & 0.24 & 0.17 & 0.13 \end{pmatrix} \quad (3)$$

According to the principle of maximum membership degree, we get the five levels of factors in the real estate market: interest rate 0.55; city residents consumer price index : 0.70; the financing of real estate development enterprises: 0.51; the completion of investment in real estate development : 0.63; the sales of commercial housing: 0.46.

According to the score, we make extreme boom = [0.9, 1], high boom = [0.5, 0.75), tiny boom = [0.25, 0.5), no boom = [0,0.25). corresponding to different warning degree, developers can adjust their investment strategy according to the score in the real estate market, and the government can also regulate the market according to the early warning degree.

Because ”extreme boom”, ”high boom”, ”tiny boom” and ”no boom” are in a large span, and it is not easy to distinguish each other under the same level of distinction. Thus, we assign scores to each level using fuzzy comprehensive clearer value. The ”extreme boom” is 90, ”high boom” is 80, ”tiny boom” is 70, and the ”no boom” is: 50. Which constitutes the matrix:

$$C = \begin{pmatrix} 90 & 80 & 70 & 50 \end{pmatrix} \quad (4)$$

Thus,

According to the same kinds of methods to randomly selected the factor levels of fifty enterprises in the real estate market, and the corresponding factors and the climate index are summarized as Table 1 and Table 2:

Table 1: A target level of climate index

No.	1	2	3	4	5	6
1	0.78	0.94	0.91	0.71	0.69	0.81
2	0.63	0.55	0.93	0.88	0.94	0.72
3	0.52	0.81	0.75	0.92	0.91	0.80
4	0.82	0.30	0.91	0.67	0.55	0.54
5	0.45	0.92	0.66	0.79	0.47	0.77
6	0.38	0.83	0.79	0.88	0.89	0.81
7	0.72	0.69	0.83	0.85	0.95	0.77
8	0.86	0.93	0.89	0.52	0.11	0.74
9	0.75	0.43	0.72	0.60	0.59	0.46
10	0.82	0.80	0.92	0.52	0.67	0.76
11	0.75	0.93	0.54	0.84	0.62	0.80
12	0.85	0.66	0.52	0.60	0.71	0.64
13	0.65	0.91	0.39	0.81	0.66	0.76
14	0.39	0.95	0.92	0.90	0.88	0.90
15	0.13	0.49	0.36	0.67	0.11	0.44
16	0.68	0.73	0.26	0.31	0.80	0.58
17	0.04	0.34	0.40	0.60	0.12	0.36
18	0.51	0.77	0.68	0.82	0.90	0.77
19	0.72	0.59	0.62	0.76	0.80	0.66
20	0.64	0.89	0.71	0.84	0.92	0.81
21	0.72	0.66	0.90	0.73	0.81	0.73
22	0.52	0.75	0.85	0.64	0.97	0.76
23	0.83	0.56	0.94	0.89	0.79	0.73
24	0.65	0.93	0.92	0.91	0.89	0.93
25	0.79	0.83	0.88	0.92	0.86	0.85
26	0.78	0.88	0.95	0.87	0.90	0.89
27	0.36	0.46	0.87	0.69	0.52	0.52
28	0.85	0.90	0.86	0.75	0.73	0.84
29	0.81	0.92	0.95	0.80	0.67	0.86
30	0.75	0.42	0.76	0.65	0.92	0.60

2.2 The System of Climate Index from the BP Neural Network

For general pattern recognition problems, it can approximate any nonlinear relation by neural network model of three layers with any degree of accuracy, so we adopt BP neural network of three layers to establish the model of the climate index in the real estate market. As it can be seen from table 1 in the analysis above, we can find the relationship between the factors and the score in the real estate market by the application of BP neural network, establishing the simple and efficient intelligent climate index evaluation system by training the network.

We put the factors of consumer price index of city residents; the financing of real estate development enterprises; the completion of investment in real estate development; the sales of commercial housing and interest rate which can affect the climate index of the real estate market as the input values of the BP neural network, and the score as the output of the neural network.

In this paper, the number of input layer neurons in the network is five, the number of output layer neurons is one. It is one of the more complex problems for the selection of hidden layer nodes. Too large or too small for the numbers of hidden layer nodes, it may lead the outputs to produce large errors for the climate index. After the adjustment of practical training and inspection, considering convergence speed and output precision of the network, we take the number of hidden layer neurons as 11. Therefore, the topology of the network is $5 \times 11 \times 1$. We use neural network tool box in the MATLAB7.0 software to set up the climate index system in the real estate market, and take fifty enterprises data for network training. An example is used to make a simulation test at last.

Table 2: Continued Table 1

No.	1	2	3	4	5	6
31	0.45	0.91	0.83	0.82	0.71	0.84
32	0.27	0.80	0.84	0.90	0.83	0.79
33	0.62	0.68	0.14	0.88	0.79	0.63
34	0.54	0.65	0.81	0.77	0.93	0.73
35	0.82	0.79	0.93	0.85	0.81	0.83
36	0.65	0.88	0.76	0.93	0.75	0.84
37	0.83	0.96	0.90	0.92	0.89	0.92
38	0.65	0.74	0.65	0.97	0.79	0.76
39	0.59	0.94	0.51	0.18	0.67	0.67
40	0.84	0.63	0.87	0.32	0.68	0.63
41	0.85	0.79	0.95	0.36	0.28	0.67
42	0.21	0.96	0.86	0.75	0.32	0.78
43	0.56	0.57	0.92	0.84	0.62	0.69
44	0.10	0.78	0.86	0.96	0.78	0.79
45	0.69	0.36	0.75	0.58	0.49	0.51
46	0.84	0.97	0.73	0.91	0.96	0.92
47	0.55	0.75	0.82	0.97	0.12	0.72
48	0.73	0.85	0.64	0.68	0.51	0.74
49	0.34	0.29	0.71	0.53	0.66	0.47
50	0.72	0.93	0.62	0.85	0.67	0.83
Test	0.55	0.70	0.51	0.63	0.46	

(1) The establishment of the climate index system from BP neural network

Build a network object. The transfer function of hidden layer neurons use tansig , and the output layer using logsig as transfer function. Use a specialized function newff provided by the Matlab neural network toolbox to set up a forward BP network, determining the structure of the network and the transfer function at the same time. The Matlab toolbox is with training function trainb. net=newff(minmax(p),[11,1],'tansig','logsig','trainb'); then set the network training parameters: the number of training epochs, the network performance target goal, the Marquardt adjustment parameters mu. Among them: net = newff said create a BP neural network;

Net means the properties and parameters of a network; Minmax said values of p for the input vector p (p), it said the matrix made by the maximum and the minimum of 9-dimension input factors. 11, 1 said the numbers of neurons in hidden layer and output layer in the system of climate index;

'tansig' means transfer function from the input layer to hidden layer using the tangent function, and 'logsig' means transfer function from hidden layer to output layer using s-shaped logarithmic function, mapping the neuron's inputs to the corresponding scope;

Trainb means using the network training algorithm. It means the neural network adjust weights and thresholds when accepting all of the training sample. In addition, mse means square error performance function when network training, and can evaluate network performance. Weights and thresholds are initialized by Network. While neural network was created by newff, weights and thresholds can be initialized automatically according to the default situation.

(2)the training of the climate index in the neural network

Trainb is realized by using the following command:

net.trainParam.show=50;
net.trainparam.lr=0.05;
net.trainparam.mc=0.9;

net.trainParam.epochs=1000;
net.trainParam.goal=1e-3;
[net,tr]=train(net,p,t);
net.trainParam.show=50 said the training numbers between two times is 50; net.trainparam.lr=0.05 said learning rate for 0.05; net.trainparam.mc=0.9 said momentum constant for 0.9; net.trainParam.epochs=1000 said the maximum of training number is 1000 ; net.trainParam.goal=1e-3 said the training accuracy target; [net,tr]=train(net,p,t) said training network, and p said the input vector of the network, t said desired output vector of the network. By the Settings above, the results of using the neural network tool in the MATLAB software to train the climate index of fifty enterprises are as follows:

Thus, after 1000 times training, network error is small for MSE = 0.00637427, and error curve is as shown in Figure. Accurate precision meets the requirements of training, and output of the network is more accurate. The output of training is: a =

Columns 1 through 14

0.7933 0.7974 0.8443 0.4975 0.6184 0.8181 0.7241 0.7512 0.4893 0.7304 0.7124 0.5670 0.7938 0.8352

Columns 15 through 28

0.4467 0.6182 0.5456 0.8007 0.6426 0.8555 0.6377 0.5810 0.7446 0.9631 0.8435 0.9434 0.6390 0.8262

Columns 29 through 42

0.8620 0.6281 0.7888 0.7964 0.9568 0.6943 0.7885 0.8670 1.0499 0.8526 0.7077 0.8508 0.7815 0.5623

Columns 43 through 50

0.8224 0.7302 0.4732 0.9981 0.8527 0.4964 0.1686 0.7611

(3) Finally, training network to achieve the goal of network convergence. [net,tr]=train(net,p,t); carry out the simulation on the sample data using the trained network a=sim(net,A). Input the test sample: A=[0.55;0.70;0.51;0.63;0.46]the result of test sample is 68.06 points. The point is close to the actual calculated value of 66.00, meeting the requirements of network , and the network is effective. 68.06 is in the [0.5, 0.75), thus, it is high boom in the real estate market.

As it is shown in figure: after training 1000 times, performance of the climate index network is achieved.

2.3 The Design of the Climate Index from the Improved BP Neural Network

Now turn the neural network trainb function into trainlm. That is to say, with improved BP neural network L - M algorithm according to parameters setting content of the traditional BP network, use MATLAB7.10.0 neural network toolbox to train the target level of fifty enterprises in the real estate market, and the results are as follows:

From the analysis and the Figures, we can see that after three times of training, the output mean square error is MSE = 6.36459e-005, error curve can be shown as below. The training output are: a =

Columns 1 through 14

0.8475 0.7578 0.7484 0.5880 0.7237 0.7816 0.7546 0.7654 0.5626 0.9052 0.7017 0.6921 0.6659 0.8482

Columns 15 through 28

0.5361 0.6624 0.5818 0.7344 0.6682 0.7572 0.7958 0.8452 0.7636 0.8239 0.7834 0.8180 0.5723 0.8078

Columns 29 through 42

0.8524 0.7297 0.8120 0.7842 0.5806 0.7615 0.8021 0.7680 0.8121 0.6895 0.8922 0.8289 0.8199 0.6666

Columns 43 through 50

0.6613 0.7581 0.5430 0.7537 0.6073 0.7150 0.5662 0.7335 Input a test sample and get a 66.20 point, the point is closer to actual value of 66.00, thus, it meets the requirements. 66.20 is in the [0.5, 0.75), thus, it is also high boom in the real estate market. We obtain the same result as the traditional BP neural network. At the same time, the system of climate index is established.

3 Comparison of the Climate Index from the Traditional BP Neural Network and the Improved L - M Algorithm

Compared with the traditional BP neural network algorithm, the network output has higher precision and faster convergence speed. The real estate market boom system with the improved BP neural network L - M algorithm optimizes the network performance further, and the output of climate index is more realistic. Using the improved intelligent evaluation tools in the system of the real estate market has strong feasibility and effective analysis results.

Figure 2 is The plot train state of the network for climate index; Figure 3 is The plot train state of LM algorithm for climate index; Figure 4 is The plot regression of the network for climate index; Figure 4(a) is the plot regression of BP network for climate index; Figure 4(b) is the plot regression of LM algorithm for climate index; Figure 5 the convergence map of BP neural network and LM algorithm of improved network for climate index; Figure 5(a) is the convergence map of BP neural network for climate index; Figure 5(b) is the convergence map of LM algorithm of improved network for climate index; Table 3 is the results of analysis of different methods between conventional BP neural network and L-M algorithm in the real estate market.

As it is shown in Table3, when the actual value is 0.6600 for both of the method for the climate index, the output of traditional BP neural network is 0.6806, while the result of improved LM-BP algorithm is 0.6620. Thus, the error of the climate index is 0.0206 from the system of traditional BP neural network, while the boom index is 0.0020 from the system of improved LM-BP algorithm. Obviously, there are differences in the output of the climate index. The improved LM-BP algorithm has smaller error and higher precision.

The game matrix payment formed by the conditions above can be shown in Table 1.

Table 3: The utility matrix of developers and home buyers in the evolutionary game

Probility		Buyers	
		R-T(r)	P-T$(1-r)$
Develpers	R-T(p)	a, b	c, d
	P-T$(1-r)$	e, f	g, h

3.1 The Built of Dynamic Equation

According to Table 1 and basic conditions for establishment of the model, when the buyers have psychological preferences of taking the strategy of reciprocal type, the utility for developers taking the strategy of reciprocal type is as follows:

$$E_H(k) = r \times a + (1-r) \times c \tag{5}$$

When developers take the strategy of profitable type, the utility is as follows:

$$E_L(k) = r \times e + (1-r) \times g \tag{6}$$

The average expected utility for developers is as follows:

$$\overline{E(k)} = p \times E_H(k) + (1-p) \times E_L(k) \tag{7}$$

according to formula (3) , the dynamic copy equation for developers taking the strategy of reciprocal type is as follows:

$$\begin{aligned} dp/dt &= p[E_H(k) - \overline{E(k)}] \\ &= p \times (1-p)[E_H(k) - E_L(k)] \\ &= p \times (1-p)[r(a-e-c+g) + (c-g)] \end{aligned} \tag{8}$$

according to formula (4) the probability of equilibrium is as follows:

$$r^* = (g-c)/[(a-e)+(g-c)] \tag{9}$$

Based on the developers taking the strategy of reciprocal type, the expected utility for buyers taking the strategy of reciprocal type is as follows:

$$E_B(G) = p \times b + (1-p) \times f \tag{10}$$

the utility of profitable pursuit for buyers is as follows:

$$E_N(G) = p \times d + (1-p) \times h \tag{11}$$

The average expected utility for buyers is as follows:

$$\overline{E(G)} = r \times E_B(G) + (1-r) \times E_N(G) \tag{12}$$

according to formula (8), the dynamic copy equation for buyers taking the strategy of reciprocal type is as follows:

$$\begin{aligned} dr/dt &= r[E_B(G) - \overline{E(G)}] \\ &= r \times (1-r)[p(b-d-f+h) + (f-h)] \end{aligned} \tag{13}$$

the probability of equilibrium is as follows:

$$p^* = (h-f)/[(b-d)+(h-f)] \tag{14}$$

In summary, the probability of evolutionary equilibrium is as follows:

$$(h-f)/[(b-d)+(h-f)], (g-c)/[(a-e)+(g-c)]$$

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3.2 Dynamic Phase Diagram Analysis for an Evolutionary Game in the Real Estate Market

First, analyze copy dynamic equation on the developers taking the strategy of reciprocal type. When $r = r^*$, dp/dt always equals to 0, that is to say, all the values p are in a steady state; when $r \rangle r^*$, two states of $p^* = 0$ and $p^* = 1$ exist, in which $p^* = 1$ is steady strategy. Similarly, when $r \langle r^*$, there are also two states of $p^* = 0$ and $p^* = 1$, in which $p^* = 0$ is the steady strategy. Circumstances phase and steady state are about three states with the changes of p as shown in Figure 1.

Similarly, we analyze the copy dynamic equation for the buyers taking the strategy of reciprocity when the developers have psychological preferences of taking the strategy of reciprocal type; when $p = p^*$, dr/dt always equals to 0, all the values of r are in a steady state; when $p\rangle p^*$, two states of $r^* = 0$ and $r^* = 1$ exist, in which $r^* = 1$ is evolutionarily steady strategy; when $p\langle p^*$, both $r^* = 0$ and $r^* = 1$ are in a steady state, in which $r^* = 0$ is the Evolutionarily steady strategy; thus, circumstances phase and steady state are about three states with the change of r as shown in Figure 2.

We use the same coordinate relations system to describe the dynamic changing proportion of copy for both developers and buyers taking the strategy of reciprocal type, as shown in Figure 3. In Figure 3, $p = 0, r = 0$ and $p = 1, r = 1$ are steady strategies. In the duplicate dynamic evolutionary game, when the initial condition is located in area II, it will converge to the stability of strategies $p = 1, r = 1$. That is to say, when the initial condition is located in area IV , it will converge to the stability of strategies $p = 0, r = 0$, on the basis of mutual benefit both the developers and buyers taking the strategy ofreciprocal type, reciprocal type. Both of the developers and buyers take the strategies ofprofitable type, profitable type. When the initial condition is located in areas I and III, the system usually can converge to the evolutionary steady strategy $p = 0, r = 0$, according to the evolution of psychology in the mutual preference.

Therefore, with the point p^*,r^* reducing, the areas I, II and III will increase, and the system will converge to the steady state of strategies $p = 1, r = 1$. The area of the probability of evolutionary steady strategy of reciprocal type, reciprocal type increases for both sides. It can also be said that when p^* and r^* are relatively small in the circumstances, both sides of the game take the reciprocal type strategies finally . In order to make p^* and r^* reducing, we should make the value of $(g-c)$ decrease, the values of $(a-e)$ and $(b-d)$ increasing, and $(h-f)$ decrease.

4 Complex Dynamics Analysis of Equation

For a deeper step analysis, we make $p = x$, $r = y$, $f-h = q$, $c-g = n$, $(b-d)+(h-f) = p$, $(a-e)+(g-c) = m$. The original equation is transformed into differential equation:

$$\begin{cases} \dot{x} = mxy + nx - mx^2y - nx^2 \\ \dot{y} = pxy + qy - pxy^2 - qy^2 \end{cases} \qquad (15)$$

4.1 The Equilibrium Point and Stability Analysis

The equilibrium points of the system is as follows: $E1(0,0)$, $E2(1,0)$, $E3(0,1)$, $E4(-q/p,-n/m)$ $E5(1,1)$.

The Jacobian matrix is:

$$J = \begin{bmatrix} my + n - 2mxy - 2nx & mx - mx^2 \\ py - py^2 & px + q - 2pxy - 2qy \end{bmatrix}$$

According to Routh-Hurwitz Criteria, the characteristic polynomial is:

$$f(\lambda) = \lambda^2 + a_1\lambda + a_2$$

the conditions for the local stability of equilibrium are $a_1 > 0$ and $a_2 > 0$.

The characteristic polynomial of equilibrium$E1(0,0)$ is:

$$f(\lambda) = (\lambda - n)(\lambda - q) = \lambda^2 - (n+q)\lambda + nq$$

, the conditions for the local stability of equilibrium$E1(0,0)$ are:$-(n+q) > 0$ and$nq > 0$;

similarly, the characteristic polynomial of equilibrium E2(0,1) is:

$$f(\lambda) = [\lambda - (m+n)](\lambda + q)$$

$$= \lambda^2 + [q - (m+n)]\lambda - q(m+n)$$

, the conditions for the local stability of equilibrium E2(0,1) are:$[q-(m+n)] > 0$ and $-q(m+n) > 0$;

the characteristic polynomial of equilibrium $E3(1,0)$ is:

$$f(\lambda) = [\lambda - (m+n)][\lambda - (p+q)]$$

$$= \lambda^2 - [(m+n)+(p+q)]\lambda + (m+n)(p+q)$$

, the conditions for the local stability of equilibrium $E3(1,0)$ are: $-[(m+n)+(p+q)] > 0$ and $(m+n)(p+q) > 0$;

The characteristic polynomial of equilibrium $E4(-q/p,-n/m)$ is

$$f(\lambda) = \lambda^2 - (\frac{mq^2 - mq}{p})(-\frac{pn^2 - pmn}{m^2})$$

. Obviously, $E4(-q/p,-n/m)$ is unstable equilibrium.

The polynomial characteristic of the equilibrium $E5(1,1)$ is:

$$f(\lambda) = [\lambda + (m+n)][\lambda + (p+q)]$$

$$= \lambda^2 + (m+n+p+q)\lambda + (m+n)(p+q)$$

, the conditions for the local stability of equilibrium $E5(1,1)$ are$(m+n+p+q) > 0$,$(m+n)(p+q) > 0$.

Explained from the economics of evolutionary game model, both developers and buyers pursue maximum profit in equilibrium point $E1(0,0)$ in the real estate market. Persons who take the strategy of reciprocal type withdraw from the market completely. If they don't quit, the reciprocal game subjects are conducting the business under the condition of the absolute loss, unless there is a national financial incentives or rewards given to the subjects who choose the mutual decision, or impose punitive measures on the actors to pursue maximum profit, otherwise, they will exit from the real estate market as rational decision makers. In the equilibrium points $E2(0,1)$ and $E3(1,0)$, a party as behavior subject take the reciprocal type strategy, and the other party take simple maximize profit strategy. In equilibrium point $E4(-q/p,-n/m)$, profit margin is zero for both behaviors, and they reach to balance temporarily. In equilibrium point $E5(1,1)$, both behaviors take mutual type strategy. Evolutionary game of reciprocal type is formed, but this is only a temporary balance. The parameters change because of the change of the external factors, which is equivalent to add disturbance in the system. Because equilibrium point $E5(1,1)$ is the most promising result for the built of reciprocal evolutionary game model. Therefore, only the stability of equilibrium point $E5(1,1)$ is studied here.

4.2 Complex Dynamic Characteristics Analysis of the Model

We study the effects of parameters on the stability of the equilibrium points. In order to identify the chaotic behavior, we present examples of bifurcation diagram, lyapunov exponent, and corresponding chaotic strange attractors with the change of parameters p,q,r,n. The initial value of x is selected as 0.88, and the initial value of y is selected as 0.98; that is to say, the initial probability to take reciprocal strategy for developers is 0.88, and the probability for buyers is 0.98.

(1) We analyze the influence of parameters on the stability of the system.

(a)Fix parameters $m=2,n=0.5,q=-3$. The initial value of x is 0.88, and the initial value of y is 0.98; Figure 4 is bifurcation diagram with the change of p for developers; Figure 5 is bifurcation diagram with the change of p for buyers; Figure 6 is the corresponding Lyapunov exponent with the change of p.

From Figure 4, in the range of $6<p<8$, the developers are steady. It means that all the developers take the reciprocity strategy.

Accordingly from Figure 5, in the range of $p<6.32$, the probability to take reciprocity strategy for buyers has been stable at the equilibrium point $(1,1)$; in the range of $p>6.32$, equilibrium point becomes unstable, and the period - doubling bifurcation phenomenon appears, namely 2 cycles; in the range of $7.08<p<7.23$, 4 cycles appear; when the value of p is larger than 7.23, the system goes into chaos, and the buyers go to a chaotic state in the real estate market.

From Figure 6, in the range of $p<6.32$, lyapunov index of the system is less than zero. The periodic bifurcation appears when p is equal to 6.32, and lyapunov index equals to zero. When p is larger than 7.23, positive lyapunov index exists, and the system goes into chaos. Obviously, lyapunov index and bifurcation diagrams are consistent.

Compare Figure 5 with Figure 4, it shows that it is earlier for developers to enter a state of chaos than buyers with the increase of value of p. From analysis on phase diagram, because $(b-d)+(h-f)=p$ in the system, the buyers show greater sensitivity than buyers under certain preconditions of$q=-(h-f)$. The significance of the system is that when the value of $q=-(h-f)$ is certain, buyers are sensitive to the value of p very muchand they are more interested in the increase of $(b-d)$. For$(b-d)$which means the income difference for the buyers to choose the mutual beneficial strategy and strategy of profit maximization, when the developers take the mutual beneficial strategy. According to the previous phase diagram, it is required to increase gradually for the value of p, thus the system will move towards the expected direction for both behaviors. That is to say, when the value of $q=-(h-f)$ is certain, the value of $(b-d)$ must increase gradually. It means that when the developers take the mutual beneficial strategythe income becomes more and more to choose the mutual beneficial strategy than the profit maximization strategy for the buyers. However, from Figure 5 and Figure 6, this theory is effective when the value of p increases in a certain range. The system will enter a chaotic state when the value of p exceeds 7.24 and $(b-d)$ exceeds 4.24. It has not exceed 4.24 of the income difference for the buyers to choose the mutual beneficial strategy and the profit maximization strategy when the system moves to the expected direction for both behaviors as mutual strategic, under the precondition of the developers taking the mutual beneficial strategy. As can be seen from Figure 4, developers have been in a state of chaos when the value is between six and eight.

That is to say, both developers and buyers are in a steady state in the range of [0,0.66] under the condition of constant fixed parameters with the increase of t. The system goes into 2 cycles bifurcation when $t=0.66$, and 4 cycles bifurcation appear when $t=0.818$. The system rushes into chaos when $t = 0.85$ at last. Namely the faster of the evolution rate, the more complicated of the real estate market. Many factors are filled with uncontrollability. It becomes more and more difficult to make right decisions for buyers. It becomes more easier to fall into a state of chaos for the market. It also can be seen that developers are always in a steady state from 0.6 to 1. This means that developers have ability to adopt the change of market because of knowledge and experience under the condition of fixed parameters. Compared with developers, buyers tend to lose control and rush into chaos because of lack of experience.

4.3 The Strange Attractor of the System

Figures below show strange attractors with varies of parameters (the decision maker's income level) under the condition of initial values $x = 0.88, y = 0.98$; Figure 16 shows the strange attractors when the values of p are $7.2; 7.5; 7.8$ respectively, under the conditions of $q = 5, m = 3.7, n = 0.4$; Figure 17 shows the strange attractors when the values of q are $-4.8; -4.6; -4.4$ respectively, under the conditions of $m = 3.7, n = 0.4, p = 7.2$; Figure 18 shows the strange attractors when the values of m are $3.7; 3.85, 3.95$ respectively, under the conditions of $n = 0.42, q = 4.8, p = 7.2$; Figure 19 shows the strange attractors when the values of n is $0.45, 0.5, 0.6$respectively, under the conditions of $m = 3.7, q = 5, p = 7.2$.

4.4 The Power Spectrum of Variables of the System

According to numerical simulation of the system, we take the cycle method to estimate the power spectrum of variables of the system, as shown from Figure 20 to Figure 24 with different parameters.

From the numerical simulation, the result shows that no matter how the initial payment matrix parameters change, namely the values of the initial parameter m, n, p and q can traverse the entire area with the passage of time, the probability will always been restricted in a certain range for behaviors in the system.

The diagram above has proved the evidence of chaos in the system: bounded-ness and ergodicity. From the external point of view, the chaotic movement is of a certain value space, and that is the chaotic attractor and the bounded-ness of chaos.

That is to say, the chaotic movement is no obvious regularity from the inside the system, and of certain value space from the view of the external point, namely the chaotic attractor and the bounded-ness of chaos. Especially, Figure 22 to Figure 24 show that power spectral density will be increased with the increase of the initial values under the conditions of the initial parameters m = 2; n = 0.5; q = 3; p = 7, namely the initial values vary from $x = 0.5; y = 0.63$ to $x = 0.88; y = 0.98$. Therefore, there are some ways to control chaos.

The characteristics of butterfly effect of chaotic movement differ from other steady movement. Slight changes of the initial values can make the adjacent orbital to separate in the exponential form after the evolution of multiple cycles. Through the analysis above, we know that the adjustment changes of the initial payment of the benefit of matrix can make the system go to a chaotic state. The sensitivity of initial values is the most obvious characteristic of chaos. We investigate how the slight changes of initial probability values effect on the system for both developers and buyers taking the strategy of reciprocity .

4.5 The Effect of Change of the Initial Probability for Buyers

Firstly, we examine how the system changes when the initial probability increases from $y = 0.3$to $y = 0.3001$ for buyers taking the mutually beneficial strategy.

In Figure 25 we can see that the varies are similar in the initial period of time, and difference increases with the increase of iteration times, which directly expresses that the system has sensitive dependence on initial values. When tiny change of the initial value is 0.0001, the change will be amplified after several iterations for buyers taking the strategy of reciprocity. At last, the scope of change is (0.25, 0.2) for the probability of buyers taking the mutually beneficial strategy. As can be seen from Figure 26, the slight varies of probability for buyers taking the strategy of reciprocity will have dramatic impact on the probability of developers taking mutually beneficial strategy, and tiny variations are also amplified at the early stage. That makes the probability of developers taking mutually beneficial strategy fluctuate within the scope of $(-1, 1)$ in the process of evolutionary game.

4.6 The Effect of Change of Initial Probability of Developers

Now we examine how the system changes when the initial probability increases from $x = 0.2$ to $x = 0.2001$ for developers taking the mutually beneficial strategy as shown below.

In Figure 27, the variables are similar in the initial period of time. Obviously the differences increase gradually with the increase of iteration times. Slight change will be amplified, when tiny changes of the initial value is 0.0001 for developers taking the strategy of reciprocity after several iterations. This makes the probability fluctuate within the scope of $(-1, 1)$ for buyers taking mutually beneficial strategy. As can be seen from Figure 28, the slight variations of probability for developers taking the strategy of reciprocity will have dramatic impact on the probability of buyers taking mutually beneficial strategy, and tiny variations are also amplified at the early stage. This makes the probability fluctuate within the scope of$(-0.4, 0.3)$ for developers taking mutually beneficial strategy.

From Figure 25 to Figure 28, it is shown that when it is in a state of chaos. Tiny changes of probability for taking the mutually beneficial strategy for behaviors will have an important effect on both of the parties, and the future evolutionary direction has a strong sensitivity to the changes in the current period. When the main body of purchasing behaviors is in this state, the subject will have difficulty in making accurate judgment for the future.

At the same time from Figure 25 to Figure 28 it can be seen that there are more effects on the developers than the buyers at the initial values, and the reason is asymmetric information. Compared with buyers, developers have more professional knowledge to analyze the market. Therefore, developers can make earlier decisions, and buyers take relative rational actions after observing the behaviors of developers. Strategies of the buyers are better than that of developers. Obviously, chaos appears and system is out of control because different parameters are taken in the system. It also draws a conclusion that not all the initial game matrix can evolve to points of stability for mutual or profit maximization. In evolutionary game, we should avoid the scope which produces chaos when initial parameters are taken. Therefore, the initial revenue is a key factor which not only decides the direction of the evolution, but also make system go into chaos. As a result, guidance is needed to introduce punitive measures in some of evolutionary game.

The sensitivity analysis shows that when the tiny changes of initial values for buyers are from 0.3 to 0.3001, the wave scope is [0,0.8895]for developers, and [0,0.1849] for buyers; when the tiny changes of initial values of developers is from 0.2 to 0.2001, the wave scope is [0,0.8316] for developers and [0,0.0216] for buyers. Therefore, when the system is in a chaotic state, whether developers or buyers having slight changes can have influence on each other. As it can be seen from the numerical simulation above, whether the tiny changes are for developers or for buyers, the effects on the developers is much greater than buyers. What indicates that when chaos occurs, it has much more serious effects on developers than buyers, which is mainly because there are the characteristics of capital intensive in the real estate market, and developers are in a dominant position, holding much more money. Every decision will have an important influence on the result. Compared with developers, the buyers are relatively scattered, holding small amount of money, and the effect of chaos is relatively weaker on them.

5 Chaos control

What should be noted is that chaotic state makes many negative impacts of the system that we need to apply some measures to delay its occurrence. In this paper, the linear feedback control is taken to delay or control the chaos, and there are three characteristics for the linear feedback control: firstly, both unstable periodic orbits and the fixed point can be controlled by this method, in other words, it can achieve control for any solution in the original system; secondly, it has the characteristics of anti-interference, so the linear feedback control can not be affected by the impact of the slight changes of parameters; finally, there are certain difficulties for some systems because of the exist of interaction of many system variables.

Firstly, we have a transformation at the equilibrium point E4 as the following rules:

$$\begin{cases} X = x + \frac{q}{p} \\ Y = y + \frac{n}{m} \end{cases} \tag{16}$$

Then the system can be rewritten as follows:

$$\begin{cases} \dot{X} = m(X - \frac{q}{p})(Y - \frac{n}{m}) - m(X - \frac{q}{p})^2(Y - \frac{n}{m}) \\ \qquad +n(X - \frac{q}{p}) - n(X - \frac{q}{p})^2 \\ \dot{Y} = p(X - \frac{q}{p})(Y - \frac{n}{m}) - p(X - \frac{q}{p})(Y - \frac{n}{m})^2 \\ \qquad +q(Y - \frac{n}{m}) - q(Y - \frac{n}{m})^2 \end{cases} \tag{17}$$

We assume that the controlled system is written as follows:

$$\begin{cases} \dot{X} = m(X - \frac{q}{p})(Y - \frac{n}{m}) - m(X - \frac{q}{p})^2(Y - \frac{n}{m}) \\ \qquad +n(X - \frac{q}{p}) - n(X - \frac{q}{p})^2 - kX \\ \dot{Y} = p(X - \frac{q}{p})(Y - \frac{n}{m}) - p(X - \frac{q}{p})(Y - \frac{n}{m})^2 \\ \qquad +q(Y - \frac{n}{m}) - q(Y - \frac{n}{m})^2 - kY \end{cases} \tag{18}$$

The Jacobian matrix of the controlled system is as follows:

$$J = \begin{bmatrix} -k & -\frac{qm}{p} - \frac{mq^2}{p^2} \\ -\frac{np}{m} - \frac{n^2p}{m^2} & -k \end{bmatrix}$$

when the characteristic polynomial is:

$$f(\lambda) = \lambda^2 + a_1\lambda + a_2$$

the conditions for the local stability of equilibrium are $a_1 > 0$ and $a_2 > 0$.

$$\begin{cases} a_1 = 2k \\ a_2 = k^2 - \frac{n^2q^2p+n^2q}{m} + npq^2 + nq \end{cases} \tag{19}$$

the local stability of equilibrium point E4 can be gained as follows:

$$\begin{cases} 2k > 0 \\ k^2 - \frac{n^2q^2p+n^2q}{m} + npq^2 + nq > 0 \end{cases} \tag{20}$$

when $q = -3, p = 5, m = 2, n = 0.5$, the control gain $k = -1$, the characteristic values are: $\lambda_1 = 1.0000 + 0.8660i, \lambda_2 = 1.0000 - 0.8660i$. At this time, the system is in a chaotic state. When we choose the control gain $k = 0.6$, the characteristic values are: $\lambda_1 = -0.6000+0.8660i, \lambda_2 = -0.6000-0.8660i$. Obviously, the system turns into a steady state. Thus, the chaos can be controlled by linear feedback control, and achieve the goal of driving the system to a steady state, inhibiting the system from going towards the desired direction.

Then we elect $k = 0.2$ and check the effect of the control.

Figure 30 is bifurcation diagram with the change of p for buyers with $m = 2; n = 0.5; p = -3$ with the linear controller. Compare Figure 30 with Figure 4, it is easy to see that the first bifurcation point is delayed from 6.328 to 6.91, and the point which rushes into chaos is delayed from 7.326 to 7.91. The effect of the control is obvious.

6 Conclusion

This research constructs an evolutionary game based on the theory of mutual harmony in the behavioral economics. It draws the conclusion that when the conditions are $r > r^*$ and $p > p^*$, the system evolves towards the expected directions, and both the developers and buyers achieve the highest degree of happiness.

Besides this, there are several useful conclusions expressed as follows:

(1) The payoff of the developers is the main factor which urges the developer groups to rush into a chaotic state; and the revenue of the buyers is the main factor which makes the buyer groups produce chaos.

(2)Chaos has more tremendous negative impact on developers than buyers, because each party can make much more sensitivity dependence on developers than buyers in chaotic state. Therefore, it is worth to be mentioned that the developers should pay much more attention on chaos control than buyers.

(3)The linear feedback control method is adopted to achieve the goal of driving the chaotic state to a steady state.

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