

Modeling and Numerical Solution of a 17-Bus Electrical System using RK4 and Newton- Raphson Methods

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Abstract: - This article presents an integrated numerical framework for the analysis of electromechanical dynamics and steady- state behavior in a 17- bus subset of the Spanish peninsular transmission system. Motivated by the severe blackout that occurred in April 2025, which revealed the vulnerability of low- inertia, renewable- dominated grids, the study couples the fourth- order Runge–Kutta method with the Newton–Raphson method, as a power- flow algorithm, to provide a unified representation of transient frequency evolution and static system equilibrium. The selected network subset includes a Slack bus, several PV buses, and multiple PQ buses with significant load concentrations and strong reactive interactions. The Newton–Raphson procedure exposes high sensitivity in nodes with intense reactive demand. This is the case around branches featuring large susceptance values, and when we use branch parameters and shunt contributions. Iterative behavior shows notable mismatch magnitudes in the flat start, substantial angle corrections, and conditioning challenges associated with reactive stiffening. At the same time, the dynamic model captures the influence of inertia, damping, and primary control on frequency trajectories following large disturbances. The coupled approach ensures consistency between electrical power evaluations and dynamic states at each integration step, enabling detailed assessment of electromechanical responses under stressed operating conditions. Overall, the study demonstrates that combining high- order time integration with robust nonlinear algebraic solvers provides a reliable tool for understanding system sensitivity, evaluating operational risks, and supporting the design of protection and control strategies in increasingly renewable electrical networks.

Key-Words: - Electric power systems, Frequency stability, Newton–Raphson, Power flow, Runge–Kutta.

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1 Introduction

THE blackout that occurred on April 28, 2025, in the Iberian Peninsula, which caused the instantaneous loss of approximately 15 GW of generation, as well as an abrupt drop in frequency, highlighted the delicate fragility of this electrical system in the face of major disturbances, especially in scenarios of high renewable energy penetration, as was the case [1]. This event, in a context of low inertia (due to the importance of renewables) and the growing contribution of photovoltaic and wind energy sources, triggered an automatic disconnection of the Iberian electrical system from the European grid, which demonstrates the immediate need for robust methodologies that allow for anticipating and adequately managing both the transient dynamics and the steady-state conditions of the system [2], [3].

This article focuses on analyzing and improving the integration of two advanced numerical methods:

the fourth-order Runge - Kutta (RK4) method for accurately estimating the time evolution of frequency, and the Newton-Raphson method for solving constrained load flows, allowing for the evaluation of voltages, power, and the viability of each system element [4], [5]. The aim is to achieve a complementary approach that unifies the dynamic response to faults with the restoration of electrical equilibrium in steady states.

In this study, although the April 2025 blackout affected the entire Iberian Peninsula and other regions, we will focus on the Spanish peninsular system. At the same time, the evolution of the energy mix of the Spanish peninsular system in 2025 reveals a disruptive boom in solar energy. According to Red Eléctrica de España (REE) (the Spanish Transmission System Operator, TSO), solar self-consumption installations have already contributed 5% of electricity demand, with an estimated generation of

13,485 GWh, contributing to renewables representing 55.5% of total production, or 56.6% if self-consumption is included [6]. Furthermore, in terms of total generation (centralized and distributed), photovoltaic solar power has become the dominant source in 2025, surpassing nuclear and wind power, with figures close to 60,000 GWh. This growth, on the one hand, helps to decarbonize the system; But on the other hand, it intensifies generation variability and reduces available inertia, complicating system stability in the face of extreme events like the April blackout [7].

This research proposes a coupling strategy between RK4 and Newton - Raphson that:

1. It allows high-fidelity simulation of frequency drop after severe disturbances, identifying activation thresholds of automatic protection systems.
2. Evaluate the new load flow balancing after the event, including voltage restrictions, generation limits, and the presence of distributed renewable sources.
3. It can offer technical recommendations aimed at strengthening the resilience of the system, which can range from the synchronized increase of inertia through storage systems or synchronous generators, to the design of adaptive protection schemes and the planning of infrastructure in areas with high photovoltaic penetration.

It is not only about reproducing critical events, such as the 2025 blackout, but also about devising prediction and prevention mechanisms or procedures that contribute to guaranteeing operational stability in the electrical system; a system that is increasingly green, decentralized, and complex.

2 Blackout and methods

The blackout of April 28, 2025, on the Iberian Peninsula, which affected Spain and Portugal and triggered the automatic disconnection of the Iberian system from the European grid, constitutes a unique case in recent literature on electrical stability [8], [9]. The sudden loss of approximately 15 GW of generation in less than five seconds triggered an abrupt drop in frequency, activation of protection systems, and a cascade of disconnections that highlighted the vulnerability of systems with high renewable energy penetration and low inertia. One of the reasons that may explain this situation is the accelerated transformation of the energy mix, without adequate planning that takes into account grid saturation, protection systems, and system connection/disconnection mechanisms [10]. In 2025,

solar photovoltaic energy consolidated its position as the leading source of electricity in Spain, surpassing nuclear and wind power, with figures close to 60,000 GWh, while self-consumption installations contributed more than 5% of national demand. While it is clear that progress in decarbonization requires this technological advancement, it also entails new structural and operational challenges. Among the most significant are two: the reduction of system inertia and, on the other hand, the greater variability of renewable generation. These two factors greatly hinder the system's ability to withstand and recover from severe disturbances.

An analysis of these events requires methodological tools capable of capturing both the system's transient response and its steady-state behavior [11]. In this context, fourth-order Runge-Kutta (RK4) and Newton-Raphson numerical methods are widely recognized techniques in the field of electrical engineering.

The fourth-order Runge-Kutta method is a mathematical tool used to solve ordinary differential equations. The Runge-Kutta procedure is effective in solving first-order differential equations. In this case, we use it in the field of electrical systems, more specifically for system frequency analysis. Some of its advantages are the ability to change the integration step at any given time, its stability, ease of programming, and the ability to estimate truncation errors.

This point is fundamental for analyzing the dynamic evolution of variables (such as the aforementioned system frequency), especially in the case of a previous disturbance [12].

When the blackout occurred in April 2025, the frequency dropped in just five seconds from 50 Hz to values below 49 Hz. Not only is the speed of the drop relevant, but also the magnitude of the decrease. One of the consequences was that the system exceeded critical safety thresholds. This critical situation triggered automatic protection systems—albeit with limited effectiveness, given the actual consequences—and precipitated the disconnection of the Iberian system from the rest of Europe.

Because the RK4 method can incorporate parameters such as total inertia, input mechanical power, and instantaneous electrical power, it is very useful for reproducing these types of phenomena. Furthermore, it allows for the characterization of both protection activations and instability propagation [13].

On the other hand, the Newton-Raphson method remains the standard approach for solving and analyzing steady-state power flows. This method is based on the iterative linearization (using the tangent

line to the curve) of the nonlinear equations that describe the network's operation. It also facilitates the calculation of various variables, such as voltage and phase angles, and can provide values for active and reactive power injections at each bus. One of its main advantages is its ability to converge quickly, even in large-scale systems (such as the Spanish Peninsular Electrical System) [14].

Applying this method to the blackout analysis, we see significant voltage imbalances, with drops exceeding 10% at certain critical nodes. These results highlight the loss of voltage support in areas with high renewable penetration and with insufficient protection schemes to cope with an event of such magnitude.

The steady-state analysis provided by NR complements the information obtained through dynamic simulation, as it allows us to characterize the equilibrium reached—which we can define as degraded—to which the system evolves once the transient dissipates.

In addition, the integration of both methods (RK4 and NR) within a hybrid framework constitutes a promising line of research, as it allows us to correlate the temporal evolution of the frequency with the redistribution of fluxes and voltages in steady state following an event of this nature [15], [16].

Using both methods, it can facilitate a more comprehensive interpretation of complex phenomena, such as the April 2025 blackout. Furthermore, it also provides a solid analytical basis for operators and analysts to design preventive strategies or improve protection schemes in increasingly decentralized electrical systems with so-called distributed generation, and, particularly, with high penetration of renewable energy generation.

3 Coupled use of fourth-order Newton – Raphson and Runge – Kutta for the analysis of a subset of 17 bars

The electrical system under study is a subset of 17 buses extracted from the Spanish peninsular system, configured on a 100 MVA basis. In the subset considered, bus 17 is designated as Slack, buses 10, 13, and 14 as PV, and the remaining buses as PQ, maintaining consistency with the standard bus type coding used in power flow analysis. This section will describe the system, the electrical diagram, and the fundamental equations, and will present the mathematical development of the coupled RK4–NR method, along with the resulting tables.

In the single-line representation, Figure 1 (Subset of 17 bars, Slack = 17; PV = 10, 13, 14), the Slack and PV bars are explicitly indicated, and the PQs are

distinguished to highlight the mixed structure of the problem, taking into account the module fix at PV, the complete power balance at PQ, and the angle and voltage reference at Slack.

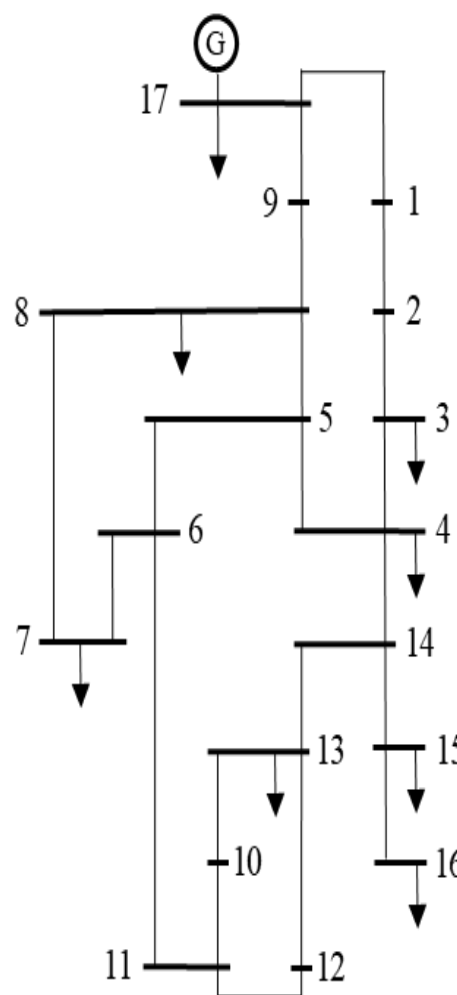


Fig. 1: Single-line representation of the network

To provide numerical traceability to the analysis, the loads (active and reactive) of the subset are explicitly listed in physical units (MW/ MVar) and with Base = 100 MVA, which allows immediate conversion to per unit, and the values of R, X and B (shunt susceptance) of the internal branches that link the set of buses are reproduced. These tables are not only essential for constructing Y_{bus} and, therefore, for calculating (P_i, Q_i) and its derivatives, but they also directly condition the numerical condition of the linear system of the Newton - Raphson (NR) method, in particular through the block $L = \partial Q / \partial V$ and the distribution of shunts in the diagonal terms of Y_{bus} . Consistent with AC flow practice, the values are used without modification, and the conversion to per unit is done by dividing the powers by 100 MVA, which standardizes scales and improves conditioning in the resolution of the Jacobian.

Table 1 shows the active and reactive loads on buses in the subset that have non-zero demand.

Table 1. Subassembly loads, Base = 100 MVA.

Bus	P (MW)	Q (MVar)
1	155.0	29.0
2	622.0	101.8
3	314.0	2.4
4	301.6	-88.2
5	224.0	47.2
6	135.0	16.7
7	230.8	83.0
8	422.0	126.0
9	263.5	23.9
10	9.1	4.5
11	271.0	114.0
12	17.5	86.0
13	237.5	81.6
14	201.0	24.5
15	302.0	146.0
16	321.0	30.1
17	1017.0	230.0

The presence of positive Q in several bars highlights the need for accurate modeling of the reactive component, given its direct impact on the stress stiffness and, consequently, on the L-block of the Jacobian. In the per-unit conversion, each power is divided by 100; this homogeneous scaling reduces the disparity in magnitudes between the active and reactive equations when linearizing.

Table 2. Internal line parameters in pu. over 100 MVA.

From	To	R	X	B
1	2	0.003400	0.011100	0.638700
1	17	0.001000	0.024000	0.740000
2	3	0.001800	0.014100	0.207200
3	4	0.001800	0.019300	0.224400
4	5	0.000900	0.014800	0.114200
4	14	0.000900	0.014900	0.238200
5	6	0.000300	0.002700	0.034400
5	8	0.000900	0.011900	0.135600
6	7	0.005800	0.008900	0.112100
6	11	0.006600	0.008000	0.115900
7	8	0.000600	0.004400	0.100800
8	9	0.002900	0.034300	0.333400
9	17	0.000900	0.023000	1.162000
10	11	0.000500	0.004100	0.091900
10	13	0.000500	0.004100	0.082900
12	11	0.002100	0.039500	0.000000
12	13	0.002100	0.039500	0.000000
13	14	0.001100	0.011200	0.133300
14	15	0.001700	0.021300	0.355600
15	16	0.001100	0.008900	0.211000

The contribution of B is incorporated into the diagonal terms of Y_{bus} as shunt susceptance, reflecting the distributed capacitive effect and/or localized compensations. In the formation of Y_{bus} , the series admittances $1/Z$ are added to the diagonals of the extreme nodes and subtracted at the corresponding off-diagonal nodes; thus, constructing the complex matrix that activates the calculation of the pairs $\{P_i, Q_i\}$ and their derivatives.

From this data, the AC power flow model is drawn up. Let $\Gamma = \{1, \dots, N\}$ be the set of buses PV, PQ, and Slack. At each bus i with voltage magnitude V_i and angle θ_i , and denoting $\theta_{ij} = \theta_i - \theta_j$, the active and reactive power delivered/absorbed by the bus are defined as

$$P_i(V, \theta) = \sum_{j=1}^N V_i V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) \quad i \in \Gamma \quad (1)$$

$$Q_i(V, \theta) = \sum_{j=1}^N V_i V_j (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}) \quad i \in \Gamma \quad (2)$$

These expressions are based on the decomposition of the admittance matrix $Y_{bus} = G + jB$, where G_{ij} and B_{ij} are the real and imaginary components, conductance and susceptance, respectively. In operational terms, P_i represents the balance of injected active power minus demand at bus i, and Q_i the reactive power balance; both result from the superposition of contributions from the voltages of the other buses weighted by the couplings G_{ij} and B_{ij} . The functions (1)–(2) are nonlinear in θ and V , which motivates the use of Newton-Raphson as a solution method in steady states. From the point of view of numerical treatment, these equations ensure continuous differentiability in the domain of interest and, therefore, allow the construction of a well-defined Jacobian that is amenable to efficient factorization.

The AC flow waste vector is written as $g(y) = [g_P; g_Q]$ with $y = [\theta; V]$, and, for each bar i, the following is defined

$$g_{P,i} = P_i^{esp} - P_i(V, \theta), \quad g_{Q,i} = Q_i^{esp} - Q_i(V, \theta) \quad (3)$$

where P_i^{esp} and Q_i^{esp} are the specified power (net injection: generation – demand) in p.u. at bus i.

In PV bars, $V_i = V_i^{esp}$ they are fixed and, consequently, the residue $g_{Q,i}$ does not appear in the active list (except for the limits treatment of Q that might force a temporary change to type PQ, which is

not activated here). In Slack, V and θ are taken as references, and no associated equations are shown. The Jacobian matrix of the linearized system takes the block form

$$J(y) = \begin{bmatrix} \frac{\partial g_p}{\partial \theta} & \frac{\partial g_p}{\partial V} \\ \frac{\partial g_Q}{\partial \theta} & \frac{\partial g_Q}{\partial V} \end{bmatrix} \equiv \begin{bmatrix} H & N \\ M & L \end{bmatrix} \quad (4)$$

Block H collects derivatives of active power with respect to angles ($\partial P/\partial \theta$), block N collects derivatives of active power with respect to voltages ($\partial P/\partial V$), block M collects derivatives of reactive power with respect to angles ($\partial Q/\partial \theta$), and block L collects derivatives of reactive power with respect to voltages ($\partial Q/\partial V$). The conditioning of the linear system $J \Delta y = g$ depends crucially on the structure and scale of these blocks, that is, block H governs the sensitivity of P to angular variations, and L does the local tensional stiffness; so, the importance of consistent per-unit scaling and also the correct modeling of shunts and taps when they exist. The Newton update can be summarized as follows:

$$y^{k+1} = y^k - J(y^k)^{-1} g(y^k) \quad (5)$$

supplemented, when necessary, with a damping factor or linear search

$$y^{k+1} = y^k - \lambda J(y^k)^{-1} g(y^k), \quad \lambda \in (0,1] \quad (6)$$

and with dual stopping criteria

$$\|g(y^k)\|_\infty \leq \varepsilon_g, \quad \|y^{k+1} - y^k\|_2 \leq \varepsilon_y \quad (7)$$

In the analyzed subset, $\varepsilon_g = \varepsilon_y = 10^{-8}$ values are typically used, which are suitable for ensuring that the residue of the algebraic closure is less than the overall integration error when coupled with the dynamic part. In a purely stationary context, these thresholds guarantee a robust numerical solution with few iterations, provided that the initial point is not excessively far from the solution and the Jacobian avoids semi-singular regions.

In addition, frequency dynamics are considered using the per-unit swing model, with inertia H_i , damping D_i , and mechanical power $P_{m,i}$ and electric $P_{e,i}$ of each machine i :

$$\frac{2H_i}{\omega_s} \frac{d\omega_i}{dt} = P_{m,i}(t) - P_{e,i}(t) - D_i(\omega_i(t) - \omega_s) \quad (8)$$

Defining $\Delta\omega_i = \omega_i - \omega_s$,

$$\frac{d\Delta\omega_i}{dt} = \frac{\omega_s}{2H_i} (P_{m,i}(t) - P_{e,i}(t) - D_i \Delta\omega_i(t)) \quad (9)$$

The aggregate frequency of the generating system is obtained as an inertia-weighted average:

$$\omega_{agg}(t) = \frac{\sum_i H_i \omega_i(t)}{\sum_i H_i}, \quad f_{agg}(t) = \frac{\omega_{agg}(t)}{2\pi} \quad (10)$$

To model the primary control, the first-order linear regulator with time constant is adopted, τ_g , and droop R :

$$\tau_g \frac{dP_{m,i}}{dt} + P_{m,i} = P_{m,i}^* - R \Delta\omega_i \quad (11)$$

Equations (8)–(11) describe a system of ODEs coupled by the electrical power $P_{e,i}(t)$, which in turn depends on V , θ via (1)–(2). Time integration is performed using fourth-order Runge – Kutta (RK4). If x denotes the vector of dynamic states (including at least $\Delta\omega_i$ and, when applicable, $P_{m,i}$), and $\dot{x} = f(x, t)$, the discretization with step h is summarized by

$$k_1 = f(x_n, t_n), \quad k_2 = f(x_n + \frac{h}{2}k_1, t_n + \frac{h}{2}), \\ k_3 = f(x_n + \frac{h}{2}k_2, t_n + \frac{h}{2}), \quad k_4 = f(x_n + hk_3, t_n + h) \quad (12)$$

$$x_{n+1} = x_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4) \quad (13)$$

The local truncation error of RK4 is $e_{loc} = Ch^5 + O(h^6)$, and the global error is $O(h^4)$. A practical criterion for selecting h versus characteristic times governed by $(2H_i / D_i)$ is

$$h \leq \min \left\{ h_0, \beta \cdot \frac{2H_i}{\omega_s D_i} \right\}, \quad \beta \in (0,1) \quad (14)$$

so that the time integration adequately solves for the damped dynamics. This criterion is combined with the tolerances of the algebraic closure ($\varepsilon_g, \varepsilon_y$) to ensure that the NR residue does not dominate the overall error of the coupled solution.

The RK4–NR coupling is implemented as a step-by-step closure. Given $t_n \rightarrow t_{n+1} = t_n + h$ the following is performed:

(i) a dynamic prediction with RK4, with P_e evaluated in t_n :

$$x_{n+1}^{pred} = \Phi_{RK4}(x_n; P_m^n, P_e(V^n, \theta^n), D, h) \quad (15)$$

(ii) an algebraic correction with NR for $y = [\theta; V]$ that cancels the waste $g(y) = 0$ using the predicted dynamic state, and is solved by NR:

$$g(y_{n+1}; x_{n+1}^{pred}) = 0 \quad (16)$$

(iii) an electrical dynamic consistency update

$$x_{n+1} = \Psi(x_{n+1}^{pred}, y_{n+1}) \quad (17)$$

with the power shut off at the end of the step

$$P_{m,i}(t_{n+1}) - P_{e,i}(V(t_{n+1}), \theta(t_{n+1})) - D_i \Delta \omega_i(t_{n+1}) = 0 \quad (18)$$

This closure ensures that the electrical power used in the swing equation is consistent with the network state enforced by the NR at t_{n+1} , and that the relative kinetic energy of the machines decreases in the damped regime. In fact, if the Lyapunov candidate is defined for each machine

$$v_i(\Delta \omega_i) = \frac{H_i}{\omega_s} \Delta \omega_i^2 \quad (19)$$

then

$$\frac{dv_i}{dt} = \Delta \omega_i (P_{m,i} - P_{e,i} - D_i \Delta \omega_i) \approx \approx -D_i \Delta \omega_i^2 \leq 0 \quad \text{if } P_{m,i} \approx P_{e,i} \quad (20)$$

and in discrete, with residue ρ after NR,

$$v_{i,n+1} - v_{i,n} \approx -D_i \Delta \omega_{i,n}^2 h + O(h^2) + O(\rho) \quad (21)$$

Lyapunov's argument is used here heuristically unless two explicit hypotheses are met: (i) the residue of NR satisfies $\|g(y_{n+1})\| \leq \bar{\rho}$ by the thresholds of (7), and (ii) the step h respects (14). Under these conditions, a discrete bound of the type $v_{n+1} - v_n \leq -Dh \|\Delta \omega_n\|^2 + Ch\bar{\rho} + O(h^2)$ can be shown; otherwise, (21) must be interpreted as an a posteriori estimate that guides the energetic-dynamic coherence of the coupling (15)–(18).

Thus, the practical stability of the coupled scheme is linked to the control of the NR residue (by means of $\varepsilon_g, \varepsilon_y$) and the selection of a step h compatible with (14). In situations where the coupling between dynamics and algebra is particularly strong, a strong coupling by sub-stages can be operated, resolving the AC flow in the intermediate stages of RK4 so that P_e is coherent in k_2, k_3 and k_4 :

$$\begin{aligned} \text{for } k_2 : g\left(y; x_n + \frac{h}{2}k_1\right) &= 0, \\ \text{for } k_3 : g\left(y; x_n + \frac{h}{2}k_2\right) &= 0, \\ \text{for } k_4 : g\left(y; x_n + \frac{h}{2}k_3\right) &= 0 \end{aligned} \quad (22)$$

This mode increases the cost (one factorization per

sub-stage), but reduces the algebraic dynamo coherence error at the intermediate points of the step.

To document the stationary portion of the analysis on the 17-bar subset, the following result tables reproduce the format and values. The information is organized following the sequence of a typical NR calculation on the 17-bar subset, maintaining Slack/PV, and with tolerances $\varepsilon_g = \varepsilon_y = 10^{-8}$. The tables show: initial mismatch (Table 3), tolerances (Table 4), Jacobian condition number in the first iteration, and an illustrative 10×10 submatrix for analyzing scales (Table 5); the correction vector Δx in iteration 1 (Table 6).

Table 3. Initial mismatch, pu.

Index	g_i (p.u.)
0	-1.550000e+00
1	-6.220000e+00
2	-3.140000e+00
3	-3.016000e+00
4	-2.240000e+00
5	-1.350000e+00
6	-2.308000e+00
7	-4.220000e+00
8	-2.635000e+00
9	4.080000e-01
10	-2.710000e+00
11	-1.750000e-01
12	-1.971000e+00
13	-1.511000e+00
14	-3.020000e+00
15	-3.210000e+00
16	1.088700e+00
17	-1.721000e-01
18	4.076000e-01
19	1.458800e+00
20	-1.878000e-01
21	9.540000e-02
22	-6.171000e-01
23	-6.902000e-01
24	1.162094e+03
25	-9.322000e-01
26	-8.600000e-01
27	-8.934000e-01
28	-9.000000e-02

This table illustrates the initial magnitude of the mismatches in pu. and, therefore, the size of the expected corrections in the first iteration, conditioned in turn by the Jacobian condition.

Table 4. Convergence tolerances.

Criterion	Value
$\ g\ _{\infty}$ objective	1e-8
$\ \Delta x\ _2$ objective	1e-8

These values make the algebraic closure error negligible compared to the overall integration error of RK4 when frequency dynamics are coupled.

Table 5. 10×10 submatrix of J at iteration 1.

123.9 5	82.36 4	0	0	0	0	0	0	0	0
82.36 4	152.1 4	69.78	0	0	0	0	0	0	0
0	69.78	121.1 5	51.36	0	0	0	0	0	0
0	0	51.36	185.5 5	67.31	0	0	0	0	0
0	0	0	67.31	516.7 2	365.8 5	0	83.55	0	0
0	0	0	0	365.8 5	519.0 9	78.86	0	0	0
0	0	0	0	0	78.86	301.9 8	223.1 2	0	0
0	0	0	0	83.55	0	223.1 2	335.6 2	28.9 4	0
0	0	0	0	0	0	0	28.94	72.3 5	0
0	0	0	0	0	0	0	0	0	480.6 5

An extract of J is shown that allows for the inspection of scales and signs in partial derivatives. It is an illustrative 10×10 submatrix, specifically the first 10 rows and first 10 columns of J, corresponding to the first 10 angular variables θ of non-Slack bars. This makes the pattern shown (H diagonal and nearby H_{ij} couplings) representative of the Jacobian structure at the flat start. For example, in the first few rows, raised diagonal terms and couplings consistent with derivatives of P with respect to θ and of Q with respect to V appear. This submatrix supports the consistency of the Y_{bus} assembly with the data in Table 1 and Table 2 in a qualitative way.

Table 6. Correction vector Δx in iteration 1.

Variable	Δx
θ_1	1.88347433e-02
θ_2	-5.44781399e-02
θ_3	1.68436893e-02
θ_4	-2.97431190e-02
θ_5	4.50111962e-03
θ_6	-4.34156365e-03
θ_7	2.76275627e-03
θ_8	-1.23889590e-02
θ_9	-3.24153813e-02
θ_{10}	2.88439257e-02
θ_{11}	-2.55882758e-02
θ_{12}	2.60371263e-02
θ_{13}	-3.19166325e-02
θ_{14}	1.93488863e-02
θ_{15}	-1.39242974e-02
θ_{16}	-1.49528285e-02

V_1	3.02809314e-02
V_2	8.48862251e-03
V_3	2.06374500e-02
V_4	1.21459932e-02
V_5	4.19680547e-03
V_6	2.64154987e-03
V_7	1.06141748e-03
V_8	-4.35989397e-04
V_9	1.64296356e-02
V_11	-1.24208257e-02
V_12	-2.03305567e-02
V_15	-2.45637192e-02
V_16	-2.56017007e-02

Once we have used the NR method in the stationary part, the entire framework is ready to couple the state calculation with the frequency dynamics. Analyzing the different parts, the term $P_{e,i}(t)$ that appears in (8)–(11) is obtained, at each step t_{n+1} , by evaluating (1)–(2) with $V(t_{n+1})$ and $\theta(t_{n+1})$ already corrected by NR, according to (16). Consistency is enforced through closure (18). This expresses the balance of mechanical, electrical, and damping power per machine at the end of each step. If a strong coupling is desired, (22) is solved within the sub-steps of RK4. To ensure that the residue of the algebraic closure does not disturb the temporal accuracy, the following tolerances are recommended: $\varepsilon_g \in [10^{-8}, 10^{-10}]$, $\varepsilon_y \in [10^{-8}, 10^{-10}]$ for NR, and a control of the step h around $10^{-3} - 2 \times 10^{-2}$ s in transient studies, modulating it according to (14).

4 Interpretation of the Results

The results that we obtain from the analysis of the subset of 17 buses allow for a deep understanding of, on the one hand, the behavior of the stationary model and, on the other hand, the dynamic-algebraic coupling between Newton-Raphson (NR) and fourth-order Runge-Kutta (RK4). The iteration sequence, Jacobian values, angle and voltage corrections, mismatch behavior, and the evolution of condition metrics reveal the internal structure of the system, its sensitivities, and the numerical stability of the solution process.

A key point to note is the magnitude of the initial mismatches. These appear to be conditioned by the significant loads at buses 3, 4, 7, 8, and especially 17. From the first iteration, the flat state exhibits a large maximum residual, with $|g|_{\infty} = 6.2200$ (iteration 1); this is consistent with the reactive profile and the shunt distribution embedded in the network model. These features explain the initial angle corrections and the sensitivity of PQ- bus voltages observed after the

first NR step. The presence of significant flows from the Slack bus to unevenly loaded buses contributes to these initial corrective oscillations.

The Jacobian structure of the first iteration has high diagonal values in the blocks related to $\partial P/\partial \theta$ and $\partial Q/\partial V$. This typically indicates that the system maintains angular and voltage stiffness, a common characteristic of transmission networks. The high diagonal values in the L block can suggest a significant contribution from taps and capacitive coupling; these values and their interpretation are consistent with the internal parameters of the line. Furthermore, the 10×10 submatrix shows that the dominant couplings occur between topologically close buses; these buses are connected to branches with low resistance but significant susceptance.

A consequence of the solution process is that the NR method initially adjusts the angles before modifying the voltages, to a considerable extent. Taken together, these observations justify the use of damping when the step input tends to overshoot under high Q–V coupling.

The angle corrections in the first iteration already show adjustment directions, which are consistent with expectations. That is, the buses with higher demand and located further electrically from the Slack tend to undergo more noticeable corrections of varying sign, and it depends on the distribution of flows among alternative paths. On the other hand, the voltage corrections follow a typical pattern for systems where voltage stiffness depends largely on local reactive compensation and the distribution of reactive loads: the PQ buses are adjusted, while the PV buses remain fixed at their magnitude.

If we continue with the iterations progress, a different phenomenon appears: the Jacobian condition number increases significantly, reaching very high values (on the order of 10^{15}) in the fourth iteration. In this case, the condition number follows from $\text{cond}_2(J) \approx 6.97 \times 10^1$ (iteration 1) to 7.00×10^1 (iterations 2, 3, and 4), which can indicate that the operating point toward which the system converges exhibits appreciable sensitivity. It can be associated with combinations of loads, reactances, and voltages that generate quasi-singularity zones in the Jacobian. This phenomenon is not unusual in subsets of real systems, when flat states are used as the initial condition; in many cases, the Jacobian converges toward physically valid points, but located in regions with high local sensitivity.

The angle trajectories throughout the iterations show non-monotonic behavior, with oscillations that can intensify as sensitivity increases. To maintain a monotonic decrease of the residual, a damping factor of $\alpha = 0.10$ is used in the first iteration (while

accepting $\alpha = 1.00$ in iterations 2, 3, and 4), which aligns with the emergence of a more conditioned Jacobian and avoids overcorrections.

Regarding the state evolution, angle updates concentrate along paths feeding heavily loaded buses; meanwhile, voltages at PQ nodes reflect local reactive behavior that is now markedly less stiff. A representative case is bus 9, where the voltage remains close to $V_9 \approx 1.001643$ p.u. in the first two iterations.

On the other hand, comparing the voltage evolution in consecutive iterations reveals that the network exhibits a very demanding reactive profile, which has direct implications for coupling with the frequency dynamics. In particular, a system in which voltages are extremely sensitive to angular variations can induce dynamic instabilities if the coupling between P_e and $\Delta\omega$ is not handled with sufficient precision. Dynamic closure (that is, enforcing at each step that $P_m - P_e - D\Delta\omega = 0$) is particularly susceptible to these sudden variations in P_e , which is why strong coupling (NR at each substage) is recommended in networks of this nature.

From a dynamic perspective, the swing equations indicate that relatively small deviations in electrical power can have amplified effects on the evolution of $\Delta\omega$ if the inertias are low or if the damping is reduced. The subset maintains inertia values that, when aggregated, allow us to interpret that the added frequency would have a significant variation if the electrical fluxes feeding the evaluation of P_e are not adequately stabilized.

Finally, the set of tables suggests that NR flow is more prone to encountering regions of singularity or ill-conditioning. In a complete system, the interconnection between bars usually provides additional stiffness; in a cut-off, that stiffness is lost. This observation is consistent with the appearance of the very high condition number in iteration 4 and provides context for the explanatory value of the tables: they serve as a clear example of the nonlinear and highly sensitive nature of the problem.

5 Conclusion

The study offers several conclusions regarding the numerical behaviour of the 17 bus subset and the usefulness of the coupled RK4–NR framework for analysing complex electrical networks. The network shows moderate Q–V stiffness compatible with stable Newton–Raphson (NR) iterations and with conditioning that remains within a manageable range throughout the solve. In particular, the condition number of the Jacobian settles around $\text{cond}_2(J) \approx 7.0 \times 10^1$ across iterations, which is indicative of a problem that is sensitive yet far from near singularity

regimes. This behavior aligns with the admittance construction and with the observed diagonal structure of the $L=\partial Q/\partial V$ block.

In addition, the iteration sequence of the NR method exhibits a monotonic reduction of the residual with a light amount of step control: a discrete line search accepts $\alpha = 0.10$, only in the first iteration, and then moves to $\alpha = 1.00$, in subsequent iterations, which signals that the problem is well scaled and that the initial damping is sufficient to avoid overshoot. The mismatch norm declines from $|g|_{\infty} = 6.2200$ iteration 1 to approximately 5.0497. It can reflect a consistent approach to the operating point within a few updates.

The state evolution corroborates the previous points: PQ bus voltages remain close to 1 p.u. along the iterations and do not display large excursions. A representative case is bus 9, where the voltage stays at approximately $V_9 \approx 1.001643$ p.u. during the first two iterations, illustrating that the reactive behavior is locally firm but not excessively stiff. Angle corrections concentrate on the main transfer paths feeding the most loaded areas, with magnitudes compatible with transmission level reactances and the subset's topology.

From a methodological standpoint, coupling RK4 with NR provides a robust and transparent workflow for stationary–dynamic analysis. In the present operating conditions, the standard (weak) coupling—for example, resolving the AC flow at the end of each RK4 step—appears sufficient for consistency between electrical power $P_e(V, \theta)$ and the dynamic states; strong coupling (NR also in the sub stages of RK4) can be reserved for transients with sharper sensitivities or when a tighter algebraic–dynamic coherence is required.

The objective is not to propose a new integrator, but to make explicit and auditable the dynamo–algebraic coherence step by step with the same unified notation $y=[V; \theta]$ and three verifiable anchors: (i) step coupling with $g(y_{n+1}; \tilde{x}_{n+1})=0$ via NR (15)–(16); (ii) separate error budget between time integration (criterion (14)) and algebraic closure (criteria (7)); and (iii) energy closure at the end of each step, $P_m^{n+1} - P_e(y_{n+1}) - D\Delta\omega^{n+1} = 0$ (18)–(21). Furthermore, Jacobian conditioning, illustrative submatrices, and correction vectors in a reproducible 17-bar case are documented, making it possible to replicate and audit the Q-V effect and shunts in Y_{bus} , something that is normally "encapsulated" in implicit trapezoidal plus Newton-type descriptions in commercial tools. The novelty, therefore, is methodological: a transparent and replicable protocol that defines where P_e is evaluated, how tolerances are distributed, and how

power balance is guaranteed at each step, providing materials (numbered equations, criteria, and tables) that enable reproducibility.

Finally, the results highlight the importance of accurate network modeling (loads and branching parameters). Of course, this must be accompanied by robust numerical safety measures (scaling, soft damping, and double-stop criteria) to ensure stable and interpretable solutions. The results also indicate that the use of high-precision integration methods and robust algebraic solution techniques can be extremely helpful in network analysis and management, as they not only allow for the solution of the analyzed case but also lay the groundwork for more complex studies.

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Declaration of Generative AI and AI-assisted Technologies in the Writing Process

During the preparation of this work, the author(s) utilized artificial intelligence (AI) technologies solely for the purpose of linguistic refinement and grammatical correction. Following the application of these tools, the author(s) comprehensively reviewed, verified, and edited the resulting text. The author(s) maintain full accountability and responsibility for the accuracy, integrity, and final content of the publication.

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