Dynamic Behaviors of N-Species Cooperation System with Distributed Delays and Feedback Controls on Time Scales

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Abstract: This paper is concerned with a n-species cooperation system with distributed delays and feedback controls on time scales. For general nonautonomous case, by using differential inequality theory and constructing a suitable Lyapunov functional, sufficient conditions which guarantee the permanence and the global attractivity of the system are obtained. For the almost periodic case, by using the Razumikhin type theorem, sufficient conditions which guarantee the existence of a positive almost periodic solution of the system are obtained. Finally, an example and numerical simulations are presented to illustrate the feasibility and effectiveness of the results.

Key–Words: Cooperation system; Permanence; Global attractivity; Almost periodic solution; Time scale.

1 Introduction

As is well known, ecosystem in the real world are continuously disturbed by unpredictable forces which can result in changes in the biological parameters such as survival rates. Of practical interest in ecology is the question of whether or not an ecosystem can withstand those unpredictable disturbances which persist for a finite period of time. In the language of control variables, we call the disturbance functions as control variables. During the last decade, dynamic behaviors such as permanence, global attractivity, periodicity and almost periodicity of different types of ecosystems with feedback control have been studied extensively; see, for example, [1-5] and the references therein.

On the other hand, in the natural world, there are many species whose developing processes are both continuous and discrete. Hence, using the only differential equation or difference equation can't accurately describe the law of their developments; see, for example, [6,7]. Therefore, there is a need to establish correspondent dynamic models on new time scales.

Recently, ecosystems with periodic coefficients on time scales received more researchers' special attention due to their applications; see, for example, [8-13] and the references therein. However, upon considering long-term dynamical behaviors, the periodic parameters often turn out to experience certain perturbations, that is, parameters become periodic up to a small error, then one has to consider the ecosystems to be almost periodic. Therefore, if we consider the effects of the environmental factors (e.g. seasonal effects of weather, food supplies, mating habits, and harvesting), the assumption of almost periodicity is more realistic, more important and more general.

To the best of the authors' knowledge, there are few papers published on the dynamic behaviors (permanence, global attractivity, almost periodicity, etc.) of ecosystems on time scales, especially for cooperative ecosystems on time scales. As we know, cooperative ecosystems is one kind of most important ecosystem in the real world.

Motivated by the above, in the present paper, we shall study a nonautonomous n-species cooperation system with distributed delays and feedback controls on time scales as follows:

$$\begin{cases} x_i^{\Delta}(t) = r_i(t)x_i(t)\Bigg[1 - c_i(t)x_i(\sigma(t)) \\ \qquad - \dfrac{x_i(t)}{a_i(t)+\sum\limits_{j=1,j\neq i}^{n} b_{ij}(t)\int_{-\theta_{ij}}^{0} B_{ij}(s)x_j(t+s)\Delta s}\Bigg] \\ \qquad -d_i(t)u_i(t)x_i(t) \\ \qquad -h_i(t)x_i(t)\int_{-\tau_i}^{0} H_i(s)u_i(t+s)\Delta s, \\ u_i^{\Delta}(t) = -\alpha_i(t)u_i(t) + \beta_i(t)x_i(t) \\ \qquad +g_i(t)\int_{-\eta_i}^{0} G_i(s)u_i(t+s)\Delta s, \end{cases} \tag{1}$$

where $x_i(i = 1, 2, \cdots, n)$ is the density of cooperation species, $u_i(i = 1, 2, \cdots, n)$ is the control variable.

Throughout this paper, we assume that

(H_1) $r_i(t), a_i(t), b_{ij}(t), c_i(t), d_i(t), h_i(t), \alpha_i(t), \beta_i(t),$

$g_i(t), i, j = 1, 2, \cdots, n$ are all continuous, real-valued functions which are bounded above and below by positive constants;

(H_2) θ_{ij}, τ_i, η_i are all positive constants, $B_{ij}(s)$, $H_i(s)$, $G_i(s)$ are all nonnegative continuous functions such that $\int_{-\theta_{ij}}^{0} B_{ij}(s)\Delta s = 1$, $\int_{-\tau_i}^{0} H_i(s)\Delta s = 1$, $\int_{-\eta_i}^{0} G_i(s)\Delta s = 1$, $i, j = 1, 2, \cdots, n$.

Let $\tau = \max\{\theta_{ij}, \tau_i, \eta_i, i, j = 1, 2, \cdots, n\}$, consider (1) together with the following initial conditions

$$\begin{aligned} &x_i(\theta) = \varphi_i(\theta) \geq 0,\ \theta \in [-\tau, 0]_{\mathbb{T}}, \varphi_i(0) > 0, \\ &u_i(\theta) = \psi_i(\theta) \geq 0,\ \theta \in [-\tau, 0]_{\mathbb{T}}, \psi_i(0) > 0, \end{aligned} \quad (2)$$

where $\varphi_i(s)$ and $\varphi_i(s)$ are continuous on $[-\tau, 0]_{\mathbb{T}}$.

For convenience, we introduce the notation

$$f^u = \sup_{t\in\mathbb{T}} f(t),\ f^l = \inf_{t\in\mathbb{T}} f(t),$$

where f is a positive and bounded function.

The organization of this paper is as follows. In section 2, we introduce some notations and definitions and state some preliminary results needed in later sections. Section 3 is devoted to studying the permanence and the global attractivity of system (1). Section 4 is devoted to studying the existence of a unique positive almost periodic solution of the system (1). An example is given in Section 5.

2 Preliminaries

Let $\mathbb{T}$ be a nonempty closed subset (time scale) of $\mathbb{R}$. The forward and backward jump operators $\sigma, \rho : \mathbb{T} \to \mathbb{T}$ and the graininess $\mu : \mathbb{T} \to \mathbb{R}^+$ are defined, respectively, by

$$\begin{aligned} \sigma(t) &= \inf\{s \in \mathbb{T} : s > t\}, \\ \rho(t) &= \sup\{s \in \mathbb{T} : s < t\}, \\ \mu(t) &= \sigma(t) - t. \end{aligned}$$

A point $t \in \mathbb{T}$ is called left-dense if $t > \inf \mathbb{T}$ and $\rho(t) = t$, left-scattered if $\rho(t) < t$, right-dense if $t < \sup \mathbb{T}$ and $\sigma(t) = t$, and right-scattered if $\sigma(t) > t$. If $\mathbb{T}$ has a left-scattered maximum m, then $\mathbb{T}^k = \mathbb{T}\backslash\{m\}$; otherwise $\mathbb{T}^k = \mathbb{T}$. If $\mathbb{T}$ has a right-scattered minimum m, then $\mathbb{T}_k = \mathbb{T}\backslash\{m\}$; otherwise $\mathbb{T}_k = \mathbb{T}$.

The basic theories of calculus on time scales, one can see [14].

A function $p : \mathbb{T} \to \mathbb{R}$ is called regressive provided $1 + \mu(t)p(t) \neq 0$ for all $t \in \mathbb{T}^k$. The set of all regressive and rd-continuous functions $p : \mathbb{T} \to \mathbb{R}$ will be denoted by $\mathcal{R} = \mathcal{R}(\mathbb{T}, \mathbb{R})$. Define the set $\mathcal{R}^+ = \mathcal{R}^+(\mathbb{T}, \mathbb{R}) = \{p \in \mathcal{R} : 1 + \mu(t)p(t) > 0, \forall\, t \in \mathbb{T}\}$.

If r is a regressive function, then the generalized exponential function e_r is defined by

$$e_r(t, s) = \exp\left\{ \int_s^t \xi_{\mu(\tau)}(r(\tau))\Delta\tau \right\}$$

for all $s, t \in \mathbb{T}$, with the cylinder transformation

$$\xi_h(z) = \begin{cases} \frac{\mathrm{Log}(1+hz)}{h}, & \text{if } h \neq 0, \\ z, & \text{if } h = 0. \end{cases}$$

Let $p, q : \mathbb{T} \to \mathbb{R}$ be two regressive functions, define

$$p\oplus q = p+q+\mu pq,\ \ominus p = -\frac{p}{1+\mu p},\ p\ominus q = p\oplus(\ominus q).$$

Lemma 1. (see [14]) *If $p, q : \mathbb{T} \to \mathbb{R}$ be two regressive functions, then*
(i) $e_0(t, s) \equiv 1$ and $e_p(t, t) \equiv 1$;
(ii) $e_p(\sigma(t), s) = (1 + \mu(t)p(t))e_p(t, s)$;
(iii) $e_p(t, s) = \frac{1}{e_p(s,t)} = e_{\ominus p}(s, t)$;
(iv) $e_p(t, s)e_p(s, r) = e_p(t, r)$;
(v) $\frac{e_p(t,s)}{e_q(t,s)} = e_{p\ominus q}(t, s)$;
(vi) $(e_p(t, s))^\Delta = p(t)e_p(t, s)$.

Lemma 2. (see [15]) *Assume that $a > 0, b > 0$ and $-a \in \mathcal{R}^+$. Then*

$$y^\Delta(t) \geq (\leq) b - ay(t),\ y(t) > 0,\ t \in [t_0, \infty)_{\mathbb{T}}$$

implies

$$y(t) \geq (\leq)\frac{b}{a}[1+(\frac{ay(t_0)}{b}-1)e_{(-a)}(t, t_0)],\ t \in [t_0, \infty)_{\mathbb{T}}.$$

Lemma 3. (see [15]) *Assume that $a > 0, b > 0$. Then*

$$y^\Delta(t) \leq (\geq) y(t)(b-ay(\sigma(t))),\ y(t) > 0,\ t \in [t_0, \infty)_{\mathbb{T}}$$

implies

$$y(t) \leq (\geq)\frac{b}{a}[1+(\frac{b}{ay(t_0)}-1)e_{\ominus b}(t, t_0)],\ t \in [t_0, \infty)_{\mathbb{T}}.$$

Let $\mathbb{T}$ be a time scale with at least two positive points, one of them being always one: $1 \in \mathbb{T}$, there exists at least one point $t \in \mathbb{T}$ such that $0 < t \neq 1$. Define the natural logarithm function on the time scale $\mathbb{T}$ by

$$L_{\mathbb{T}}(t) = \int_1^t \frac{1}{\tau}\Delta\tau,\ t \in \mathbb{T} \cap (0, +\infty).$$

Lemma 4. (see [16]) *Assume that $x : \mathbb{T} \to \mathbb{R}^+$ is strictly increasing and $\widetilde{\mathbb{T}} := x(\mathbb{T})$ is a time scale. If $x^\Delta(t)$ exists for $t \in \mathbb{T}^k$, then*

$$\frac{\Delta}{\Delta t} L_{\mathbb{T}}(x(t)) = \frac{x^\Delta(t)}{x(t)}.$$

Lemma 5. (see [14]) *Assume that $f, g : \mathbb{T} \to \mathbb{R}$ are differentiable at $t \in \mathbb{T}^k$, then $fg : \mathbb{T} \to \mathbb{R}$ is differentiable at t with*

$$\begin{aligned}(fg)^\Delta(t) &= f^\Delta(t)g(t) + f(\sigma(t))g^\Delta(t)\\ &= f(t)g^\Delta(t) + f^\Delta(t)g(\sigma(t)).\end{aligned}$$

Let $C = C([-\tau, 0]_{\mathbb{T}}, \mathbb{R}^n), H^* \in \mathbb{R}^+$. Denote $C_{H^*} = \{\varphi, \varphi \in C, \|\varphi\| < H^*\}$, $S_{H^*} = \{x, x \in \mathbb{R}^n, \|x\| < H^*\}$, $\|\varphi\| = \sup\limits_{\theta \in [-\tau,0]_{\mathbb{T}}} |\varphi(\theta)|$.

Consider the system

$$x^\Delta = f(t, x), \tag{3}$$

where $f(t, \phi)$ is continuous in $(t, \phi) \in \mathbb{R} \times C$ and almost periodic in t uniformly for $\phi \in C_{H^*}, C_{H^*} \subseteq C$. $\forall \alpha > 0, \exists L(\alpha) > 0$ such that $|f(t, \phi)| \le L(\alpha)$, as $t \in \mathbb{T}, \phi \in C_\alpha$.

In order to investigate the almost periodic solution of system (3), we introduce the associate product system of system (3)

$$x^\Delta = f(t, x),\ y^\Delta = f(t, y). \tag{4}$$

Lemma 6. (see [17]) *Assume that there exists a Lyapunov function $V(t, x, y)$ defined on $[0, +\infty)_{\mathbb{T}} \times S_{H^*} \times S_{H^*}$, which satisfies the following conditions:*

(1) *$\alpha(|x-y|) \le V(t, x, y) \le \beta(|x-y|)$, where $\alpha(s)$ and $\beta(s)$ are continuous, increasing and positive definite;*

(2) *$|V(t, x_1, y_1) - V(t, x_2, y_2)| \le \omega(|x_1 - x_2| + |y_1 - y_2|)$, where $\omega > 0$ is a constant;*

(3) *$D^+V^\Delta_{(2.2)}(t, x, y) \le -\lambda V(t, x, y)$, where $\lambda > 0$ is a constant.*

Moreover, assumes that (3) has a solution $\xi(t)$ such that $\|\xi\| \le H < H^*$ for $t \in [t_0, +\infty)_{\mathbb{T}}$. Then system (3) has a unique almost periodic solution which is uniformly asymptotic stable.

Definition 7. *A positive bounded solution $(x_1(t), x_2(t), \cdots, x_n(t), u_1(t), \cdots, u_n(t))$ of (1) is said to be globally attractive if for any other positive bounded solution $(y_1(t), y_2(t), \cdots, y_n(t), v_1(t), \cdots, v_n(t))$ of (1), the following equality holds:*

$$\lim_{t\to+\infty}\left[\sum_{j=1}^n |x_j(t) - y_j(t)| + \sum_{j=1}^n |u_j(t) - v_j(t)|\right] = 0.$$

3 Permanence and attractivity

Assume that the coefficients of (1) satisfy

(H_3) $r_i^l > \frac{r_i^u}{a_i^l} M_{1i} + (d_i^u + h_i^u) M_{2i},\ i = 1, 2, \cdots, n.$

Theorem 8. *Let $(x_1(t), x_2(t), \cdots, x_n(t), u_1(t), \cdots, u_n(t))$ be any positive solution of system (1) with initial condition (2). If $(H_1) - (H_3)$ hold, then system (1) is permanent, that is, any positive solution $(x_1(t), x_2(t), \cdots, x_n(t), u_1(t), \cdots, u_n(t))$ of system (1) satisfies*

$$m_{1i} \le \liminf_{t\to+\infty} x_i(t) \le \limsup_{t\to+\infty} x_i(t) \le M_{1i}, \tag{5}$$

$$m_{2i} \le \liminf_{t\to+\infty} u_i(t) \le \limsup_{t\to+\infty} u_i(t) \le M_{2i}, \tag{6}$$

especially if $m_{1i} \le x_i(t_0) \le M_{1i}$, $m_{2i} \le u_i(t_0) \le M_{2i}, i = 1, 2, \cdots, n$, then

$$m_{1i} \le x_i(t) \le M_{1i},\ m_{2i} \le u_i(t) \le M_{2i},$$
$$t \in [t_0, +\infty)_{\mathbb{T}},\ i = 1, 2, \cdots, n,$$

where

$$M_{1i} = \frac{r_i^u}{r_i^l c_i^l},\ M_{2i} = \frac{(\beta_i^u + g_i^u)M_{1i}}{\alpha_i^l},$$
$$m_{1i} = \frac{r_i^l - \frac{r_i^u}{a_i^l}M_{1i} - (d_i^u + h_i^u)M_{2i}}{r_i^u c_i^u},$$
$$m_{2i} = \frac{(\beta_i^l + g_i^l)m_{1i}}{\alpha_i^u}.$$

Proof. Assume that $(x_1(t), x_2(t), \cdots, x_n(t), u_1(t), \cdots, u_n(t))$ be any positive solution of system (1) with initial condition (2). From the ith equation of system (1), we have

$$\begin{aligned}x_i^\Delta(t) &\le r_i(t)x_i(t)(1 - c_i(t)x_i(\sigma(t)))\\ &\le x_i(t)(r_i^u - r_i^l c_i^l x_i(\sigma(t))).\end{aligned} \tag{7}$$

By Lemma 3, we can get

$$\limsup_{t\to+\infty} x_i(t) \le \frac{r_i^u}{r_i^l c_i^l} := M_{1i}.$$

Then, for arbitrary small positive constant $\varepsilon > 0$, there exists a $T_1 > 0$ such that

$$x_i(t) < M_{1i} + \varepsilon,\ \forall t \in [T_1, +\infty]_{\mathbb{T}}.$$

From the $n + i$th equation of system (1), when $t \in [T_1, +\infty)_{\mathbb{T}}$,

$$\begin{aligned}u_i^\Delta(t) &= -\alpha_i(t)u_i(t) + \beta_i(t)x_i(t)\\ &\quad + g_i(t)x_i(t)\int_{-\eta_i}^0 G_i(s)u_i(t+s)\Delta s\end{aligned}$$

$$< \quad -\alpha_i^l u_i(t) + (\beta_i^u + g_i^u)(M_{1i} + \varepsilon),$$

Let $\varepsilon \to 0$, then

$$u_i^{\Delta}(t) \leq -\alpha_i^l u_i(t) + (\beta_i^u + g_i^u) M_{1i}. \tag{8}$$

By Lemma 2, we can get

$$\limsup_{t \to +\infty} u_i(t) = \frac{(\beta_i^u + g_i^u) M_{1i}}{\alpha_i^l} := M_{2i}.$$

Then, for arbitrary small positive constant $\varepsilon > 0$, there exists a $T_2 > T_1$ such that

$$u_i(t) < M_{2i} + \varepsilon, \ \forall t \in [T_2, +\infty]_{\mathbb{T}}.$$

On the other hand, from the ith equation of system (1), when $t \in [T_2, +\infty)_{\mathbb{T}}$,

$$\begin{aligned} & x_i^{\Delta}(t) \\ = \ & r_i(t)x_i(t)\Bigg[1 - c_i(t)x_i(\sigma(t)) \\ & - \frac{x_i(t)}{a_i(t) + \sum\limits_{j=1, j\neq i}^{n} b_{ij}(t)\int_{-\theta_{ij}}^{0} B_{ij}(s)x_j(t+s)\Delta s}\Bigg] \\ & -d_i(t)u_i(t)x_i(t) \\ & -h_i(t)x_i(t)\int_{-\tau_i}^{0} H_i(s)u_i(t+s)\Delta s, \\ > \ & x_i(t)[r_i^l - \frac{r_i^u}{a_i^l}(M_{1i} + \varepsilon) - r_i^u c_i^u x_i(\sigma(t)) \\ & -(d_i^u + h_i^u)(M_{2i} + \varepsilon)]. \end{aligned}$$

Let $\varepsilon \to 0$, then

$$\begin{aligned} x_i^{\Delta}(t) \quad \geq \quad & x_i(t)[r_i^l - \frac{r_i^u}{a_i^l} M_{1i} - r_i^u c_i^u x_i(\sigma(t)) \\ & -(d_i^u + h_i^u) M_{2i}]. \end{aligned} \tag{9}$$

By Lemma 3, we can get

$$\liminf_{t \to +\infty} x_i(t) = \frac{r_i^l - \frac{r_i^u}{a_i^l} M_{1i} - (d_i^u + h_i^u) M_{2i}}{r_i^u c_i^u} := m_{1i}.$$

Then, for arbitrary small positive constant $\varepsilon > 0$, there exists a $T_3 > T_2$ such that

$$x_i(t) > m_{1i} - \varepsilon, \ \forall t \in [T_3, +\infty]_{\mathbb{T}}.$$

From the $n+i$th equation of system (1), when $t \in [T_3, +\infty)_{\mathbb{T}}$,

$$\begin{aligned} u_i^{\Delta}(t) \quad = \quad & -\alpha_i(t)u_i(t) + \beta_i(t)x_i(t) \\ & +g_i(t)x_i(t)\int_{-\eta_i}^{0} G_i(s)u_i(t+s)\Delta s \\ > \quad & -\alpha_i^u u_i(t) + (\beta_i^l + g_i^l)(m_{1i} - \varepsilon). \end{aligned}$$

Let $\varepsilon \to 0$, then

$$u_i^{\Delta}(t) \geq -\alpha_i^u u_i(t) + (\beta_i^l + g_i^l) m_{1i}. \tag{10}$$

By Lemma 2, we can get

$$\liminf_{t \to +\infty} u_i(t) = \frac{(\beta_i^l + g_i^l) m_{1i}}{\alpha_i^u} := m_{2i}.$$

Then, for arbitrary small positive constant $\varepsilon > 0$, there exists a $T_4 > T_3$ such that

$$u_i(t) > m_{2i} - \varepsilon, \ \forall t \in [T_4, +\infty]_{\mathbb{T}}.$$

In special case, if $m_{1i} \leq x_i(t_0) \leq M_{1i}$, $m_{2i} \leq u_i(t_0) \leq M_{2i}$, by Lemma 2 and Lemma 3, it follows from (7)-(10) that

$$\begin{aligned} & m_{1i} \leq x_i(t) \leq M_{1i}, \ m_{2i} \leq u_i(t) \leq M_{2i}, \\ & t \in [t_0, +\infty)_{\mathbb{T}}, \end{aligned}$$

This completes the proof. □

Theorem 9. *Assume that all the conditions of Theorem 2.1 hold, further assume that*

(H_4) $\dfrac{r_i^l}{a_i^u + \sum\limits_{j=1, j\neq i}^{n} b_{ij}^u M_{1j}} - \beta_i^u > 0$;

(H_5) $\alpha_i^l - d_i^u > 0$;

(H_6) *There exist positive constants $k > 1$ and $\gamma > 0$ such that $\Pi > \gamma$, where*

$$\begin{aligned} \Pi = \min_i \Bigg[\Bigg(& \frac{r_i^l}{a_i^u + \sum\limits_{j=1, j\neq i}^{n} b_{ij}^u M_{1j}} \\ & -\beta_i^u \Bigg) m_{1i}, (\alpha_i^l - d_i^u) \Bigg] \\ & -k \sum_{i=1}^{n} \Bigg[\sum_{j=1, j\neq i}^{n} \frac{r_i^u b_{ij}^u M_{1i} M_{1j}}{(a_i^l + \sum\limits_{j=1, j\neq i}^{n} b_{ij}^l m_{1j})^2} \\ & +h_i^u + g_i^u \Bigg]. \end{aligned}$$

Then the solution of system (1) is globally attractive.

Proof. Assume that $z_1(t) = (x_1(t), x_2(t), \cdots, x_n(t), u_1(t), \cdots, u_n(t))$ and $z_2(t) = (y_1(t), y_2(t), \cdots, y_n(t), v_1(t), \cdots, v_n(t))$ be two positive bounded solutions of system (1). It follows from (5)-(6) that for sufficient small positive constant ε_0 $(0 < \varepsilon_0 < \min\limits_{1 \leq i \leq n} \{m_{1i}, m_{2i}\})$, there exists a $T > 0$ such that

$$\begin{aligned} & m_{1i} - \varepsilon_0 < x_i(t), y_i(t) < M_{1i} + \varepsilon_0, \\ & m_{2i} - \varepsilon_0 < u_i(t), v_i(t) < M_{2i} + \varepsilon_0, \end{aligned} \tag{11}$$

where $t \in [T,+\infty)_{\mathbb{T}}, i = 1,2,\cdots,n$, and

$$\frac{r_i^l}{a_i^u + \sum\limits_{j=1,j\neq i}^{n} b_{ij}^u (M_{1j}+\varepsilon_0)} - \beta_i^u > 0. \tag{12}$$

Since $x_i(t), y_i(t), i = 1,2,\cdots,n$ are positive, bounded and differentiable functions on $\mathbb{T}$, then there exists a positive, bounded and differentiable function $m(t), t \in \mathbb{T}$, such that $x_i(t)(1+m(t)), y_i(t)(1+m(t)), i = 1,2,\cdots,n$ are strictly increasing on $\mathbb{T}$. By Lemma 4 and Lemma 5, we have

$$\begin{aligned}
&\frac{\Delta}{\Delta t} L_{\mathbb{T}}(x_i(t)[1+m(t)]) \\
&= \frac{x_i^{\Delta}(t)}{x_i(t)} + \frac{x_i(\sigma(t))m^{\Delta}(t)}{x_i(t)[1+m(t)]}, \\
&\frac{\Delta}{\Delta t} L_{\mathbb{T}}(y_i(t)[1+m(t)]) \\
&= \frac{y_i^{\Delta}(t)}{y_i(t)} + \frac{y_i(\sigma(t))m^{\Delta}(t)}{y_i(t)[1+m(t)]},\ i = 1,2,\cdots,n.
\end{aligned}$$

Here, we can choose a function $m(t)$ such that $\frac{|m^{\Delta}(t)|}{1+m(t)}$ is bounded on $\mathbb{T}$, that is, there exist two positive constants $\zeta > 0$ and $\xi > 0$ such that $0 < \zeta < \frac{|m^{\Delta}(t)|}{1+m(t)} < \xi$, $\forall t \in \mathbb{T}$.

Set

$$\begin{aligned}
V(t) &= \sum_{i=1}^{n} V_i(t), \\
V_i(t) &= |e_{-\delta}(t,T)|(|L_{\mathbb{T}}(x_i(t)(1+m(t))) \\
&\quad - L_{\mathbb{T}}(y_i(t)(1+m(t)))| + |u_i(t)-v_i(t)|),
\end{aligned}$$

where $\delta \geq 0$ is a constant (if $\mu(t) = 0$, then $\delta = 0$; if $\mu(t) > 0$, then $\delta > 0$). It follows from the mean value theorem of differential calculus on time scales for $t \in [T,+\infty)_{\mathbb{T}}$,

$$\begin{aligned}
&\frac{1}{M_{1i}+\varepsilon_0}|x_i(t)-y_i(t)| \\
\leq\ & |L_{\mathbb{T}}(x_i(t)(1+m(t))) - L_{\mathbb{T}}(y_i(t)(1+m(t)))| \\
\leq\ & \frac{1}{m_{1i}-\varepsilon_0}|x_i(t)-y_i(t)|,\ i = 1,2,\cdots,n. \quad (13)
\end{aligned}$$

Now, we divide the proof into two cases.

Case I. If $\mu(t) > 0$, set $\delta > \max\{(r_i^u c_i^u + \frac{\xi}{m_{1i}})M_{1i}, \gamma\}$ and $1-\mu(t)\delta < 0$. Calculating the upper right derivatives of $V_i(t)$ along the solution of system (1), it follows from (11)-(13), $(H_4) - (H_6)$ that for $t \in [T,+\infty)_{\mathbb{T}}$,

$$\begin{aligned}
&D^+ V_i^{\Delta}(t) \\
=\ & |e_{-\delta}(t,T)|\mathrm{sgn}(x_i(t)-y_i(t))\Big[\frac{x_i^{\Delta}(t)}{x_i(t)} - \frac{y_i^{\Delta}(t)}{y_i(t)} \\
& + \frac{m^{\Delta}(t)}{1+m(t)}\Big(\frac{x_i(\sigma(t))}{x_i(t)} - \frac{y_i(\sigma(t))}{y_i(t)}\Big)\Big] \\
& -\delta|e_{-\delta}(t,T)||L_{\mathbb{T}}(x_i(\sigma(t))(1+m(\sigma(t)))) \\
& -L_{\mathbb{T}}(y_i(\sigma(t))(1+m(\sigma(t))))| \\
& +|e_{-\delta}(t,T)|\mathrm{sgn}(u_i(t)-v_i(t))(u_i^{\Delta}(t)-v_i^{\Delta}(t)) \\
& -\delta|e_{-\delta}(t,T)||u_i(\sigma(t))-v_i(\sigma(t))| \\
\leq\ & |e_{-\delta}(t,T)|\Big[-\frac{r_i(t)}{a_i(t)+\sum\limits_{j=1,j\neq i}^{n} b_{ij}(t)(M_{1j}+\varepsilon_0)} \\
& \times|x_i(t)-y_i(t)| \\
& + \sum_{j=1,j\neq i}^{n} \frac{r_i(t)b_{ij}(t)(M_{1i}+\varepsilon_0)}{(a_i(t)+\sum\limits_{j=1,j\neq i}^{n} b_{ij}(t)(m_{1j}-\varepsilon_0))^2} \\
& \times\int_{-\theta_{ij}}^{0} B_{ij}(s)|x_j(t+s)-y_j(t+s)|\Delta s \\
& +r_i(t)c_i(t)|x_i(\sigma(t))-y_i(\sigma(t))| \\
& +d_i(t)|u_i(t)-v_i(t)| + h_i(t)\int_{-\tau_i}^{0} H_i(s) \\
& \times|u_i(t+s)-v_i(t+s)|\Delta s \\
& +\frac{m^{\Delta}(t)}{1+m(t)}\frac{-x_i(\sigma(t))|x_i(t)-y_i(t)|}{x_i(t)y_i(t)} \\
& +\frac{m^{\Delta}(t)}{1+m(t)}\frac{|x_i(\sigma(t))-y_i(\sigma(t))|}{y_i(t)}\Big] \\
& -\delta|e_{-\delta}(t,T)||L_{\mathbb{T}}(x_i(\sigma(t))(1+m(\sigma(t)))) \\
& -L_{\mathbb{T}}(y_i(\sigma(t))(1+m(\sigma(t))))| \\
& +|e_{-\delta}(t,T)|[-\alpha_i(t)|u_i(t)-v_i(t)| \\
& +\beta_i(t)|x_i(t)-y_i(t)| \\
& +g_i(t)\int_{-\eta_i}^{0} G_i(s)|u_i(t+s)-v_i(t+s)|\Delta s] \\
& -\delta|e_{-\delta}(t,T)||u_i(\sigma(t))-v_i(\sigma(t))| \\
\leq\ & -|e_{-\delta}(t,T)|\Big[\Big(\frac{r_i^l}{a_i^u+\sum\limits_{j=1,j\neq i}^{n} b_{ij}^u(M_{1j}+\varepsilon_0)} \\
& -\beta_i^u\Big)(m_{1i}-\varepsilon_0)|L_{\mathbb{T}}(x_i(t)(1+m(t))) \\
& -L_{\mathbb{T}}(y_i(t)(1+m(t)))| \\
& +(\alpha_i^l - d_i^u)|u_i(t)-v_i(t)|\Big] + |e_{-\delta}(t,T) \\
& \times|\Big[\sum_{j=1,j\neq i}^{n} \frac{r_i^u b_{ij}^u(M_{1i}+\varepsilon_0)(M_{1j}+\varepsilon_0)}{(a_i^l+\sum\limits_{j=1,j\neq i}^{n} b_{ij}^l(m_{1j}-\varepsilon_0))^2} \\
& \times\int_{-\theta_{ij}}^{0} B_{ij}(s)|L_{\mathbb{T}}(x_j(t+s)(1+m(t+s))) \\
& -L_{\mathbb{T}}(y_j(t+s)(1+m(t+s)))|\Delta s
\end{aligned}$$

$$+h_i^u\int_{-\tau_i}^{0}H_i(s)|u_i(t+s)-v_i(t+s)|\Delta s$$
$$+g_i^u\int_{-\eta_i}^{0}G_i(s)|u_i(t+s)-v_i(t+s)|\Delta s\bigg].$$

Since $V_i(t,z_1(t+s),z_2(t+s))<kV_i(t,z_1(t),z_2(t))$, $s\in[-\tau,0]_{\mathbb{T}},k>1$ is a constant, then

$$\begin{aligned}&D^+V_i^\Delta(t)\\ \leq\ & -\min\bigg[\bigg(\frac{r_i^l}{a_i^u+\sum\limits_{j=1,j\neq i}^{n}b_{ij}^u(M_{1j}+\varepsilon_0)}-\beta_i^u\bigg)\\ &\times(m_{1i}-\varepsilon_0),(\alpha_i^l-d_i^u)\bigg]V_i(t)\\ &+k\bigg[\sum_{j=1,j\neq i}^{n}\frac{r_i^ub_{ij}^u(M_{1i}+\varepsilon_0)(M_{1j}+\varepsilon_0)}{(a_i^l+\sum\limits_{j=1,j\neq i}^{n}b_{ij}^l(m_{1j}-\varepsilon_0))^2}\\ &+h_i^u+g_i^u\bigg]V_i(t).\end{aligned}$$

Therefore,

$$\begin{aligned}&D^+V^\Delta(t)=\sum_{i=1}^{n}D^+V_i^\Delta(t)\\ \leq\ & -\bigg(\min_i\bigg[\bigg(\frac{r_i^l}{a_i^u+\sum\limits_{j=1,j\neq i}^{n}b_{ij}^u(M_{1j}+\varepsilon_0)}-\beta_i^u\bigg)\\ &\times(m_{1i}-\varepsilon_0),(\alpha_i^l-d_i^u)\bigg]\\ &-k\sum_{i=1}^{n}\bigg[\sum_{j=1,j\neq i}^{n}\frac{r_i^ub_{ij}^u(M_{1i}+\varepsilon_0)(M_{1j}+\varepsilon_0)}{(a_i^l+\sum\limits_{j=1,j\neq i}^{n}b_{ij}^l(m_{1j}-\varepsilon_0))^2}\\ &+h_i^u+g_i^u\bigg]\bigg)V(t)\\ \leq\ & -\gamma V(t).\end{aligned}\tag{14}$$

By the comparison theorem and (14), we have

$$\begin{aligned}V(t)\ &\leq\ |e_{-\gamma}(t,T)|V(T)\\ &<\ 2\bigg(\frac{M_{1i}+\varepsilon_0}{m_{1i}-\varepsilon_0}+M_{2i}+\varepsilon_0\bigg)|e_{-\gamma}(t,T)|,\end{aligned}$$

that is,

$$\begin{aligned}&|e_{-\delta}(t,T)|(|L_{\mathbb{T}}(x_i(t)(1+m(t)))\\ &-L_{\mathbb{T}}(y_i(t)(1+m(t)))|+|u_i(t)-v_i(t)|)\\ &<2\bigg(\frac{M_{1i}+\varepsilon_0}{m_{1i}-\varepsilon_0}+M_{2i}+\varepsilon_0\bigg)|e_{-\gamma}(t,T)|,\end{aligned}$$

then

$$\begin{aligned}&\frac{1}{M_{1i}+\varepsilon_0}|x_i(t)-y_i(t)|+|u_i(t)-v_i(t)|\\ &<2\bigg(\frac{M_{1i}+\varepsilon_0}{m_{1i}-\varepsilon_0}+M_{2i}+\varepsilon_0\bigg)\\ &\times|e_{(-\gamma)\ominus(-\delta)}(t,T)|.\end{aligned}\tag{15}$$

Since $1-\mu(t)\delta<0$ and $0<\gamma<\delta$, then $(-\gamma)\ominus(-\delta)<0$. It follows from (15) that

$$\lim_{t\to+\infty}|x_i(t)-y_i(t)|=0,\ \lim_{t\to+\infty}|u_i(t)-v_i(t)|=0.$$

Case II. If $\mu(t)=0$, set $\delta=0$, then $\sigma(t)=t$ and $|e_{-\delta}(t,T)|=1$. Calculating the upper right derivatives of $V(t)$ along the solution of system (1), it follows from (11)-(13), $(H_4)-(H_6)$ that for $t\in[T,+\infty)_{\mathbb{T}}$,

$$\begin{aligned}&D^+V^\Delta(t)\\ =\ & \mathrm{sgn}(x_i(t)-y_i(t))\bigg(\frac{x_i^\Delta(t)}{x_i(t)}-\frac{y_i^\Delta(t)}{y_i(t)}\bigg)\\ &+\mathrm{sgn}(u_i(t)-v_i(t))(u_i^\Delta(t)-v_i^\Delta(t))\\ \leq\ & -|x_i(t)-y_i(t)|\bigg[r_i(t)c_i(t)\\ &+\frac{r_i(t)}{a_i(t)+\sum\limits_{j=1,j\neq i}^{n}b_{ij}(t)(M_{1j}+\varepsilon_0)}\bigg]\\ &+\sum_{j=1,j\neq i}^{n}\frac{r_i(t)b_{ij}(t)(M_{1i}+\varepsilon_0)}{(a_i(t)+\sum\limits_{j=1,j\neq i}^{n}b_{ij}(t)(m_{1j}-\varepsilon_0))^2}\\ &\times\int_{-\theta_{ij}}^{0}B_{ij}(s)|x_j(t+s)-y_j(t+s)|\Delta s\\ &+d_i(t)|u_i(t)-v_i(t)|\\ &+h_i(t)\int_{-\tau_i}^{0}H_i(s)|u_i(t+s)-v_i(t+s)|\Delta s\\ &+[-\alpha_i(t)|u_i(t)-v_i(t)|+\beta_i(t)|x_i(t)-y_i(t)|\\ &+g_i(t)\int_{-\eta_i}^{0}G_i(s)|u_i(t+s)-v_i(t+s)|\Delta s]\\ \leq\ & -\bigg[r_i^lc_i^l+\frac{r_i^l}{a_i^u+\sum\limits_{j=1,j\neq i}^{n}b_{ij}^u(M_{1j}+\varepsilon_0)}\\ &-\beta_i^u\bigg](m_{1i}-\varepsilon_0)|L_{\mathbb{T}}(x_i(t)(1+m(t)))\\ &-L_{\mathbb{T}}(y_i(t)(1+m(t)))|-(\alpha_i^l-d_i^u)|u_i(t)-v_i(t)|\\ &+\sum_{j=1,j\neq i}^{n}\frac{r_i^ub_{ij}^u(M_{1i}+\varepsilon_0)}{(a_i^l+\sum\limits_{j=1,j\neq i}^{n}b_{ij}^l(m_{1j}-\varepsilon_0))^2}\end{aligned}$$

$$
\begin{aligned}
&\times \int_{-\theta_{ij}}^{0} B_{ij}(s)|x_j(t+s)-y_j(t+s)|\Delta s \\
&+h_i^u \int_{-\tau_i}^{0} H_i(s)|u_i(t+s)-v_i(t+s)|\Delta s \\
&+g_i^u \int_{-\eta_i}^{0} G_i(s)|u_i(t+s)-v_i(t+s)|\Delta s] \\
\leq\ & -\min\Bigg[\Bigg(r_i^l c_i^l + \frac{r_i^l}{a_i^u + \sum\limits_{j=1,j\neq i}^{n} b_{ij}^u (M_{1j}+\varepsilon_0)} \\
&-\beta_i^u\Bigg)(m_{1i}-\varepsilon_0), (\alpha_i^l - d_i^u)\Bigg] V_i(t) \\
&+k\Bigg[\sum_{j=1,j\neq i}^{n} \frac{r_i^u b_{ij}^u (M_{1i}+\varepsilon_0)(M_{1j}+\varepsilon_0)}{(a_i^l + \sum\limits_{j=1,j\neq i}^{n} b_{ij}^l (m_{1j}-\varepsilon_0))^2} \\
&+h_i^u + g_i^u\Bigg] V_i(t).
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
&D^+V^{\Delta}(t) = \sum_{i=1}^{n} D^+ V_i^{\Delta}(t) \\
\leq\ & -\Bigg(\min_i \Bigg[\Bigg(r_i^l c_i^l + \frac{r_i^l}{a_i^u + \sum\limits_{j=1,j\neq i}^{n} b_{ij}^u (M_{1j}+\varepsilon_0)} \\
&-\beta_i^u\Bigg)(m_{1i}-\varepsilon_0), (\alpha_i^l - d_i^u)\Bigg] \\
&-k\sum_{i=1}^{n}\Bigg[\sum_{j=1,j\neq i}^{n} \frac{r_i^u b_{ij}^u (M_{1i}+\varepsilon_0)(M_{1j}+\varepsilon_0)}{(a_i^l + \sum\limits_{j=1,j\neq i}^{n} b_{ij}^l (m_{1j}-\varepsilon_0))^2} \\
&+h_i^u + g_i^u\Bigg]\Bigg) V(t) \\
\leq\ & -\gamma V(t). \qquad (16)
\end{aligned}
$$

By the comparison theorem and (16), we have

$$
\begin{aligned}
V(t) &\leq |e_{-\gamma}(t,T)|V(T) \\
&< 2\Bigg(\frac{M_{1i}+\varepsilon_0}{m_{1i}-\varepsilon_0} + M_{2i}+\varepsilon_0\Bigg)|e_{-\gamma}(t,T)|,
\end{aligned}
$$

that is,

$$
\begin{aligned}
&|L_{\mathbb{T}}(x_i(t)(1+m(t))) - L_{\mathbb{T}}(y_i(t)(1+m(t)))| \\
&+|u_i(t)-v_i(t)| \\
&< 2\Bigg(\frac{M_{1i}+\varepsilon_0}{m_{1i}-\varepsilon_0} + M_{2i}+\varepsilon_0\Bigg)|e_{-\gamma}(t,T)|,
\end{aligned}
$$

then

$$
\begin{aligned}
&\frac{1}{M_1+\varepsilon_0}|x_i(t)-y_i(t)| + |u_i(t)-v_i(t)| \\
&< 2\Bigg(\frac{M_{1i}+\varepsilon_0}{m_{1i}-\varepsilon_0} + M_{2i}+\varepsilon_0\Bigg)|e_{-\gamma}(t,T)|. \qquad (17)
\end{aligned}
$$

It follows from (17) that

$$
\lim_{t\to+\infty}|x_i(t)-y_i(t)| = 0,\ \lim_{t\to+\infty}|u_i(t)-v_i(t)| = 0.
$$

From the above discussion and Definition 7, we can see that the solution of system (1) is globally attractive. This completes the proof. □

4 Almost periodic solution

The aim of this section is to investigate the positive almost periodic solution of system (1), to do so, we further assume that system (1) satisfies

(H_7) $r_i(t), a_i(t), b_{ij}(t), c_i(t), d_i(t), h_i(t), \alpha_i(t), \beta_i(t), g_i(t), i, j = 1, 2, \cdots, n$ are all continuous, real-valued positive almost periodic functions.

The relevant definitions and the properties of almost periodic functions, see [18,19].

Let $S(\mathbb{T})$ be the set of all solutions $(x_1(t), x_2(t), \cdots, x_n(t), u_1(t), \cdots, u_n(t))$ of system (1) satisfying $m_{1i} \leq x_i(t) \leq M_{1i},\ m_{2i} \leq u_i(t) \leq M_{2i}$ for all $t \in \mathbb{T}, i = 1, 2, \cdots, n$.

Lemma 10. $S(\mathbb{T}) \neq \emptyset$.

Proof. By Theorem 8, we see that for any $t_0 \in \mathbb{T}$ with $m_{1i} \leq x_i(t_0) \leq M_{1i},\ m_{2i} \leq u_i(t_0) \leq M_{2i}$, system (1) has a solution $(x_i(t), u_i(t))$ satisfying $m_{1i} \leq x_i(t) \leq M_{1i}$, $m_{2i} \leq u_i(t) \leq M_{2i}$, $t \in [t_0, +\infty)_{\mathbb{T}}$. Since $r_i(t), a_i(t), b_{ij}(t), c_i(t), d_i(t), h_i(t), \alpha_i(t), \beta_i(t), g_i(t)$, $\sigma(t)$ are almost periodic, there exists a sequence $\{t_n\}$, $t_n \to +\infty$ as $n \to +\infty$ such that $r_i(t+t_n) \to r_i(t), a_i(t+t_n) \to a_i(t), b_{ij}(t+t_n) \to b_{ij}(t), c_i(t+t_n) \to c_i(t), d_i(t+t_n) \to d_i(t), h_i(t+t_n) \to h_i(t), \alpha_i(t+t_n) \to \alpha_i(t), \beta_i(t+t_n) \to \beta_i(t), g_i(t+t_n) \to g_i(t), \sigma_i(t+t_n) \to \sigma_i(t)$ as $n \to +\infty$ uniformly on $\mathbb{T}$.

We claim that $\{x_i(t+t_n)\}$ and $\{u_i(t+t_n)\}$ are uniformly bounded and equi-continuous on any bounded interval in $\mathbb{T}$.

In fact, for any bounded interval $[\alpha, \beta]_{\mathbb{T}} \subset \mathbb{T}$, when n is large enough, $\alpha + t_n > t_0$, then $t + t_n > t_0, \forall t \in [\alpha, \beta]_{\mathbb{T}}$. So, $m_{1i} \leq x_i(t+t_n) \leq M_{1i},\ m_{2i} \leq u_i(t+t_n) \leq M_{2i}$ for any $t \in [\alpha, \beta]_{\mathbb{T}}$, that is, $\{x_i(t+t_n)\}$ and $\{u_i(t+t_n)\}$ are uniformly bounded. On the other hand, $\forall t_1, t_2 \in [\alpha, \beta]_{\mathbb{T}}$, from the mean value theorem of differential calculus on time scales, we have

$$
\begin{aligned}
&|x_i(t_1+t_n) - x_i(t_2+t_n)| \\
&\leq r^u M_{1i}[1 + \frac{M_{1i}}{a_i^l} + c_i^u M_{1i} + (d_i^u + h_i^u) M_{2i}]
\end{aligned}
$$

$$\begin{aligned}&\times|t_1-t_2|, \qquad (18)\\ &|u_i(t_1+t_n)-u_i(t_2+t_n)|\\ &\le (\alpha_i^u M_{2i}+(\beta_i^u+g_i^u)M_{1i})|t_1-t_2|. \qquad (19)\end{aligned}$$

The inequalities (18) and (19) show that $\{x_i(t+t_n)\}$ and $\{u_i(t+t_n)\}$ are equi-continuous on $[\alpha,\beta]_{\mathbb{T}}$. By the arbitrary of $[\alpha,\beta]_{\mathbb{T}}$, the conclusion is valid.

By Ascoli-Arzela theorem, there exists a subsequence of $\{t_n\}$, we still denote it as $\{t_n\}$, such that

$$x_i(t+t_n)\to p_i(t), u_i(t+t_n)\to u_i(t),$$

as $n\to+\infty$ uniformly in t on any bounded interval in $\mathbb{T}$.

For any $\theta\in\mathbb{T}$, we can assume that $t_n+\theta\ge t_0$ for all n, and let $t\ge 0$, integrate both equations of system (1) from $t_n+\theta$ to $t+t_n+\theta$, then by using the Lebesgues dominated convergence theorem, we can obtain

$$\begin{aligned}&p_i(t+\theta)-p_i(\theta)\\ =&\int_\theta^{t+\theta}\Big\{r_i(\omega)x_i(\omega)\Big[1-\\ &\frac{x_i(\omega)}{a_i(\omega)+\sum\limits_{j=1,j\neq i}^n b_{ij}(\omega)\int_{-\theta_{ij}}^0 B_{ij}(s)x_j(\omega+s)\Delta s}\\ &-c_i(\omega)x_i(\sigma(\omega))\Big]-d_i(\omega)u_i(\omega)x_i(\omega)\\ &-h_i(\omega)x_i(\omega)\int_{-\tau_i}^0 H_i(s)u_i(\omega+s)\Delta s\Big\}\Delta\omega,\\ &q_i(t+\theta)-q_i(\theta)\\ =&\int_\theta^{t+\theta}[-\alpha_i(\omega)u_i(\omega)+\beta_i(\omega)x_i(\omega)\\ &+g_i(\omega)\int_{-\eta_i}^0 G_i(s)u_i(\omega+s)\Delta s]\Delta\omega.\end{aligned}$$

This means that $(p_i(t),q_i(t))$ is a solution of system (1), and by the arbitrary of θ, $(p_i(t),q_i(t))$ is a solution of system (1) on $\mathbb{T}$. It is clear that

$$\begin{aligned}&m_{1i}\le p_i(t)\le M_{1i},\\ &m_{2i}\le q_i(t)\le M_{2i},\ \forall t\in\mathbb{T}.\end{aligned}$$

This completes the proof. □

Theorem 11. *Assume that $(H_1)-(H_7)$ hold, then system (1) has a unique positive almost periodic solution which is globally attractive.*

Proof. Consider the associated product system of (1),

$$\begin{cases} x_i^\Delta(t) = r_i(t)x_i(t)\Big[1-\\ \qquad\frac{x_i(t)}{a_i(t)+\sum\limits_{j=1,j\neq i}^n b_{ij}(t)\int_{-\theta_{ij}}^0 B_{ij}(s)x_j(t+s)\Delta s}\\ \qquad -c_i(t)x_i(\sigma(t))\Big]-d_i(t)u_i(t)x_i(t)\\ \qquad -h_i(t)x_i(t)\int_{-\tau_i}^0 H_i(s)u_i(t+s)\Delta s,\\ u_i^\Delta(t) = -\alpha_i(t)u_i(t)+\beta_i(t)x_i(t)\\ \qquad +g_i(t)\int_{-\eta_i}^0 G_i(s)u_i(t+s)\Delta s,\\ y_i^\Delta(t) = r_i(t)y_i(t)\Big[1-\\ \qquad\frac{y_i(t)}{a_i(t)+\sum\limits_{j=1,j\neq i}^n b_{ij}(t)\int_{-\theta_{ij}}^0 B_{ij}(s)y_j(t+s)\Delta s}\\ \qquad -c_i(t)y_i(\sigma(t))\Big]-d_i(t)v_i(t)y_i(t)\\ \qquad -h_i(t)y_i(t)\int_{-\tau_i}^0 H_i(s)v_i(t+s)\Delta s,\\ v_i^\Delta(t) = -\alpha_i(t)v_i(t)+\beta_i(t)y_i(t)\\ \qquad +g_i(t)\int_{-\eta_i}^0 G_i(s)v_i(t+s)\Delta s,\end{cases} \qquad (20)$$

Let $z(t)=(z_1(t),z_2(t))$ be a positive solution of product system (20), where

$$\begin{aligned}z_1(t)&=(x_1(t),\cdots,x_n(t),u_1(t),\cdots,u_n(t)),\\ z_2(t)&=(y_1(t),\cdots,y_n(t),v_1(t),\cdots,v_n(t)).\end{aligned}$$

Define $z_{1i}(t)=(x_i(t),u_i(t)), z_{2i}(t)=(y_i(t),v_i(t))$.

By using the same Lyapunov functional in Section 3. Set

$$\begin{aligned}V(t,z_1(t),z_2(t))&=\sum_{i=1}^n V_i(t,z_{1i}(t),z_{2i}(t)),\\ V_i(t,z_{1i}(t),z_{2i}(t))&=|e_{-\delta}(t,T)|(|L_{\mathbb{T}}(x_i(t)(1+m(t)))\\ &\quad -L_{\mathbb{T}}(y_i(t)(1+m(t)))|\\ &\quad +|u_i(t)-v_i(t)|),\end{aligned}$$

and $|z_1(t)-z_2(t)|=\sum\limits_{i=1}^n(|x_i(t)-y_i(t)|+|u_i(t)-v_i(t)|)$.

It follows from (13) that

$$\begin{aligned}&\min\{\frac{1}{M_{1i}+\varepsilon_0},1\}|e_{-\delta}(t,T)|(|x_i(t)-y_i(t)|\\ &+|u_i(t)-v_i(t)|)\\ \le\ & V_i(t,z_{1i}(t),z_{2i}(t))\\ \le\ & \max\{\frac{1}{m_{1i}-\varepsilon_0},1\}|e_{-\delta}(t,T)|(|x_i(t)-y_i(t)|\\ &+|u_i(t)-v_i(t)|),\end{aligned}$$

then

$$\begin{aligned}
&\min_i\{\frac{1}{M_{1i}+\varepsilon_0},1\}|e_{-\delta}(t,T)| \\
&\times\sum_{i=1}^{n}(|x_i(t)-y_i(t)|+|u_i(t)-v_i(t)|) \\
\leq\;& V(t,z_1(t),z_2(t)) \\
\leq\;& \max_i\{\frac{1}{m_{1i}-\varepsilon_0},1\}|e_{-\delta}(t,T)| \\
&\times\sum_{i=1}^{n}(|x_i(t)-y_i(t)|+|u_i(t)-v_i(t)|).
\end{aligned}$$

that is

$$\begin{aligned}
&\min_i\{\frac{1}{M_{1i}+\varepsilon_0},1\}|e_{-\delta}(t,T)|(|z_1(t)-z_2(t)|) \\
\leq\;& V(t,z_1(t),z_2(t)) \\
\leq\;& \max_i\{\frac{1}{m_{1i}-\varepsilon_0},1\}|e_{-\delta}(t,T)|(|z_1(t)-z_2(t)|).
\end{aligned}$$

Therefore, condition (1) in Lemma 6 is satisfied.

Since

$$\begin{aligned}
&|V_i(t,z_{1i}(t),z_{2i}(t))-V_i(t,\tilde{z}_{1i}(t),\tilde{z}_{2i}(t))| \\
=\;& |e_{-\delta}(t,T)|\big||L_{\mathbb{T}}(x_i(t)(1+m(t))) \\
&-L_{\mathbb{T}}(y_i(t)(1+m(t)))|+|u_i(t)-v_i(t)| \\
&-|L_{\mathbb{T}}(\tilde{x}_i(t)(1+m(t))) \\
&-L_{\mathbb{T}}(\tilde{y}_i(t)(1+m(t)))|-|\tilde{u}_i(t)-\tilde{v}_i(t)|\big| \\
\leq\;& |L_{\mathbb{T}}(x_i(t)(1+m(t)))-L_{\mathbb{T}}(\tilde{x}_i(t)(1+m(t)))| \\
&+|u_i(t)-\tilde{u}_i(t)| \\
&+|L_{\mathbb{T}}(y_i(t)(1+m(t))) \\
&-L_{\mathbb{T}}(\tilde{y}_i(t)(1+m(t)))|+|v_i(t)-\tilde{v}_i(t)| \\
\leq\;& \max\{\frac{1}{m_{1i}-\varepsilon_0},1\}(|x_i(t)-\tilde{x}_i(t)| \\
&+|u_i(t)-\tilde{u}_i(t)| \\
&+|y_i(t)-\tilde{y}_i(t)|+|v_i(t)-\tilde{v}_i(t)|),
\end{aligned}$$

then

$$\begin{aligned}
&|V(t,z_1(t),z_2(t))-V(t,\tilde{z}_1(t),\tilde{z}_2(t))| \\
\leq\;& \max_i\{\frac{1}{m_{1i}-\varepsilon_0},1\}\sum_{i=1}^{n}(|x_i(t)-\tilde{x}_i(t)| \\
&+|u_i(t)-\tilde{u}_i(t)| \\
&+|y_i(t)-\tilde{y}_i(t)|+|v_i(t)-\tilde{v}_i(t)|) \\
=\;& \max_i\{\frac{1}{m_1-\varepsilon_0},1\}(|z_1(t)-\tilde{z}_1(t)| \\
&+|z_2(t)-\tilde{z}_2(t)|).
\end{aligned}$$

Therefore, condition (2) in Lemma 6 holds.

By the proof of Theorem 9. Calculating the upper right derivatives of $V(t,z_1(t),z_2(t))$ along the solution of system (20), it follows from (14) and (16) that for $t\in[T,+\infty)_{\mathbb{T}}$,

$$D^+V^{\Delta}(t,z_1(t),z_2(t))\leq-\gamma V(t,z_1(t),z_2(t)).$$

Therefore, condition (3) in Lemma 6 is satisfied.

From the above discussion, we can see that all conditions in Lemma 6 hold. By Lemma 6 and Lemma 10, system (1) has a unique almost periodic solution which is uniformly asymptotic stable. Together with Theorem 8 and Theorem 9 that system (1) has a unique positive almost periodic solution which is globally attractive. This completes the proof. □

5 Example and simulations

Consider the following system on time scales ($n=2$)

$$\begin{cases}
x_1^{\Delta}(t) = (0.8+0.2\sin\sqrt{2}t)x_1(t)[1- \\
\quad \frac{x_1(\sigma(t))}{(4.5+0.5\cos t)+0.01\int_{-\theta_{12}}^{0}B_{12}(s)x_2(t+s)\Delta s} \\
\quad -x_1(t)-0.2u_1(t) \\
\quad -0.01\int_{-\tau_1}^{0}H_1(s)u_1(t+s)\Delta s], \\
u_1^{\Delta}(t) = -(0.4+0.1\cos\sqrt{3}t)u_1(t) \\
\quad +(0.015+0.005\sin\sqrt{2}t)x_1(t) \\
\quad +0.01\int_{-\eta_1}^{0}G_1(s)u_1(t+s)\Delta s, \\
x_2^{\Delta}(t) = (0.85+0.15\sin t)x_2(t)[1- \\
\quad \frac{x_2(\sigma(t))}{(4+\cos t)+0.02\int_{-\theta_{21}}^{0}B_{21}(s)x_1(t+s)\Delta s} \\
\quad -x_2(t)-0.2u_2(t) \\
\quad -0.02\int_{-\tau_2}^{0}H_2(s)u_2(t+s)\Delta s], \\
u_2^{\Delta}(t) = -(0.45+0.05\cos t)u_2(t) \\
\quad +(0.012+0.002\sin t)x_2(t) \\
\quad +0.02\int_{-\eta_2}^{0}G_2(s)u_2(t+s)\Delta s.
\end{cases} \tag{21}$$

By a direct calculation, we can get

$$\begin{aligned}
&r_1^u=1, r_1^l=0.6, a_1^u=5, a_1^l=4, \\
&b_{12}^u=b_{12}^l=0.01, c_1^u=c_1^l=1, d_1^u=d_1^l=0.2, \\
&h_1^u=h_1^l=0.01, \alpha_1^u=0.5, \alpha_1^l=0.3, \\
&\beta_1^u=0.02, \beta_1^l=0.01, g_1^u=g_1^l=0.01, \\
&r_2^u=1, r_2^l=0.7, a_2^u=5, a_2^l=3, \\
&b_{21}^u=b_{21}^l=0.02, c_2^u=c_2^l=1, d_2^u=d_2^l=0.2, \\
&h_2^u=h_2^l=0.02, \alpha_2^u=0.5, \alpha_2^l=0.4, \\
&\beta_2^u=0.014, \beta_2^l=0.01, g_2^u=g_2^l=0.02,
\end{aligned}$$

then

$$\begin{aligned}
&M_{11}=1.6667, M_{21}=0.1667, \\
&m_{11}=0.1483, m_{21}=0.0059, \\
&M_{12}=1.4286, M_{22}=0.1214, \\
&m_{12}=0.1971, m_{22}=0.0118,
\end{aligned}$$

and

(H_3) $r_1^l - \frac{r_1^u}{a_1^l} M_{11} + (d_1^u + h_1^u) M_{21} = 0.2183,$
$r_2^l - \frac{r_2^u}{a_2^l} M_{12} + (d_2^u + h_2^u) M_{22} = 0.2505.$

(H_4) $\frac{r_1^l}{a_1^u + b_{12}^u M_{12}} - \beta_1^u = 0.0997,$
$\frac{r_2^l}{a_2^u + b_{21}^u M_{11}} - \beta_2^u = 0.1251;$

(H_5) $\alpha_1^l - d_1^u = 0.1, \alpha_2^l - d_2^u = 0.2;$

(H_6) Let $k = 1.1$, then $\Pi = 0.0074$. One can choose $\gamma \in (0, 0.0074]$.

From the above, we can see that the conditions (H_1)–(H_7) hold. According to Theorem 11, system (21) has a unique positive almost periodic solution which is globally attractive.

Let $\theta_{12} = \theta_{21} = \eta_1 = \eta_2 = +\infty$. If $\mathbb{T} = \mathbb{R}$, set $B_{12}(s) = B_{21}(s) = G_1(s) = G_2(s) = e^s$; if $\mathbb{T} = \mathbb{Z}$, set $B_{12}(s) = B_{21}(s) = G_1(s) = G_2(s) = (\frac{1}{2})^s$. The dynamics simulation of system (21), see Figures 1 and 2.

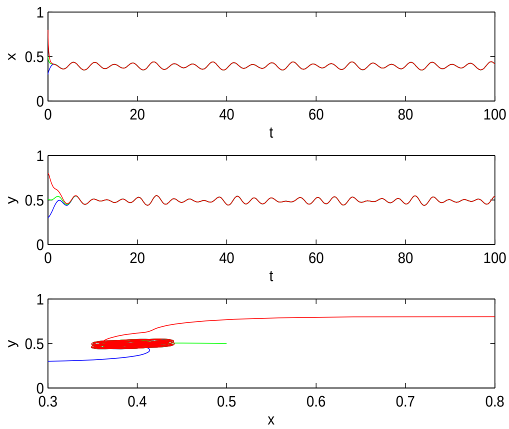

Figure 1: Let $\mathbb{T} = \mathbb{R}$, dynamics behavior of system (21) with the initial conditions $(x(0), y(0)) = \{(0.3, 0.3), (0.5, 0.5), (0.8, 0.8)\}$.

6 Conclusion

This paper studied a n-species cooperation system with distributed delays and feedback controls on time scales. For general nonautonomous case, by using differential inequality theory and constructing a suitable Lyapunov functional, sufficient conditions which guarantee the permanence and the global attractivity of the system are obtained. For the almost periodic case, by using the Razumikhin type theorem, sufficient conditions which guarantee the existence of a positive almost periodic solution of the system are obtained. The methods used in this paper are completely new, and the methods can be applied to study other dynamic systems.

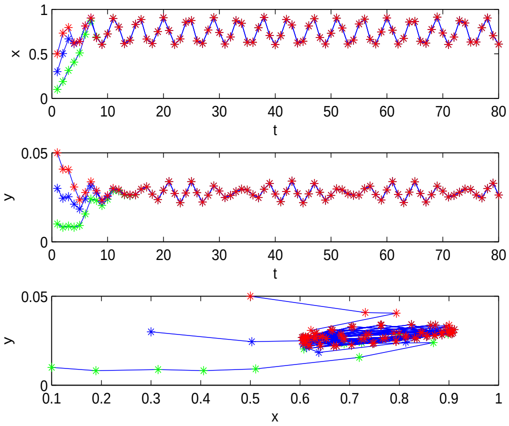

Figure 2: Let $\mathbb{T} = \mathbb{Z}$, dynamics behavior of system (21) with the initial conditions $(x(1), y(1)) = \{(0.1, 0.01), (0.3, 0.03), (0.5, 0.05)\}$.

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References:

[1] Q. Liu, R. Xu, Persistence and global stability for a delayed nonautonomous single-species model with dispersal and feedback control, Diff. Equ. Dyn. Syst., 11(3-4) (2003) 353-367.

[2] F. Chen, Positive periodic solutions of neutral Lotka-Volterra system with feedback control, Appl. Math. Comput., 162(3) (2005) 1279-1302.

[3] F. Yin, Y. Li, Positive periodic solutions of a single species model with feedback regulation and distributed time delay, Appl. Math. Comput., 153 (2004) 475-484.

[4] F. Yang, D. Jiang, Existence and global attractivity of positive periodic solution of a Logistic growth system with feedback control and deviating arguments, Ann. Diff. Equ., 17(4) (2001) 337-384.

[5] Y. Xia, J. Cao, H. Zhang, F. Chen, Almost periodic solutions in n-species competitive system

with feedback controls, J. Math. Anal. Appl., 294 (2004) 504-522.

[6] V. Spedding, Taming natures numbers, New Scientist, 2404 (2003) 28-31.

[7] R. McKellar, K. Knight, A combined discrete-continuous model describing the lag phase of Listeria monocytogenes, Int. J. Food Microbiol, 54(3) (2000) 171-180.

[8] M. Hu, L. Wang, Positive periodic solutions for an impulsive neutral delay model of single-species population growth on time scales, WSEAS Trans. Math., 11(8) (2012) 705-715.

[9] M. Hu, H. Lv, Almost periodic solutions of a single-species system with feedback control on time scales, Adv. Diff. Equ., 2013, 2013:196.

[10] Z. Liu, Double periodic solutions for a ratio-dependent predator-prey system with harvesting terms on time scales, Discrete Dyn. Nat. Soc., Volume 2009, Article ID 243974.

[11] M. Fazly, M. Hesaaraki, Periodic solutions for predator-prey systems with Beddington-DeAngelis functional response on time scales, Nonlinear Anal. Real., 9(3) (2008) 1224-1235.

[12] L. Bi, M. Bohner, M. Fan, Periodic solutions of functional dynamic equations with infinite delay, Nonlinear Anal. Theor., 68(5) (2008) 1226-1245.

[13] X. Chen, H. Guo, Four periodic solutions of a Generalized delayed predator-prey system on time scales, Rocky Mountain J. Math., 38(5) (2008) 1307-1322.

[14] M. Bohner, A. Peterson, Dynamic equations on time scales, An Introduction with Applications, Boston: Birkhauser, 2001.

[15] M. Hu, L. Wang, Dynamic inequalities on time scales with applications in permanence of predator-prey system, Discrete Dyn. Nat. Soc., Volume 2012, Article ID 281052.

[16] D. Mozyrska, D. F. M. Torres, The natural logarithm on time scales, J. Dyn. Syst. Geom. Theor., 7 (2009) 41-48.

[17] R. Yuan, Existence of almost periodic solution of functional differential equations, Ann. Diff. Equ., 7(2) (1991) 234-242.

[18] Y. Li, C. Wang, Uniformly almost periodic functions and almost periodic solutions to dynamic equations on time scales, Abstr. Appl. Anal., Volume 2011, Article ID 341520.

[19] A.M. Fink, Almost Periodic Differential Equation, Springer-Verlag, Berlin, Heidleberg, New York, 1974.