

Optimal Control of Mobile Robot's Trajectory Movement

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Abstract: This paper is devoted to the development of an optimal control system of trajectory movement of a mobile robot. Synthesis of the trajectory control law is based on the methods of optimal control and asymptotical methods of singular perturbation theory. Singular perturbation theory's methods are used for reducing the order of the solving problem and it helps to simplify the realization of the control law. Proposed system has two control loops: position control loop and velocity control loop. Based on this, the structure of the control system is represented by two modules: the reference and the executive. In this work the results of experimental verification of the trajectory control system at the mobile robot Rover5 with the tank-type chassis are presented.

Key-Words: trajectory control, quadratic performance index, optimal control, mobile robot, tracked chassis.

1 Introduction

Autonomous mobile robots are increasingly being implemented in industry, scientific and research projects, and many other areas where the direct participation of man is undesirable or not possible. The problem of mobile robot (MR) control is relevant over the years because of the wide range of practical applications and theoretical problems associated with it. To confirm this fact we should notice that there is a great number of works in the field of motion control of MR, published in recent decades. Their reviews can be found in [1-10].

For trajectory movement control problem the main task is to minimize the MR's current position's deviation from the position specified by a reference trajectory. Thus, as a rule, simple nonlinear models are operated, which characterize only the kinematic relationship between the current MR's motion parameters and parameters predetermined by a reference trajectory. At this point it is possible to mark out two basic approaches to the synthesis of the trajectory control. The first approach is based on linear control design methods and tangent linearization of system model about the reference trajectory. At that, a pole placement method is usually used to adjust controller parameters [3,6-9]. The second approach is based on the feedback linearization or nonlinear control design methods, according to which controller's parameters are

calculated using the method of Lyapunov functions [2-5,10].

Despite the efficiency of these approaches, the point of using optimal control methods for the synthesis of trajectory control system is actual. It is possible to note several works [11-13], where for setting-up the parameters of control law it is proposed to solve some optimization problems. At that, the quality of tracking a reference trajectory is determined by some criterion, and this criterion should be optimized. As a result are obtained the control laws which are determined by selected criteria and methods of optimization.

This work is devoted to synthesis of optimal control system of MR's trajectory movement. The solution of this problem is based on the optimal control methods and singular perturbation method [14]. The last one is based on the order reduction of the problem and a simplified representation of controller gives us a possibility to simplify implementation of the control law.

The remaining sections of the paper proceed as follows: Section 2 describes the mathematical model of the control object and the trajectory control problem statement; Section 3 is devoted to the problem of synthesis of optimal control system of MR's trajectory movement; Section 4 describes the structure of the trajectory control system; the results of experimental tests of the designed system are shown in Section 5; some conclusions are shown in Section 6.

2 Formulation of a trajectory control problem

2.1 MR's mathematical model

The movement of mobile robot with a tank-type chassis in a horizontal plane is reviewing (Fig.1). MR's kinematics model is [15,16]:

$$\begin{cases} \frac{dX}{dt} = V \cos \varphi = \frac{V_1 + V_2}{2} \cos \varphi, \\ \frac{dY}{dt} = V \sin \varphi = \frac{V_1 + V_2}{2} \sin \varphi, \\ \frac{d\varphi}{dt} = \omega = \frac{V_2 - V_1}{2d}, \end{cases} \quad (1)$$

where X, Y – is the current position of the MR in the earth coordinate system; V_1, V_2, V – linear velocities of left and right tracks and of the MR, correspondingly; ω – MR's angular velocity; φ – the angle between the vector V and the axis X .

The equations of the dynamics of the MR's movement in the horizontal plane including the dynamics of robot's motors (it is assumed that both motors are completely identical) are [15,16]:

$$\begin{cases} m \frac{dV}{dt} = \frac{c_m \cdot n}{r} (I_1 + I_2), \\ J \frac{d\omega}{dt} = \frac{c_m \cdot n \cdot d}{r} (I_2 - I_1), \\ L \frac{dI_1}{dt} = -R_m \cdot I_1 - \frac{c_e \cdot n}{r} (V + d \cdot \omega) + U_1, \\ L \frac{dI_2}{dt} = -R_m \cdot I_2 - \frac{c_e \cdot n}{r} (V - d \cdot \omega) + U_2, \end{cases} \quad (2)$$

where m, J – mass and moment of inertia of the robot; n – gear ratio; r – radius of the driving-wheel; c_m – motor torque constant; $I_i, i=1,2$ – current in the armature winding of the motor; U_i – voltage in the motor armature; R_m, L – resistance and inductance of the armature winding of the motor; c_e – motor e.m.f. constant.

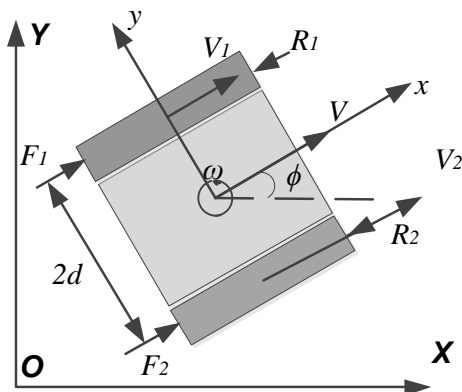


Fig.1 The MR's movement on a plane.

2.2 Statement of the MR's trajectory movement control problem

We suppose that there is a given trajectory $p_{ref}(t) = (X_{ref}(t), Y_{ref}(t), \varphi_{ref}(t))$. MR's trajectory-following errors (Fig. 2) is defined as [3-8]

$$\begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix} = \begin{pmatrix} \cos \varphi & \sin \varphi & 0 \\ -\sin \varphi & \cos \varphi & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} X_{ref} - X \\ Y_{ref} - Y \\ \varphi_{ref} - \varphi \end{pmatrix}. \quad (3)$$

Differentiating (3), we obtain the nonlinear model of tracking errors dynamics of MR's movement on a reference trajectory:

$$\begin{cases} \frac{de_1}{dt} = \omega_{ref} \cdot e_2 - V + V_{ref} \cdot \cos e_3, \\ \frac{de_2}{dt} = -\omega_{ref} \cdot e_1 + V_{ref} \cdot \sin e_3, \\ \frac{de_3}{dt} = \omega_{ref} - \omega. \end{cases}$$

Linearization of these equations about the reference trajectory (i.e with $e_i = 0, i=1,2,3$) leads to a system

$$\begin{cases} \frac{de_1}{dt} = \omega_{ref} \cdot e_2 + u_1, \\ \frac{de_2}{dt} = -\omega_{ref} \cdot e_1 + V_{ref} \cdot e_3, \\ \frac{de_3}{dt} = u_2, \end{cases} \quad (4)$$

where is denoted $u_1 = V_{ref} - V, u_2 = \omega_{ref} - \omega$.

The problem is in searching of control inputs u_1 and u_2 which would be achieved with a minimum of the integral quadratic cost function

$$\rho = \frac{1}{2} \int_0^{\infty} (q_1 e_1^2 + q_2 e_2^2 + q_3 e_3^2 + r_1 u_1^2 + r_2 u_2^2) dt. \quad (5)$$

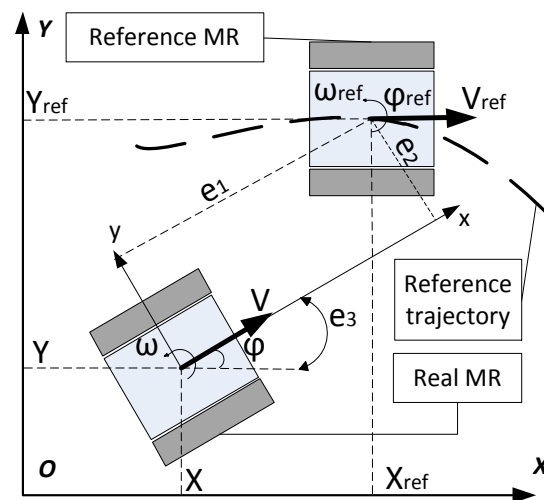


Fig. 2 Statement of the problem of MR's trajectory movement control.

Having known the control inputs u_1 and u_2 the required robot's velocities values are calculated

$$V_{req} = V_{ref} - u_1, \quad \omega_{req} = \omega_{ref} - u_2.$$

The next step is to stabilize the linear and angular velocities of the MR in the neighborhood of the required values V_{req} and ω_{req} . This problem can also be formulated as an optimal control problem, i.e. minimize the quadratic cost function

$$\rho_f = \frac{1}{2} \int_0^{\infty} (q_V \tilde{V}^2 + q_\omega \tilde{\omega}^2 + r_{U1} U_1^2 + r_{U2} U_2^2) dt, \quad (6)$$

on the trajectories of the system (2), written in deviations from the nominal mode $\tilde{V} = V - V_{req}$, $\tilde{\omega} = \omega - \omega_{req}$.

Thus, the trajectory movement control problem is reduced to two sub-problems. The first one concerns with the synthesis of the position control loop based on the solution of the optimization problem (4), (5). The second sub-problem is connected with the synthesis of the velocity (linear and angular) control loop of MR's movement, based on the solution of the optimization problem (2), (6). It should be noticed that if V_{req} and ω_{req} are constants, the first sub-problem is time-invariant, otherwise it's time-varying, the second sub-problem is time-invariant.

3 Solution of the trajectory movement control problem

3.1 Synthesis of position control loop

Position control loop is designed on the basis of the solution of the optimization problem (4), (5), which can be conveniently written in vector-matrix form:

$$\rho_K = \frac{1}{2} \int_0^{\infty} (e^T Q_K e + u_K^T R_K u_K) dt \rightarrow \min_{u_K}, \quad (7)$$

$$Q_K = \text{diag}(q_1, q_2, q_3) \geq 0, \quad R_K = \text{diag}(r_1, r_2) > 0,$$

$$e = \begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix}, \quad u_K = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix},$$

with the constraint

$$\frac{de}{dt} = A_K(t)e + B_K u_K, \quad (8)$$

$$A_K(t) = \begin{pmatrix} 0 & \omega_{ref}(t) & 0 \\ -\omega_{ref}(t) & 0 & V_{ref}(t) \\ 0 & 0 & 0 \end{pmatrix}, \quad B_K = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix},$$

where "T" – the sign of transpose; "diag(...)" – is a

diagonal matrix with the given elements on the main diagonal.

The solution of (7), (8) under the condition of controllability of matrix pair $(A(t), B)$ (which holds for all V_{ref}, ω_{ref} , except $V_{ref} = \omega_{ref} = 0$) is [17] (hereinafter, the time dependence is omitted for simplicity)

$$u_K = -R_K^{-1} B_K^T P_K e = G_K e,$$

$$G_K = -R_K^{-1} B_K^T P_K = \begin{pmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \end{pmatrix},$$

where P_K is a symmetric positive definite solution of the Riccati matrix differential equation

$$\frac{dP_K}{dt} = -A_K^T P_K - P_K A_K + P_K B_K^T R_K^{-1} B_K P_K - Q_K,$$

$$P_K(t = t_f \rightarrow \infty) = 0.$$

Finally, for the required values of MR's linear and angular velocities, we have:

$$V_{req} = V_{ref} - g_{11}e_1 - g_{12}e_2 - g_{13}e_3,$$

$$\omega_{req} = \omega_{ref} - g_{21}e_1 - g_{22}e_2 - g_{23}e_3.$$

3.2 Synthesis of velocity control loop

The task of velocity control loop synthesis we will formulate in vector-matrix form:

$$\rho = \frac{1}{2} \int_0^{\infty} (x^T Q x + u^T R u) dt \rightarrow \min_u, \quad (9)$$

$$Q = \text{diag}(q_V, q_\omega, 0, 0) \geq 0,$$

$$R = \text{diag}(r_{U1}, r_{U2}) > 0,$$

with the constraint

$$\frac{dx}{dt} = Ax + Bu, \quad x(0) = x_0, \quad (10)$$

where is denoted

$$x = \begin{pmatrix} V \\ \omega \\ I_1 \\ I_2 \end{pmatrix}, \quad u = \begin{pmatrix} U_1 \\ U_2 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ b_1 & 0 \\ 0 & b_1 \end{pmatrix},$$

$$A = \begin{pmatrix} 0 & 0 & a_1 & a_1 \\ 0 & 0 & -a_2 & a_2 \\ -a_3 & -a_4 & -a_5 & 0 \\ -a_3 & a_4 & 0 & -a_5 \end{pmatrix},$$

$$a_1 = \frac{c_m \cdot n}{r \cdot m}, \quad a_2 = a_1 \cdot \frac{d \cdot m}{J}, \quad a_3 = \frac{c_e \cdot n}{r \cdot L},$$

$$a_4 = a_3 \cdot d, \quad a_5 = \frac{R_m}{L}, \quad b_1 = \frac{1}{L}.$$

If the system (10) is controllable and observable then the optimal control problem (9), (10) has the unique solution [17]

$$u = -R^{-1} B^T P x = G x, \quad (11)$$

where P is a solution of the Riccati matrix algebraic equation

$$0 = -A^T P - PA^T + PB^T R^{-1} B P - Q.$$

The main disadvantages of such a solution are, firstly, calculating difficulties connected with the solution of the Riccati's matrix equation in real time, and secondly, the need to measure all the components of the state vector for evaluation of optimal control.

In this case, the second aspect limits the use of the optimal control law (11) more acutely, because, usually, robots are equipped with sensors of motor's angular velocity, but not the current sensors.

Let's assume, that MR's structure is able to measure only the angular velocities of the motors ω_{r1} and ω_{r2} . From (1) it is easy to show that

$$\omega_{r1} = \frac{n}{r}(V + d\omega), \quad \omega_{r2} = \frac{n}{r}(V - d\omega). \quad (12)$$

Then the state-space equation (10) is supplemented by the output equation

$$y = Cx, \\ y = \begin{pmatrix} \omega_{r1} \\ \omega_{r2} \end{pmatrix}, \quad C = \frac{n}{r} \begin{pmatrix} 1 & d & 0 & 0 \\ 1 & -d & 0 & 0 \end{pmatrix}.$$

To overcome this limitation it is suggested to use the suboptimal control, which is an asymptotic approximation to the original optimal control law (11) [15]. The suboptimal control synthesis's method is based on a singularly perturbed presentation of the system and its subsequent reduction [14]. In this case state variables are usually divided into "slow" x_1 (slowly changing), and "fast" x_2 (rapidly changing). Usually variables that characterize the trajectory displacement are considered to be slow, and the variables that describe the internal processes in the system are considered to be fast. In such a situation, it is logical to define as slow the variables of linear and angular robot's velocities, and as fast – the currents in the motors of the robot. As a result, the system (10) can be rewritten in singular perturbed form

$$\frac{dx_1}{dt} = A_{12}x_2, \quad x_1(0) = x_{10}, \\ \varepsilon \cdot \frac{dx_2}{dt} = A_{21}x_1 + A_{22}x_2 + B_2u, \quad x_2(0) = x_{20}, \quad (13) \\ y = C_1x_1,$$

where

$$x_1 = \begin{pmatrix} V \\ \omega \end{pmatrix}, \quad x_2 = \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}, \quad u = \begin{pmatrix} U_1 \\ U_2 \end{pmatrix}, \quad C_1 = \frac{n}{r} \begin{pmatrix} 1 & d \\ 1 & -d \end{pmatrix},$$

$$A_{12} = \begin{pmatrix} a_1 & a_1 \\ -a_2 & a_2 \end{pmatrix}, \quad A_{21} = \begin{pmatrix} -\bar{a}_3 & -\bar{a}_4 \\ -\bar{a}_3 & \bar{a}_4 \end{pmatrix},$$

$$A_{22} = \begin{pmatrix} -\bar{a}_5 & 0 \\ 0 & -\bar{a}_5 \end{pmatrix}, \quad B_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

$$\bar{a}_3 = \frac{c \cdot n}{r}, \quad \bar{a}_4 = \bar{a}_3 \cdot d, \quad \bar{a}_5 = R_m, \quad \varepsilon = L.$$

If A_{22} is nonsingular, then for sufficiently small ε the vector of slow variables x_1 are approximated (up to $O(\varepsilon)$, O – Landau's symbol) by the vector x_s , which is defined by slow system [14]

$$\frac{dx_s}{dt} = A_s x_s + B_s u_s, \quad x_s(0) = x_{s0}, \quad (14)$$

$$y = C_1 x_s, \quad A_s = -A_{12} A_{22}^{-1} A_{21}, \quad B_s = -A_{12} A_{22}^{-1} B_2.$$

Let's solve the problem of minimizing the performance index for the slow system (14):

$$\rho_s = \frac{1}{2} \int_0^\infty (x_s^T Q_s x_s + u_s^T R u_s) dt \rightarrow \min_{u_s}$$

$$Q_s = \text{diag}(q_1, q_2) \geq 0.$$

The solution of this problem has such a form:

$$u_s = -R^{-1} B_s^T P_s = G_s x_s, \quad (15)$$

$$0 = -A_s^T P_s - P_s A_s + P_s B_s^T R^{-1} B_s P_s - Q_s.$$

It is important to notice a peculiarity of solutions (11) and (15), that with the ε small enough and Hurwitz matrix A_{22} the asymptotic equality is held [14]

$$u = u_s + O(\varepsilon) = G_s x_s + O(\varepsilon). \quad (16)$$

The task of the implementation of feedback components of the vector x_1 remains unsolved. It is convenient to use the expression for output of slow system (13), whence can be expressed

$$x_s = C_{11}^{-1} y.$$

Substituting this expression in (11) and (16), we finally have

$$u = G_s C_{11}^{-1} y + O(\varepsilon). \quad (17)$$

The formulas (15)-(17) are the mathematical basis for the synthesis of sub-optimal MR's velocity control loop that is necessary for the calculation to solve the reduced problem.

The fact that under the condition of stability of the slow and fast subsystems, closed-loop original system is stable for all values of the perturbation in some interval $(0, \varepsilon^*)$ turned out to be helpful [14, 15]. The critical value ε^* deals as a index of the robust properties of the closed-loop system, i.e. the more is ε^* , the more robust is a system. Some methods for ε -bound computing are shown in [14].

4 Structure of the trajectory movement control system

The separation of the control problem into two sub-problems (position and velocity control loops) leads to an obvious structure of trajectory movement control system (Fig. 3).

The proposed structure of control system consists of two main modules: reference module and executive module. The reference module is placed on the PC. It contains the trajectory generator, position control loop and errors computing unit. The executive module includes velocity control loop and estimator of the robot's position. It is placed on MR's board.

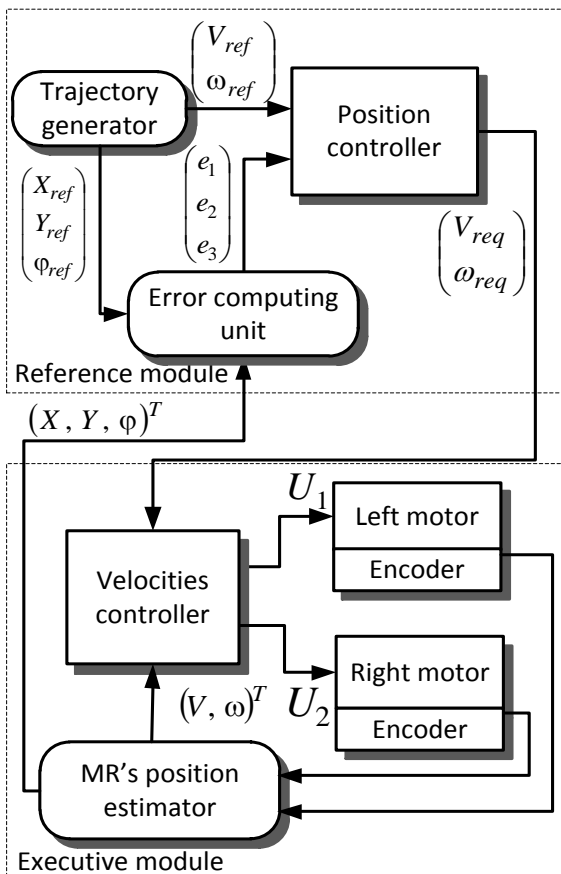


Fig. 3 Structural diagram of the MR's control system.

4.1 MR's trajectory generator

The robot's movement along the reference trajectory is defined by the coordinates X_{ref} , Y_{ref} of an arbitrary robot's point M. Projections of the point's M velocity on the fixed axes are continuous functions of time:

$$V_{Xref} = \frac{dX_{ref}}{dt}, V_{Yref} = \frac{dY_{ref}}{dt}.$$

The considered MR is a nonholonomic system. So, there is a nonholonomic constraint which can be written in the following form [1]

$$-V_X \sin \varphi + V_Y \cos \varphi = 0.$$

This constraint doesn't allow setting arbitrarily the angular coordinate φ of robot, which in such a situation must be a solution of the differential equation [1]

$$\frac{d\varphi_{ref}}{dt} = \frac{-V_{Xref} \sin \varphi + V_{Yref} \cos \varphi}{b_M \cos \gamma}, \quad (18)$$

$$\varphi_{ref}(0) = \varphi_0,$$

where $b_M \cos \gamma$ and $b_M \sin \gamma$ – constant coordinates of a point M in the coordinate system associated with the robot. Integrating (18), we find the law for φ_{ref} changes.

4.2 MR's position estimator

The assessment of a current MR's position is calculated according the measurements of MR's motors angular velocities ω_{r1} and ω_{r2} on the left and right boards. Further, from (12) linear and angular velocities are calculated:

$$V = \frac{r}{2 \cdot n} (\omega_{r1} + \omega_{r2}), \quad \omega = \frac{r}{2 \cdot d \cdot n} (\omega_{r1} - \omega_{r2}).$$

The current position of the robot is determined from (1), by using the Euler approximation:

$$X = X_p + V \cdot \cos \varphi_p \cdot T_s,$$

$$Y = Y_p + V \cdot \sin \varphi_p \cdot T_s,$$

$$\varphi = \varphi_p + \omega \cdot T_s,$$

where T_s is a sample period, index «p» matches a value corresponding to the last sampling point.

5 Experimental test of control system

5.1 Description of the experimental MR

Object of the study is a MR, built on the tracked chassis Rover5 by DAGU (Fig. 4). Power section of robot includes two electric drives based on the DC. As a control unit Arduino UNO R3 controller based on ATmega328 microcontroller by Atmel is used. To control the robot motors Rover5 Explorer PCB board from DAGU is used. It is equipped with two FET H-bridges rated at 4A, as well as built-in charging circuitry of NiMh and NiCd batteries. Wireless communication interface is implemented through the usb-programmable wireless module Wixel by Pololu Corporation, which connected with the Arduino board via Wixel Shield.

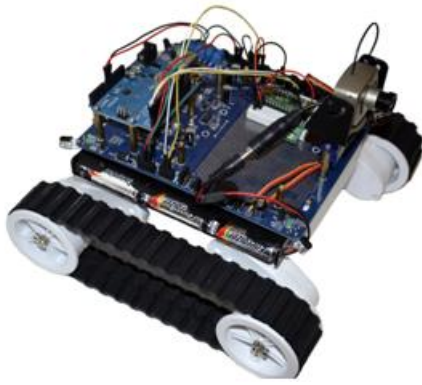


Fig. 4 MR Rover5.

Linear dimensions of the robot Rover5: track length $l=0.245$, m; driving wheel's radius $r=0.03$, m; robot's mass $m=0.83$, kg; distance from the center of the robot to the track $d=0.19$, m. Motor parameters: rated voltage: $U_n=7.2$, V; armature resistance $R_m=100$, Ohm; armature inductance $L=0.0123$, H; gear ratio $n=86.8$. For measuring the motor's speed MR Rover5 is equipped with two quadrature encoders (Fig. 5). Encoder's resolution is 1000 pulses for 3 turns of the driving wheel.

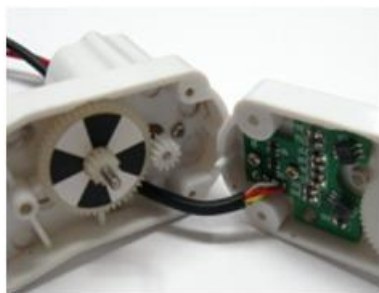


Fig. 5 Rover5 quadrature encoder.

5.2 The experimental results

A circle was taken as a test trajectory. At the initial time the robot is at the point with coordinates (0,0), the trajectory starts to move from the point (0,0.25).

The frequency of harmonic signals $X_{ref}(t)$ and $Y_{ref}(t)$ is set so that the trajectory $p_{ref}(t)$ forms a circle for 40 seconds.

The experimental results on a real Rover5 robot are shown in Figure 6-8, where are shown reference and real Rover5's trajectories, required V_{req} and real V linear velocities, required ω_{req} and real ω robot's angular velocities.

The results show us that the system has an acceptable quality. Robot enters the reference trajectory for 8 seconds, then deviations are $|e_x| \leq 0.022$, m, $|e_y| \leq 0.008$, m, $|e_\varphi| \leq 0.05$, rad, the relative trajectory tracking error doesn't exceed 9%.

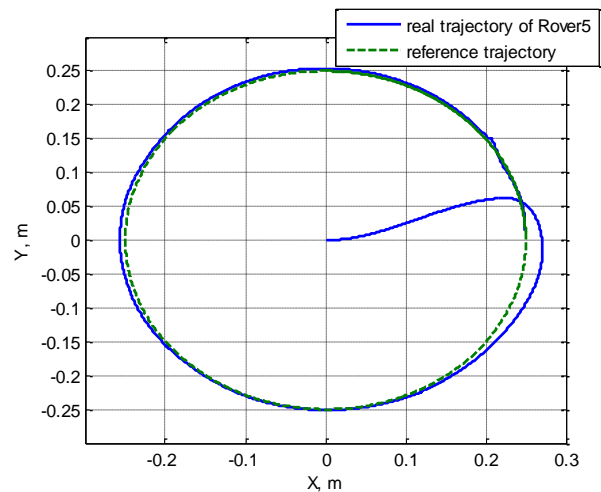


Fig. 6 The experimental results: reference and real trajectory of MR Rover5.

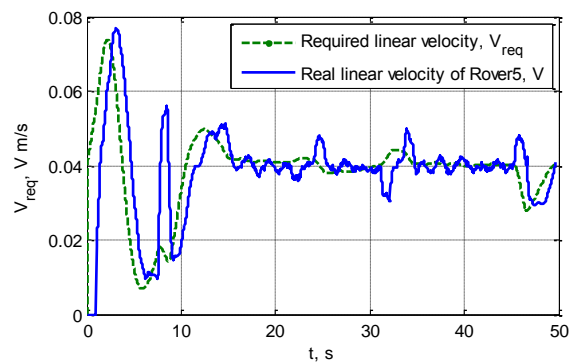


Fig. 7 The experimental results: required and real linear velocity of MR Rover5.

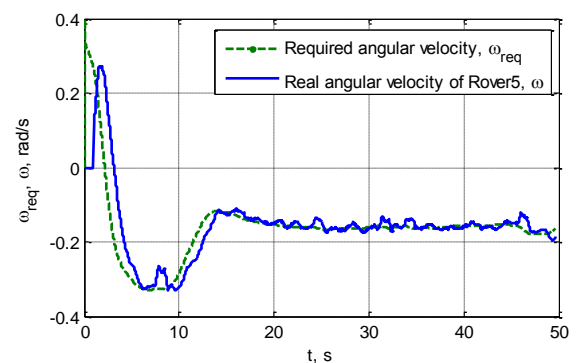


Fig. 8 The experimental results: required and real angular velocity of MR Rover5.

6 Conclusions

In this work the problem of the mobile tank-type chassis robot's optimal trajectory movement control system design is considered. The proposed system consists of two modules (reference and executive) and has, accordingly, two control loops: the first – position control loop, the second – velocity control loop.

The synthesis of trajectory control loop is based on solving the linear-quadratic optimization problem for tracking error. Its mathematical model is the tangent linearization of the MR's tracking error dynamical system about the reference trajectory. In general case, the control law is a tracking error feedback with time-varying coefficients. Non-stationary nature of the problem is determined by the time-varying values of reference linear and angular velocities of the robot. In such a case when these velocities are constant, the problem becomes stationary and the feedback's coefficients also become time-invariant.

The designing of MR's velocity (linear and angular) control loop is also based on solving the linear-quadratic optimization problem. To simplify this problem, from the point of calculation and implementation of optimal control it is proposed to use its asymptotic approximation obtained by singular perturbation method.

Experimental verification of developed MR's trajectory movement control system was performed on Rover5 robot with tank-type chassis. The results of experiment confirmed the acceptable quality of control system.

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