Research on price game process and wavelet chaos control of three oligarchs' insurance market in China

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Abstract: Based on the actual data, this paper shows Chinese insurance market monopoly status through the Market Concentration Rate, and established the repeated price game model for three oligarchs with delayed decision according to Bertrand model. Discussed the two main goals of insurance companies faced in the game process: the expansion of profit and market share. Through the application of nonlinear dynamics theory, this paper makes the analysis and dynamic simulation of the game model, studies the existence and stability of equilibrium point of the system, reveals the dynamic evolution process of market under the different price adjustment speed and the different concern degree of profit. Then have numerical simulation to the system, and gives the complex dynamic characteristics, such as Lyapunov exponent, strange attractor, sensitivity to initial value. Due to wavelet function has good time-frequency localization ability; this paper introduces different wavelet functions to have the chaos control to the system, results shows that wavelet function has a good convergence effect to the chaotic state of the system. The research results of this paper have very good guidance on macro adjustment and control of the insurance market.

Key–Words: Price game, Three oligarchs, Market share, Delayed decision, Chaos control

1 Introduction

After decades of development, insurance market has become an important constituent part of states' financial. Chinese insurance market has entered into the situation that oligopoly possess the market, and compete in different insurance product market. It has become the most important issue that how to occupy initiative in the complicated competition, and have the upper hand in the process of game. This issue will surely impact the Chinese insurance companies' development and decision-making. In recent years, Chinese domestic insurance business has developed rapidly, since 2001, the growth rate of coverage has keep larger than 9% for 10 years, in 2010 year, this rate is increased by 13.83% year-on-year, and in 2011 the insurance industry accounted gross revenue for 3.4%.

In the development process of Chinese insurance, we notice a series of features, the most of which is the oligopoly. From the Market Concentration Ratio point of view, Chinese insurance market concentration degree showed a downward trend in recent years, Even so, in 2011, the index is still up to 71.11%. According to the type division on industrial monopoly and competition by professor Bain, Chinese insurance market belongs to highly concentrated oligopoly market. The market share of three major companies in Chinese insurance market: China Life, Pacific insurance and Ping An insurance is shown in table 1:

Year	2004	2005	2006	2007
Market Share(%)	83.15	77.03	77.55	70.17
Year	2008	2009	2010	2011
Market Share(%)	65.47	62.63	57.17	57.11

Table 1: Market share of top three insurance companies in 2004-2011

China Life, Pacific insurance and Ping An insurance has occupy the absolute number of market share, this shows that the domestic insurance market has obviously oligopoly characteristic. Generally speaking, in the process of game between oligarchs, the oligarch pay the most attention on the game price and the market share, their final purpose is to increase their market share and the maximum gain an advantage through the strategy formulation of price adjustment. In the process of strategy formulation and price adjustment, anyone of the oligarchs make a change on the market strategy will have a huge influence on several other oligarch's market income and the corresponding competition results. Furthermore, in the course of the game, any strategy change will have an impact on the existing state of the system, shift the existing equilibrium state, and even make the system enter into uncontrollable state. Therefore, in order to maximize the proceeds of game, the accuracy of oligarchs' decision-making, and the whole insurance market's stability in the process of competitive, it has an important economic and practical significance to have the research on the process of the oligarchs' price game.

Due to it can explain the running state of the system and have the corresponding chaos control accurately, chaos theory has a very good application in the analysis process of the system in recent years. In the analysis process of economic system, bifurcation theory which background is the difference equations had been introduced first, theoretical analysis ability is good either. It has become a key problem in the process of economics research.

Many scholars have the research on the price and the characteristics of the insurance market within oligarchs' game. Dragone D (1) imported the external environment effect factors into the original oligopoly game model, and gained a stochastic optimal control model, with which to have the complex dynamic analysis to the pollution abatement. Dubiel-Teleseynski T (2) had the research on special situations of adjusting players and diseconomies of scale, to analyze the nonlinear dynamics characteristics in the process of heterogeneous duopoly game with numerical simulation. Junhai Ma etc. (2-5) introduced the delay decision to the duopoly and triopoly repeated price game model, discussed the complex dynamic characteristic in the Chinese insurance industry and cold rolled steel market, which has an important practical significance for the actual operation of the market. Woo-Sik Son (6), researched control method to the chaos situation in complex economic model, adjusted the chaos system return to the steady state through the feedback and delayed feedback control method, and has the important guiding significance to the development of related theories. Elsadany, A.A etc (7, 8) studied on the running state of the complex economic system under different conditions, and made great achievements. Z Chen etc (10) studied the important role of chaos theory in cryptography, and established two-level chaos-based video cryptosystem.

This paper establish three oligopoly game model based on classic Bertrand model and the actual situation of Chinese insurance market. Combined with the factor of market share, this model have the detail analysis to the influence that the change of price adjustment and market share to the game process. Study the effect of oligarch's decision to the actual state of the market in different purposes, in the chaos control aspect, we introduce two wavelet functions to make the system regress convergence.

2 Establishment of the Model

2.1 Establishment of three oligopoly game model

The game process of the insurance market, is different from the product market, which is based on the price as variables. Therefore, in this paper, we choose classic Bertrand model as basic decision model to conduct the process. In Bertrand model, q, p represent the output and price respectively. In this model, we assume that q,p represent turnover and transaction price of some kind of insurance products respectively.

Assume that: in a certain insurance products' market, there exist three oligarchs to occupy the market.

Decision making occurs in discrete time period t=1,2,. $p_i, q_i \geq 0 (i = 1, 2, 3)$ are the price and the quantity demanded of three oligarchs' certain products, $C_i = c_i q_i (i = 1, 2, 3)$ is the cost function of three oligarchs' certain products.

Establish their inverse demand function of the three oligarchs based on the assumption of Bertrand model.

$$\begin{cases} q_1 = a_1 - b_1 p_1 + d_1 p_2 + e_1 p_3 \\ q_2 = a_2 - b_2 p_2 + d_2 p_3 + e_2 p_1 \\ q_3 = a_3 - b_3 p_3 + d_3 p_1 + e_3 p_2 \end{cases} \tag{1}$$

$d_1, d_2, d_3 > 0$,$e_1, e_2, e_3 > 0$ is substitute coefficient,$b_1, b_2, b_3 > 0$. Based on the classic Bertrand model, the company's product output, shows an inverse relationship with its product, shows proportional relationship with the price of competitors, therefore, the coefficients b_i, d_i, e_i, (i = 1,2,3)represent the corresponding proportion between different oligarchs. a_i represents a fixed turnover of insurance companies.

The profits of insurance companies, is composed by the total revenue and total cost generally, which is the different between total revenue and total cost. We can obtain the profit function of the three oligarchs' game process with the function: $\pi_i = TR_i - C_i = p_i \cdot q_i - c_i \cdot q_i$:

Maximize the profits is the purpose of oligarchs in the process of choosing price strategy, that is, make the derivative of profit function is 0, $\frac{\partial \pi_i}{\partial p_i} = 0$. Thus we can obtain the reaction function of an oligopoly in a certain period of time, that is in this time, the optimal price strategy applied by one oligopoly reflect on the strategies can be adopt by other competitors, and this is the Nash equilibrium. But in the actual market operation process, game between oligarchy is ongoing, so the strategy is a long-term dynamic process, its behavior should not only has the adaptability, but also should have a long memory, so the economic system should carry on the overall adjustment based on Marginal Income Ratio. In the evolution study of the economic system, many scholars have corresponding research(3-5), in this paper we improve the dynamical evolution equation according to the actual situation of insurance companies, and its dynamic repeated adjustment game model is

$$p_i(t+1) = p_i(t) + \alpha_i p_i(t)\frac{\partial \pi_i(t)}{\partial p_i(t)}(i = 1, 2, 3) \quad (2)$$

α_i is the price adjustment speed of oligarch i. As can be seen in model (2), the insurance product prices of next period is influenced by the price of this period and marginal profit. If the marginal profit is positive, the profit will increase with the risen of price, so in order to maximum the profit, the price of next period should rise; conversely, if the marginal profit is negative, as, the profit will reduce with the risen of price, so the prise of next period should reduce accordingly.

In the game process of oligarchs, there are so many target should be noticed, besides profit, the market share is another important issue which oligarchs pay most attention on. Formulate rational game strategy to raise their market share, is an important way in the oligopoly market to increase its own competitive advantage and the brand value.

Based on the previous scholars' research, we innovation in the add market share factor to have a improvement to the model (2) innovatively. Increase the oligarch company's competitive advantage with the increase of its market share.

$$\begin{cases} \Delta q_1 = q_1 - (q_2 + q_3) = (a_1 - a_2 - a_3) \\ +(-b_1 - e_2 - d_3)p_1 + (d_1 + b_2 - e_3)p_2 \\ +(e_1 - d_2 + b_3)p_3 \\ \Delta q_2 = q_2 - (q_1 + q_3) = (a_2 - a_1 - a_3) \\ +(b_1 + e_2 - d_3)p_1 + (-d_1 - b_2 - e_3)p_2 \\ +(d_2 - e_1 + b_3)p_3 \\ \Delta q_3 = q_3 - (q_1 + q_2) = (a_3 - a_1 - a_2) \\ +(b_1 + d_3 - e_2)p_1 + (e_3 - d_1 + b_2)p_2 \\ +(-d_2 - e_1 - b_3)p_3 \end{cases} \quad (3)$$

The change of insurance companys' market share Δq_i, is the corresponding change of oligarch's turnover under the condition of total market turnover remains unchanged . Therefore, in this paper, the changes in market share is represented through the difference between one oligarch's turnover and other two oligarchs'. The oligarch's market turnover increased, shows that the market share rising, and Δq_i also corresponding bigger, conversely, Δq_i decreasing.

Similarly, in order to maximum the profit, the oligarchs also want to reach Nash equilibrium in the game process, that is: $\frac{\partial \Delta q_i(t)}{\partial p_i(t)} = 0$, and the corresponding dynamic repeated adjustment game model is

$$p_i(t+1) = p_i(t) + \alpha_i p_i(t)\frac{\partial \Delta q_i(t)}{\partial p_i(t)}(i = 1, 2, 3) \quad (4)$$

Thus, combine (2), (4), we can get three oligarchs' game model:

$$\begin{aligned} p_i(t+1) &= p_i(t) + \alpha_i p_i(t)\{\omega_i[\xi_i \tfrac{\partial \pi_i(t)}{\partial p_i(t)} \\ &+ (1-\xi_i)\tfrac{\partial \pi_i(t-1)}{\partial p_i(t-1)}] + (1-\omega_i)[\eta_i \tfrac{\partial \Delta q_i(t)}{\partial p_i(t)} \\ &+ (1-\eta_i)\tfrac{\partial \Delta q_i(t-1)}{\partial p_i(t-1)}]\}(i = 1, 2, 3) \end{aligned} \quad (5)$$

α_i is the price adjustment speed, ω_i is the attention degree of the three oligarchs to profit. ξ_i and η_i is the different attention degree of the three oligarchs to current marginal profit and current market share, current weight coefficient.$0 \leq \xi_i, \eta_i \leq 1$,when $\xi_i = 1$ or $\eta_i = 1$, means not to take the delay decision.

2.2 Analysis of the three oligopoly game model

Have the calculation and analysis to the three oligarchs' game model, For the convenience of solving, suppose that:$x_i(t+1) = p_i(t)$, will (3), (4) into model (5), and the system (5) changed into:

$$\begin{cases} p_1(t+1) = p_1(t) + \alpha_1 p_1(t)\{\omega_1[\xi_1(a_1 \\ -2b_1p_1(t) + d_1p_2(t) + e_1p_3(t) + b_1c_1) \\ +(1-\xi_1)(a_1 - 2b_1x_1(t) + d_1x_2(t) + e_1x_3(t) \\ +b_1c_1)] + (1-\omega_1) * (-b_1 - e_2 - d_3)\} \\ p_2(t+1) = p_2(t) + \alpha_2 p_2(t)\{\omega_2[\xi_2(a_2 \\ -2b_2p_2(t) + d_2p_3(t) + e_2p_1(t) + b_2c_2) \\ +(1-\xi_2)(a_2 - 2b_2x_2(t) + d_2x_3(t) + e_2x_1(t) \\ +b_2c_2)] + (1-\omega_2) * (-d_1 - b_2 - e_3)\} \\ p_3(t+1) = p_3(t) + \alpha_3 p_3(t)\{\omega_3[\xi_3(a_3 \\ -2b_3p_3(t) + d_3p_1(t) + e_3p_2(t) + b_3c_3) \\ +(1-\xi_3)(a_3 - 2b_3x_3(t) + d_3x_1(t) + e_3x_2(t) \\ +b_3c_3)] + (1-\omega_3) * (-d_2 - e_1 - b_3)\} \end{cases} \quad (6)$$

Calculate the one periodic equilibrium point, we obtain eight fixed points of the system,
$E_1 = (\varepsilon_1, \varepsilon_1)$,$\cdots$,$E_8 = (\varepsilon_8, \varepsilon_8)$:

$\varepsilon_1 = (0,0,0)$, $\varepsilon_2 = (\frac{W_1}{2b_1\omega_1},0,0)$, $\varepsilon_3 = (0,\frac{W_2}{2b_2\omega_2},0)$, $\varepsilon_4 = (0,0,\frac{W_3}{2b_3\omega_3})$

$$\varepsilon_5 = (\frac{2b_2\omega_2 W_1 + d_1\omega_1 W_2}{4b_1b_2\omega_1\omega_2 - d_1e_2\omega_1\omega_2}, \frac{2b_1\omega_1 W_2 + e_2\omega_2 W_1}{4b_1b_2\omega_1\omega_2 - d_1e_2\omega_1\omega_2}, 0)$$

$$\varepsilon_6 = (\frac{2b_3\omega_3 W_1 + e_1\omega_1 W_3}{4b_1b_3\omega_1\omega_3 - d_3e_1\omega_1\omega_3}, 0, \frac{2b_1\omega_1 W_3 + d_3\omega_3 W_1}{4b_1b_3\omega_1\omega_3 - d_3e_1\omega_1\omega_3})$$

$$\varepsilon_7 = (0, \frac{2b_3\omega_3 W_2 + d_2\omega_2 W_3}{4b_2b_3\omega_2\omega_3 - d_2e_3\omega_2\omega_3}, \frac{2b_2\omega_2 W_3 + e_3\omega_3 W_2}{4b_2b_3\omega_2\omega_3 - d_2e_3\omega_2\omega_3})$$

$$\varepsilon_8 =$$

$$(\frac{2b_2\omega_2\omega_3(4b_2b_3-d_2e_3)W_1+\omega_1\omega_3d_1(4b_2b_3-d_2e_3)W_2+2b_2\omega_1\omega_2(2b_2e_1+d_1d_2)W_3}{\omega_1\omega_2\omega_3[(4b_2b_3-d_2e_3)(4b_1b_2-d_1e_2)-(2b_2e_1+d_1d_2)(2b_2d_3+e_2e_3)]},$$

$$\frac{W_2}{2b_2\omega_2} + (\frac{2b_2\omega_2\omega_3(2b_2e_3+d_2d_3)W_1+\omega_1\omega_3d_1(2b_2e_3+d_2d_3)W_2+2b_2\omega_1\omega_2(2b_2d_2+d_2e_1)W_3}{\omega_1\omega_2\omega_3[(4b_2b_3-d_2e_3)(4b_1b_2-d_1e_2)-(2b_2e_1+d_1d_2)(2b_2d_3+e_2e_3)]}),$$

$$\frac{2b_2\omega_2\omega_3(2b_2d_3+e_2e_3)W_1+[\omega_1\omega_3d_1(2b_2d_3+e_2e_3)+\omega_1\omega_3e_3(4b_1b_2-d_1e_2)]W_2}{\omega_1\omega_2\omega_3[(4b_2b_3-d_2e_3)(4b_1b_2-d_1e_2)-(2b_2e_1+d_1d_2)(2b_2d_3+e_2e_3)]} + \frac{2b_2\omega_1\omega_2(4b_1b_2-d_1e_2)W_3}{\omega_1\omega_2\omega_3[(4b_2b_3-d_2e_3)(4b_1b_2-d_1e_2)-(2b_2e_1+d_1d_2)(2b_2d_3+e_2e_3)]})$$

Where:

$$\begin{cases} W_1 = \omega_1(a_1+b_1c_1)+(1-\omega_1)(-b_1-e_2-d_3) \\ W_2 = \omega_2(a_2+b_2c_2)+(1-\omega_2)(-b_2-e_3-d_1) \\ W_3 = \omega_3(a_3+b_3c_3)+(1-\omega_3)(-b_3-e_1-d_2) \end{cases}$$

$E_1 = (\varepsilon_1,\varepsilon_1)$,$\cdots$, $E_7 = (\varepsilon_7,\varepsilon_7)$ contains 0 elements, have no practical significance, so should not be considered. The Jacobin matrix on fixed point $E_8 = (\varepsilon_8,\varepsilon_8)$ is

$$\begin{bmatrix} s_1 & \alpha_2p_2\omega_2\xi_2e_2 & \alpha_3p_3\omega_3\xi_3d_3 & 1 & 0 & 0 \\ \alpha_1p_1\omega_1\xi_1d_1 & s_2 & \alpha_3p_3\omega_3\xi_3e_3 & 0 & 1 & 0 \\ \alpha_1p_1\omega_1\xi_1e_1 & \alpha_2p_2\omega_2\xi_2d_2 & s_3 & 0 & 0 & 1 \\ \alpha_1p_1\omega_1(1-\xi_1)(-2b_1) & \alpha_2p_2\omega_2(1-\xi_2)e_2 & \alpha_3p_3\omega_3(1-\xi_3)d_3 & 0 & 0 & 0 \\ \alpha_1p_1\omega_1(1-\xi_1)d_1 & \alpha_2p_2\omega_2(1-\xi_2)(-2b_2) & \alpha_3p_3\omega_3(1-\xi_3)e_3 & 0 & 0 & 0 \\ \alpha_1p_1\omega_1(1-\xi_1)e_1 & \alpha_2p_2\omega_2(1-\xi_2)d_2 & \alpha_3p_3\omega_3(1-\xi_3)(-2b_3) & 0 & 0 & 0 \end{bmatrix}^T$$

And

$$\begin{aligned} s_1 &= 1+\alpha_1\{\omega_1[\xi_1(a_1-4b_1p_1+d_1p_2+e_1p_3+b_1c_1) \\ &+ (1-\xi_1)(a_1-2b_1X_1+d_1X_2+e_1X_3+b_1c_1)] \\ &+ (1-\omega_1)(-b_1-e_2-d_3)\} \end{aligned}$$

$$\begin{aligned} s_2 &= 1+\alpha_2\{\omega_2[\xi_2(a_2-4b_2p_2+d_2p_3+e_2p_1+b_2c_2) \\ &+ (1-\xi_2)(a_2-2b_2X_2+d_2X_3+e_2X_1+b_2c_2)] \\ &+ (1-\omega_2)(-b_2-e_3-d_1)\} \end{aligned}$$

$$\begin{aligned} s_3 &= 1+\alpha_3\{\omega_3[\xi_3(a_3-4b_3p_3+d_3p_1+e_3p_2+b_3c_3) \\ &+ (1-\xi_3)(a_3-2b_3X_3+d_3X_1+e_3X_2+b_3c_3)] \\ &+ (1-\omega_3)(-b_3-e_1-d_2)\} \end{aligned}$$

We know that E_8 is the Nash equilibrium of the system, but its stability need to meet certain conditions.

3 Numerical simulate and analysis

In order to have a detail observation of the characteristics of the complex dynamic system, we conduct the numerical simulation to the system (5), and obtain corresponding complex dynamic characteristic figures. For the convenience, we make the parameter as:

$$
\begin{aligned}
&a_1 = 1.1375, b_1 = 0.625, d_1 = 0.3, e_1 = 0.15,\\
&c_1 = 3.75, \omega_1 = 0.8, \xi_1 = 0.5, \eta_1 = 0.6,\\
&a_2 = 1, b_2 = 1, d_2 = 0.3, e_2 = 0.4,\\
&c_2 = 3, \omega_2 = 0.5, \xi_2 = 0.7, \eta_2 = 0.7\\
&a_3 = 0.8, b_3 = 0.8, d_3 = 0.5, e_3 = 0.1,\\
&c_3 = 3, \omega_3 = 0.625, \xi_3 = 0.6, \eta_3 = 0.5
\end{aligned}
$$

Based on the actual situation of Chinese insurance companies, we set the parameters, and have the numerical simulate to system (5).

3.1 Effects of the price adjustment speed to the system status

Consider the bifurcation phenomenon of final result and corresponding Lyapunov exponent figure, and other figures such as singular attactor and initial value sensitivity, which is caused by the price adjustment speed α_i (i = 1,2,3).

Therefore, we have the analysis to the complex dynamic characteristic of system (5), as shown in figure 1-figure 3.

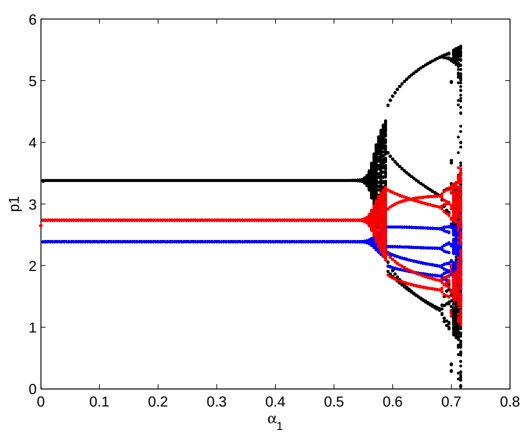

Figure 1: Bifurcation when a_1 changes and $\alpha_2 = 0.55$,$\alpha_3 = 0.5$

From the figures above, we noticed that different price adjustment speed of different oligarchs will make the system have unique way entering into the chaos state. The bifurcation diagram shows the path that the system entering into chaos state, while the

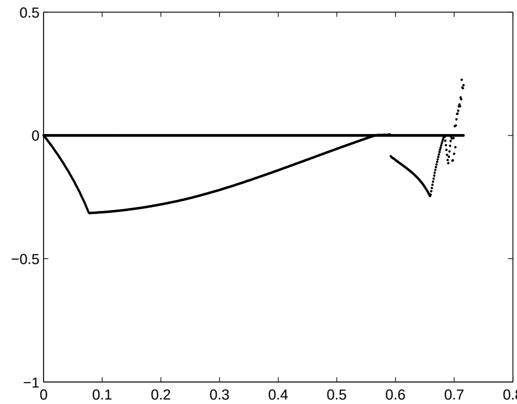

Figure 2: Lyapunov exponent when a_1 changes and $\alpha_2 = 0.55$,$\alpha_3 = 0.5$

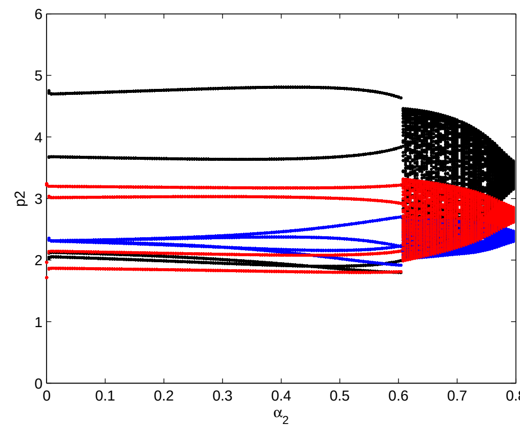

Figure 3: Bifurcation when a_2 changes and $\alpha_1 = 0.6$,$\alpha_3 = 0.5$

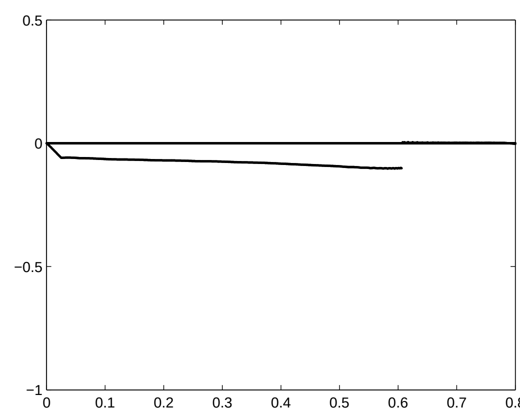

Figure 4: Lyapunov exponent when a_2 changes and $\alpha_1 = 0.6$,$\alpha_3 = 0.5$

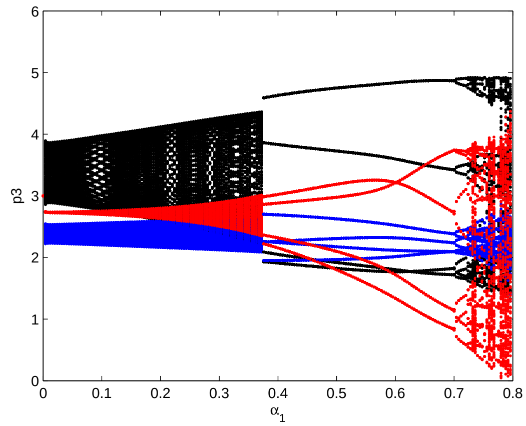

Figure 5: Bifurcation when a_3 changes and $\alpha_1 = 0.6$,$\alpha_2 = 0.55$

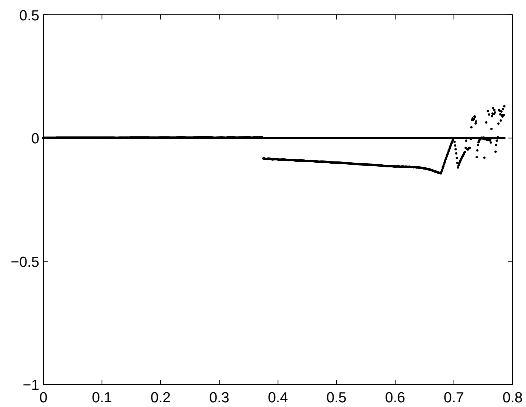

Figure 6: Lyapunov exponent when a_3 changes and $\alpha_1 = 0.6$,$\alpha_2 = 0.55$

Lyapunov index diagram has the corresponding performance to the bifurcation diagram. Lyapunov index shows the average convergence or divergence index rate of approximate orbit in phase space. When the Lyapunov index is larger than 0, the system is in chaos state.

As shown in figure 1, the change of price adjustment α_1 lead to complex dynamic behavior of the system, when $\alpha_1 < 0.562$, the system is in stable state, and have a unique solution, with the increase of α_1 , the system enter into chaos state first, and then change into period-doubling bifurcation state again, eventually lead to chaos. From the corresponding Lyapunov exponent in Figure 2, we can have a check on it, when $\alpha_1 < 0.562$, the Lyapunov exponent is less than zero all along, and when $\alpha_1 > 0.562$, the Lyapunov exponent large than zero first, and then change to less than zero again, when $\alpha_1 > 0.7$, the system enter into chaos state, these is coincide with the complex dynamic characteristic which is shown in bifurcation figure. Also, in figure 2, when $\alpha_2 \leq 0.608$, the system is in period-doubling bifurcation state, and the three oligarchs' market is in the state of orderly and controlled competition, when$\alpha_2 > 0.608$, the system enter into the state of chaos. In figure 3, when $0.375 < \alpha_3 \leq 0.73$, the system is in period-doubling bifurcation state, the market is in the process of orderly competition, and can be controlled. On the outside of the range, the system is in chaos state, which can also be test and verified by the Lyapunov exponent shown in figure 4 and figure 6.

At this time, three oligarchs' market is in chaos state, and the result of any oligarchs decision is unpredictable, any minor changes of initial state will have an tremendous impact on the final result, and this is also the state which should be avoided in the process of competitive.

3.2 Effects of the price adjustment speed to the system status

Considering the influence of market share on the running state of three oligarchs' insurance market. In this paper, the parameter $\omega_i (i = 1, 2, 3)$ express the three insurance companies' attention degree to the profit, then the attention degree to market share is $(1 - \omega_i)$, so we have an analysis on the complex dynamic characteristic on system (6) which is caused by ω_i .

Figure 4 shows the change status of system (6) when $\alpha_1 = 0.6, \alpha_2 = 0.75, \alpha_3 = 0.5$, $\omega_2 = 0.5, \omega_3 = 0.625$, and ω_1 changes from 0 to 1.

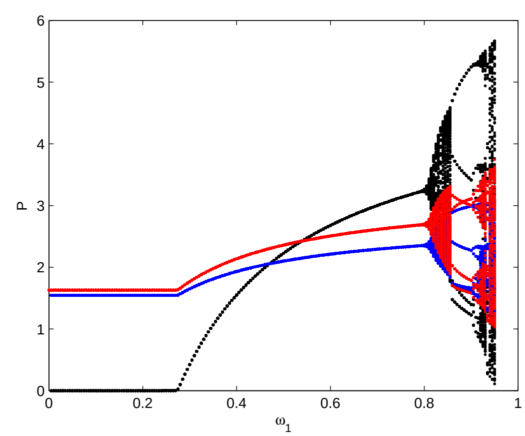

Figure 7: Bifurcation when ω_1 changes and $\alpha_1 = 0.6, \alpha_2 = 0.75, \alpha_3 = 0.5, \omega_2 = 0.5, \omega_3 = 0.625$

From the result above, we found that when $\omega_1 < 0.795$, the system is in stable state, and have a unique solution, the system has practical implications when

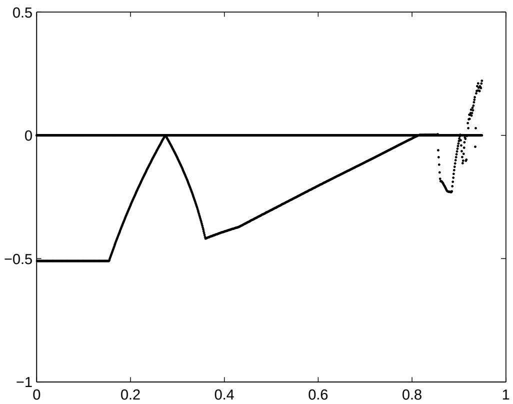

Figure 8: Lyapunov exponent when ω_1 changes and $\alpha_1 = 0.6, \alpha_2 = 0.75, \alpha_3 = 0.5, \omega_2 = 0.5, \omega_3 = 0.625$

$0.274 < \omega_1 < 0.795$. With the increasing of ω_1 , the system enter into chaos state first, then immediately reentered the period doubling bifurcation state, and eventually into chaos, we can verify it with the corresponding Lyapunov index. This shows that, in the process of three oligarchs' game, attention degree of market share is the key factors about the result of the game. In the course of the actual operation of insurance market, oligarchs should not unduly concerned about corporate profits, but should pay more attention to the market share(In system (6), the attention degree of oligarch 1 to the profit should be less than 79.5%), otherwise it will face the situation that market shocks, unpredictable, which is an undesirable result. Correlation figures of other two oligarchs can also be obtained in the same way, this paper will not repeat give.

3.3 Singular attractor and initial sensitivity of system

In the phase space, the chaotic motion corresponds to trajectories which have a random distribution in a certain region. Figure 5 has shown the singular attactor of the system when the system is in the state of $\alpha_1 = 0.58, \alpha_2 = 0.45, \alpha_3 = 0.6$ and $\alpha_1 = 0.58, \alpha_2 = 0.45, \alpha_3 = 0.5$.

An obvious feature of chaotic attractor is exponential separation and approaching to the attractor point, which shows that chaos system is sensitive to initial conditions. We have the analysis on the initial value sensitivity of system, as shown in figure 6.

When $\alpha_1 = 0.8, \alpha_2 = 0.65, \alpha_3 = 0.5$, increase the initial value $(p_1(0), p_2(0), p_3(0)) = (3, 3, 3)$ by 0.000001 respectively. We can find that, although other conditions haven't change, in the initial iterative process, two orbits almost coincidence, and with the increase of iteration times, the space between the two orbits divide more and more large. This illustrate that in chaos state, tiny change of oligarchs' initial decision, will have a significant impact on the result after many iteration, which led to the uncertainty of the results. So we must have an control to the change of price adjustment speed, maintain it in a certain range, and make the system controllability in the decision-making process, prevent the three oligarchs' insurance system from entering into uncontrollable condition in the game process, keep the insurance market in benign competitive situation.

Therefore, to avoid the three oligarchs' insurance

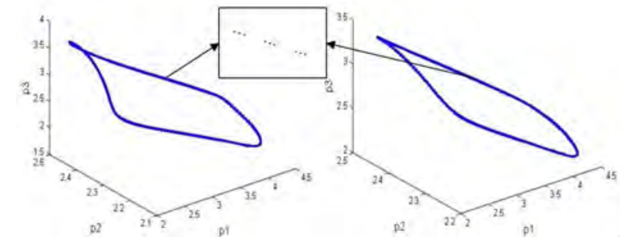

Figure 9: Singular attractor when $\alpha_1 = 0.58, \alpha_2 = 0.45, \alpha_3 = 0.6$ and $\alpha_1 = 0.58, \alpha_2 = 0.45, \alpha_3 = 0.5$

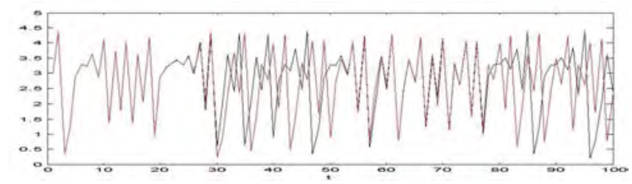

Figure 10: Initial value sensitivity when $p_1(0) = 3$ changed into 3.000001

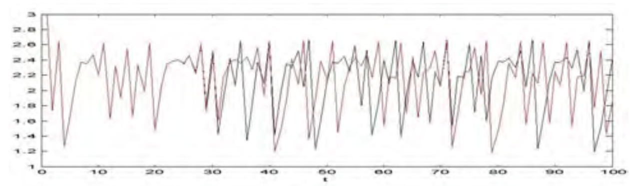

Figure 11: Initial value sensitivity when $p_2(0) = 3$ changed into 3.000001

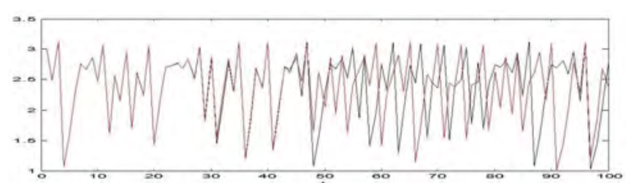

Figure 12: Initial value sensitivity when $p_3(0) = 3$ changed into 3.000001

market entering into the chaos state. It is necessary for us to strengthen the insurance market macro-control, with the implement of a series of policy and market regulation methods, to guarantee the price adjustment speed can stabilize in a certain range.

4 Chaos control

From the analysis above, we found that when three oligarchs are taking one periodic delay strategy, and with the consideration of market share factors, the changes of price adjustment speed and attention degree of market share will lead the system into chaos state. At this time, the three oligarchs' insurance market is in a state of disorder, the competition between each other will be unpredictable, its profits cannot be controlled. Therefore, in order to ensure the orderly operation of the insurance market and the competition situation of predictability, it is necessary to control the chaos state.

Conventional chaos control methods mainly include the feedback and non feedback control, feedback control is to stabilize the unstable periodic orbits of chaotic attractor through feedback part information of the output signal. But due to the time-delay problem, there exits a period time delay while correcting the deviations, which makes the feedback may result in the system inaccurate and unstable. The system (6) is established based on actual oligopoly Chinese insurance market which exist one periodic time delay, so there will be such risks if feedback control method is adopt.

Wavelet function has good time-frequency localization ability, and wavelet bases can form unconditional bases of commonly used space. Due to the two important properties of wavelet function, it can have fast convergence control on the divergent system. This paper introduces the wavelet function to have the chaos control on the system (6), two wavelet function: $y = ke^{-0.1p(t)^2}$ and $y = k(1-p(t)^2)e^{-p(t)^2}$, it can be easily verified that these two functions are wavelet function, $p(t)$ is the price of insurance products, k is the control factor.

Establish the wavelet control system (7) based on $y = ke^{-0.1p(t)^2}$:

$$\begin{cases} p_1(t+1) = (p_1(t) + \alpha_1 p_1(t)\{\omega_1[\xi_1(a_1 \\ -2b_1p_1(t) + d_1p_2(t) + e_1p_3(t) + b_1c_1) \\ +(1-\xi_1)(a_1 - 2b_1x_1(t) + d_1x_2(t) + e_1x_3(t) \\ +b_1c_1)] + (1-\omega_1)(-b_1 - e_2 - d_3)\}) \\ *(ke^{-0.1p(t)^2}) \\ p_2(t+1) = p_2(t) + \alpha_2 p_2(t)\{\omega_2[\xi_2(a_2 \\ -2b_2p_2(t) + d_2p_3(t) + e_2p_1(t) + b_2c_2) \\ +(1-\xi_2)(a_2 - 2b_2x_2(t) + d_2x_3(t) + e_2x_1(t) \\ +b_2c_2)] + (1-\omega_2)(-d_1 - b_2 - e_3)\} \\ p_3(t+1) = p_3(t) + \alpha_3 p_3(t)\{\omega_3[\xi_3(a_3 \\ -2b_3p_3(t) + d_3p_1(t) + e_3p_2(t) + b_3c_3) \\ +(1-\xi_3)(a_3 - 2b_3x_3(t) + d_3x_1(t) + e_3x_2(t) \\ +b_3c_3)] + (1-\omega_3)(-d_2 - e_1 - b_3)\} \end{cases} \tag{7}$$

When $\alpha_1 = 0.8, \alpha_2 = 0.55, \alpha_3 = 0.5$, the system is in chaos state, take the system parameter values into the wavelet control system (7), we can observe the system with the change process of control factor k .

Simply, the system with the control function $y = k(1-p(t)^2)e^{-p(t)^2}$ can be observed, as shown in figure 7:

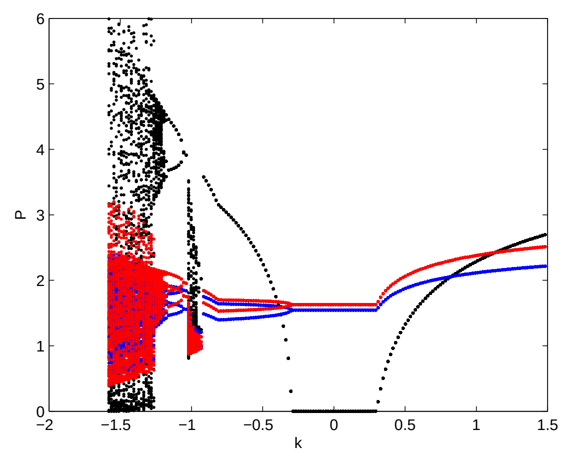

Figure 13: Chaos control to the system when wavelet function is $y = ke^{-0.1p(t)^2}$

Through the chaos control of the system, it can be noticed that when the control function is $y = ke^{-0.1p(t)^2}$ and $-0.87 < k < 1.5$, the system enter into the stable state, simply, when the control function is $y = k(1-p(t)^2)e^{-p(t)^2}$ and $-0.85 < k < 0.9$, return to stable state either, the chaotic state of the system has been controlled. This shows that the wavelet function has a good control effect to chaotic state of the system.

At the same time, we find that different wavelet

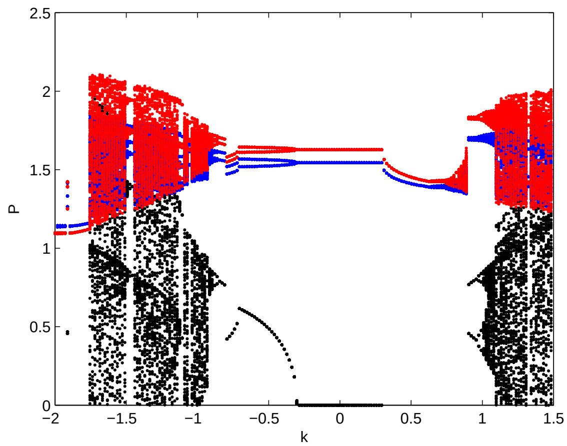

Figure 14: Chaos control to the system when wavelet function is $y = k(1 - p(t)^2)e^{-p(t)^2}$

function has different convergence interval, therefore the enterprises' operators should properly select wavelet function of the control system in actual problems, and select the appropriate factor.

5 Conclusion

With the detailed analysis of Chinese insurance market's oligopoly situation, this paper have the consideration on the profits and market share factors in the process of competitive innovatively, and establish the three oligarchs' game model which has one time delay strategy, have the theory analysis and numerical simulate to the model, then have the chaos control with wavelet function to the model, draw the following conclusions.

1. In a oligarchs' game system, the change of different price adjustment speed, has different influence to the path that the system enter into chaotic state. In this paper, the system has different way entering into chaotic state, with the increase of α_1, the solution enter into the period-doubling bifurcation and then into chaos state, and another situation is the system is in chaos state first, then period-doubling bifurcation, and eventually enter into the chaotic state, etc. Therefore, in the process of market monitoring and macro-control, market regulator should pay more attention to the influence of the oligarchs' price adjustment speed to the system state.

2. In the game process of the insurance market, the market share has an important influence to the oligarchs' game process and result, if oligarchs emphasis on profit too much and dispire market share in the process of strategy formulation, will cause the market enters into chaotic state.

3. The feedback control strategy has certain inaccuracy on systems with time delay for chaos control, this paper introduce wavelet function to control chaotic systems, the results shows that wavelet function has a good convergence control to the chaotic system, different wavelet function has different control ability to the system and different stability intervals. The managers of enterprises should choose suitable wavelet function for chaos control in actual decision making process.

From the result of this paper, the government need a vigorous macroeconomic regulation and control on insurance market price adjustment speed, oligarchs should pay attention to market share factor to avoid insurance market enter into chaotic state. At present Chinese insurance market main occupied by the China life insurance, Pacific insurance and Ping An insurance company, as large state-owned enterprise, the three companies can make rapid reaction to the price adjustment speed and insurance market macro-control policy, which will guarantee the sound development and benign competitive situation of the whole Chinese insurance market.

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References:

[1] D. Dragone, L. Lambertini, G. Leitmann and A. Palestini, A stochastic optimal control model of pollution abatement, *Nonlinear Dynamics and Systems Theory*. 10, 2010, pp. 117–124.

[2] T. Dubiel-Teleseynski, Nonlinear dynamics in a heterogeneous duopoly game with adjusting players and diseconomies of scale[J], *Communications in Nonlinear Science and Numerical Simulation*. 16(1), 2011, pp. 296–308.

[3] Junhai Ma, Xiaosong Pu, Complex Dynamics in Nonlinear Triopoly Market with Different Expectations, *Discrete Dynamics in Nature and Society*. Article ID 902014, 2011.

[4] Junling Zhang, Junhai Ma, Research on the Price Game and the Application of Delayed Decision in Oligopoly Insurance Market, *Nonlinear Dynamics*. 70, 2012, pp. 2327–2341.

[5] Zhihui Sun, Junhai Ma, Complexity of triopoly price game in Chinese cold rolled steel market, *Nonlinear Dynamics*. 67, 2012, pp. 2001–2008.

[6] Woo-Sik. Son, Young-Jai. Park, Delayed feedback on the dynamical model of a financial system[J], *Chaos, Solitons & Fractals*. 44, 2011, pp. 208–217.

[7] A.A. Elsadany, Dynamics of a delayed duopoly game with bounded rationality[J], *Mathematical and Computer Modeling*. 52(9/10), 2010, pp. 1479–1489.

[8] H.N. Agiza, A.A. Elsadany, Chaotic dynamics in nonlinear duopoly game with heterogeneous players[J], *Applied Mathematics and Computation*. 149(3), 2004, pp. 843–860.

[9] L. Chen, G.R. Chen, Controlling chaos in an economic model[J], *Physica A*. 374, 2007, pp. 349–358.

[10] Z. Chen, W.H. Ip, C.Y. Chan and K.L. Yung, Two-level chaos-based video cryptosystem on H.263 codec, *Nonlinear Dynamics*. 62, 2010, pp. 647–664.

[11] Z. Chen, W.H. Ip, C.Y. Chan and K.L. Yung, Two-level chaos-based video cryptosystem on H.263 codec, *Nonlinear Dynamics*. 62, 2010, pp. 647–664.

[12] J.G. Du, T. Huang and Z. Sheng, Analysis of decision-making in economic chaos control, *Nonlinear Analysis. Real World Applications*. 10(4), 2009, pp. 2493–2501.

[13] G.A. Leonov, M.M. Shumafov, Stabilization of controllable linear systems, *Nonlinear Dynamics and Systems Theory*. 10(3), 2010, pp. 235–268.

[14] K. Howra, J.M. Vahid, K. Hamed, et al, Cournot games with linear regression expectations in oligopolistic markets, *Mathematics and Computers in Simulation*. 80, 2010, pp. 1874–1885.

[15] A. Dubovik, M.C.W. Janssen, Oligopolistic competition in price and quality, *Games and Economic Behavior*. 75, 2012, pp. 120–138.

[16] W.C. Chen, Dynamics and control of a ?nancial system with time-delayed feedbacks, *Chaos, Solitons & Fractals*. 37(4), 2008, pp. 1198–1207.

[17] Yuanyuan. Chen, Jiang. Yu, Chengjun. Sun, Stability and Hopf bifurcation analysis in a three-level food chain system with delay, *Chaos, Solitons & Fractals*. 31, 2007, pp. 683–694.

[18] Marek. Szydlowski, Adam. Krawiec, The stability problem in the KaldorCKalecki business cycle model, *Chaos, Solitons & Fractals*. 25, 2005, pp. 299–305.

[19] S.Z. Hassan, On delayed dynamical duopoly[J], *Applied Mathematics and Computation*. 151(1), 2004, pp. 275–286.

[20] Y.Q. Fan, T. Xie, J.G. Du, Complex dynamics of duopoly game with heterogeneous players: A further analysis of the output model, *Applied Mathematics and Computation*. 218, 2012, pp. 7829–7838.

[21] W. Xu, J.H. Ma, Study on the dynamic model of a duopoly game with delay in insurance market, *WSEAS Transactions on Mathematics*. 11(7), 2012, pp. 615–624.

[22] G.H. Wang, J.H. Ma, A study on the complexity of multi-enterprise output game in supply chain, *WSEAS Transactions on Mathematics*. 11(4), 2012, pp. 334–344.

[23] H.W. Wang, J.H. Ma, Chaos control and synchronization of a fractional-order autonomous system, *WSEAS Transactions on Mathematics*. 11(8), 2012, pp. 716–727.