$$|T_L| \leq T_{L_{max}} = 1.2T_{L_0}$$

Therefore, h in Eq. (A-1) is function to the slope of the switching manifold λ and the boundary layer δ.

In sliding mode controller design, we mainly concern in calculating suitable value for the gain k after a proper selection to the switching function $s(x)$ (by proper we mean that the origin is an asymptotically stable after the state reaches the switching manifold $s(x) = 0$). Now, if we set the permissible error and λ as in the following

$$e_{per.} = 0.05 \text{ deg.} = \frac{\pi}{3600}\, rad, \qquad \lambda = 25$$

then from (24), we have

$$\delta = \lambda * e_{per.} = \frac{\pi}{144} \Rightarrow |e_1| \leq e_{per.}$$

Accordingly, to find the gain k, we first compute h as follows:

$$\max|F| \leq 1.2\big(F_{co} + \sigma_o(2\delta + 1)\big) = 20.84$$

$$\Rightarrow h = 20.84 + 2.4 + 0.24 * 12\pi + 0.24 * 25 * 2\frac{\pi}{144} = 32.55$$

and then for $\beta = 1.25$, we get

$$k = \alpha * 1.25 * 32.55 = 42\,, \quad \alpha > 1$$

Also, from Eq. (20), γ equal to

$$\gamma = \frac{144}{\pi} tan \frac{\pi}{2.5} = 141$$

Finally, the sliding mode controller to the servo actuator is

$$\left.\begin{array}{l} u_{approx.} = -\frac{84}{\pi} tan^{-1}(141 * s) \\ s = (\dot{x} - \dot{x}_d) + 25 * (x - x_d) \end{array}\right\} \qquad \text{(A-2)}$$

The sliding mode controller will be able to prevent the state leaves the positively invariant set Δ_δ, which means that the error $(x - x_d)$ is less than the permissible limit that was specified earlier.

Appendix (B)

In this case we consider the same desired position and velocity as in Eq. (29) with the following initial condition

$$x = 0.035\, rad, \qquad \dot{x} = 0\ rad/sec.$$

$$\Rightarrow e(0) = (e_1, e_2) = (0.035, 0)$$

Also, consider the same switching function as in case one ($s = e_2 + 25e_1$). Then, the invariant set is given by

$$\Theta = \{x \in \mathcal{R}^2 : 0 \leq s(t) < 0.875\,, |e_1(t)| \leq 0.035\} \qquad \text{(B-1)}$$

In addition we have

$$|e_2(t)| \leq 1.75$$

$$\Rightarrow \max|\dot{x}| = \max|e_2| + \max|\dot{x}_d| = 2.75\, rad/sec.$$

Then $\max|F|$ can be estimated as

$$\max|F| \leq 1.2(F_{co} + 2.75 * \sigma_o) = 22.2$$

As in the first case, h is equal to

$$h = 22.2 + 2.4 + 0.24 * 12\pi + 0.24 * 25 * 1.75 = 44.15$$

The sliding mode controller gain k from Eq. (6) is taken equal to

$$k = 45 > h$$

Finally, the sliding mode controller for the second case is given by

$$\left.\begin{array}{l} u = -45 * \text{sgn}(s) \\ s = (\dot{x} - \dot{x}_d) + 25 * (x - x_d) \end{array}\right\} \qquad \text{(B-2)}$$

If the state initiated inside the positively invariant set as given in (B-1), the sliding mode controller will regulate the error state to the origin irrespective to the uncertainty and the non-smooth components in the servo actuator model.