Modified design method of an optimal control system for precision motor drive

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Abstract: The paper presents an improved design method of an optimal control system of a linear object with elastic coupling. The proposed method of the optimal control system design implies the selection of desired stability degree instead of the penalty matrixes selection procedure for the quadratic functional. The main idea of the method is the new state matrix utilization, which has its eigenvalues at the specified distance to the right from the eigenvalues of the original state matrix. Thereupon the closed loop state feedback system matrix eigenvalues can be assigned at that specified distance to the left from imaginary axis of the complex plane, in other words the desired stability degree of the system can be achieved. The proposed method of the control algorithm design is demonstrated on the control system of the electric drive with two-mass mechanism (plant). Discrete optimal control system was synthesized. Plant's characteristic was evaluated from bode plot obtained during identification experiment. Unavailable or immeasurable plants state was estimated by reduced-order observer.

Key–Words: Optimal control system, discrete control system, reduced-order observer, two-mass mechanism, state regulator, modal control, degree of stability

1 Introduction

Let us consider an electromechanical system including a controlled voltage converter, an electrical drive and a mechanism as a two-mass design scheme. This system provides the reliable description of the position control processes for both azimuthal and elevation axes of a ground telescope rotary support [1–5].

Normally such electromechanical system is described with the system of ordinary differential equations

$$\begin{cases} T_{inv}\dfrac{d\Omega_0}{dt} = k_{inv}u - \Omega_0 \\ T_e\dfrac{dM}{dt} = \beta(\Omega_0 - \Omega_1) - M \\ J_1\dfrac{d\Omega_1}{dt} = M - M_{12} \\ \dfrac{dM_{12}}{dt} = k_s(\Omega_1 - \Omega_2) \\ J_2\dfrac{d\Omega_2}{dt} = M_{12} \\ \dfrac{d\alpha_1}{dt} = \Omega_1 \end{cases} \quad (1)$$

where T_{inv} – time constant of invertor, Ω_0 – no load speed, k_{inv} – gain coefficient of the invertor, T_e – electromagnetic time constant of motor, β – stiffness of the speed-torque characteristic of electric drive, M – motor torque, Ω_1 - angular velocity of the first mass, M_{12} - twist torque between the first and the second mass, k_s – stiffness coefficient of the two-mass mechanism, Ω_2 - angular velocity of the second mass, α_1 - rotation angle of the first mass.

Time constants T_{inv} and T_e are insignificant, therefore the system can be simplified as follow

$$\begin{cases} M = au - \beta\Omega_1 \\ J_1\dfrac{d\Omega_1}{dt} = M - M_{12} \\ \dfrac{dM_{12}}{dt} = k_s(\Omega_1 - \Omega_2) \\ J_2\dfrac{d\Omega_2}{dt} = M_{12} \\ \dfrac{d\alpha_1}{dt} = \Omega_1 \end{cases} \quad (2)$$

where $a = k_{inv}\beta$.

Block scheme of the plant simplified according to system (2) presents on figure 1.

By using state-space approach, model of the plant can be represented by system

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u. \\ y = \mathbf{C}\mathbf{x} \end{cases} \quad (3)$$

where $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are system, input and output matrixes respectively and state-variables vector of the considered system $\mathbf{x} = \begin{bmatrix}\Omega_1 & M_{12} & \Omega_2 & \alpha_1\end{bmatrix}^T$.

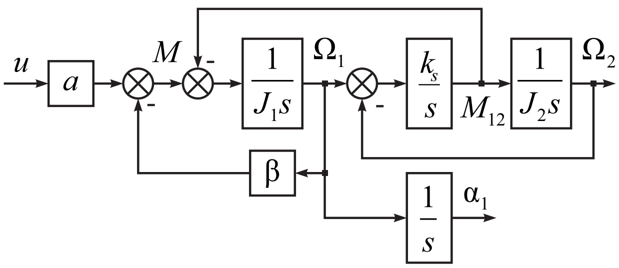


Figure 1: Simplified block scheme of the plant

2 Forced motion robust control

Typical operation mode of a telescope rotary support is reproduction of the targeting signal, which can be approximated by piecewise linear function [6]. The model of such signal is described as follows.

$$\begin{cases}\dot{\xi} = \mathbf{G}\xi \\ g = \mathbf{H}\xi\end{cases} \tag{4}$$

where

$$\mathbf{G} = \begin{bmatrix}0 & 1\\0 & 0\end{bmatrix} \quad \mathbf{H} = \begin{bmatrix}1 & 0\end{bmatrix}$$

The control problem can be solved with forced motion robust (isodromic) control implementation [7]. Design principles of the mathematical model for such system described in [8, 9]. The main idea is to use the model of an external action as part of the regulator, as follows

$$\dot{\mathbf{z}} = \mathbf{G}z + \mathbf{L}(g - y) = \mathbf{G}\mathbf{z} + \mathbf{L}\varepsilon \tag{5}$$

where g – target signal, $\varepsilon = (g - y)$ – error signal, and matrix $\mathbf{L}$ should be selected to provide complete state controllability of matrixes pair $(\mathbf{G}, \mathbf{L})$. Thus model of the combined object (CO) will take the form (6)

$$\begin{cases}\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u \\ \dot{\mathbf{z}} = \mathbf{G}\mathbf{z} + \mathbf{L}g - \mathbf{L}\mathbf{C}\mathbf{x} \\ y = \mathbf{C}\mathbf{x}\end{cases} \tag{6}$$

or

$$\begin{cases}\dot{\mathbf{x}}_n = \mathbf{A}_n\mathbf{x}_n + \mathbf{B}_n u + \mathbf{B}_g g \\ y_n = \mathbf{C}_n\mathbf{x}_n\end{cases} \tag{7}$$

where $\mathbf{x}_n = [\mathbf{x}^T\ \mathbf{z}^T]^T$ – state-space variables vector of the CO including external action model [9] and

$$\mathbf{A}_n = \begin{bmatrix}\mathbf{A} & \mathbf{0}\\ -\mathbf{L}\mathbf{C} & \mathbf{G}\end{bmatrix} \quad \mathbf{B}_n = \begin{bmatrix}\mathbf{B}\\ 0\end{bmatrix}$$

$$\mathbf{B}_g = \begin{bmatrix}\mathbf{0}\\ \mathbf{L}\end{bmatrix} \quad \mathbf{C}_n = \begin{bmatrix}\mathbf{C}\\ 0\end{bmatrix}^T$$

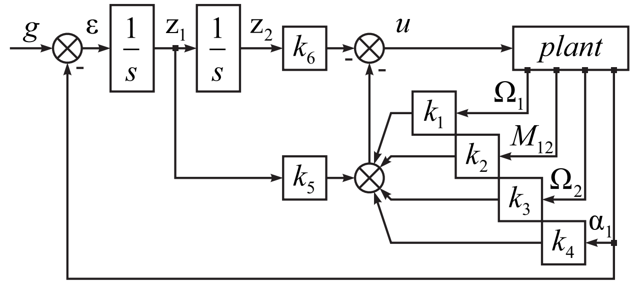
are state, input, reference signal input, output matrixes of CO respectively.

The state-regulator was used (8). It provides stability of closed loop system and zero steady-state error assuming disturbance absence [7].

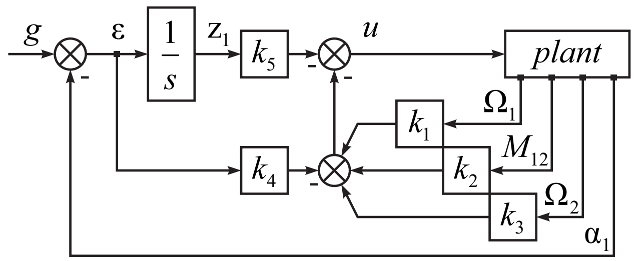
$$u = -\mathbf{K}\mathbf{x}_n \tag{8}$$

The block scheme of such contol system considering on figure 2a.

Order of the controller can be reduced by utilization of the plant's integrator with output signal α_1. Thus block scheme of the closed loop control system is modified as presented of figure 2b.



(a)



(b)

Figure 2: Block scheme of the close loop control system

One can choose the state feedback matrix $\mathbf{K}$ to make the set of closed loop eigenvalues equal the specified set of eigenvalues. The equation of the characteristic polynomial for $\mathbf{A}_n - \mathbf{B}_n\mathbf{K}$ to the characteristic polynomial having roots equal to the specified or desired set of eigenvalues is the most common way to obtain this matrix [10–12].

As a rule the specified set of eigenvalues is presented by standard polynomials. In that case the difficult is to give preference to one or another polynomial type, despite the desirable response speed is provided by anyway. It should be noted that the synthesis of the state feedback matrix based on the modal control is followed by changing of all the eigenvalues of the state matrix, including those ones which determines rapidly damped components of the transient. So far as these components have lack of influence on the general transient shape, one can say that superfluous coercing influences are implemented to the plant.

3 Synthesis of the optimal control system with guaranteed degree of stability

Another method of evaluation is by means of minimization of quadratic or cost functional [12–14]

$$J = \int_0^\infty (\mathbf{x}_n^T \mathbf{Q} \mathbf{x}_n + u^T \mathbf{R} u) \mathrm{d}t \tag{9}$$

In this case large absolute eigenvalues of the plant state matrix are changing insufficiently in a closed loop system; therefore it can be said that control is carried out with less coercing influence.

Implementation of a quadratic optimal control was restricted by two reasons. The first one was the difficulty of the Riccati equation algebraic evaluation, which is now eliminated by contemporary mathematical software [15]. The second one is the indeterminacy of state penalty matrix $\mathbf{R}$ and input penalty matrix $\mathbf{Q}$ selection. In case of ever positive semi definite matrix $\mathbf{Q}$ and positive definite matrix $\mathbf{R}$ the quadratic optimal control provides stability of the closed loop system [10, 12]. However, quality factors of the optimal system are strongly depended of a specific selection of these matrixes. The selection method is time-consuming. To avoid this, the following approach is proposed.

Its known [16], that if the quadratic matrixes are linked by some function $M = f(N)$ eigenvalues of these matrixes are linked by the same function

$$s_i^M = f(s_i^N) \quad i = 1 \ldots n. \tag{10}$$

Considering this, during the synthesis of the optimal control for the system (7) it is proposed to replace the matrix $\mathbf{A}_n$ by (11)

$$\mathbf{A}_{ns} = \mathbf{A}_n + \eta \mathbf{I} \tag{11}$$

where $\mathbf{I}$ – identity matrix, η is the desired stability degree of matrix $\mathbf{F}_n = \mathbf{A}_n - \mathbf{B}_n\mathbf{K}$.

For the case of unconditioned positive definite matrixes of penalty, $\mathbf{Q}$ and $\mathbf{R}$, which can be identity matrixes for example, the Riccati equation (12) should be solved

$$\mathbf{P}\mathbf{B}_n\mathbf{R}^{-1}\mathbf{B}_n^T\mathbf{P} - \mathbf{Q} - \mathbf{A}_{ns}^T\mathbf{P} - \mathbf{P}\mathbf{A}_{ns} = 0. \tag{12}$$

Then the state feedback matrix can be found as follows

$$\mathbf{K} = \mathbf{R}^{-1}\mathbf{B}_n^T\mathbf{P}. \tag{13}$$

The procedure for solving the Riccati equation (12) in MATLAB as follows

$$\mathrm{K} = \mathrm{lqr}(\mathrm{A_{ns}}, \mathrm{B_n}, \mathrm{Q}, \mathrm{R})$$

And the closed loop state feedback system matrix can be evaluated as

$$\mathbf{F}_{ns} = \mathbf{A}_{ns} - \mathbf{B}_n\mathbf{K}. \tag{14}$$

All its eigenvalues will be disposed to the left from imaginary axis of the complex plane.

According to (11) the state feedback system matrix (14) can be rewritten as

$$\mathbf{F}_{ns} = \mathbf{A}_n + \eta\mathbf{I} - \mathbf{B}_n\mathbf{K} = \mathbf{F}_n + \eta\mathbf{I}. \tag{15}$$

and

$$\mathbf{F}_n = \mathbf{F}_{ns} - \eta\mathbf{I}. \tag{16}$$

Matrix (15) eigenvalues are disposed to the left from imaginary axis of the complex plane, therefore matrix (16) eigenvalues will be displaced to the left from image axis at the interval not less than η.

As a result, indeterminate method of penalty matrixes selection for the optimal control is replaced by selection of the optimal system stability degree, which has obvious physical sense: it defines response speed of the optimal system.

In real devices control algorithm is executed in the microcontroller program. Hence, state, input and output matrixes transforms as shown below [17]

$$\mathbf{A}_{nd} = e^{\mathbf{A}_n T_d}, \quad \mathbf{B}_{nd} = \int_0^{T_d} e^{\mathbf{A}_n \sigma}\mathbf{B}_n \mathrm{d}\sigma, \quad \mathbf{C}_{nd} = \mathbf{C}_d$$

where T_d - sample time of the discrete system.

MATLAB function to convert model from continuous to discrete time is

$$\mathrm{Sysd} = \mathrm{c2d}(\mathrm{Sysc}, \mathrm{Td})$$

here Sysd, Sysc – discrete and continuous state-space systems respectively.

In the case of the discrete system equation (11) transforms as follows (17)

$$\mathbf{A}_{nsd} = \frac{\mathbf{A}_{nd}}{e^{-\eta T_d}} = \frac{\mathbf{A}_{nd}}{r} \tag{17}$$

where r - desired degree of stability of the discrete system $\mathbf{F}_{nd} = \mathbf{A}_{nd} - \mathbf{B}_{nd}\mathbf{K}_d$.

In MATLAB the state feedback matrix $\mathbf{K}_d$ for discrete system is calculated as follows

$$\mathrm{K_d = dlqr(A_{nsd}, B_{nd}, Q, R)}$$

4 Control system with observer

In case of immeasurable states of the plant (3) a reduced-order observer is usually used and the optimal control (8) transforms as follows [9]

$$u = -\mathbf{K}\mathbf{x}_n = \mathbf{N}_1\mathbf{y} + \mathbf{N}_2\widetilde{\mathbf{w}} \tag{18}$$

here $\mathbf{y} = \mathbf{C}_m\mathbf{x}_n$ - vector of measured state variables, $\widetilde{\mathbf{w}} = \mathbf{T}\mathbf{x}_n$ - vector of observed state variables, $\mathbf{N}_1 = \begin{bmatrix} n_{11} & n_{12} \end{bmatrix}$ - vector of coefficients of measured state variables, $\mathbf{N}_2$ – vector of coefficients of observed state variables.

State-space representation of the observer has the following form

$$\begin{cases} \dot{\widetilde{\mathbf{w}}} = \mathbf{A}_o\widetilde{\mathbf{w}} + \mathbf{B}_o u + \mathbf{R}_o\mathbf{y} \\ u = \mathbf{N}_1\mathbf{y} + \mathbf{N}_2\widetilde{\mathbf{w}} \end{cases} \tag{19}$$

Here state matrix $\mathbf{A}_o$ of the observer is selected randomly with only requirement that all its eigenvalues must be negative and $\mathbf{B}_o = \mathbf{T}\mathbf{B}_n$.

Vectors $\mathbf{N}_1$ and $\mathbf{N}_2$ can be found from

$$\begin{bmatrix} \mathbf{N}_1 & \mathbf{N}_2 \end{bmatrix} = -\mathbf{K} \begin{bmatrix} \mathbf{C}_m \\ \mathbf{T} \end{bmatrix}^{-1} \tag{20}$$

where $\mathbf{C}_m = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ and matrix $\mathbf{T}$ is a solution of the Lyapunov equation

$$\mathbf{A}_o\mathbf{T} - \mathbf{T}\mathbf{A}_n + \mathbf{R}_o\mathbf{C}_m = 0. \tag{21}$$

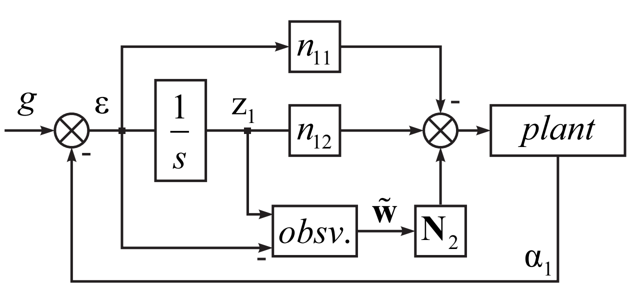


Figure 3: Block diagram of control system

Form (22) of observer can be obtained by substitution the second equation into the first.

$$\begin{cases} \dot{\widetilde{\mathbf{w}}} = \mathbf{A}_{on}\widetilde{\mathbf{w}} + \mathbf{R}_{on}\mathbf{y} \\ u = \mathbf{N}_1\mathbf{y} + \mathbf{N}_2\widetilde{\mathbf{w}} \end{cases} \tag{22}$$

where $\mathbf{A}_{on} = (\mathbf{A}_o + \mathbf{B}_o\mathbf{N}_2)$, $\mathbf{R}_{on} = (\mathbf{R}_o + \mathbf{B}_o\mathbf{N}_1)$.

Thus observer has only one input vector – $\mathbf{y}$.

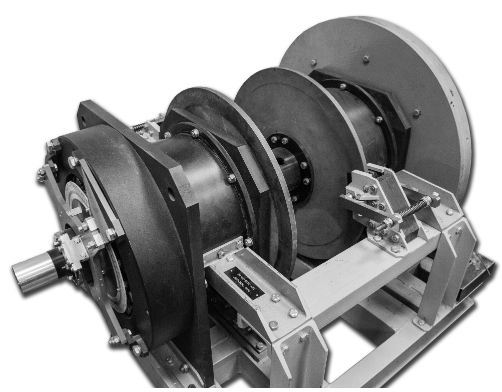
Block scheme of the closed-loop control system is presented on figure 3.

5 Experiment

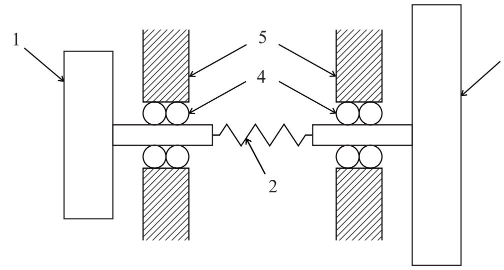
Testing of the proposed synthesis method of the optimal control was carried out on a laboratory bench with a two-mass motor drive mechanism. General view and schematic diagram of the electromechanical laboratory bench are shown in figure 4.

On figure 4 b) two shafts are fixed in two supports – 5 using bearing assemblies 4. The first shaft – 1 contains three-phase motor produced by Ruchservomotor and incremental optical encoders from Renishaw. The payload prototype mounted on the second shaft – 3 formed as a stack of rings that allows changing its inertia.

Both shafts are connected by a special coupling which can modify the torsional rigidity of the compound. It should be noted that the position encoder is located only on the first mass. Moreover, the coulomb friction unequally distributed along the shaft.



(a) general view of the electromechanical laboratory bench



(b) schematic diagram of the electromechanical laboratory bench

Figure 4: Laboratory bench

Since the absence of plants actual parameters, at the synthesis stage we will use mathematical model obtained from experimental frequency response. Frequency response of the electric drive with two-mass mechanism illustrated on figure 5, where curve 1 represents the frequency response of the plant with first mass angular velocity as an output signal.

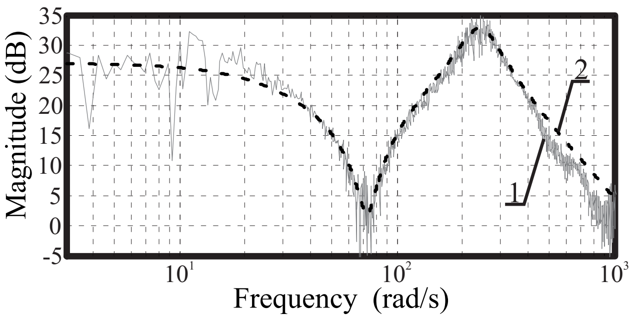


Figure 5: Frequency response of the electric drive with two-mass mechanism

Approximating the frequency response by lines with slopes multiples of $20dB/dec$, we obtain the model of the object in transfer function form (5)

$$W(s) = \frac{\Omega_1(s)}{u(s)} = \frac{k(\tau^2 s^2 + 2\zeta\tau s + 1)}{(T_1 s + 1)(T_2^2 s^2 + 2\xi T_2 s + 1)(T_3 s + 1)} \tag{23}$$

where $k = 22.4$, $\tau = 0.014\text{s}$, $\zeta = 0.078$, $\xi = 0.206$, $T_1 = 0.038\text{s}$, $T_2 = 0.004\text{s}$, $T_3 = 0.004\text{s}$.

Frequency response of the model represented by curve 2 on figure 5. One can see that frequency response of the plant and the model coincide sufficiently which mean adequate approximation of the experimental characteristics. By using MATLAB function

$$\begin{bmatrix}\text{A} & \text{B} & \text{C}\end{bmatrix} = \text{ss(W(s))}$$

and expanding the state variables vector by the angle of the first mass, we obtain the plant's model in matrix form

$$\mathbf{x} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & \alpha_1 \end{bmatrix}^T$$

$$\mathbf{A} = \begin{bmatrix} -379 & -182 & -131 & -47.5 & 0 \\ 512 & 0 & 0 & 0 & 0 \\ 0 & 256 & 0 & 0 & 0 \\ 0 & 0 & 64 & 0 & 0 \\ 0 & 51.2 & 2.26 & 16.6 & 0 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 64 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

$$\mathbf{C} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

In considered case the state feedback matrix $\mathbf{K}$ in equation (8) was evaluated at the degree of stability value $\eta = 19\text{s}^{-1}$.

Eigenvalues of the closed loop system (16) are equal to

$$\mathbf{s_{F_n}} = \begin{bmatrix} -38 \\ -45.7 + 8.8i \\ -45.7 - 8.8i \\ -91 + 2.6i \\ -91 - 2.6 \\ -267 \end{bmatrix}^T . \tag{24}$$

The closest to the imaginary axes real eigenvalue which is equal to -38 determine the stability degree of the system. The stability degree is not less than the specified.

In considered case only one state variable was measured – α_1. Moreover a controller can use the output signal of the integrator z_1 added to the error channel [9].

Vectors of measured and observed states variables in the control law (18) take the form

$$\mathbf{y} = \begin{bmatrix} \alpha_1 & z_1 \end{bmatrix}^T, \widetilde{\mathbf{w}} = \begin{bmatrix} \widetilde{w}_1 & \widetilde{w}_2 & \widetilde{w}_3 & \widetilde{w}_4 \end{bmatrix}^T.$$

Figure 6 a) presents the tracking error transient of the system with the optimal state regulator and reduced order observer in the course of reproducing of input signal increasing linearly with the speed of $1°/s$, where curve 1 represent the transient process of the model, curve 2 represent the transient process of the system with the real control object.

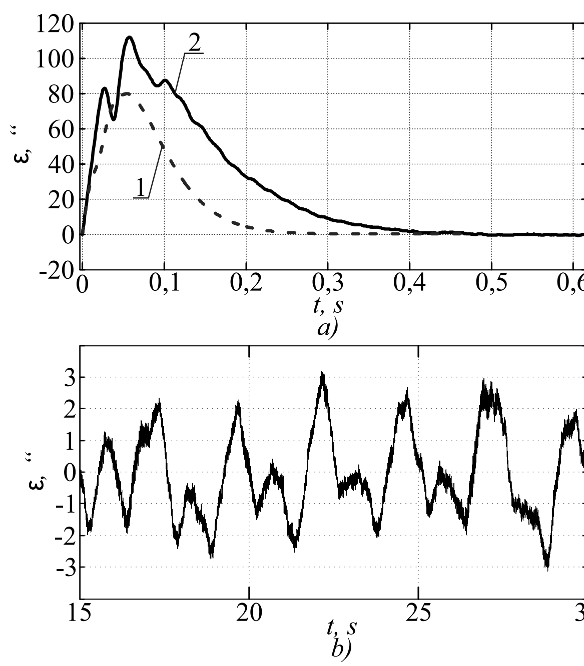


Figure 6: Results of experiment

Faster transition process in the system is caused by disregard of dry friction.

Figure 6 b) depicts the steady-state tracking error chart in case of the same input signal. Maximal error value is equal to $3.2''$, root-means-square error is equal to $1.4''$.

6 Conclusion

The disadvantage of the standard optimal control synthesis is the indetermination of the penalty matrixes selection in the quadratic functional. The positive defined penalty matrix provides only stability of the system. To provide the desirable response speed it is necessary to iterate over a significant amount of penalty matrixes.

The proposed modification of the quadratic optimal control allows limiting oneself to the selection of only one value which determines the stability degree in the closed loop system.

Experimental verification of the proposed procedure on the control system of the electric drive with two-mass mechanism showed good results in practical terms.

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