Finite Frequency H- /H∞ Stability of Singularly Perturbed Systems

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Abstract: - In this paper, the problem of finite frequency H- /H∞ control for singularly perturbed systems based on GKYP lemma is studied. The objective of the H- /H∞ control problem is to design a controller such that the resulting closed-loop system is stable, and the transfer function is bounded real for singularly perturbed systems at low and high frequencies. By employing GKYP lemma, respectively, on the slow and fast subsystem, the problems of the reduced order subsystems are solved in terms of linear matrix inequalities (LMIs). The two frequency- scale solution for the full-order SPS constructed in this paper uses the solutions of two well-defined lower-order problems, and therefore it is numerically better conditioned. An iterative algorithm for the computation of the BMIs is presented. The effectiveness of the proposed method is demonstrated through comparing with positive real control design method.

Keywords: - Singularly Perturbed Systems (SPS); H- /H∞ control; finite frequency; Kalman-Yakubovich-Popov (KYP) lemma; linear matrix inequalities (LMI).

1 Introduction

The problem of control design for singularly perturbed systems has attracted the attention of many researchers for many years [1-2]. Singularly perturbed systems also known as multiple time-scale dynamic systems normally occur due to the presence of small "parasitic" parameters, typically small time constants, masses, etc. In state space, such systems are commonly modeled using the mathematical framework of singular perturbations, with a small parameter, say, determining the degree of separation between the "slow" and "fast" modes of the system [3].

On the other hand, the concept of H- /H∞ has played an important role in control and system theory [4-5-6]. In the past years, the problem of H- /H∞ control has received much attention [7-8]. Oloomi and Sawan [7] studied the suboptimal matching problem for SISO two frequency scale systems and obtained a suboptimal H∞ solution through solving the model matching problems for low and high frequency models.

One of the most fundamental results relation frequency domain and time domain, is the Kalman-Yakubovič-Popov (KYP) [9-10-11] lemma, which establishes the equivalence between a frequency domain inequalities (FDI) and a linear matrix inequality (LMI). As the extension of the standard KYP lemma, the generalized KYP (GKYP) lemma is introduced by Iwasaki et al. [12-13], that provides an LMI characterization of frequency domain inequalities in finite frequency range.

Mei et al [14] studied for H∞ Control of SPS by a GKYP lemma based approach. [3] Studied the finite frequency strictly positive real (FFSPR) control for singularly perturbed systems, and obtained the sufficient conditions which were given in terms of nonlinear matrix inequalities

In this paper, we consider the H-/H∞ control problem for SPS in finite frequency ranges, and design a controller which is also singularly perturbed to satisfy different frequency-domain specifications. By introducing H- index and H∞ norm performan ce of transfer function matrices at different frequency band, such that SPS has good dynamic, good robustness and good sensor noise rejection. Finally, the comparison with H-/H∞ Control in the example will show the superiority of our results.

This study is organized as follows: In Section 2 we give the problem formulation and some necessary preliminaries are presented. In section 3 presents FFH-/H∞ property analysis for slow and fast subsystem of SPS. In Section 4, simulation example is shown. And finally the paper is concluded by brief conclusion in Section 5.

Notation: For a matrix M, its transpose and complex conjugate transpose are denoted by M^T and M^*, respectively. $M \succ 0$ and $M \prec 0$ denote positive definiteness and negative definiteness.

2 Problem Formulations

Consider a Singularly perturbed system (SPS) described by:

$$\begin{cases} E_\varepsilon \dot{X}(t) = AX(t) + BU(t) \\ Y(t) = CX(t) + DU(t) \end{cases} \quad (1)$$

Where $E_\varepsilon, A \in \Re^{2,2}$, $B \in \Re^{2,1}$, $C \in \Re^{1,2}$, $D \in \Re^{1,1}$, , $X(t) \in \Re^{2,1}$ is a state vector, $U(t) \in \Re^{1,1}$ is a control input and $Y(t) \in \Re^{1,1}$ is an output.

$$X = \begin{bmatrix} X1 \\ X2 \end{bmatrix},\ E_\varepsilon = \begin{bmatrix} I_{n1} & 0 \\ 0 & \varepsilon I_{n2} \end{bmatrix},\ A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

$$B = \begin{bmatrix} B1 \\ B2 \end{bmatrix}, \quad C = \begin{bmatrix} C1 & C2 \end{bmatrix}$$

ε is a small perturbation parameter.
Decomposing the SPS (1), we get the slow and fast subsystems as follows:

- Slow subsystem

$$\begin{cases} \dot{X}_s = A_0 X_s + B_0 U_s \\ Y_s = C_0 X_s + D_0 U_s \end{cases} \quad (2)$$

- Fast subsystem

$$\begin{cases} \varepsilon \dot{X}_f = A_{22} X_f + B_2 U_f \\ Y_f = C_2 X_f + D_2 U_f \end{cases} \quad (3)$$

Where

$A_0 = A_{11} - A_{12} A_{22}^{-1} A_{21}$, $B_0 = B_1 - A_{12} A_{22}^{-1} B_2$

$F_0 = F_1 - A_{12} A_{22}^{-1} F_2$, $C_0 = C_1 - C_2 A_{22}^{-1} A_{21}$

$D_0 = D - C_2 A_{22}^{-1} B_2$, $D_2 = D - D_0$

$X_s \in \Re^{1,1}$ is the state vector of the slow subsystem

$X_f \in \Re^{1,1}$ is the state vector of the fast subsystem

U_s is the Control of the slow subsystem

U_f is the Control of the fast subsystem

For SPS, the transfer function can be written as sum of two transfer function matrices in two different frequency scales, s and εs, corresponding to the time scales t and t/ε [Luigi Glielmo] [15-16], that is

$$G(s) = G_s(s) + G_f(\varepsilon s) \quad (4)$$

Where

$$G_s(s) = C_0 (sI - A_0)^{-1} B_0 + D_0 \quad (5)$$

$$G_f(\varepsilon s) = G_f(p) = C_2 (pI - A_{22})^{-1} B_2 + D_2 \quad (6)$$

Here the transfer functions $G_s(s)$ and $G_f(p)$ are called the low-frequency and high-frequency approximations of $G(s,\varepsilon)$.

Definition

- we define the finite frequency H- index [Chen, Patton, and Liu(1996) and Chen and Patton (1999)] and the finite frequency H∞ norm[T Iwasika] as

$$\inf\left(\|G(s)\|_-\right) \succ \beta \quad \forall|\omega| \le \omega_l$$

$$\sup\left(\|G(s)\|_\infty\right) \prec \gamma \quad \forall|\omega| \ge \omega_h$$

where $\beta \succ 0$, $\gamma \succ 0$ are scalar, and ω_l, ω_h represent the low and high frequency.

Now the finite frequency H- /H∞ for SPS (1) to be addressed in this paper can be formulated as follows: for a given SPS (1), find a state feedback controller

$$U = KX + U_c$$

(7)

Where K is the state feedback gain vector and U_c is the compensation control.

Such that, for sufficient small parameter ε , the closed-loop System

$$\begin{aligned} E_\varepsilon \dot{X}(t) &= A_c X(t) + BU_c(t) \\ Y(t) &= C_c X(t) + DU_c(t) \end{aligned} \quad (8)$$

is stable and finite frequency bounded real.
Where $A_c = A + BK$, $C_c = C + DK$

Lemma. Let complex matrices A, B, a Hermitian matrix Θ and a positive scalar ω_l be given. Then the following statements are equivalent:

i) $$\begin{bmatrix} (j\omega I - A)^{-1} B \\ I \end{bmatrix}^* \Theta_l \begin{bmatrix} (j\omega I - A)^{-1} B \\ I \end{bmatrix} \prec 0 \quad (9)$$

Where:

$$\omega \in \Omega_l, \Omega_l := \{\omega \in \Re / \det(j\omega I - A) \ne 0, |\omega| \le \omega_l\}$$

ii) There exist $n \times n$ Hermitian matrices P_l and Q_l satisfying $Q_l \succ 0$ and

$$\begin{bmatrix} A & B \\ I & 0 \end{bmatrix}^* \begin{bmatrix} -Q_l & P_l \\ P_l & \omega_l^2 Q_l \end{bmatrix} \begin{bmatrix} A & B \\ I & 0 \end{bmatrix} + \Theta_l \prec 0 \quad (10)$$

The finite frequency condition in lemma can be generalized to the case where the frequency range is any interval of the form $|\omega| \ge \omega_h$, [17].

Corollary. Let complex matrices A, B, a Hermitian matrix Θ and a positive scalar ω_h be given. Then the following statements are equivalent:

i) $$\begin{bmatrix}(j\omega I-A)^{-1}B\\ I\end{bmatrix}^*\Theta_h\begin{bmatrix}(j\omega I-A)^{-1}B\\ I\end{bmatrix}\prec 0 \qquad (11)$$

Where:

$$\omega\in\Omega_h,\Omega_h:=\{\omega\in\Re/\det(j\omega I-A)\neq 0,|\omega|\geq\omega_h\}$$

ii) There exist $n\times n$ Hermitian matrices P_h and Q_h satisfying $Q_h\succ 0$ and

$$\begin{bmatrix}A & B\\ I & 0\end{bmatrix}^*\begin{bmatrix}Q_h & P_h\\ P_h & -\omega_h^2 Q_h\end{bmatrix}\begin{bmatrix}A & B\\ I & 0\end{bmatrix}+\Theta_h\prec 0 \qquad (12)$$

The Schur complement

The Schur complement converts a class of convex nonlinear inequalities that appears regularly in control problems to an LMI. We have [18]

$$\begin{bmatrix}Q & S\\ S^T & R\end{bmatrix}\prec 0 \quad\Leftrightarrow\quad \begin{cases}R\prec 0\\ Q-SR^{-1}S^T\prec 0\end{cases} \qquad (13)$$

Or

$$\begin{bmatrix}Q & S\\ S^T & R\end{bmatrix}\succ 0 \quad\Leftrightarrow\quad \begin{cases}R\succ 0\\ Q-SR^{-1}S^T\succ 0\end{cases} \qquad (14)$$

Where Q and R are symmetric

Proof

We put $M=\begin{bmatrix}Q & S\\ S^T & R\end{bmatrix}$

Set $U=R^{-1}S^T$

$$\begin{bmatrix}I & U^T\\ 0 & I\end{bmatrix}\begin{bmatrix}Q & S\\ S^T & R\end{bmatrix}\begin{bmatrix}I & 0\\ U & I\end{bmatrix}=\begin{bmatrix}Q+SU+U^TS^T+U^TRU & S+U^TR\\ S^T+RU & R\end{bmatrix}$$

$$S+U^TR=S-SR^{-1}R=0$$
$$SU+U^TS^T=SR^{-1}S^T-SR^{-1}S^T=0$$
$$Q+U^TRU=Q-SR^{-1}RR^{-1}S^T=Q-SR^{-1}S^T$$

Is obtained

$$\begin{bmatrix}Q-SR^{-1}S^T & 0\\ 0 & R\end{bmatrix}$$

The matrix $Q-SR^{-1}S^T$ is called the schur complement of R in M.

Then:

$$M \text{ is négative} \quad\Leftrightarrow\quad \begin{cases}R\prec 0\\ Q-SR^{-1}S^T\prec 0\end{cases}$$

$$M \text{ is positive} \quad\Leftrightarrow\quad \begin{cases}R\succ 0\\ Q-SR^{-1}S^T\succ 0\end{cases}$$

3 H- /H∞ Control of singularly perturbed system (SPS)

3.1 The slow subsystem and the associated FFH- control in the low frequency domain

For slow subsystem (2), considering the following problem: find K_s, such that for a given ω_l, the closed-loop subsystem

$$\begin{cases}\dot{X}_s(t)=A_{0c}X(t)+B_0U_{sc}(t)\\ Y_s=C_{0c}X(t)+D_0U_{sc}(t)\end{cases} \qquad (15)$$

is stable and FF H-.

Where $A_{0c}=A_0+B_0K_s$, $C_{0c}=C_0+D_0K_s$, $U_{sc}(t)=U_s-K_sX_s(t)$

The following theorem gives a sufficient condition for the slow subsystem (15) to be internally stable and FF H- .

Theorem1. For given ω_l, if there exist matrices $P_l=P_l^T$, $Q_l\succ 0$, and $X_l\succ 0$ such that the following matrix equalities satisfied:

$$A_{0c}^TX_l+X_lA_{0c}\prec 0 \qquad (16)$$

$$\begin{bmatrix}A_{0c} & B_0\\ I & 0\end{bmatrix}^*\begin{bmatrix}-Q_l & P_l\\ P_l & \omega_l^2Q_l\end{bmatrix}\begin{bmatrix}A_{0c} & B_{\grave{a}}\\ I & 0\end{bmatrix}+\Theta_l\prec 0 \qquad (17)$$

Then closed-loop system (15) is internally stable and FFH-. Theorem gives a sufficient condition for the existence of static feedback gain that achieves internally stable and FFH- property for the transfer function of the closed-loop system.

- Definition of H- index

$$\inf\left(\|G(s)\|_-\right)\succ\beta \quad \forall|\omega|\leq\omega_l \qquad (18)$$

Equivalent to $G_s^*(j\omega)\times G_s(j\omega)\succ\beta^2I$ (19)

i.e.

$$(C_0(j\omega I-A_0)^{-1}B_0+D_0)^*(C_0(j\omega I-A_0)^{-1}B_0+D_0)\succ\beta^2I \qquad (20)$$

It is could be rewritten the following form

$$\begin{bmatrix}(j\omega I-A_0)^{-1}B_0\\ I\end{bmatrix}^*\begin{bmatrix}C_0 & D_0\\ 0 & I\end{bmatrix}^*\begin{bmatrix}-I & 0\\ 0 & \beta^2I\end{bmatrix}\begin{bmatrix}C_0 & D_0\\ 0 & I\end{bmatrix}\begin{bmatrix}(j\omega I-A_0)^{-1}B_0\\ I\end{bmatrix}\prec 0 \qquad (21)$$

Then

$$\Theta_l=\begin{bmatrix}-C_0^TC_0 & -C_0^TD_0\\ -D_0^TC_0 & -D_0^TD_0+\beta^2I\end{bmatrix}\prec 0 \qquad (22)$$

(17) is equivalent to

$$\begin{bmatrix} A_{0c} & B_0 \\ I & 0 \end{bmatrix}^* \begin{bmatrix} -Q_l & P_l \\ P_l & \omega_l^2 Q_l \end{bmatrix} \begin{bmatrix} A_{0c} & B_à \\ I & 0 \end{bmatrix} + \begin{bmatrix} -C_{0c}^T C_{0c} & -C_{0c}^T D_0 \\ -D_0^T C_{0c} & -D_0^T D_0 + \beta^2 I \end{bmatrix} \prec 0$$

$$\begin{bmatrix} -A_{0c}^T Q_l A_{0c} + A_{0c}^T P_l + P_l A_{0c} + \omega_l^2 Q_l & -A_{0c}^T Q_l B_0 + P_l B_0 \\ * & -B_0^T Q_l B_0 \end{bmatrix} + \begin{bmatrix} -C_{0c}^T C_{0c} & -C_{0c}^T D_0 \\ -D_0^T C_{0c} & -D_0^T D_0 + \beta^2 I \end{bmatrix} \prec 0$$

$$\begin{bmatrix} -A_{0c}^T Q_l A_{0c} + A_{0c}^T P_l + P_l A_{0c} + \omega_l^2 Q_l - C_{0c}^T C_{0c} & -A_{0c}^T Q_l B_0 + P_l B_0 - C_{0c}^T D_0 \\ * & -B_0^T Q_l B_0 - D_0^T D_0 + \beta^2 I \end{bmatrix} \prec 0 \quad (23)$$

The condition is given in terms of BMIS which is presented like this:

$$\min_{P_l, Q_l, X_l} \{\mu\}$$

Subject to:

$$A_{0c}^T X_l + X_l A_{0c} \prec \mu I \quad (24)$$

$$\begin{bmatrix} -A_{0c}^T Q_l A_{0c} + A_{0c}^T P_l + P_l A_{0c} + \omega_l^2 Q_l - C_{0c}^T C_{0c} - \mu I & -A_{0c}^T Q_l B_0 + P_l B_0 - C_{0c}^T D_0 \\ * & -B_0^T Q_l B_0 - D_0^T D_0 + \beta^2 I - \mu I \end{bmatrix} \prec 0 \quad (25)$$

3.2 The fast subsystem and the associated FFH∞ control in the high frequency domain

For fast subsystem (3), considering the following problem: find K_f, such that for a given ω_h, the closed-loop subsystem

$$\begin{cases} \varepsilon \dot{X}_f(t) = A_{cf} X_f(t) + B_2 U_{fc}(t) \\ Y = C_{cf} X_f(t) + D_2 U_{fc}(t) \end{cases} \quad (26)$$

is stable and FFH∞.

Where $A_{cf} = A_{22} + B_2 K_f$, $C_{cf} = C_2 + D_2 K_f$, $U_{fc}(t) = U_f - K_f X_f(t)$

The following theorem gives a sufficient condition for the fast subsystem (26) to be internally stable and FFH∞.

Theorem2. For given ω_h, if there exist matrices $P_h = P_h^T$, $Q_h \succ 0$, and $X_h \succ 0$ such that the following matrix equalities satisfied:

$$A_{cf}^T X_h + X_h A_{cf} \prec 0 \quad (27)$$

$$\begin{bmatrix} A_{cf} & B_2 \\ I & 0 \end{bmatrix}^* \begin{bmatrix} -Q_h & P_h \\ P_h & \omega_h^2 Q_h \end{bmatrix} \begin{bmatrix} A_{cf} & B_2 \\ I & 0 \end{bmatrix} + \Theta_h \prec 0 \quad (28)$$

Then closed-loop system (26) is internally stable and FFH∞.

- Definition of H∞ norm

$$\sup\left(\|G(s)\|_\infty \right) \prec \gamma \quad \forall |\omega| \geq \omega_h \quad (29)$$

Equivalent to $G_f^*(j\omega) \times G_f(j\omega) \prec \gamma^2 I$ (30)

i.e.

$$(C_2(j\omega I - A_{22})^{-1} B_2 + D_2)^* (C_2(j\omega I - A_{22})^{-1} B_2 + D_2) \prec \gamma^2 I \quad (31)$$

It is could be rewritten the following form

$$\begin{bmatrix} (j\omega I - A_{22})^{-1} B_2 \\ I \end{bmatrix}^* \begin{bmatrix} C_2 & D_2 \\ 0 & I \end{bmatrix}^* \begin{bmatrix} I & 0 \\ 0 & -\gamma^2 I \end{bmatrix} \begin{bmatrix} C_2 & D_2 \\ 0 & I \end{bmatrix} \begin{bmatrix} (j\omega I - A_{22})^{-1} B_2 \\ I \end{bmatrix} \prec 0 \quad (32)$$

Then

$$\Theta_h = \begin{bmatrix} C_2^T C_2 & C_2^T D_2 \\ D_2^T C_2 & D_2^T D_2 - \gamma^2 I \end{bmatrix} \prec 0 \quad (33)$$

(28) is equivalent to

$$\begin{bmatrix} A_{cf}^T Q_h A_{cf} + A_{cf}^T P_h + P_h A_{cf} - \omega_h^2 Q_h + C_2^T C_2 & A_{cf}^T Q_h B_2 + P_h B_2 + C_2^T D_2 \\ * & B_2^T Q_h B_2 + D_2^T D_2 - \gamma^2 I \end{bmatrix} \prec 0 \quad (34)$$

$$\begin{bmatrix} A_{cf}^T Q_h A_{cf} & A_{cf}^T Q_h B_2 \\ * & B_2^T Q_h B_2 \end{bmatrix} + \Pi_f \prec 0 \quad (35)$$

$$\Pi_f + \begin{bmatrix} A_{cf}^T Q_h \\ B_2^T Q_h \end{bmatrix} Q_h^{-1} \begin{bmatrix} Q_h A_{cf} & Q_h B_2 \end{bmatrix} \prec 0 \quad (36)$$

Using Schur complementary Lemma, Then we have

$$\begin{bmatrix} \Pi_f & \begin{bmatrix} A_{cf}^T Q_h \\ B_2^T Q_h \end{bmatrix} \\ * & -Q_h \end{bmatrix} \prec 0 \quad (37)$$

Where

$$\Pi_f = \begin{bmatrix} A_{cf}^T P_h + P_h A_{cf} - \omega_h^2 Q_h + C_2^T C_2 & P_h B_2 + C_2^T D_2 \\ * & D_2^T D_2 - \gamma^2 I \end{bmatrix}$$

Then

$$\begin{bmatrix} A_{cf}^T P_h + P_h A_{cf} - \omega_h^2 Q_h + C_2^T C_2 & P_h B_2 + C_2^T D_2 & A_{cf}^T Q_h \\ * & D_2^T D_2 - \gamma^2 I & B_2^T Q_h \\ * & * & -Q_h \end{bmatrix} \prec 0 \qquad (38)$$

$$\Psi_f + \begin{bmatrix} C_2^T \\ D_2^T \\ 0 \end{bmatrix} I^{-1} \begin{bmatrix} C_2 & D_2 & 0 \end{bmatrix} \prec 0 \qquad (39)$$

By using Schur complement Lemma, we can obtain

$$\begin{bmatrix} \psi_f & \begin{bmatrix} C_2^T \\ D_2^T \\ 0 \end{bmatrix} \\ * & -I \end{bmatrix} \prec 0 \qquad (40)$$

Where $\begin{bmatrix} A_{cf}^T P_h + P_h A_{cf} - \omega_h^2 Q_h & P_h B_2 & A_{cf}^T Q_h \\ * & -\gamma^2 I & B_2^T Q_h \\ * & * & -Q_h \end{bmatrix} \prec 0$

Then (38) equivalent to

$$\begin{bmatrix} A_{cf}^T P_h + P_h A_{cf} - \omega_h^2 Q_h & P_h B_2 & A_{cf}^T Q_h & C_2^T \\ * & -\gamma^2 I & B_2^T Q_h & D_2^T \\ * & * & -Q_h & 0 \\ * & * & * & -I \end{bmatrix} \prec 0 \qquad (41)$$

Resolution by approach LMI :

- The resolution of the previous problem can be obtained by the resolution of a problem of programming by LMI.

Step1. Choose an initial K_f; solve the following convex optimization problem:

$$\min_{P_h, Q_h, X_h} \{\mu\}$$

Subject to:

$$A_{cf}^T X_h + X_h A_{cf} \prec \mu I \qquad (42)$$

$$\begin{bmatrix} A_{cf}^T P_h + P_h A_{cf} - \omega_h^2 Q_h - \mu I & P_h B_2 & A_{cf}^T Q_h & C_2^T \\ * & -\gamma^2 I - \mu I & B_2^T Q_h & D_2^T \\ * & * & -Q_h - \mu I & 0 \\ * & * & * & -I - \mu I \end{bmatrix} \prec 0 \qquad (43)$$

are hold. If $\mu \le 0$, then problem is solved; otherwise go to step 2.

Step2. With the obtained matrices P_h Q_h, X_h, solve the above optimization with respect to K_s. Again if $\mu \le 0$, the problem is solved, otherwise, go to Step 1.

Assuming, for the moment, that we have successfully designed K_s and K_f a composite controller is formed as:

$$K = \begin{bmatrix} K_s & K_f \end{bmatrix} \qquad (44)$$

It follows from (15), (26) and previous lemma that, for sufficient small ε, $U = KX + U_c$ is an internally stabilizing controller and preserved FFH-/H∞ at low and high frequencies.

4 Simulation

In this section a numerical example is given to demonstrate the effectiveness of the proposed method. Consider the singularly perturbed system:

$$\begin{cases} E_\varepsilon \dot{X}(t) = \begin{bmatrix} 2 & 1 \\ -1 & -2 \end{bmatrix} X + \begin{bmatrix} 2 \\ -0.5 \end{bmatrix} U \\ Y = \begin{bmatrix} 1 & 1 \end{bmatrix} X(t) - 0.5U \end{cases} \qquad (45)$$

Using Matlab LMI control toolbox,

- A solution to ((24-25) and (42-43)) is obtained as follows:

For $\omega_l = 3$, following theorem 1, one get

K_s = -20.3668, Q_l = 0.0053 ,

P_l = -0.5185 , X_l = 20.4428

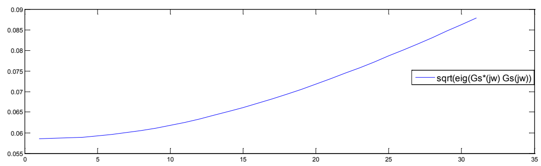

Fig 1 singular value of the slow subsystem

$$\beta = 0.0332 \quad , \quad \inf\left(sqrt(eig(G_s^*(j\omega) G_s(j\omega))\right) = 0.0586$$

$$\beta \prec \inf\left(\|G(s)\|_-\right)$$

For $\omega_h = 100$, following theorem 2, one get

$K_f = 1.0125$, $Q_h = 1.2051$,

$P_h = 0.6266$, $X_h = 2.9441$

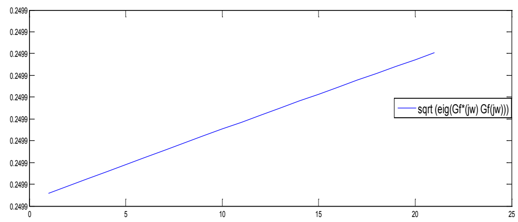

Fig 2 singular value of the fast subsystem

$$\gamma = 0.2500 \quad , \quad \sup\left(sqrt(eig(G_f^*(j\omega)G_f(j\omega))\right) = 0.2499$$

$$\sup\left(\quad \|G(s)\|_\infty \right) \prec \gamma$$

The composite state feedback controller can be obtained:

$$K = \begin{bmatrix} -20.3668 & 1.0125 \end{bmatrix}$$

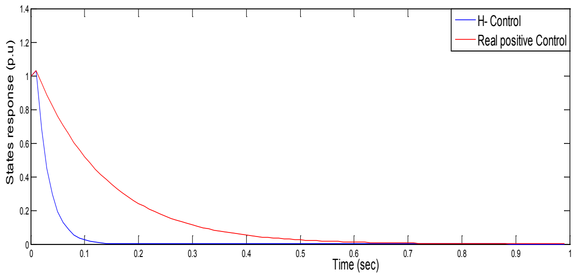

Fig 3. States response of Xs (H-) and Xs (Positive Real)

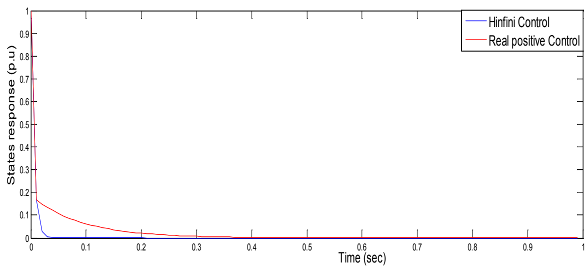

Fig 4. States response of Xf (Hinfini) and Xf (Positive Real)

A comparison of state responses between H-/H∞ and Positive Real control are shown in Figs. 3 and 4, respectively. It can be observed state responses are stabilized by the controller designed via the LMI approach to the finite frequency H-/H∞ control of SPS better than the real positive control.

5 Conclusion

In this paper, we have studied the problem finite frequency H-/H∞ control for singularly perturbed systems (SPS) by using the generalized KYP lemma. The sufficient conditions for the existence of an H-/H∞ control are derived based on generalized KYP lemma and given in terms of linear matrix inequalities (LMIs). A controller which is singularly perturbed is also explicated constructed through designing its fast and slow parts. The result is applicable to deal with the H-/H∞ control problem in different frequency ranges to make system better dynamic, a numerical example has been given to illustrate the effectiveness of the proposed method.

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