On the Fuzzy Control of Nonlinear Discrete Systems Affine in the Control

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Abstract: - In this paper, we focus on modeling and control of discrete nonlinear systems affine in control. A comparative study, in a discrete time, between two control strategies is developed to learn the input properties of fuzzy systems of Takagi-Sugeno constant to conclusions.

Key-Words: - Nonlinear system affine in the control system of Takagi-Sugeno, input-output representation, inversion, linearization input-output fuzzy optimization H_∞ stability, fuzzy controller, the control signal, internal model, discrete system.

1 Introduction

As The guarantee the desired properties for a physical process, requires the development of a control law [1],[2]. In this sense, an alternative is to design a mathematical model of the process to be controlled, exploited the one hand for the synthesis of the controller and the other for the simulation of the performance obtained in closed loop [3]. In a non-linear and linear unlike the automatic, there is no universal techniques able to present general results for the analysis and synthesis systems and nonlinear controllers [4],[5],[6].

The input-output models are of major interest in the problems of identification and fuzzy control [7]. On the one hand, they have a much simpler structure than the models of state and secondly, they have a direct relationship between the command and output. This allows the construction of an inverse system and thus solving the problem of trajectory tracking.

In this paper, we restrict ourselves to discrete nonlinear systems affine in control. We propose to apply the fuzzy input-output representation of discrete nonlinear systems in the description of nonlinear systems considered in Takagi-Sugeno constant finding in *TSK*, a decomposition of the overall fuzzy system in elementary fuzzy subsystems, each defined on an elementary fuzzy mesh which facilitates the study of complex fuzzy systems, is performed allowing the development of the reversing mechanism of the fuzzy system.

These proposed steps will be exploited in the implementation of two methods of control in the regulation of a nonlinear system ("Duffing forced-oscillation"), the first is based on the fuzzy linearization technique discrete input-output (*I.O.D.F.L*), about second, it incorporates the fuzzy internal model control (*F.I.M.C*). A comparative study is subsequently performed to highlight the benefit provided, views stabilization and improvement of performance by each of the two approaches for the considered system.

2 Representation of fuzzy systems discrete nonlinear affine in control

Suppose a *TSK* type fuzzy system [8], multi-input single-output *M.I.S.O* with n entries $e_1,\cdots,e_n$ and the reponse s. The fuzzy system is

then represented by a collection of rules, of the form (1):

$$\begin{aligned} &R^{(i_1,\cdots,i_n)}: \\ &\text{If } e_1 \text{ is } A_1^{i_1} \text{ and} \cdots \text{and } e_n \text{ is } A_n^{i_n} \\ &\text{Then } s=\Theta\left(i_1,\cdots,i_n\right) \end{aligned} \quad (1)$$

with : $A_k^{i_k}, k=1,\cdots,n$ is the $i_{k^{ième}}$ is the fuzzy symbol associated with the variable e_k.

$\Theta\left(i_1,\cdots,i_n\right)$ real constant in the conclusion of the rule indexed $\left(i_1,\cdots,i_n\right)\in I_k$.

If L_k fuzzy symbols are vague as $I_k=\{1,2,\cdots,L_k\}$ to describe e_k the complete rule base is then composed of $L=\prod_{k=1,\cdots,n} L_k$ rules.

For inputs $e=\left[e_1,e_2,\cdots,e_n\right]^T\in\Re^n$, the output generated by the fuzzy system is given by (2):

$$s=\frac{\sum_{(i_1,\cdots,i_n)\in I}\theta^{(i_1,\cdots,i_n)}\left(e\right)\Theta\left(i_1,\cdots,i_n\right)}{\sum_{(i_1,\cdots,i_n)\in I}\theta^{(i_1,\cdots,i_n)}\left(e\right)} \quad (2)$$

with:

$\theta^{(i_1,\cdots,i_n)}\left(e\right)$ represents the degree of truth of the premise of the rule $R^{(i_1,\cdots,i_n)}$.
That:

$$\theta^{(i_1,\cdots,i_n)}\left(e\right)=\prod_{k=1}^{n}\mu_{A_k^{ik}}\left(e_k\right) \quad (3)$$

Suppose a partition strict universe of discourse of inputs with triangular membership functions uniformly distributed, then:

$$\sum_{(i_1,\cdots,i_n)\in I}\theta^{(i_1,\cdots,i_n)}\left(e\right)=1;\ e\in\Re^n \quad (4)$$

In this case, equation (2) becomes:

$$s=\sum_{(i_1,\cdots,i_n)\in I}\theta^{(i_1,\cdots,i_n)}\left(e\right)\Theta\left(i_1,\cdots,i_n\right) \quad (5)$$

A *TSK* fuzzy system of multi-input multi-output *M.I.M.O* concluded steady with n inputs $e_1,\cdots,e_n$ and m outputs $s_1,\cdots,s_n$ can be represented by a collection of m fuzzy systems in multi-input single-output of the form (1). Representation of a process by a mathematical model, essential for the synthesis of control laws, is characterized by recurrent equations linear or nonlinear linking inputs, outputs and states.

Consider the state space of discrete nonlinear system given by (6):

$$\begin{cases} X\left(k+1\right)=f\left[X\left(k\right),u\left(k\right)\right] \\ y(k)=h\left[X(k)\right] \end{cases} \quad (6)$$

The state representation of a nonlinear discrete system given by the expression (6) has a critical need to approximate $2n+1$ nonlinear functions (the n non-linear functions of the vector field f, the n non linear functions of the vector field g and the non linear function h), overcome these constraints frequently encountered, a transformation of the state representation (6) in an input-output representation in order to exhibit an explicit relationship between output and control is proposed. Discrete processes, studied in this section are assumed to have had at least one equilibrium point originally chosen [1],[8],[9] where relations between inputs, outputs and states are expressed by fuzzy rules.
In this sense, we are interested in establishing a direct relationship between the input and output system (6):

$$\begin{aligned} &\frac{d}{du}\left[h\circ f^{(\eta)}\left(x,u\right)\right]=0 \\ &\forall\left(x,u\right)\in\Re^n\times\Re \text{ et } \eta\le r-1 \end{aligned} \quad (6.a)$$

and

$$\frac{d}{du}\left[h\circ f^{(r)}\left(x,u\right)\right]\neq 0 \quad (6.b)$$

with: $f^{(i)}$ is the $i^{éme}$ iterative composition of f. The delay between input $u\left(k\right)$ and the output $y\left(k\right)$ is indicated by the relative degree r, this

means that the measured input at a time k only affects the system output after r time units.

Assuming that the system considered is minimum phase, the output $y(k)$ is estimated at sampling instants taken successively until the appearance of the input $u(k)$, then an explicit the binder inlet to the outlet of the system (6) can be established by (7):

$$\begin{aligned} y(k) &= h\left[x(k)\right] \\ y(k+1) &= h \circ f\left[x(k)\right] \\ &\vdots \\ y(k+r-1) &= h \circ f^{(r-1)}\left[x(k)\right] \end{aligned} \tag{7.a}$$

The recursive relationship expressing output $y(k+r)$ and the input $u(k)$ is then obtained by the following equation (7.b):

$$y(k+r) = h \circ f^{(r)}\left[x(k), u(k)\right] \tag{7.b}$$

If $\dfrac{dy(k+r)}{dx(k)}$ is non-singular at equilibrium (chosen in this part, the source), one can express the state $x(k)$ based on past inputs and outputs [10],[11] [15],[19]. The system output can be rewritten as (8):

$$y(k+r) = \Psi_d\left[\mathrm{E}_1(k), u(k)\right] \tag{8.a}$$

where:

$$\mathrm{E}_1(k) = \begin{bmatrix} y(k), \cdots, y(k-n+1), \\ u(k-1), \cdots, u(k-m+1) \end{bmatrix} \tag{8.b}$$

$$m \le n$$

and Ψ_d non linear function.

The basic idea of the performance input-output fuzzy discrete I.O.F.D.R is expressed in the nonlinear function Ψ_d given by (8) with a *TSK*-type fuzzy system whose rule base is a collection of rules of the form (9):

$$\begin{aligned} &R^{(i_1,\cdots,i_n,j_1,\cdots,j_m)}: \text{If } y(k) \text{ is } A_1^{i_1} \text{ and } \cdots \text{and} \\ &y(k-n+1) \text{ is } A_n^{i_n} \text{ and } u(k) \text{ is } B_1^{j_1} \\ &\text{and} \cdots \text{and } u(k-m+1) \text{ is } B_m^{j_m} \\ &\text{Then } y_f(k+r) = \Theta_{fd}(i_1,\cdots,i_n,j_1,\cdots,j_m) \end{aligned} \tag{9}$$

with:

$A_\ell^{i_\ell}$ and $B_v^{i_v}, \ell = 1,\cdots n$ and $v = 1,\cdots,m$ the symbols are associated respectively with fuzzy variables $y(k-\ell+1)$ and $u(k-v+1)$ and $\Theta_{fd}(i_1,\cdots,i_n,j_1,\cdots,j_m)$ is the conclusion at real constant associated with the rule indexed $(i_1,\cdots,i_n,j_1,\cdots,j_m)$.

If L_ℓ and V_v the symbols are associated respectively with the fuzzy inputs $y(k-\ell+1)$ and $u(k-v+1)$ defined on the universe of discourse, then the basis of regulatory system is composed of $\Lambda = \prod_{\ell=1}^{n} L_\ell \prod_{v=1}^{m} V_v$ fuzzy rules where $i_\ell \in I_\ell = \{1,\cdots,L_\ell\}$ and $i_v \in J_v = \{1,\cdots,V_v\}$, $\ell = 1,\cdots,n$ and $v = 1,\cdots m.$

The output of the fuzzy system can then be expressed by (10):

$$y_f(k+r) = \sum_{(i_1,\cdots,i_n,j_1,\cdots,j_n)} \theta^{(i_1,\cdots,i_n,j_1,\cdots,j_n)}\left(\mathrm{E}(k)\right). \Theta(i_1,\cdots,i_n,j_1,\cdots,j_n) \tag{10.a}$$

Where: $\mathrm{E}(k) \in \Re^{n+m}$ is the input vector and $\theta(\mathrm{E})$ is the vector of degrees of validity of the premises of rules such as:

$$\mathrm{E}(k) = \left[y(k),\cdots,y(k-n+1),u(k),\cdots,u(k-m+1)\right]$$

(10.b) For an optimal approximation of Ψ_d fuzzy noted Ψ_{df}, the input-output relation (8) can be rewritten as (11):

$$y(k+r) = \Psi_{df}\left[\mathrm{E}(k)\right] + \Delta \tag{11}$$

where: Δ the approximation error is bounded by a positive δ constant such that $|\Delta| \le \delta$. In the discrete case and for a reference

trajectory $y_{des}(k)$, the system remains stable closed-loop control law $u(k)$ for a given, if the output of the system satisfies (12):

$$\lim_{k\to\infty} e_0(k) = \lim_{k\to\infty}\left(y_{des}(k) - y(k)\right) = 0 \qquad (12)$$

The trajectory tracking is checked if the nonlinear system ensures the controllability condition of output throughout the desired path, in other words:

$$\frac{dy(k+r)}{du(k)} = \frac{d\Psi_d}{du(k)} \neq 0 \qquad (13)$$

We are interested, then, to develop a method for decomposing a fuzzy system overall elementary fuzzy subsystems each defined on a mesh elementary fuzzy. This decomposition ensures that the output of a fuzzy system overall is equal at every instant, the output generated by an elementary subsystem, which will subsequently facilitate the inversion problem of model.

Suppose a *TSK* type fuzzy system [1],[12],[4] of the form (1) having n entries e_n, the fuzzy system (1) is defined on input universe of discourse E^n given by (13) :

$$\mathrm{E}^n = \mathrm{E}_1 \times \cdots \times \mathrm{E}_n = \bigcup_{i_p=1}^{L_p-1}\left[a_p^{i_p}, a_p^{i_p+1}\right];\ p = 1,\cdots,n \qquad (13)$$

The system consists of $\prod_{p=1}^{n}\left(L_p - 1\right)$ in fuzzy systems as each is defined by 2^n fuzzy rules, so the generated output of the overall fuzzy system (14) is rewritten by the following expression (15):

$$\begin{aligned} &R_{(i_1,\cdots,i_n)}^{(v_1,\cdots,v_n)}: \\ &If\ e_1\ is\ A_1^{i_1+v_1}\ and \cdots and\ e_n\ is\ A_n^{i_n+v_n} \\ &Then\ s_{(i_1,\cdots,i_n)} = \Theta\left(i_1+v_1,\cdots,i_n+v_n\right); \\ &v_p = 0,1;\ p = 1,\cdots,2 \end{aligned} \qquad (14)$$

$$s = s_{(i_1,\cdots,i_n)} = \sum_{(v_1,\cdots,v_n)\in\{0,1\}^n} \theta^{(i_1+v_1,\cdots,i_n+v_n)}(e).\Theta\left(i_1+v_1,\cdots,i_n+v_n\right) \qquad (15)$$

with:

$a_p^{i_p}$ is the modal value of the fuzzy symbol $A_p^{i_p}$ such that $\mu_{A_p^{ip}}\left(a_p^{i_p}\right) = 1$

It is proposed to transform a fuzzy subsystem to n inputs and one output *M.I.S.O* in a subsystem in an input and an output *S.I.S.O* The principle is to transfer $n-1$ input variables e_{n-1} premises to the conclusions of rules. This procedure aims to reduce the number of fuzzy rules for each subsystem and spend a *TSK* fuzzy system with multiple inputs to a fuzzy system to one, allowing procedure then the reversing mechanism of fuzzy model.

For a system with n inputs the output of the subsystem (15) can be rewritten as a following vector (16):

$$s_{(i_1,\cdots,i_n)} = \theta^{(i_1,\cdots,i_n)}(e).\mathrm{T}\left(i_1,\cdots,i_n\right) = \sum_{(v_1,\cdots,v_n)\in\{0,1\}^n} \theta^{(i_1+v_1,\cdots,i_n+v_n)}(e).\Theta\left(i_1+v_1,\cdots,i_n+v_n\right) \qquad (16)$$

The analytical expression of the output (16) above developed, can be examined by a sub-fuzzy system. The output of the fuzzy subsystem, taking into account the equation (16). For a fuzzy system to n inputs the output generated, obtained following the parameterization figure 1 of fuzzy symbols can be expressed by the following generalized related (17):

$$s_{(i_1,\cdots,i_n)} = \sum_{(\tau_1,\cdots,\tau_n)\in} \frac{1}{2^n\prod_{b=1}^{n}\delta_b^{i_b}} \sum_{(v_1,\cdots,v_n)\in} (-1)^{(1+v_1)\tau_1+\cdots+(1+v_n)\tau_n}. \left[(e_1)^{\tau_1} + \left((-1)^{v_1} c_1^{i_1} + \delta_1^{i_1}\right)^{(1-\tau_1)}\right]\cdots. \left[(e_n)^{\tau_n} + \left((-1)^{v_n} c_n^{i_n} + \delta_n^{i_n}\right)^{(1-\tau_n)}\right].\Theta\left(i_1+v_1,\cdots,i_n+v_n\right) \qquad (17)$$

If we assume $P1=(e_1,\cdots,e_{n-1})$ the vector consists of $n-1$ inputs, the output expression can be rewritten as follows:

$$s_{(i_1,\cdots,i_n)} = \sum_{v_n\in\{0,1\}} \Theta_{(i_1,\cdots,i_n)}(i_n+v_n,P1).\mu_{A_n^{i_n}}(e_n) \quad (18.a)$$

with:

$$\begin{aligned}&\Theta_{(i_1,\cdots,i_n)}(i_n+v_n,P1)=\\&\sum_{(v_1,\cdots,v_{n-1})\in}\xi^{(i_1+v_1,\cdots,i_{n-1}+v_{n-1})}(P1).\\&\Theta(i_1+v_1,\cdots,i_{n-1}+v_{n-1},i_n+v_n);\\&v_n\in\{0,1\}\end{aligned} \quad (18.b)$$

From this transformation, for each elementary mesh, there are only two rules rather than 2^n. We can then write:

$$\text{If } e_n \text{ is } A_n^{i_n} \text{ then } s_{(i_1,\cdots,i_n)}=\Theta_{(i_1,\cdots,i_n)}(i_n,P1) \quad (19.a)$$

$$\text{If } e_n \text{ is } A_n^{i_n+1} \text{ then } s_{(i_1,\cdots,i_n)}=\Theta_{(i_1,\cdots,i_n)}(i_n+1,P1) \quad (19.b)$$

It is of interest to specify that the transformations of each output generated by the corresponding sub fuzzy systems are of major interest for the realization of the inversion of the fuzzy model appropriate. The problem of followed trajectory lies in the determination of an input (control) capable of returning the output from one system to the monitoring of the reference trajectory. To solve this problem, we assume that a system can reverse [13],[15],[16],[17], in the presence of the trajectory eligible to provide input, applied to the system , leads to a convergence of its output to the reference trajectory.

In general, for an affine fuzzy system as e_n given by the form (19), if (20) is verified then the fuzzy inverse model is given analytically by the relation (20):

$$\frac{ds_{(i_1,\cdots,i_n)}}{de_n}\neq 0 \quad (20)$$

The fuzzy model is then given by the analytic (21) which is an exact reversal of the model:

$$e_n=\frac{-\Psi_1^{(i_1,\cdots,i_n)}(P1)}{\Psi_2^{(i_1,\cdots,i_n)}(P1)}+\frac{s_{(i_1,\cdots,i_n)}}{\Psi_2^{(i_1,\cdots,i_n)}(P1)} \quad (21)$$

This inversion is actually conditioned by the ownership of the input e_n to the corresponding mesh.

2 Regulations of Nonlinear Discrete Systems Affine in Control

Deal with the complexities presented by nonlinear dynamical systems, several nonlinear control strategies have been developed, we are interested in this work to the law of input-output linearization and fuzzy control internal model in a discreet [3],[4],[5],[6],[9],[11],[12].

2.1 Input-Output Fuzzy Discrete Linearization

The input-output linearization *I.O.F.D.L* is particularly suitable for systems affine in control, [17],[18],[19],[20]. In general, one can transform a class of nonlinear systems in a discrete class of linear systems through discrete linearization *I.O.F.D.L*. In this case, the transformed linear system can be controlled by current methods of automatic (discrete pole placement, internal model control, etc ...).

The principle of *I.O.F.D.L* is to determine a control law $u(k)$ such that the closed loop is stable and the system output $y(k)$ converges to the desired trajectory $y_{des}(k)$.

According to the principle of *I.O.F.D.L*, introducing a new input $v(k)$ to obtain the control law $u(k)$ [6],[7],[12] rewritten by the expression (22):

$$u(k)=\Psi_d^{-1}\left[P1,v(k)\right] \quad (22)$$

as:

$$\frac{d\Psi_d}{du}\neq 0 \quad (23)$$

Then we can write (23):

$$y(k+r)=v(k) \tag{24}$$

To solve the problem of instability, a pole placement strategy can be adopted to impose a dynamic closed loop. The new input v of the linearized system is then chosen as follows (25):

$$v(k)= y_{des}(k+r)+\sum_{j=0}^{r-1}\eta_j\left[y_{des}(k+j)-y(k+j)\right]= y_{des}(k+r)+\eta^T e \tag{25}$$

with:

$$\eta^T=[\eta_0,\cdots,\eta_{r-1}],\ e(k)=\left[e_0(k),\cdots,e_0(k+r-1)\right]^T$$
such as: $e_0(k)=y_{des}(k)-y(k)$ (26)

while the coefficients $\eta_0,\cdots,\eta_{r-1}$ are selected so that the solutions of the polynomial (27) are inside the unit circle:

$$z+\eta_{r-1}z^{r-1}+\cdots+\eta_0=0 \tag{27}$$

We can conclude that:

$$e_0(k+r)+\eta^T e(k)=0;\ \lim_{k\to\infty}e_0(k)=0 \tag{28}$$

The control law $u(k)$ of thus developed is able to ensure convergence to zero of the tracking error trajectory $e_0(k)=y_{des}(k)-y(k)$. These steps can be developed in a fuzzy, which is replaced by the nonlinear function Ψ_d and Ψ_{fd} the functions Ψ_1 and Ψ_2 functions and are analytic functions for expressing the output of the fuzzy system activated on the mesh, it is then input-output fuzzy discrete linearization (*I.O.F.D.L*) [15],[19]. The control law $u(k)$ can be rewritten as (29):

$$u(k)=\frac{-\Psi_{fd1}}{\Psi_{fd2}}+\frac{v(k)}{\Psi_{fd2}} \tag{29}$$

For $v(k)$ is designed according to the equation (22), the dynamic equation error is given by (30):

$$e_0(k+r)=-\eta^T e-\Delta \tag{30.a}$$

or in a matrix form:

$$e_0(k+1)=Ae-B\Delta \tag{30.b}$$

with:

$$A=\begin{bmatrix}0 & 1 & 0 & \cdots & 0\\ 0 & 0 & 1 & \cdots & 0\\ \cdots & \cdots & \cdots & \cdots & \cdots\\ -\eta_0 & -\eta_1 & -\eta_2 & \cdots & -\eta_{r-1}\end{bmatrix} \tag{30.c}$$

$$B^T=\begin{bmatrix}0 & 0 & \cdots & 1\end{bmatrix} \tag{30.d}$$

The figure.7. illustrates the delay between the output $y(k)$ and output $y(k+r)$. The control loop nonlinear system by the law of *I.O.F.D.L*. However, the control law (29) does not ensure the robustness of the control structure proposed in the presence of uncertainties Δ. Actually, the development of *I.O.F.D.L* is synthesized from the nominal fuzzy model without taking into account uncertainty Δ.

The variance between the actual output and the fuzzy output is obtained by approximation given by the expression (31):

$$y(k+r)=y_f(k+r)+\Delta \tag{31}$$

as: $|\Delta|<\delta,\ \delta>0$

The *I.O.F.D.L* is increased by an additive component u_A, we propose to synthesize by the optimization method H_∞ [9],[10],[11],[18] and is then written:

$$u(k)=-\frac{\Psi_{fd_1}}{\Psi_{fd_2}}+\frac{1}{\Psi_{fd_2}}v(k)-\frac{1}{\Psi_{fd_2}}u_A(k)$$

(32)

In this case, the dynamic equation of the tracking error reference becomes:

$$e_0(k+r) = -\eta^T e(k) + u_A - \Delta \qquad (33.a)$$

or in a matrix form:

$$e(k+1) = Ae(k) + Bu_A - B\Delta \qquad (33.b)$$

and A are B as defined in (30.c) and (30.d).

The principle of the control additive is shown in Fig. 2.

2.2 Robustification by the component H_∞

The transfers function between the uncertainties and represented by Δ the tracking error trajectory $e_0(k)$ without the addition of a component of robustification, can be determined from the dynamic equation (31). If we define $C = [1 \quad 0 \quad \cdots \quad 0] \in \Re^r$ so that $e_0(k) = Ce(k)$ we obtain:

$$H_{e_0\Delta}(z) = -C(zI - A)^{-1} B \qquad (34.a)$$

The objective is to determine the additive component $u_A(k)$ so as to modify the transfer $H_{e_0\Delta}(z)$ that:

$$\left\| H_{e_0\Delta}(z) \right\| \le \varepsilon \qquad (34.b)$$

With: $\varepsilon > 0$
Where:

$$\left\| H_{e_0\Delta}(z) \right\|_\infty = \sup_{\Delta \in L_2} \frac{\|e_0\|_2}{\|\Delta\|_2} \qquad (34.c)$$

The component u_A is defined as:

$$u_A = Ge(k) \qquad (35)$$

The error equation becomes:

$$e(k+1) = (A + BG)e(k) - B\Delta = \tilde{A}e - B\Delta \qquad (36)$$

The transfer function between the uncertainties and the tracking error is expressed finally by (37):

$$H_{e_0\Delta}(z) = -C(zI - \tilde{A})^{-1} B \qquad (37)$$

The component M is synthesized via an optimization H_∞ using the modified Riccati inequality (38) [14],[11]:

$$A^T \Omega A - P + Q + C^T C - A^T \Omega B \left(\frac{1}{1 + B^T \Omega B} \right) B^T \Omega A \le 0 \qquad (38.a)$$

with:

$$\Omega = \left(P^{-1} - \varepsilon^{-2} BB^T \right)^{-1} \qquad (38.b)$$

that:

P is a symmetric positive definite matrix that, if it exists:

$$Q > 0 \text{ and } \varepsilon^2 - B^T PB > 0 \qquad (38.c)$$

Then the quadratic function:

$$\Lambda(k) = \frac{1}{2} e^T(k).P.e(k) \qquad (38.d)$$

is a Lyapunov function [3],[9],[13] which satisfies the constraint mitigation given for the relation (38) and the transfer function between Δ and $e_0(k)$ given by (34).

With $u_A(k)$ expressed by the relation with (38.e) below:

$$u_A(k) = \mathrm{M}e(k) = -\frac{B^T \Omega A}{1 + B^T \Omega B} e(k) \qquad (38.e)$$

2.3 Fuzzy Discrete Internal Model Control *F.D.I.M.C*

The structure of the *F.D.I.M.C* is given in Fig.3. In our study, the process is to order a *TSK* model is obtained by the development of sub-fuzzy systems, in each elementary mesh, the overall fuzzy system and the fuzzy correction corresponding to the inverse model is adopted achieved by applying the principle of reverse

previously developed. If a fuzzy model of the nonlinear process and its inverse can be determined, the *F.I.M.C* can be implemented [12],[13],[20],[17].

3 Application

We propose control the electrical system of nonlinear oscillatory "Duffing forced-oscillation" treated in [21] by both techniques *I.O.F.D.L* and *F.D.I.M.C*, described by the following dynamic equations:

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = -0.1x_2(t) - x_1^3 + 12\cos(t) + u(t) \\ y(t) = x_1(t) \end{cases} \tag{39}$$

The relative degree r of the system (39) is equal his order n, such as:

$$r = n = 2 \tag{40}$$

The fuzzy model of the TSK system is given by a collection of rules of the form:

$R^{(i_1,i_2,i_3)}$:

If: $y(k)$ is $A_1^{i_1}$ and $y(k+1)$ is $A_1^{i_2}$ then

$$y(k+2) = \Theta_{fd}(i_1,i_2,i_3) + u(k); i_1 = 1,\cdots 6; i_2 = 1,2. \tag{41}$$

Consider the basic rules, illustrated in Table 1 according to the description of the fuzzy model of our system:

Table 1 Base of Fuzzy System Rules of TSK

	A_1^1	A_1^2	A_1^3	A_1^4	A_1^5	A_1^6
A_2^1	$A_3^1 : -2.2$ $A_3^2 : 0.06$	$A_3^1 : -2$ $A_3^2 : 0.5$	$A_3^1 : -1.1$ $A_3^2 : 1.4$	$A_3^1 : -0.9$ $A_3^2 : 1.1$	$A_3^1 : -0.5$ $A_3^2 : 1$	$A_3^1 : -0.3$ $A_3^2 : 1.6$
A_2^2	$A_3^1 : -1.4$ $A_3^2 : 0.5$	$A_3^1 : -1.6$ $A_3^2 : 0.7$	$A_3^1 : -0.8$ $A_3^2 : 0.9$	$A_3^1 : -0.5$ $A_3^2 : 1.1$	$A_3^1 : -0.4$ $A_3^2 : 1$	$A_3^1 : -0.1$ $A_3^2 : 0.9$

The fuzzy model is identified around the desired trajectory $y_{des}(k) = 2\sin(k)$. The universe of discourse of inputs $y(k)$, $y(k+1)$ and $u(k)$ are:

$$\left[y_{min}(k), y_{max}(k)\right] = \left[y_{min}(k+1), y_{max}(k+1)\right] = [-3,3]; \left[u(k), u(k)\right] = [-1.5,1.5] \tag{42}$$

illustrated in the Fig 4.

The discretisation of the input-output representation of the system refers to the Euler approximation [22] and the developed results in [23] are given by (43):

$$y(k+2) = -0.9y(k+1) + 0.1y(k) - Ty^3(k) + 12T\cos(k) + Tu(k) \tag{43}$$

Simulation result of open loop system is presented in Fig. 5 following which correspond to the discretized system response to a step $T = 0.1s$ and error $e_0(k) = y_{des}(k) - y(k)$.

We notice the difference between the system output and the desired output. To minimize the error between the reference output and the output of open loop system is proposed to apply the one hand, the technique of input-output linearization discreet, and secondly, the internal model control *I.M.C* as being of approaches to the regularization of nonlinear systems.

3.1 Application of the *I.O.F.D.L*

To apply the *I.O.F.D.L* we first develop the signal $v(k)$, we then selected as pole placement $z_1 = 0.78$ and $z_2 = 0.36$. In developing control additive u_A, is chosen $\varepsilon = 2.5$, the selected matrix $Q = 0.001I$ (I is the identity matrix of order two), then there is a symmetric positive definite matrix P :

$$P = \begin{bmatrix} 1.1305 & 0.04 \\ 0.21 & 1.71 \end{bmatrix}$$

Table 2. Includes the various measured quantities.

Table 2 Robustification by component H_∞

	without u_A	$\varepsilon = 2.5$
$\left\|H_{e_0\Delta}(z)\right\|_\infty$	5.241	1.201
$\left\|e_0\right\|_\infty$	8.521	0.69
G	-	$\begin{bmatrix}0.313 & -1.421\end{bmatrix}$

Table 2 shows the influence of additive u_A on performences control system, in effect, the error $e_0(k)$ is limited by the presence of u_A.

3.2 Application of the *F.D.I.M.C*

In this part, it is intended to apply the *I.M.C* in the regulation of nonlinear system considered. In order to apply the *I.M.C* is used to elaborate the inverse fuzzy model required in the synthesis of the regulator.The decomposition of the overall fuzzy system proposed has five fuzzy subsystems set of 5 basic meshes: $(1,1,1),(2,1,1),(3,1,1),(4,1,1)$ and $(5,1,1)$.

Let $P1$ le vecteur composé des entrées $e_1 = y(k)$ et $e_2 = y(k+1)$. The fuzzy system's output is $s = y_f(k+2)$.

The output of the subsystem defined on the fuzzy elementary mesh (i_1,i_2,i_3) is given by the following expression:

$$s(i_1,i_2,i_3) = \sum_{v3\in\{0,1\}} \Theta_{(i_1,i_2,i_3)}(i_3+v_3,P1).\mu_{A_3^{i_3}}(e_3) = \left(\sum_{v3\in\{0,1\}}\left(\frac{\sum_{(v_1,v_2)\in\{0,1\}^2}\xi^{(i_1+v_1,i_2+v_2)}(P1).}{\Theta(i_1+v_1,i_2+v_2,i_3+v_3)}\right)\right).\mu_{A_3^{i_3}}(e_3)$$
$$e_1 = y(k), e_2 = y(k+1) \text{ and } e_3 = u(k) \quad (43)$$

with:

$$\Theta_{(i_1,i_2,i_3)}(i_3+v_3,P1) = \sum_{(v_1,v_2)\in}\xi^{(i_1+v_1,i_2+v_2)}(P1).\Theta(i_1+v_1,i_2+v_2,i_3+v_3)$$
$$v_3\in\{0,1\} \quad (44)$$

From this transformation, for each elementary mesh, there are only two rules rather than 2^3.For example for the mesh $(3,1,1)$ of the output generated by appropriate fuzzy system can be expressed by the relation (45) as follows:

$$s_{(3,1,1)} = u(k)\begin{bmatrix}-0.024y(k)y(k+1)-0.068y(k)-\\0.009y(k+1)-0.097\end{bmatrix} +0.102y(k)+0.013y(k+1)+0.036y(k)y(k+1)+0.145 \quad (45)$$

By use of the reversing mechanism fuzzy rules of the model are transformed into rules of order. To construct the inverse model (46), we must ensure that:

$$\frac{dy(k+2)}{du(k)} \neq 0 \quad (46)$$

3.3 Simulation results

Simulation results of the closed loop system are obtained by Fig. 6, 7, 9 and 10 following. For $\varepsilon = 2.5$ the Fig 7, and 8 show respectively the reference trajectory and the output of the system. We chose, first, as the model under fuzzy system whose entries belong to the mesh $(3,1,1)$, the response's system and the evolution of the drive are presented in Fig 28 and 29 following, then, is implanted under the five fuzzy systems developed the overall fuzzy system and we consider the minimal error between the output of the trial and the output nonlinear models *I.M.C* in the closed loop. The system output is now tainted by a disturbance equal to a unit step applied at time $t = 10s$. Simulations are performed if the model is the subsystem of the mesh and if we take into account all sub fuzzy systems of the overall system shown respectively in Fig 8, 11 and 33. The simulation results obtained by applying the two control techniques *I.O.F.D.L* and *F.D.I.M.C*, have emphasized the interest shown, no stability guarantee view for the controlled system. The application of the law control additive u_A, in the case of *I.O.F.D.L*, introduced improvements to the trajectory

tracking system, by minimizing the error $e_0(k)$ between the system response and the desired output. However, due to external disturbances on the output, improving system performance is obtained by *F.D.I.M.C* while it is not guaranteed by the technique of *I.O.F.D.L* which required optimization H_∞.

4 Conclusion

We underlined the performance monitoring undertaken by the two methods of regulation adopted on the oscillatory system. However, the *I.O.F.D.L*, despite the satisfactory results presented point of view trajectory tracking, it cannot guarantee the performance maintains the controlled system such external disturbances. While the structure maintains *F.D.I.M.C* gave a satisfactory performance despite all the considered approximations and external disturbance on the output.

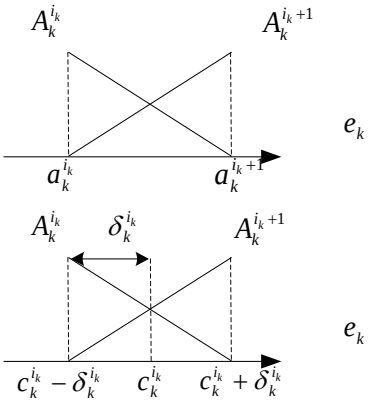

Fig.1 Parameterization of Fuzzy Symbols Inputs

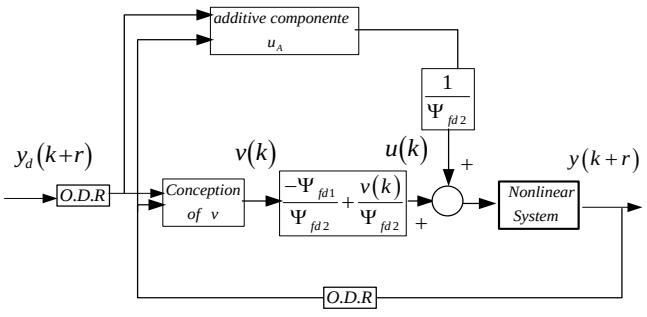

Fig.2 Robustification de la L.E.S.D.F

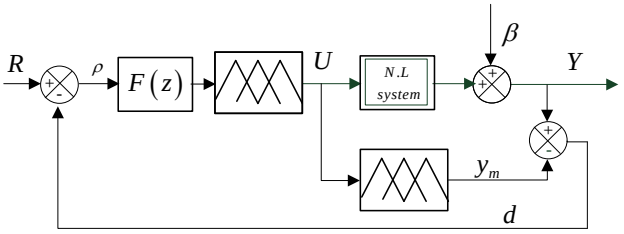

Fig.3 Structure of the Fuzzy Internal Model Control *F.I.M.C* for Nonlinear Systems

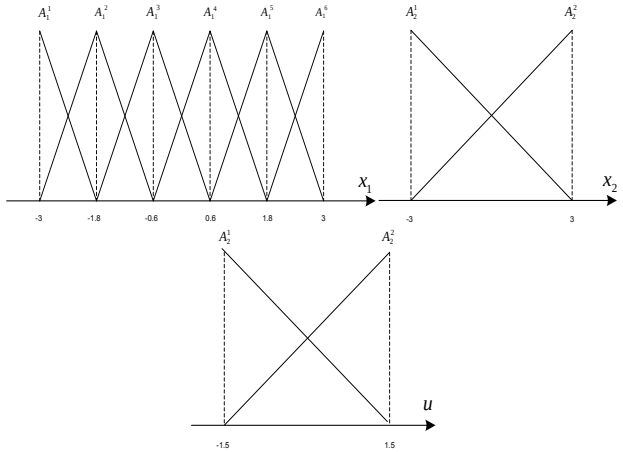

Fig.4 Fuzzy Partition of Universes of Discourse

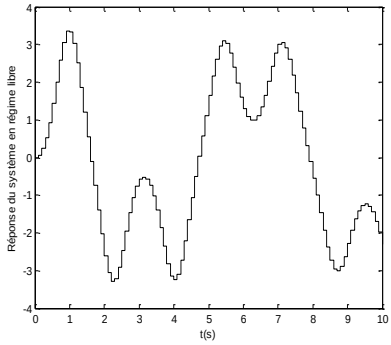

Fig.5. Response of the System in the Open Loop

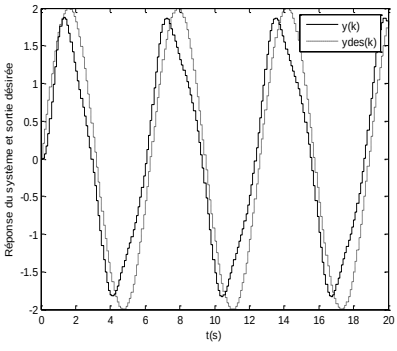

Fig.6. Reference Trajectory and Output of the Closed Loop System without uA

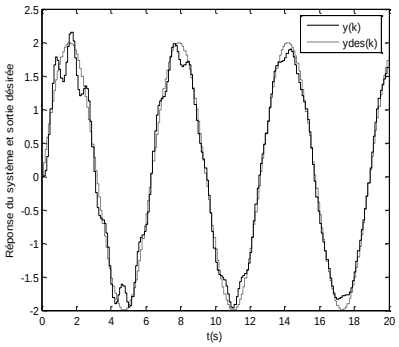

Fig.7. Reference Trajectory and Output of the System for $\varepsilon = 2.5$

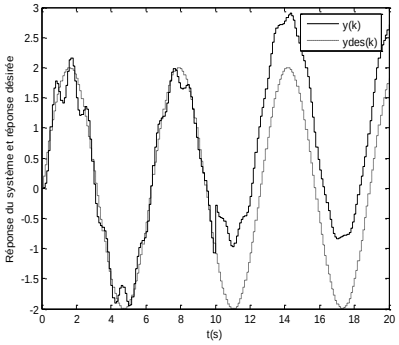

Fig.8. Reference Trajectory and Output of the System for $\varepsilon = 2.5$ in presence of disturbance

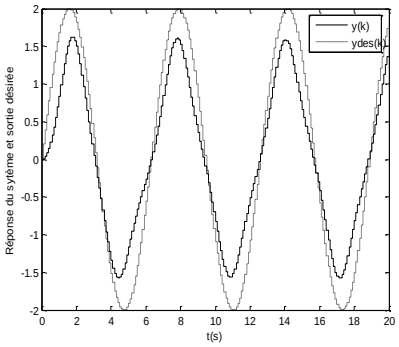

Fig.9 Reference Trajectory and the System's Output for the Mesh Subsystem $(3,1,1)$

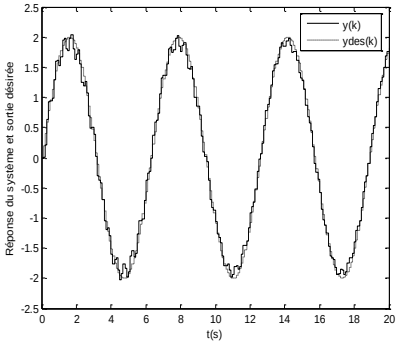

Fig.10. Reference Trajectory and the System's Output for all Fuzzy Subsystems

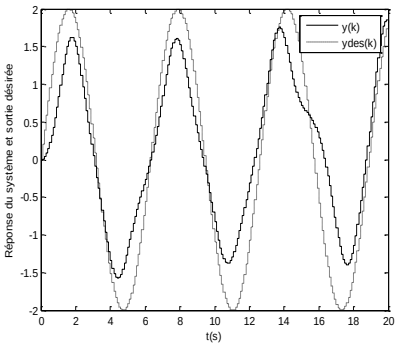

Fig.11. Reference Trajectory and the System's Output for the Mesh Subsystem $(3,1,1)$ in Presence of Disturbance

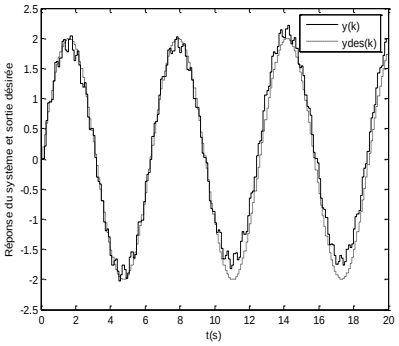

Fig.12. Reference Trajectory and System's Output for all Fuzzy Subsystems in Presence of Disturbance

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