

Fuel Minimization of a Moving Vehicle in Suburban Traffic

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Abstract: In this paper we study how a driver could use traffic light information in order to adapt driving speed profile to save fuel. The mission is given by a final destination to reach (through a set of traffic lights) within a specific deadline and the objective is to minimize the fuel consumption. We assume that the speed between each traffic light is constant and we do not take into account the effects of acceleration and gear shifting. Also, we use an existing model for the fuel consumption which depends quadratically on the speed of the vehicle. For simple cases (one traffic light), we derive analytical results using optimization theory and the Karush-Kuhn-Tucker (KKT) necessary conditions for optimality. For more complex and realistic cases, we use Dijkstra's shortest path algorithm to discretize our decision problem. By "setting nodes" at each distance where there is a traffic light, we can model a realistic situation with an equivalent discrete graph with nonnegative edge costs. Then Dijkstra's algorithm helps to find the shortest path through this set of nodes. The results indicate that the best strategy is to drive as close to an optimal speed as possible, which is dictated by the vehicle's characteristics, while avoiding the traffic light blocks. One block look ahead strategies have also shown in simulations to be effective in particular realistic scenarios. It is also shown that, depending on the speed limit, minimal fuel and minimal time strategies may be quite similar.

Key-Words: Fuel minimization, Traffic light information, Look ahead strategy, KKT, Dijkstra's algorithm, Optimization

1 INTRODUCTION

1.1 Motivation

The major influence of road transport in global CO_2 emissions and the rise in oil prices are good motivations to develop new solutions to save fuel. Over the last decades, engineers have been inventing new technologies to improve vehicle efficiency and reduce CO_2 emissions. As the automobile industry has done a lot of effort to minimize its impact on the environment, the advances in traffic control systems may also play an important role. For instance, [24] and [25] used optimal control theory to suggest a control model which reduces traffic congestion and therefore CO_2 emissions. Research on communication between vehicles and traffic lights has also been of great interest. [16] studied the possibility of wireless communications where vehicles would be equipped with radio interfaces. In the same way, [26] has investigated the use of bluetooth technology for realizing a communication network between cars. Communications between vehicles could have diverse applications: safety, environment, traffic smoothness. A study suggests the use

of smart phones to provide advice to drivers to adapt their speed in order to save fuel [15]. They use two different approaches: in the first one, a smart phone collects data about the traffic light and gives a recommended speed to the driver in order not to hit the red traffic light. In the second approach, the smart phone collects data about the traffic flow and advises the driver. [13] and [14] investigate how Vehicle-to-Infrastructure communications could be used to improve intersection safety. [17] showed that wireless communications between vehicles and traffic lights would permit to significantly reduce the waiting time, the number of stops and the CO_2 emissions. Therefore, within few years, with the improvement of new technologies, it is not unlikely that vehicles will be able to have easily access in real time to traffic light state (timing and distance data). On board computers could determine the optimal speed profile (with respect to time or fuel minimization for example) which would be communicated to the driver.

1.2 Optimal driving in the literature

Related work about optimal driving policy has been the subject of lot of work in the literature. [12] used optimization theory to find the best speed profile to minimize work and fuel consumption along a route. [19] and [21] derived optimal driving policy (with respect to trip time and fuel consumption) for a truck traveling on a leveled road. Their method uses predictive cruise control: it is based on look ahead information, meaning that they need in advance the information about the road topography. Predictive cruise control has been studied for truck [22], hybrid vehicle [23], and for a car [4]. [4] used traffic light information to determine (with computer simulations) the best speed profile minimizing trip time and fuel consumption (their primary optimization variable was trip time). [27] derived an intelligent driver model based on interaction between vehicles and traffic lights. Their model showed a fuel consumption economy of 25% on some urban routes.

1.3 Fuel consumption model in the literature

Since the 80's, lot of work has been done to model the fuel consumption as a function of speed. In the literature, we can find two kinds of methods to derive fuel consumption models [8]. The first one expresses the fuel consumption as a function of the cruise speed, the idle consumption and the number of stops. These characteristics can be found experimentally. The other method uses the change in kinetic energy during the trip to determine the fuel consumption. Akcelik (1983) detailed these two models and showed that they give similar results [8]. Akcelik and Bayley derived in 1981 a model for the instantaneous fuel consumption [8]. In their model, the instantaneous fuel consumption depends quadratically on the instantaneous speed and acceleration with coefficients which depend on the physical characteristics of the vehicle. They also suggested a way to measure these coefficients. In our paper, we will use the model of [1]. He expressed the fuel consumption as a function of speed (quadratic function) with constant physical characteristics such as the engine friction, the tire rolling resistance, the air resistance, the effects of the brakes and the vehicles accessories. The values of these constants are specific for each vehicle and we will use the data given in [1] for an average powered car.

Many studies used control theory to show that the maximum fuel efficiency of a motor vehicle is realized at constant speed [10, 11, 12]. This optimal speed depends on the characteristics of the vehicle (mass, shape, equipments) but for an average powered vehicle it is about 50 mph and the fuel efficiency de-

creases significantly for lower speed [1]. Therefore, the driver behavior inside a suburban traffic plays a significant role in the fuel consumption. For example, [10] showed that an eco-driver could save 5.8% of fuel compared to a driver who would not receive any advice. Knowing that there are more than 330,000 in the US [18], the influence of traffic lights is important to consider.

1.4 Objective

In this paper, we investigate how a driver could use the traffic light information (timing and distance data) in order to adapt his speed profile to reduce his fuel consumption. We assume that the vehicle can receive real time data from the traffic lights (with wireless communication). The contribution of this paper is that it presents analytical results for the one traffic light problem and numerical simulations for more realistic cases. By the term "optimal speed", or V_0 , we will refer to the least fuel consuming speed (or, max Miles Per Gallon) at a constant speed when there are no traffic lights and final time constraints. When lights or/and final time constraints are present we will use the term "optimal speed profile" to indicate the most fuel efficient strategy.

For the one traffic light problem, we investigate the optimal speed profile for different cases depending on the value of the optimal speed V_0 (which is determined by the model of fuel consumption chosen) and of the state of the traffic light. The objective function (function to minimize) is the fuel consumption and is subject to speed constraints. For realistic cases, we use Dijkstra's shortest path algorithm to discretize our decision problem. By "setting nodes" at each distance where there is a traffic light, we can model a realistic situation with an equivalent discrete graph with non-negative edge costs. Each node represents a set of coordinates (time and distance from the origin) and the weight between two nodes is the fuel consumption to go from one node to another node. Dijkstra's algorithm finds the shortest path to go from a source to a destination and therefore it gives the optimal speed profile with respect to fuel minimization. We applied this approach to both fuel minimization and time minimization problems. In the time minimization problem, the weight between two nodes is the time needed to link these two nodes. We also compare the optimization policy found by Dijkstra's algorithm with the one step ahead policy (minimization at each step). The one step ahead optimization would be applied in a case where a vehicle could only get the information about the next traffic light. We observed that in certain cases (if the final time is free), the optimal speed profile found with Dijkstra's algorithm and the one found

with one step ahead optimization are the same. This is interesting for two reasons. First, Dijkstra's algorithm is computationally expensive as opposed to the one step ahead optimization. Second, Dijkstra's algorithm requires to know all the information about the traffic lights (timing and distances data) whereas one step ahead optimization only needs the information of the next traffic light. However, the lack of information about the future in the one step ahead policy may prevent the driver from arriving at the destination before the deadline.

The results for the one traffic light problem and the simulations for the realistic cases agree. The optimal strategy is to drive the closest possible to the optimal speed V_0 while avoiding to stop at a traffic light.

1.5 Paper structure

In Section 2, we formulate the problem and detail the model used for the fuel consumption. In Section 3, we derive the results for the zero and one traffic light problem. In Section 4, we describe Dijkstra's algorithm and show how it can be used to provide optimization policies. Finally, the results are presented at the end of Section 4. The last section gives the conclusions and future works.

2 PROBLEM FORMULATION

In this section, we formulate the problem and detail the assumptions made. For the fuel consumption, we use an existing model and assume its reliability. We describe the significance of each term and show the existence of an optimal speed V_0 .

2.1 Problem definition

Our objective is to minimize the fuel consumption of a vehicle driving in a network of traffic lights while reaching a destination within a specific deadline. As suggested in the introduction, we consider that the vehicle can receive the information about traffic lights. We will not focus on the technological aspects of this communication. We want to determine the optimal speed profile along the trip, taking into account the traffic lights. We use a simple model of instantaneous fuel consumption and do not take into account the effects of acceleration and gear shifting. Moreover, we assume that the vehicle drives at constant speed between each traffic light.

2.2 Fuel consumption model

The first task was to model the consumption as a function of speed. There exist a lot of models in the literature [1, 8] and most of them seem to agree on a cubic consumption model:

$$\frac{dC}{dt} = a_0 + a_1V + a_2V^2 + a_3V^3 \quad (KJ/hr) \quad (1)$$

In this equation, C represents the fuel consumption in kilojoule (or milliliter). Therefore, along a trip we want to minimize

$$C = \int (a_0 + a_1V + a_2V^2 + a_3V^3) dt \quad (KJ) \quad (2)$$

If V is constant (between two traffic lights), we can easily integrate (2) and therefore it is the same to minimize

$$C = (a_0 + a_1V + a_2V^2 + a_3V^3) \frac{d}{V}$$

Here, d is the distance to travel and t is the time to cover this distance. Thus, we want to minimize $C(V)$, where

$$C(V) = \left(\frac{a_0}{V} + a_1 + a_2V + a_3V^2 \right) d \quad (KJ) \quad (3)$$

It is also convenient to have an expression for the consumption per unit of distance, so we define C_d as $C_d = C/d$, i.e.,

$$C_d(V) = \frac{a_0}{V} + a_1 + a_2V + a_3V^2 \quad (KJ/mile) \quad (4)$$

This is essentially the inverse of Miles Per Gallon. To determine the coefficients a_0 , a_1 , a_2 and a_3 we use the model of [1]. For an average powered car (AVPWR) at constant speed, [2] models the consumption as

$$C_d(V) = \frac{\alpha_{fpwr} V_{gear} + \alpha_{acc}}{V} + \alpha_{tire} + \alpha_{air} V^2 \quad (5)$$

We note that in this model the coefficient in front of V is null ($a_2 = 0$).

- The first term $\frac{\alpha_{fpwr} V_{gear}}{V}$ takes into account the friction of the engine, V_{gear} can be seen as the average speed in gear used. Although we do not take into account gear shifting, a good approximation of V_{gear} is $V_{gear} = 55$ mph (miles per hour).
- The second term $\frac{\alpha_{acc}}{V}$ takes into account the consumption of all the accessories (such as air conditioning, lights, audio system, brakes system, etc).

- The third term α_{tire} takes into account the resistance of tires which induces drag.
- The last term $\alpha_{air}V^2$ takes into account the drag generated by the particular shape of the car.

Typical values of constants are shown in table (6).

α_{fpwr}	α_{acc}	α_{tire}	α_{air}
1692	6750	316	0.403

(6)

We can see in Fig. 1 that for a given vehicle (with the above characteristics), there exists an optimal speed which minimizes the fuel consumption C_d ; we will call it V_0 . In this case, V_0 is roughly equal to 50 mph, which is consistent with the reality.

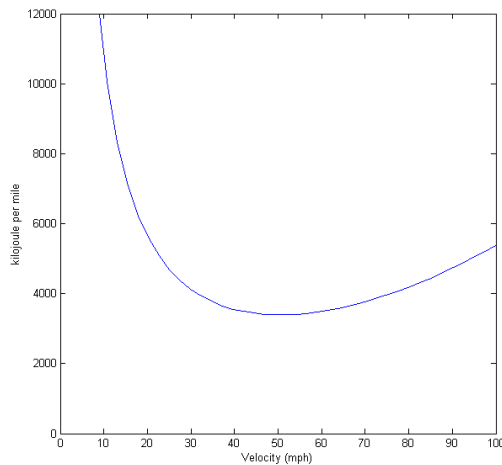


Figure 1: Consumption C_d as a function of speed

In Fig. 2 (released by [31]), we can see the results of a study done by the US Department of Energy [30]. We observe that the maximum fuel efficiency of the cars tested is realized around 40 mph and 60 mph, which confirms the reliability of the model we use.

3 ZERO AND ONE TRAFFIC LIGHT PROBLEMS

In this section, we derive the KKT necessary conditions for optimality. Then, we want to determine the optimal speed profile in the case of zero and one traffic light. For each case, we consider a random situation (with a random speed profile), then we express analytically the fuel consumption as a function of the problem parameters and we find the minimum of this function. Depending on the different cases, variables are subject to constraints, which brings us to use the KKT conditions. The results depend on the optimal

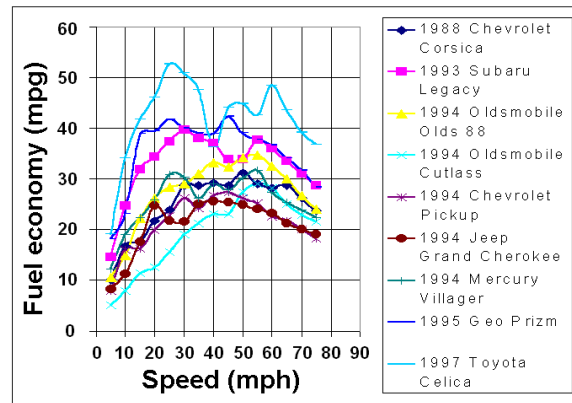


Figure 2: Fuel economy as a function of speed for 9 different cars (study of [30])

speed, the time duration of the red and green traffic lights and the distance from the origin to the traffic light.

3.1 No traffic light

Before dealing with the one traffic light problem, let us see what happens in the simplest possible case without any traffic light. One wants to reach a given distance D within a maximum time t_f while minimizing the fuel consumption. This is obvious, nonetheless we have to notice that there are two different cases. First, if V_0 is such that $V_0 \geq D/t_f$ the driver should drive at the optimal speed $V = V_0$ to minimize the fuel consumption. The second case to consider is when V_0 is not large enough to reach the distance D before the final time t_f . In this case, the driver should go faster than V_0 but at the slowest possible speed (to minimize fuel consumption), i.e., $V = D/t_f$. In summary, either we have enough time to reach the distance D with V_0 or we drive at speed D/t_f (Fig. 3).

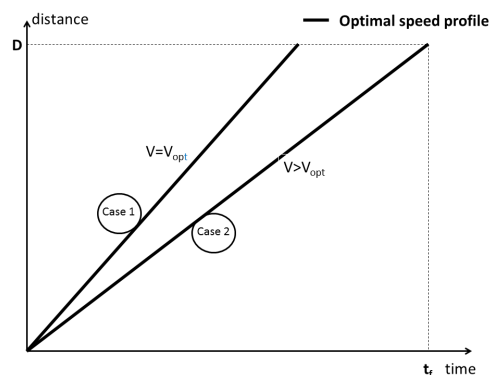


Figure 3: Summary of the two different cases for no traffic light

3.2 One traffic light

Now, we consider the case of one traffic light (red between t_1 and t_2) which is situated at the distance d of the starting point (Fig. 4).

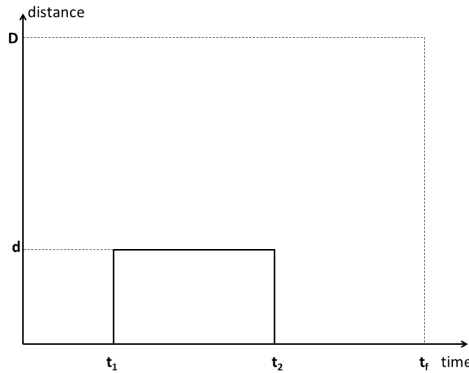


Figure 4: One traffic light at distance d

Once again, there are two different cases depending on the value of V_0 . The easiest case to deal with is when the optimal speed is “large enough”. In fact, if $V_0 \geq d/t_1$, then one does not hit the traffic light, and the optimal speed profile is obvious: $V = V_0$ (Fig. 5). The case when $V_0 < d/t_1$ is more complicated

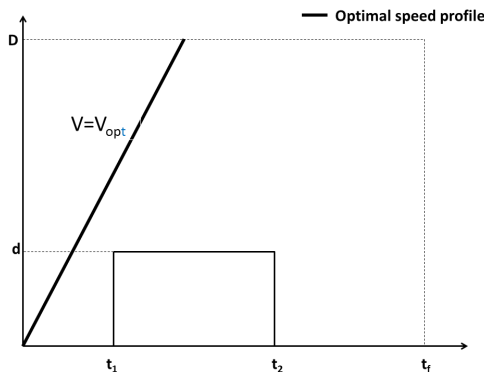


Figure 5: Optimal speed profile if $V_0 \geq d/t_1$

and needs to be broken in three sub-cases. In the first sub-case, we determine the optimal speed profile in a situation in which the driver will be fast enough not to hit the traffic light. In the second sub-case, the driver will hit the red traffic light and will have to stop until it turns green. In the last sub-case the driver will go beyond the red traffic light (in other words, he will hit the green traffic light).

Let us call V_1 the speed from the origin to distance d and V_2 the speed from d to D . The three different cases described above are $V_1 \geq d/t_1$, $d/t_2 \leq V_1 \leq d/t_1$ and $V_1 \leq d/t_2$.

3.2.1 First sub-case: $V_1 \geq d/t_1$

First we deal with the case when one does not hit the traffic light. We take a random speed profile (with constant speed between the starting point and the traffic light and between the traffic light and D) where we choose not to cross the traffic light (Fig. 6). We

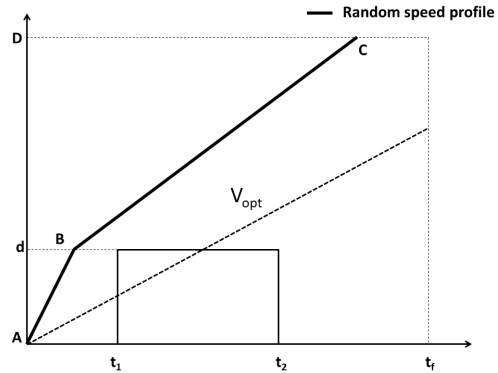


Figure 6: Random speed profile if $V_1 \geq d/t_1$

can show by a simple analysis that the optimal speed profile (with the same shape as in Fig. 6) is the one shown in Fig. 7. In fact, in the first section (from the origin to distance d), the optimal strategy is to hit the corner of the traffic light because $V_0 \leq d/t_1$. In the second section (from distance d to distance D), there are two possibilities: either the driver should drive at V_0 or he should reach the final distance D at the final time t_f . In Fig. 7, V_0 is not large enough so that the driver should adopt the strategy of “case 2” illustrated in Fig. 3. Thus, the optimal speed profile is described in Fig. 7.

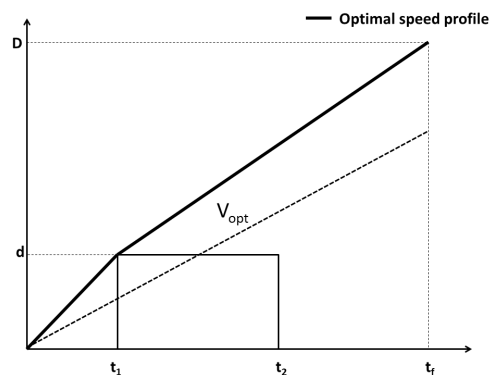


Figure 7: Optimal path if the traffic light is not crossed

3.2.2 Second sub-case: $d/t_2 \leq V_1 \leq d/t_1$

Now, we deal with the second sub-case when one hits the traffic light (we remind to the reader that we are still in the case where $V_0 < d/t_1$). We take a random

speed profile where we choose to hit the traffic light at $t = t_s$ (Fig. 8):

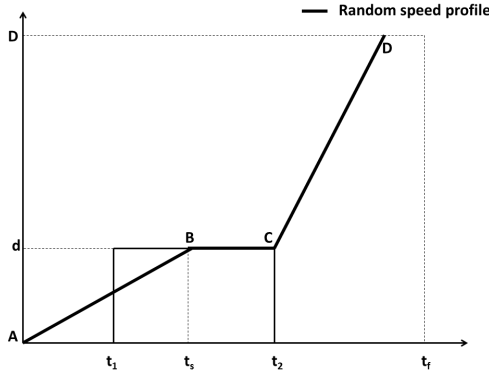


Figure 8: Random speed profile if $d/t_2 \leq V_1 \leq d/t_1$

The optimal speed profile from C to D is known: according to the section 3.1, it is $V = V_0$ if $V_0 \geq \frac{D-d}{t_f-t_2}$ and $V = \frac{D-d}{t_f-t_2}$ otherwise. Therefore, it remains to determine the best speed profile from A to C. To do so, we break the objective function (the function to minimize) into two parts. The first will take into account the consumption from A to B and the second the consumption from B to C. To determine the value of these consumption, we use (2):

$$\begin{aligned} C_{AC} &= C_{AB} + C_{BC} \\ &= (a_0 + a_1V_1 + a_2V_1^2 + a_3V_1^3)t_s \\ &\quad + a_0(t_2 - t_s) \\ &= (a_1V_1 + a_2V_1^2 + a_3V_1^3)t_s + a_0t_2 \\ &= (a_1 + a_2V_1 + a_3V_1^2)d + a_0t_2 \\ &= (a_2V_1 + a_3V_1^2)d + a_1d + a_0t_2 \end{aligned}$$

In these equations, V_1 represents the speed from A to B. Since a_1d and a_0t_2 are positive constants, the function to minimize is:

$$f(V_1) = a_2V_1 + a_3V_1^2$$

We want to minimize $f(V_1)$ (V_1 is the control variable) but we have to remember that $f(V_1)$ is subject to some constraints. In fact, we are in the case where we chose to hit the traffic light so V_1 must be greater than d/t_2 and less than d/t_1 . Finally, the minimization problem to solve is:

$$\begin{cases} \min & a_2V_1 + a_3V_1^2 \\ \text{s.t.} & d/t_2 \leq V_1 \leq d/t_1 \end{cases} \quad (7)$$

Since a_2 and a_3 are positive, the function $f(V_1)$ is increasing on the interval $(\frac{d}{t_2}, \frac{d}{t_1})$ and so it is minimized

in $V_1 = d/t_2$. In other words, one has to hit the end of the red traffic light (Fig. 9).

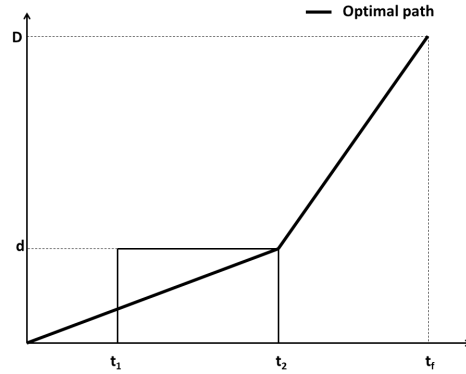


Figure 9: Optimal speed profile if we hit the traffic light and if $d/t_2 \leq V_0 \leq d/t_1$

3.2.3 Third sub-case: $V_1 \leq d/t_2$

In the last sub-case, one goes beyond the red traffic light and reaches the distance d during a green traffic light at $t = t_3$. From now on, we consider a random speed profile with $V_1 \leq d/t_2$ (Fig. 10). This time we

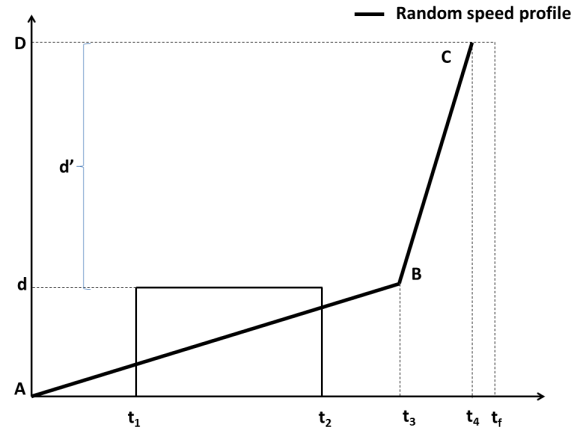


Figure 10: Random speed profile if $V_1 \leq d/t_2$

will take $a_2 = 0$ (like in the model of [2]) to simplify equations but the results would be unchanged even if a_2 was different from 0. First, let us note that the optimal speed V_0 verifies $\frac{dC_d(V)}{dV}(V_0) = 0$. Therefore, using (4), $-a_0/V_0^2 + 2a_3V_0 = 0$ or multiplying by V_0^2 :

$$-a_0 + 2a_3V_0^3 = 0 \quad (8)$$

To determine the fuel consumption to minimize, we break the objective function into two parts and use (3). V_1 is the speed from A to B and V_2 is the speed from

B to C.

$$C_{AC} = C_{AB} + C_{BC} \\ = (a_0/V_1 + a_1 + a_3V_1^2)d \\ + (a_0/V_2 + a_1 + a_3V_2^2)d'$$

But $V_1 = \frac{d}{t_3}$ and $V_2 = \frac{d'}{t_4-t_3}$ so, after simplifications,

$$C_{AC} = a_1d + a_1d' + a_0t_4 + a_3 \left(\frac{d^3}{t_3^2} + \frac{d'^3}{(t_4-t_3)^2} \right)$$

But the quantity $a_1d + a_1d'$ is constant and positive, so minimizing C_{AC} is the same as minimizing

$$C = a_3 \left(\frac{d^3}{t_3^2} + \frac{d'^3}{(t_4-t_3)^2} \right) + a_0t_4 \quad (9)$$

We want to minimize (9) with respect to t_3 and t_4 , but there are some constraints on t_3 and t_4 . In fact, $t_2 \leq t_3$, $t_3 \leq t_4$ and $t_4 \leq t_f$. Therefore, the minimization problem to solve is:

$$\begin{cases} \min & a_3 \left(\frac{d^3}{t_3^2} + \frac{d'^3}{(t_4-t_3)^2} \right) + a_0t_4 \\ & t_2 - t_3 \leq 0 \quad (\lambda_1) \\ \text{s.t.} & t_3 - t_4 \leq 0 \quad (\lambda_2) \\ & t_4 - t_f \leq 0 \quad (\lambda_3) \end{cases} \quad (10)$$

λ_1 , λ_2 and λ_3 are the lagrange multipliers associated to the constraints $t_2 - t_3 \leq 0$, $t_3 - t_4 \leq 0$ and $t_4 - t_f \leq 0$ (for more details see the book [3]). Then we can form the Lagrangian:

$$L(t_3, t_4, \lambda_1, \lambda_2, \lambda_3) = a_3 \left(\frac{d^3}{t_3^2} + \frac{d'^3}{(t_4-t_3)^2} \right) + a_0t_4 \\ + \lambda_1(t_2 - t_3) + \lambda_2(t_3 - t_4) \\ + \lambda_3(t_4 - t_f) \quad (11)$$

Therefore, KKT conditions for system (10) are (according to [3]):

$$\begin{cases} a_3 \left(\frac{-2d^3}{t_3^3} + \frac{2d'^3}{(t_4-t_3)^2} \right) - \lambda_1 + \lambda_2 = 0 \\ -2a_3 \frac{d'^3}{(t_4-t_3)^3} + a_0 - \lambda_2 + \lambda_3 = 0 \\ \lambda_1(t_2 - t_3) = 0 \\ \lambda_2(t_3 - t_4) = 0 \\ \lambda_3(t_4 - t_f) = 0 \\ \lambda_i \geq 0 \quad i = 1, 2, 3 \\ t_2 - t_3 \leq 0 \\ t_3 - t_4 \leq 0 \\ t_4 - t_f \leq 0 \end{cases}$$

By substituting $\frac{d}{t_3} = V_1$ and $\frac{d'}{t_4-t_3} = V_2$ in the first two equations, it is equivalent to:

$$\begin{cases} -2a_3(V_1^3 - V_2^3) - \lambda_1 + \lambda_2 = 0 \\ -2a_3V_2^3 + a_0 - \lambda_2 + \lambda_3 = 0 \\ \lambda_1(t_2 - t_3) = 0 \\ \lambda_2(t_3 - t_4) = 0 \\ \lambda_3(t_4 - t_f) = 0 \\ \lambda_i \geq 0 \quad i = 1, 2, 3 \\ t_2 - t_3 \leq 0 \\ t_3 - t_4 \leq 0 \\ t_4 - t_f \leq 0 \end{cases} \quad (12)$$

Solutions of KKT equations system We need to remind the reader that if we can find $(V_1, V_2, \lambda_1, \lambda_2$ and $\lambda_3)$ which verify all the equations of (12), then (V_1, V_2) is the optimal speed profile to minimize fuel consumption. First we note that $t_3 \neq t_4$ (otherwise V_2 would be infinite) so that the fourth equation of system (12) implies $\lambda_2 = 0$. Then the first equation becomes

$$\lambda_1 = 2a_3(V_2^3 - V_1^3) \quad (13)$$

The nonnegativity of λ_1 requires $V_2 \geq V_1$, in other words there does not exist optimal speed profile with $V_2 < V_1$. From now on, we assume $V_2 \geq V_1$.

Let us determine the value of V_1 and V_2 . If we look at the fifth equation of (12), there are two cases: $\lambda_3 = 0$ or $t_4 = t_f$. If $\lambda_3 = 0$, the second equation implies $-2a_3V_2^3 + a_0 = 0$, which means that $V_2 = V_0$ (using (8)). Now, if we examine (13), there are again two cases: either $V_2 > V_1$, or $V_2 = V_1$. If $V_2 > V_1$, then $\lambda_1 \neq 0$ and so the third equation of (12) tells us $t_2 = t_3$.

Finally, there are 4 possible optimal speed profiles (depending on the different characteristics d, d', t_2, t_f and on the optimal speed V_0) that we can summarize in the following schemes:

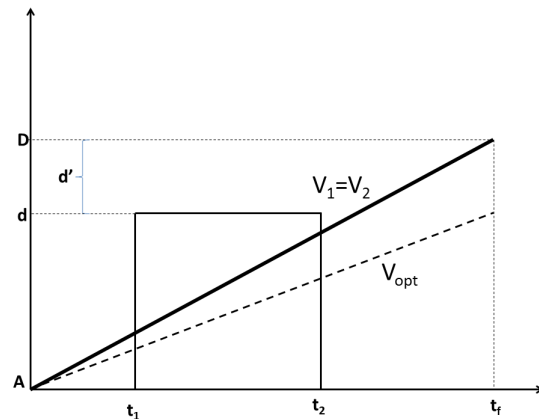


Figure 11: Optimal speed profile solution 1 where $V_1 = V_2 = V > V_0, t_4 = t_f$

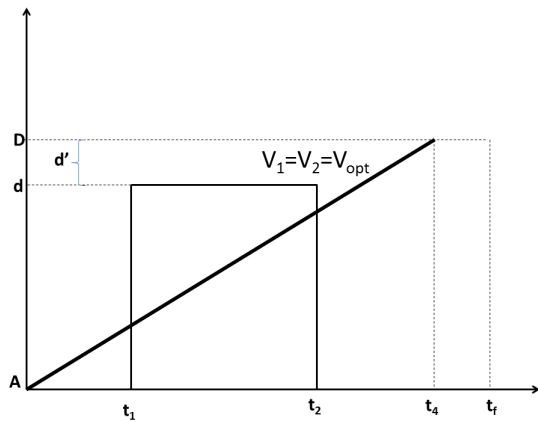


Figure 12: Optimal speed profile solution 2 where $V_1 = V_2 = V = V_0$, $t_4 < t_f$

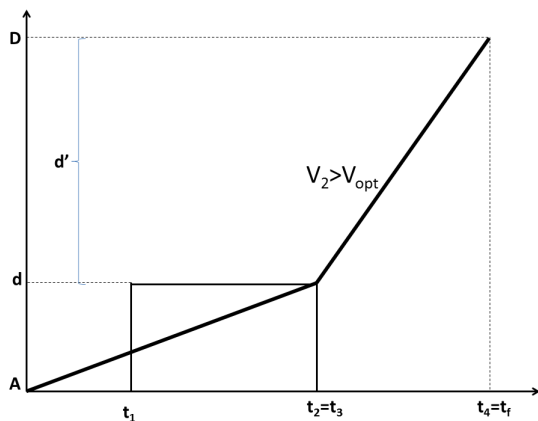


Figure 13: Optimal speed profile solution 3 where $V_1 = \frac{d}{t_2}$, $V_2 = \frac{d'}{t_f - t_2}$, $t_2 = t_3$, $t_4 = t_f$

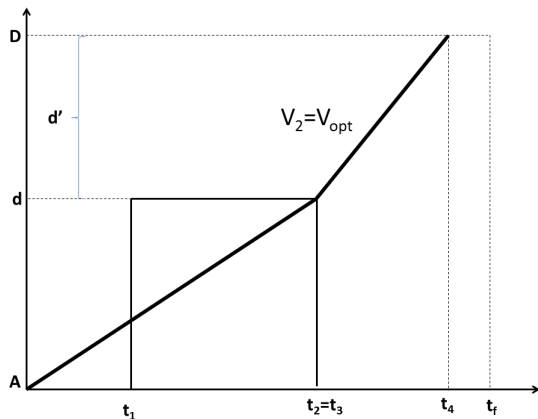


Figure 14: Optimal speed profile solution 4 where $V_1 = \frac{d}{t_1}$, $V_2 = \frac{d'}{t_4 - t_2}$, $t_2 = t_3$, $t_4 < t_f$

3.2.4 Summary of the one traffic light problem

As previously seen, one has to consider the optimal solution which depends on timing data, distance data and the optimal speed. There are two possible scenarios along with the corresponding optimal solutions. If $V_0 \geq d/t_1$, the optimal strategy is to drive at speed V_0 (case 1). If $V_0 < d/t_1$, unless we can reach the final distance without hitting the traffic light (case 2.a), the best strategy is either to pass the traffic light just before it turns to red, or to drive slowly enough to hit the end of the red traffic light (case 2.b). In case 2.b, one will have to compute the value of the fuel consumption in the two cases to determine which path is the best (it will depend on the values of t_1 , t_2 , d and t_f). We remark that waiting at a traffic light is never the best strategy.

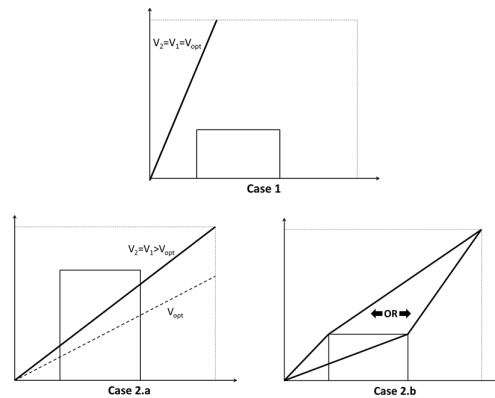


Figure 15: Summary for the one traffic light problem

4 USING DIJKSTRA'S ALGORITHM TO FIND THE OPTIMAL SPEED PROFILE

For more than one traffic light, the situation becomes too complex and we cannot use analytical expressions anymore. That is why we use discretization to create a graph that reflects the relevant decision problem which can be solved using Dijkstra's algorithm.

4.1 Definition

Dijkstra's algorithm is an algorithm which finds the shortest path (from a source to every other nodes) for a graph with nonnegative edge costs [5]. In the way we model our problem, we use Dijkstra's algorithm between a source to a destination to get the optimal speed profile and its total cost. It is run in polynomial time with respect to the number of nodes.

4.2 Method

In this section, we show how we use Dijkstra’s algorithm to determine the optimal speed profile through a set of traffic lights (with respect to fuel minimization). We still consider the same problem as before but with many traffic lights. We call d_i the distance of each traffic lights from the origin. The method we use is to discretize the time by setting nodes at each d_i ’s. We also set a node at the origin (source) and at the distance D (destination). The cost between two nodes is the value of fuel consumption to go from one node to the other node. Let us illustrate this idea with a simple example: one traffic light (red between t_1 and t_2) situated at distance d_1 from the origin (Fig. 16). In Fig.

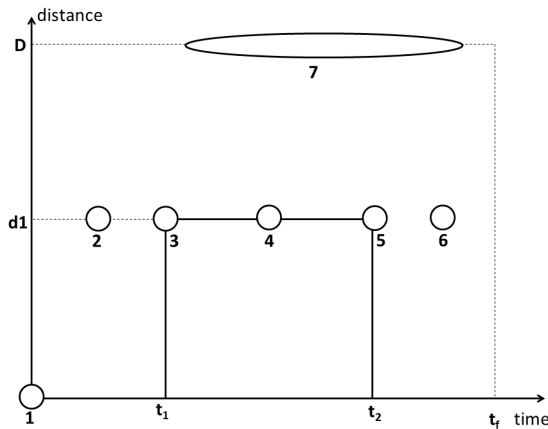


Figure 16: Discretization by setting nodes

16, the circles represent the nodes we decided to set. Node 1 is the origin and node 7 is the destination. The key in Dijkstra’s algorithm is to set up the relations (nodes cannot be linked to every other nodes) and the costs between the different nodes. For example, in this situation, node 1 can be directly linked to every other nodes except node 7 (it has to be linked with an intermediate node). Nodes 2, 5 and 6 can only go to node 7. Node 3 (which is at the beginning of the red traffic light) can go to node 5 (in this case the driver waits at the traffic light) or node 7 (in this case the driver passes just before the traffic light turns to red). Node 4 can only go to node 5 (if one hits a red traffic light, he or she has to wait till it turns green). The cost between two nodes (which can be linked) is set using (3). For example, the cost between node 1 and node 4 is $C(V_{14})$, where V_{14} is the speed to go from node 1 to 4. Then, by considering these relations between nodes, we can draw a graph with nonnegative edge costs and use Dijkstra’s algorithm to find the shortest path between the origin and the destination. The equivalent graph of our example is illustrated in Fig. 17. In this figure, the labels C_{ij} represent the costs from node i to node j : $C_{ij} = C(V_{ij})$. For numerical

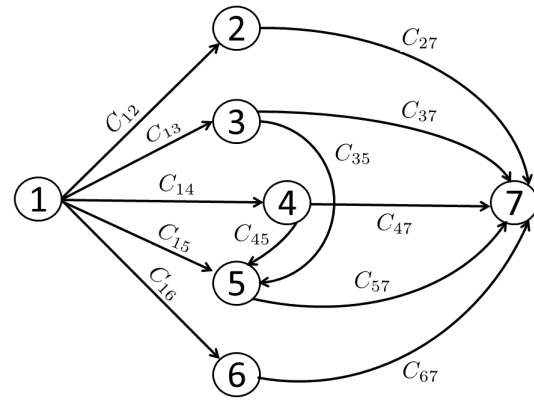


Figure 17: Graph corresponding to Fig. 16

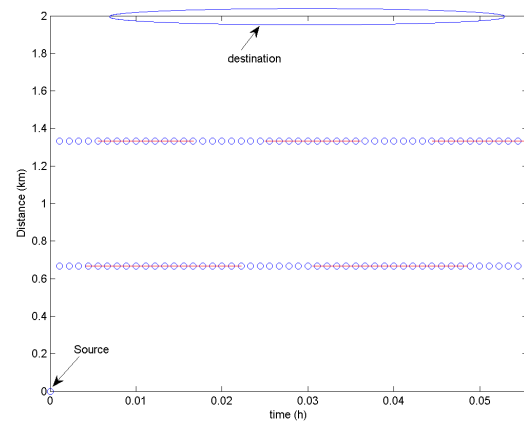


Figure 18: Nodes and red traffic lights

simulations (with more than one traffic light), we put many nodes at each d_i ’s to better discretize the problem and have more precise results.

4.3 Results

4.3.1 Two traffic lights

To illustrate the method on simple situations, we took the case of two sets of traffic lights situated at distance d_1 and d_2 . The objective is to reach the distance $D = 2$ kilometers. We set hundred nodes at d_1 and d_2 plus one node at the origin and one node at the destination. We notice that the last node does not have a fixed position (as opposed to every other nodes). In fact its position depends on the position of the second last node. However the cost between the last node and the other nodes is fixed.

In Fig. 18 the blue circles represent the nodes (which will be hidden on the next pictures) and the red lines represent the red traffic lights. The results in three different traffic light schedules are presented in

the following figures. The straight dot lines represents V_0 , the plain horizontal lines represent the red traffic light and the plain non horizontal lines represents the optimal speed profile calculated with Dijkstra's algorithm.

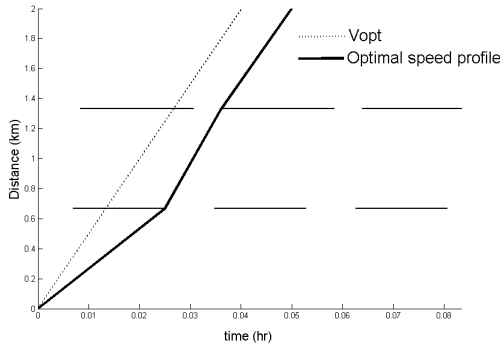


Figure 19: Optimal speed profile for Case 1

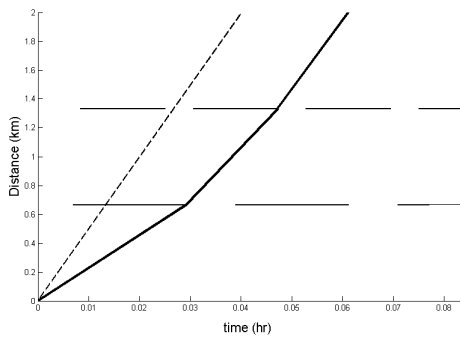


Figure 20: Optimal speed profile for Case 2

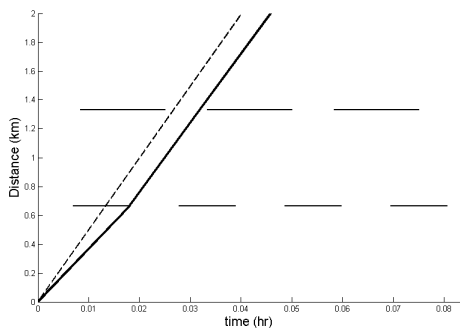


Figure 21: Optimal speed profile for Case 3

We can see that the results of Section 3.2 exhibit similar patterns as in these cases: the best strategy is always not to cross the traffic lights; either we ride at the optimal speed or we hit a “corner” at a traffic light. For example, in case 3, the calculated strategy is to hit the end of the first red traffic light and then to drive at

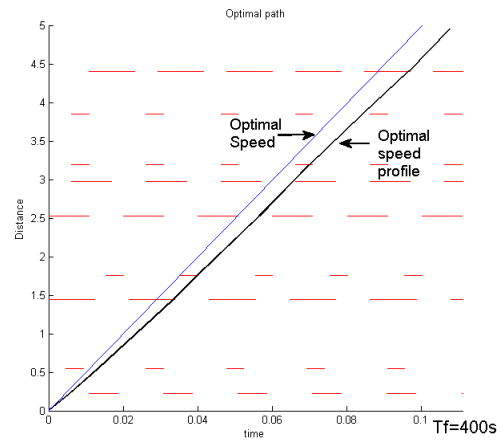


Figure 22: Optimal path in a real case

V_0 during the second and third section. We also note that the best strategy in the the third section is always to drive at V_0 , which makes sense because there are not constraints anymore generated by traffic lights.

4.3.2 Realistic case

We used exactly the same method on a realistic situation. For the partition (length and distance) of the traffic lights we used the data of [4]. Data timing and distance represent a real situation in the city of Greenville, SC. The distance D is equal to 5 kilometers to reach in less than 400 seconds. We put two hundred of nodes per sections (distances where there is a set of traffic lights), the calculated optimal path is shown in Fig. 22. Once again we remark that we never cross a single traffic light. The overall strategy appears to be to drive close to V_0 while avoiding to stop at traffic lights. This roughly means that at each sections, the driver should drive at V_0 if it does not make him hit a red traffic light, and if it does, he should hit one of the corners of the traffic light. We note here that “hitting the corners” is not always the optimal profile as it appeared in the cases of the two lights, although it is true for the majority of the sections.

4.3.3 Comparison with minimal time problem

It is possible to apply exactly the same method to determine the optimal speed profile minimizing time (with a speed limit). In the algorithm, we modified some of the relations between nodes (because of the speed limit) and we also changed the costs (the cost between two nodes is the time to go from one node to the other). We performed simulations on three different cases (with three different speed limits). In Fig. 23, 24 and 25, the dot line represents the speed limit

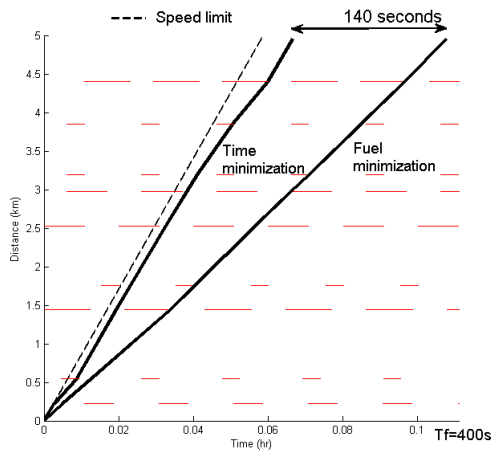


Figure 23: Fuel minimization VS Time minimization with $V_{limit}=85$ mph

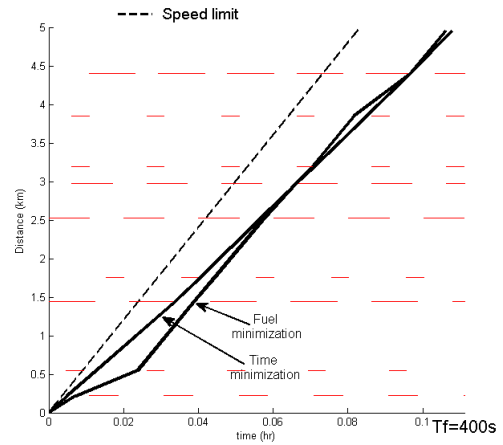


Figure 25: Fuel minimization VS Time minimization with $V_{limit}=60$ mph

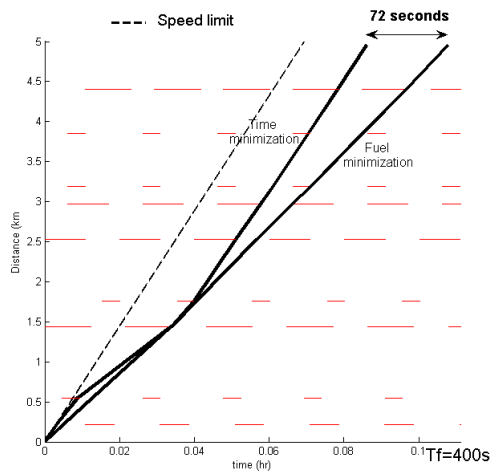


Figure 24: Fuel minimization VS Time minimization with $V_{limit}=72$ mph

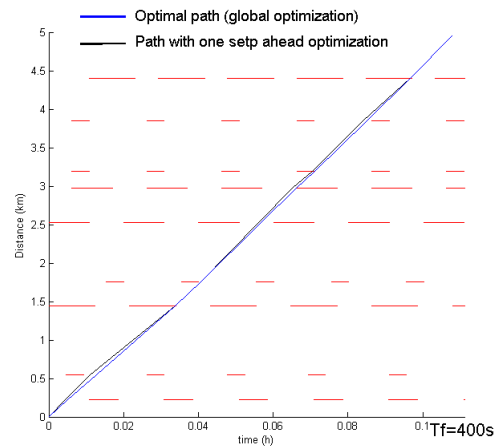


Figure 26: Comparison of optimal speed profile with one-step ahead optimization

and the plain lines represent the time and fuel minimization speed profiles. In the first case ($V_{limit}=85$ mph, Fig. 23), the fuel minimization speed profile is 17% more fuel economic than the time minimization one. In the second case ($V_{limit}=72$ mph, Fig. 24), the fuel minimization path is 8% more fuel economic. We note that when the speed limit decreases, the two speed profiles tend to be the same (Fig. 25, $V_{limit}=60$ mph).

4.3.4 Comparison with one step ahead problem

Using Dijkstra’s algorithm requires to know all the data about the traffic lights network (times and distances of the traffic lights) at the beginning of the trip. Even if we know these data, they could change over time and so the optimal speed profile predicted with Dijkstra’s algorithm could actually not be opti-

mal. Therefore we wanted to determine what would be the speed profile if we only knew the information one traffic light ahead. We performed a simulation for the realistic situation and it turns out that the one step ahead optimization speed profile is pretty close to the overall optimization realized with Dijkstra’s algorithm (Fig. 26).

4.3.5 Modified method

As we already said, the main problem with Dijkstra’s algorithm is the running time. For the previous simulations, we put hundred of nodes per set of traffic light. The results showed that we never cross a single red traffic light, that means that we would not need to put nodes where there is a red traffic light. Therefore we modified our method by setting nodes at the extremities of each red traffic lights and in the mid-

dle of each green traffic light. We can see in Fig. 27 the positions of the nodes and traffic lights. By choosing the appropriate positions for the nodes to set up, we reduced the total number of nodes and the algorithm runs much faster (about 30 seconds instead of 5 minutes for the method with hundred nodes per set of traffic lights). Even though the modified method is less precise (because less discretized), it gives rapid results.

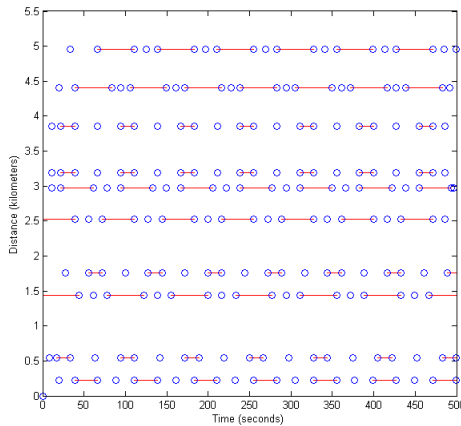


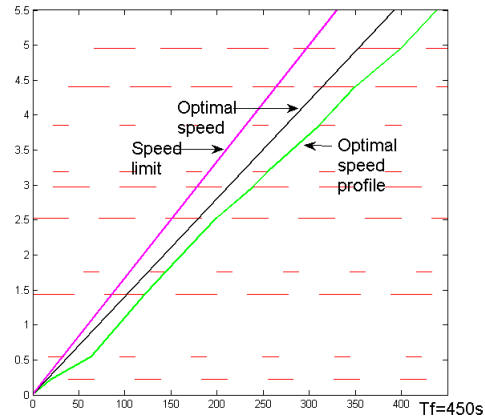
Figure 27: Setting of nodes and traffic lights

We ran different simulations for different values of speed limits and final time. We also compared the overall optimization (Dijkstra’s algorithm) with the one step ahead optimization. For example, we can see in Fig. 28 that the two optimizations give the same results.

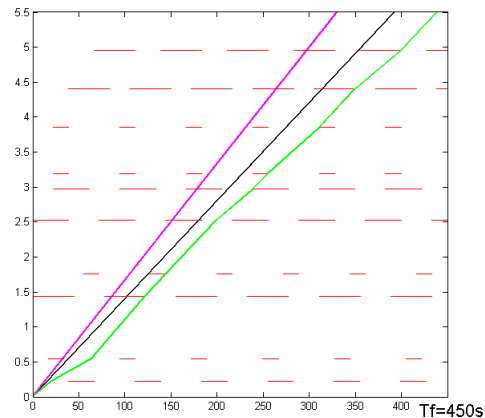
However, sometimes the one step ahead optimization fails due to the lack of information about the future. For example, let us consider a case with a speed limit equal to 90 mph. If the final time is 450 seconds the optimal speed profile is the same with the two different optimizations (shown in Fig. 29). But if the final time is 400 seconds, the one step ahead optimization does not allow the driver to reach the destination before the final time (Fig. 30). We note that 90 mph is obviously not a realistic speed limit but we took this case to illustrate a possible drawback of the one step ahead method.

4.3.6 Impact of final time on fuel consumption

For each traffic light configuration, there exists a minimum final time below which we cannot reach the destination on time. For example, for the realistic case already studied, this minimum final time is about 300 seconds for a speed limit of 90 mph and 400 seconds for a speed limit of 60 mph (see Fig. 31). The evolution of the fuel consumption with respect to the final



Overall optimization



One step ahead optimization

Figure 28: Comparison with a speed limit of 60 mph

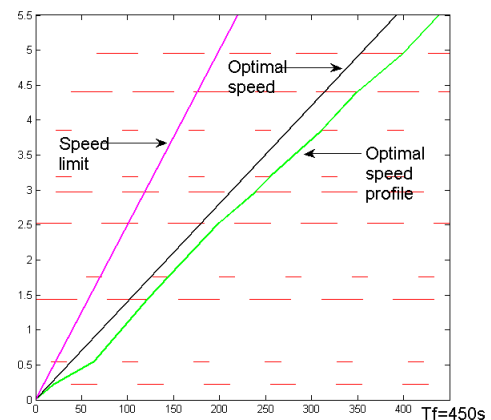
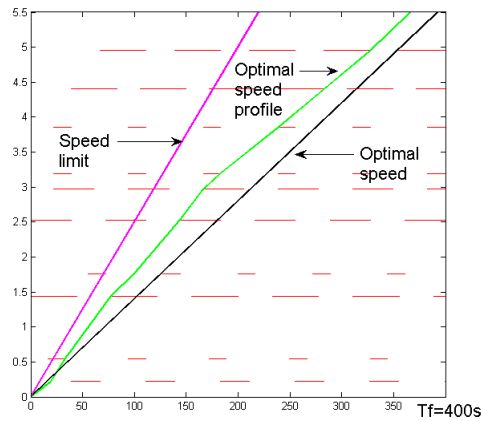
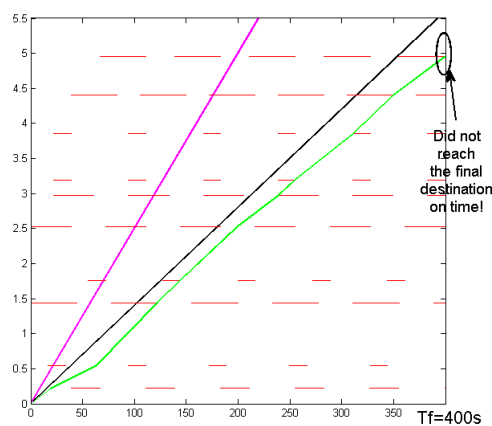


Figure 29: Optimal speed profile (for both optimizations) with $t_f = 450$ seconds



Overall optimization



One step ahead optimization

Figure 30: Comparison of with a speed limit of 90 mph

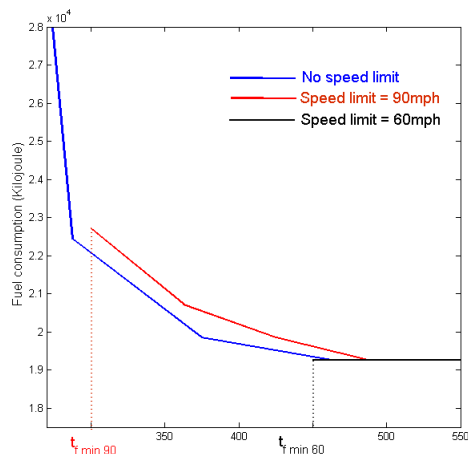


Figure 31: Influence of final time for different speed limits

time depends on the relative difference between the optimal speed and the speed limit. The first case to consider is the no speed limit case. In this case, the fuel consumption decreases and tends to an asymptotic value (blue line in Fig. 31). The minimum time for which this asymptotic value is obtained is the optimal final time: the consumption and the final time are minimum. When the speed limit is significantly higher than the optimal speed, The fuel consumption decreases and tends to the same value as in the no speed limit case (see the red line in Fig. 31, for a speed limit of 90 mph and an optimal speed of 50 mph). When the optimal speed and the speed limit are about the same, the fuel consumption is constant and is about the asymptotic value of the no speed limit case. When the speed limit is lower than the optimal speed, the fuel consumption is constant and its value is higher than the no speed limit case.

5 CONCLUSIONS AND FUTURE WORK

We developed analytical results for simple cases, and a method using Dijkstra’s shortest path algorithm for more complicated ones, to determine the most fuel efficient speed profile of a vehicle through a set of traffic lights. The running time of this method depends on the number of nodes used to discretize the problem and we provided simple ways to obtain good approximations for realistic scenarios. Using this basic method, we also demonstrated that the one step ahead policy performs quite well in relation to the true optimal for the realistic benchmark scenario studied. We further studied the minimal time problem and how the speed limit affects its solution relative to the minimal fuel optimization. Several opportunities for future research on this topic could be based on this initial work. We highlight some of these in what follows.

From our numerical calculations, the optimal strategy (given a fixed number of traffic lights with known schedules) appears to be: driving the closest possible to the optimal speed V_0 while hitting the corners of the traffic lights. Although we obtained a proof of the suggested optimal strategy for simple cases, it remains to fully investigate under what conditions on the timing and spacing distribution of the lights this policy is the best.

In this paper, we assumed that the traffic light timing data had fixed values. An interesting problem would be to take into account the uncertainty due to the traffic light timing, by considering time duration of the red and green traffic lights as random variables with known statistics. In cases where the vehicle does not have exact information about the traffic lights

we could use one step ahead optimization or n -steps ahead optimization (depending on the amount of information that we can obtain). It also remains to be seen how well such strategies perform.

Finally, a more accurate analysis should also take into account the effects of gear shifting and acceleration. For example, [1] proposed a model which takes into account gear shifting and the effects of the brakes. A model of fuel consumption rate that includes acceleration is of the form

$$f_t(t) = \alpha + \left[\beta_1 R_T(t) v(t) \right]_{\{R_T(t)>0\}} + \left[\beta_2 \frac{M}{1000} a^2(t) v(t) \right]_{\{a(t)>0, R_T(t)>0\}}, \quad (14)$$

where $R_T(t)$ is the total *tractive* force required to move the car forward; the operator $[\cdot]_{\{B\}}$ on \mathbf{A} works as follows: $[\mathbf{A}]_{\{B\}} = \mathbf{A}$ if B is true; otherwise $[\mathbf{A}]_{\{B\}} = 0$. $R_T(t)$ is given by

$$R_T(t) = b_1 + b_2 v^2(t) + \frac{M}{1000} a(t) + g \frac{M}{1000} \frac{G}{100}, \quad (15)$$

as described in [9]. In Equations (14) and (15), α , M , G , β_i , and b_i , $i = 1, 2$ are constants related to specific cars. Our ongoing research using Equation (14) to estimate the fuel consumption gives a sample plot of minimum fuel consumption for a car travelling within a fixed distance where there are two traffic lights. Fig. 32 shows optimal trajectory of driving from initial velocity $v_0 = 0$ to final velocity $v_f = 0$, where the two pink blocks are representing two red signals. Fig. 32 is obtained by numerically solving

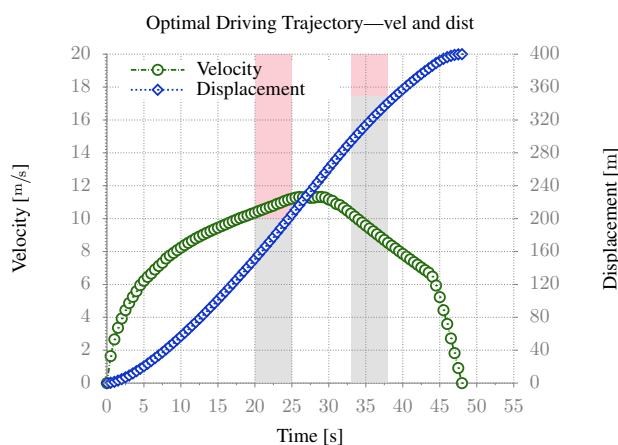


Figure 32: A velocity and displacement profile when acceleration taken into account

an associated optimal control problem to minimize

the fuel consumption, as further discussed in [20]. Optimizing fuel consumption of a travelling car under more complicated traffic conditions is the subject of our ongoing research.

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