Shooting Control Algorithm Based on Emendation of Model Error of Assumed Relative Target Motion

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Abstract: - Model error of assumed relative target motion is the major factor to influence calculation precision of firing data of fire control system for modern antiaircraft artillery, and is most difficult to overcome, so shooting control algorithm featured on Resemble Closed-loop emendation of firing data and Future Airspace Window System (FAWS) is proposed. Taking example for anti-ship missile with lengthways zigzag maneuver, the mathematical model of calculating emendation value of firing data by Resemble Closed-loop method is established, and the validity of adopting FAWS is analyzed. Monte Carlo simulation is completed under different conditions, whose results validate the feasibility of the algorithm, which provides an effective measure to overcome model error of assumed relative target motion.

Key-words: -Automatic Control; Antiaircraft Artillery; Fire Control System; Model Error of Assumed Relative Target Motion; Resemble Closed-loop Fire Correction; FAW

1 Introduction

For the antiaircraft artillery systems, the pills are not dominated by the fire control system any more during the time of projectile flight from the launch point to the target, so it is necessary to make a basic assumption of target motion when computing firing data. Model error of assumed relative target motion is the position error between true target position and the position hitting the target, which is also named as future position, predicted by a certain model of assumed relative target motion. By all appearances, this error is only determined by model of assumed relative target motion. In fact, the assumption of target will follow a straight-line and non-acceleration flight-path is adopted by a majority of fire control systems, which simplifies the problem of prediction [1,2]. Although the describing of target motion is more accurate, it is adventuresome to adopt acceleration assumption, which is maybe just the opposite to what one wishes [2]. The model error of assumed relative target motion exists inevitably unless we know the target motion in advance, and this error is the major factor to influence calculation precision of firing data of fire control system for modern antiaircraft artillery, and is most difficult to overcome. This difficult problem is researched by lots of scholars, but it is not yet solved faultlessly [3-6]. A method of resemble closed-loop emendation of firing data is used to eliminate prediction error of future position which is caused by assumption of target motion in [6,7], and also it is indicated that the redeeming precision of this method for disciplinarian error of firing data is better than that of the best large-scale closed-loop fire correction. FAW shooting method is presented to solve the problem of hardly precise prediction of future position of maneuver target of fire control system, through establishing future airspace close in the future position, in order to cover with the target motion extension as best as one can [8,9]. This method can increase the kill probability, because it enlarges effective antipersonnel area.

In allusion to missiles with zigzag maneuver mode, integrating the advantages of the two shooting methods, this paper puts forward a shooting control algorithm based on emendation of model error of assumed relative target motion, which can enhance the adaptive ability to intercept missiles with zigzag maneuver for mode antiaircraft artillery, consequently it minishes or overcomes model error of assumed relative target motion. The rest of the paper is organized as follow. Section 2 discusses the theory of resemble closed-loop emendation of firing data. The solution to model error of assumed relative target motion with zigzag maneuver is presented in Section 3. Section 4 discusses the theory of FAWS. The effectiveness of this method is demonstrated by simulated examples as given in Section 5. Concluding remarks are provided in Section 6.

2 Theory of Resemble Closed-loop Emendation of Firing Data

The implement process of resemble closed-loop emendation is described as follow [7]. After the effective firing data were exported and stored by fire control computer, the inverse solution of firing data of present position is made based on tracking target flight-path. Taking pill flight time as condition, the firing data before pill flight time is inquired from the storage sequence of firing data, the error between the two kinds of firing data is defined as error of firing data, which is regarded as redeeming reference value. The inverse solution of flight-path is the core step. The whole process is automatically accomplished by fire control system dynamically, unlike large-scale closed-loop fire correction, which cannot correct the metrical deviation of the shell-target but ball firing course.

A basic assumption condition for this resemble closed-loop emendation method is that the true value of flight-path is defined as target position obtained by detector equipment or target present position calculated by target flight-path filtering module. And this assumption is certificated in National Shooting Range test [7].

This resemble closed-loop emendation method is used to eliminate disciplinarian error of firing data during a period of time. For disciplinarian error, Least Square method can be used to fit, and then its bias value is predicted to emendate the firing data outputted by fire control computer.

3 Solution to Model Error of Assumed Relative Target Motion with Zigzag Maneuver

In order to evade the interception of ship-air missile or naval antiaircraft artillery, anti-ship missiles usually carry through zigzag maneuver in lateral or lengthways direction, and the latter maneuver is propitious to triumphantly sally. Therefore, the latter maneuver is more effective than the former maneuver [10]. Suppose that anti-ship missile flies in zigzag maneuver in lengthways direction, at the cycle of T , namely, zigzag maneuver is in altitudinal direction of the geodetic rectangular coordinates, so the sequence of altitudinal error takes on seasonal variety. When the error sequence is fitting on polynomial curve relative time t by least square method, it is need to solve the two questions: the first is length of fitting zone of time t ; the second is the degree of polynomial curve. Via the analysis of simulation data, one single cycle and three degree polynomial curve satisfy the request. The reasons are explained as follow:

(1) The variational disciplinarian of maneuver range in each cycle T is same, the data in one single cycle contain its whole characteristic. If fitting data are from several cycles, it need to enhance the degree of polynomial curve, but the fitting precision is not evidently ameliorated. For example, the degree of polynomial curve is seven at least when selecting data from two cycles, figure 1 shows its fitting error and the error of three degree curve in a single cycle. Without loss of generality, only selecting the fitting error in a single cycle ($T = 3s$) to compare, the average error of absolute value of the two methods are 8.76 and 7.99 meter, respective. It shows that the fitting effect of the two methods is close.

(2) The fitting effect when the degree is below three is bad, and to compare with curve of high-degree, the fitting effect of three degree curve is enough. As shown in figure 2, with the cycle of $T = 3s$, during the time from $0s$ to $3s$, the error is called fitting error, which is defined that the error between the fitting data and relative fitting value in

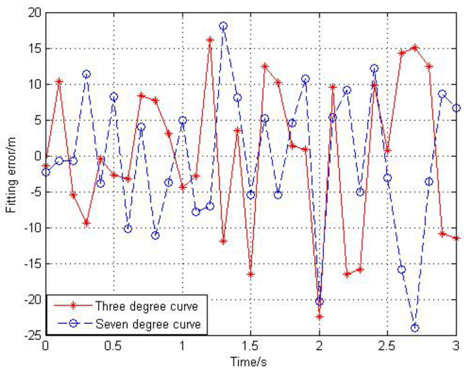


Fig.1 Fitting error of different degree curves

the fitting curve. The average of absolute value of the three methods are 5.99、4.84 and 26.45 meter, respective. During the time from $3s$ to $6s$, the error is called emendation error, which is defined that the error between the true value and the emendation value in the fitting curve, the average are 7.38、7.69 and 25.66 meter, respective. As we can see, the effect of two degree curve is worst, although the fitting error of seven degree curve is smaller than that of three degree curve, its emendation error is bigger, and the emendation error is more suitable to reflect fitting effect.

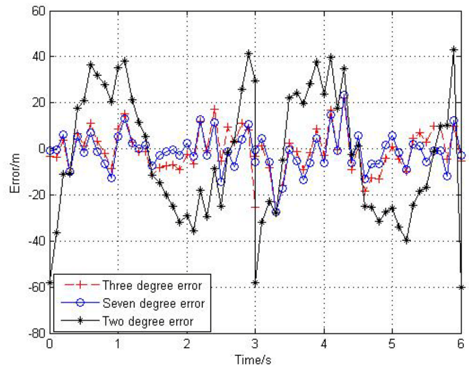


Fig.2 Error of different degree curves

Three degree polynomial curve can be calculated by the following equation (1)

$$\Delta h = a_{h1}t^3 + a_{h2}t^2 + a_{h3}t + a_{h4} \quad (1)$$

Suppose that the fitting time zone is ΔT, with T_0 as its start time, T is the cycle of zigzag maneuver. Note that the pill flight time is t_f, then when calculating the emendation value, independent variable of time can be given by equation (2)

$$t = \left(\Delta T + t_f\right) - \left[\left(\Delta T + t_f - T_0\right)/T\right] \times T \quad (2)$$

Where $[a]$ rounds the elements of a to the nearest integers less than or equal to a.

4 Future Airspace Window Shooting System

As shown from the frontal simulation results, the emendation error is still not small enough. Through establishing future airspace close in the future position, FAW Shooting mode can increase the kill probability, because it enlarges effective antipersonnel area. So a strategy of adopting FAW shooting mode on the basis of fire correction is put forward.

The rectangular coordinates X is usually given as follow [11]: the coordinate origin is the ideal future position O, with axis x_3 is in the direction of relative velocity of shell-target. axis x_1 is vertical to axis x_3 in up-direction of plumb plane. axis x_2 is vertical to plane x_1Ox_3 with the right-handed coordinate system, here, plane x_1Ox_2 is prediction plane of pill impacting.

Theorem 1: suppose that pill obeys Gaussian dispersion on the prediction plane of pill impacting, and Σ is its known positive definiteness covariance matrix. Given the number of pill distribution centers m, let the pill distribution centers $\overline{X_i}$ be configured by the curve of $\left\|Q^{-1}X\right\| = \sqrt{2}$ in the form of equation (3), and with equal fired pills, so elliptic future airspace window will be established with pill configuring center is planar [8].

$$\overline{X_i} = \left(\overline{x_i} \quad \overline{y_i}\right)^T = Q\left[r_m\cos\left(\frac{2\pi}{m}i\right) \quad r_m\sin\left(\frac{2\pi}{m}i\right)\right]^T \quad (3)$$

there into, $i = 1,2,...,m$, $r_m = \sqrt{2}$, $Q = Udiag\left(\lambda_1, \lambda_2\right)$, λ_1^2, λ_2^2 are the Eigenvalue of Σ, with unitary matrix U satisfies the equation $U^*\Sigma^{-1}U = diag\left(\lambda_1^{-2} \quad \lambda_2^{-2}\right)$.

The firing data of each antiaircraft artillery is calculated in the geodetic rectangular coordinates, but FAW is configured in the rectangular coordinates X, so the relative distance between future position and each pill distribution center will be converted into the geodetic rectangular coordinates when (3) is to be applied. The approximate calculation equation is given by equation (4)

$$\begin{cases} \Delta X = \Delta D\cdot\cos\varepsilon\cdot\sin\beta + L_\beta\cdot\cos\beta - L_\varepsilon\cdot\sin\varepsilon\cdot\sin\beta \\ \Delta Y = \Delta D\cdot\cos\varepsilon\cdot\cos\beta - L_\beta\cdot\sin\beta - L_\varepsilon\cdot\sin\varepsilon\cdot\cos\beta \\ \Delta Z = \Delta D\cdot\sin\varepsilon + L_\varepsilon\cdot\cos\varepsilon \end{cases} \quad (4)$$

Where L_β and L_ε denote linear discrepancy, they are calculated by equation (3), inclined range error ΔD obeys Gaussian dispersion $N\left(0, \sigma_D^2\right)$.

5 Monte Carlo Simulation Validation

The validity of the presented fire control algorithm is simulated by Monte Carlo method, the emluator chart is shown in figure 3. Its implement

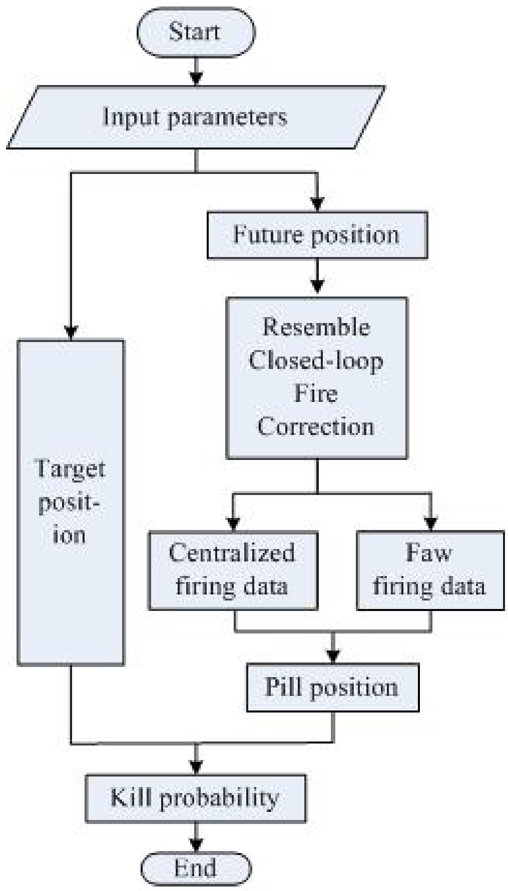


Fig.3 Emluator chart

process is as follow: firstly, target zigzag maneuver flight-path is generated to obtain target position

(X_t, Y_t, Z_t); suppose that target flies in straight-line and constant velocity flight-path, and the target future position is calculated. Then correcting the target future position by resemble closed-loop emendation method, after that, the firing data of centralized shooting mode and FAW shooting mode are calculated, respectively. Through extrapolating the ballistic trajectory, the pill position (X_p, Y_p, Z_p) is also obtained. Finally, judging whether the target will be hit by a certain piece of pill according to the pill position and target position, so the number of pills hitting the target is collected, and the kill probability is figured out on the basis of exponential destroying law.

The judging method whether the pill will hit the target is described as follow: suppose target is a sphere at the radius r, if the distance d between pill and the target center which is calculated by the equation $d=\sqrt{(X_p-X_t)^2+(Y_p-Y_t)^2+(Z_p-Z_t)^2}$ satisfying $d \le r$, well then the pill hits the target. Given the average hit numbers ω of pills required for destroying the target, in the *ith* simulation, the number of pill hitting target is N_i, then the kill probability of this firing can be calculated by

$$H(i)=1-\left(1-\frac{1}{\omega}\right)^{N_i} \tag{5}$$

Suppose the whole number of simulation is M, so the average kill probability H by Monte Carlo method is

$$H=\sum_{i=1}^{M} H(i)/M \tag{6}$$

Some of the common parameters relevant to our simulation are: the cycle of zigzag maneuver is $3s(9\sim 12s)$; the emulational interval is $0.2s$; target position obeys Gaussian dispersion $N(0,\sigma_t^2)$ in X axis and Y axis, with $\sigma_t = 5m$, zigzag maneuver is in Z axis, with the maneuver amplitude A; there is 6 antiaircraft artilleries, each firing 20 pills, and the pills obey Gaussian dispersion $N(0,\sigma_p^2)$ in the three axis's of geodetic rectangular coordinates, with $\sigma_p = 3m$. The sphere radius r of target is $5m$, the average hit numbers ω of pills required for destroying the target is three. The shooting parameter of FAW is defined as one-half the longer axis and the shorter axis of ellipse, which are noted as r_a, r_b, respective. Because the target is only maneuvering in Z axis, so the $\Delta X, \Delta Y$ in equation (4) can not be added, here FAW shooting is equivalent beeline shape of distributed shooting. The number M of Monte Carlo simulation is 5000. For ease of writing, the centralized shooting mode without resemble closed-loop emendation is noted as Shooting mode 1, the centralized shooting mode with resemble closed-loop emendation is noted as Shooting mode 2, FAW shooting mode with resemble closed-loop emendation is noted as Shooting mode 3.

Scenario 1): $r_a = r_b = 0$, the amplitude of the zigzag maneuver is $20m$.

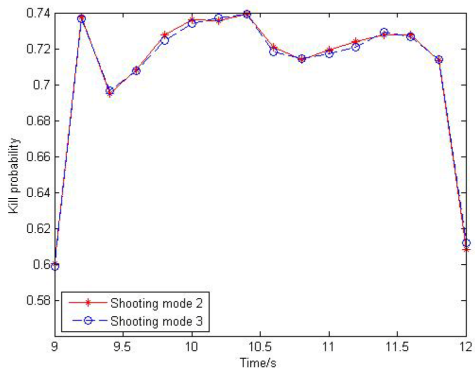
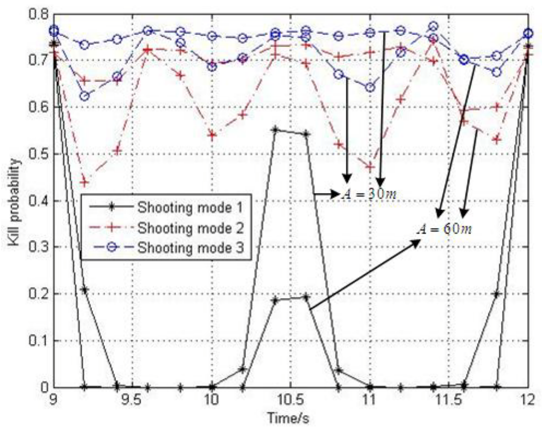


Fig.4 Kill probability

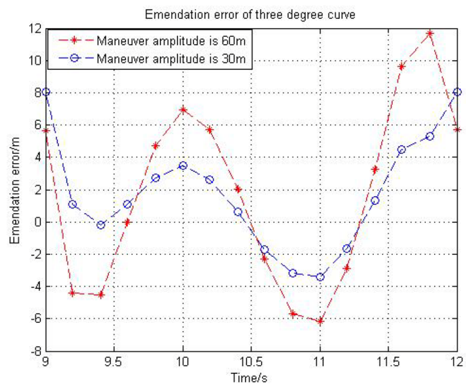
In figure 4, the average of kill probability of Shooting mode 2 and Shooting mode 3 are 0.6860 and 0.6854, respective, the relative error between them is only -0.09%. when $r_a = r_b = 0$, excluding the influence of random factor, the effect of Shooting mode 3 is the same with that of Shooting mode 2, because here Shooting mode 3 degenerates Shooting mode 2.

Scenario 2): $r_a = 4m, r_b = 10m$, the amplitude of the zigzag maneuver is $30m$ or $60m$, respective.

In table 1, the relative error is denoted the relative error of the kill probability between Shooting mode 3 and Shooting mode 2. As we can see from figure 5, the conclusions are made as follow:



(a) Kill probability



(b) Emendation error

Fig.5 Simulation result of Scenario 2

Table1 Average kill probability of Scenario 2

Maneuver amplitude	Shooting mode 1	Shooting mode 2	Shooting mode 3	Relative error
$A = 30m$	0.1915	0.6770	0.7424	9.66%
$A = 60m$	0.1157	0.6239	0.7174	14.98%

(1) resemble closed-loop emendation shooting mode can increase the kill probability for target with zigzag maneuver;

(2) on the basis of resemble closed-loop emendation shooting mode, FAW shooting mode can still increase more kill probability, the reason is that resemble closed-loop emendation shooting mode can not eliminate assumed error completely, but can decrease the error remarkably, as shown in figure 5(b), so FAW enlarges the effective antipersonnel area, which can increase the kill probability;

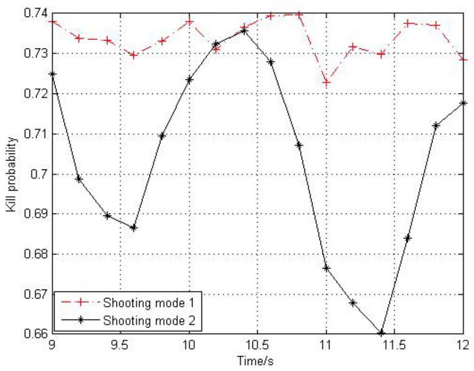
(3) when the amplitude of the zigzag maneuver enlarges from 30m to 60m, the effect of resemble closed-loop emendation of firing data mode is descended, well then the advantages of FAW is more obvious, the relative error of kill probability between Shooting mode 3 and Shooting mode 2 increases from9.66% to14.98%, as shown in table 1;

(4) when the amplitude of the zigzag maneuver is 60m, the kill probability of Shooting mode 3 and Shooting mode 2 are fluctuant without exception, but the fluctuation of Shooting mode 2 is bigger than that of Shooting mode 3. For example, taking the kill probability above 0.6 as standard to fire, the firing time of Shooting mode 2 will be not continuous, however Shooting mode 3 can fire in real time, so Shooting mode 3 increases the effective firing time.

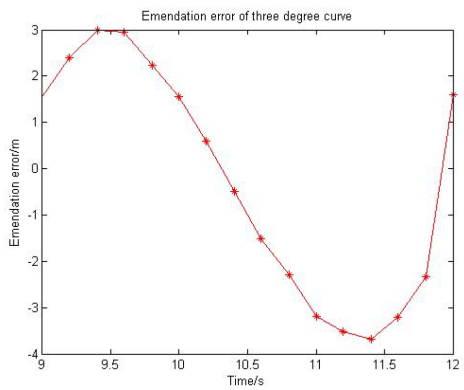
Scenario 3): the amplitude of the zigzag maneuver is $0m$.

In figure 6, the average of kill probability of Shooting mode 2 and Shooting mode 1 are 0.7047 and 0.7358, respectively. The average of kill probability, from Shooting mode 2 to Shooting mode 1, has been improved by 4.41%.

As we can see, when the amplitude of the zigzag maneuver is $0m$, resemble closed-loop emendation mode increases the firing error by three degree curve fitting on the contrary, as given in figure 6 (b), which results in decrease of kill probability of Shooting mode 2. It is shown that the error solution is important for resemble closed-loop emendation mode. It is necessary to make characteristic of error clear, otherwise, the effect is just the opposite to what one wishes.



（a）Kill probability



（b）Emendation error

Fig.6 Simulation result of Scenario 3

Scenario 4): the amplitude of the zigzag maneuver is $40m$, the shooting parameters of FAW is $r_a = 4m, r_b = 7m$ 、 $r_a = 4m, r_b = 14m$ or $r_a = 4m$, $r_b = 21m$, respective.

As we can see from figure 7 and table 2, although the kill probability of Shooting mode 3 is higher than that of Shooting mode 2 under conditions of three different shooting parameters of FAW without exception, they have differences.

(1) when the shooting parameter r_b of FAW is $14m$, the kill probability of Shooting mode 3 is smooth and higher than that of Shooting mode 2 during the whole flight-path; when the shooting parameter r_b of FAW is $21m$, although it is smooth, the kill probability of Shooting mode 3 is worse than that of Shooting mode 2 during part flight-path; when the shooting parameter r_b of FAW is $7m$, the kill probability of Shooting mode 3 is obviously fluctuant;

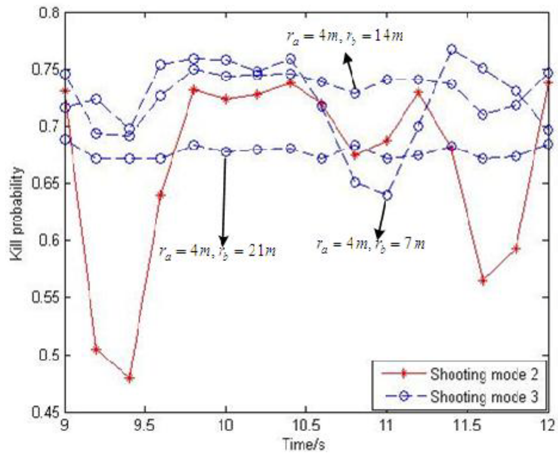


Fig.7 Simulation result of Scenario 4

Table2 Average kill probability of Scenario 4

Shooting parameters	Shooting mode 3	Shooting mode 2	Relative error
$r_a = 4m, r_b = 7m$	0.7115	0.6569	8.31%
$r_a = 4m, r_b = 14m$	0.7291	0.6569	10.99%
$r_a = 4m, r_b = 21m$	0.6715	0.6569	2.22%

(2) Looking at the aspect of the average kill probability of the whole flight-path, when the shooting parameter r_b of FAW is $14m$, the effect is optimal. When the number of pills is definite, if the shooting parameter of FAW is too big, the kill probability will be decreased because the pill distribution is sparse in the airspace, this is also the reason that the kill probability is decrescent when the shooting parameter r_b of FAW is $21m$;

(3) Based on the above analysis, there should exist optimal matching relation between the target

maneuver characteristic and FAW shooting parameter, the effect of FAW shooting method will not be exerted incisively and vividly no matter what the FAW shooting parameter is too big or small.

6 Conclusion

For interception of zigzag maneuvering target, shooting control strategy featured on Resemble Closed-loop emendation of firing data and Future Airspace Window Shooting mode is proposed, the former can minish the error of firing data, and the latter can enlarge the effective distribution area of pills. Integrating the advantages of the two shooting methods can improve the interception probability and overcome the influence of model error of assumed relative target motion on effectiveness of antiaircraft artillery system. Monte Carlo simulation results validated this method.

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