Quantification of Uncertainty Cost Functions for Controllable Solar Power Modeling

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Abstract: - Navigating the secheduling of generation resources of energy in power systems marked by a significant presence of renewable generation involves intricate optimization challenges. The conventional tools for resolving such challenges include programming techniques and heuristic approaches, both contingent upon a precisely articulated target function for optimization. Traditional optimization tools rely on precisely defined target functions, but the evolving landscape of power systems introduces complexity, especially with unpredictable behaviors of renewable sources. The research specifically quantifies penalty costs associated with photovoltaic (PV) generators, employing probabilistic methods for a robust mathematical analysis. The developed analytical model enhances adaptability in economic dispatch problems, considering uncertainty in decision-making. Validation using Monte Carlo simulation emphasizes uncertainty in PV generation and highlights the advantages of the proposed analytic model. The quadratic form of the model aligns coherently with simulation outcomes, contributing significantly to understanding uncertainty quantification in solar power modeling. The research aims to refine controllable solar power models, establish robust uncertainty cost functions, and improve the accuracy of economic dispatch strategies. Ultimately, this work promotes the seamless integration of solar energy into diverse and dynamic energy grids.

Key-Words: - Controllable Renewable Generation, Montecarlo Simulations, Power Systems, Uncertainty Quantification, Solar Power

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1 Introduction

The need for advanced modeling techniques in the energy dispatch optimization in power systems, particularly in the context of a substantial presence of renewable generation, is due to uncertainty quantification approaches in order to solve this optimization problem, [1]. Heuristic algorithms have served as tools to address these uncertainty complexities, in order to get the a robust energy dispatch, [2]. In these issues is relevant to both relying on precisely defined target functions for optimization and using uncertainty model techniques for handling the variability of renewable primary sources (wind speed and solar irradiation), [3]. The modern power systems in need of trace of uncertainty, with high penetration of renewables, are characterized by complexity due to variability of the sources with inherently unpredictable behaviors. In this way, the power system operator are in the need of having robust modeling strategies, [2]. Thus, this study and mathematical proposed approach intend to delve into the intricacies of controllable solar power technologies (solar generators with back-ups for guaranteed a range of power in a time instance), specifically focusing on the quantification of

penalty costs associated with photovoltaic (PV) generators in two conditions, underestimation and overestimation of the available power, [3].
A review of the existing literature and high penetartion of solar technologies in several countries, reveals a increasing recognition of the challenges posed by uncertainty in renewable energy systems with a big capacity inside of the power systems, particularly in the economic dispatch of power. Traditional optimization techniques have been employed, often relying on deterministic models that struggle to capture the stochastic nature of renewable resources, [4]. In response to this need, our study proposes a rigorous mathematical analysis that leverages the uses of probabilistic methods, aiming a contrast to traditional deterministic models, [5].
The importance of incorporating uncertainty quantification into modeling frameworks are recent advancements highlights, emphasizing the need for robust cost functions to enhance the accuracy of economic dispatch strategies, [6]. A comprehensive review of literature in the adaptability and robustness indicates a need of shift toward a paradigm integrating controllable renewable systems into economic dispatch target functions, [7], [8]. This shift aims to enhance the operator process for decision-making, considering the evolving nature of power systems with the inclusion of renewable sources, [8].
To validate our proposed approach, based in previous developments ([9], [10], [11], [12]), we draw upon Monte Carlo simulation techniques, emphasizing the uncertainty associated with PV generation, especially in the context of the whole posiibilities capacity of energy storage, [9], [10]. The proposed formulation is a new development in simulation-based validation methodologies, with an emphasis on the importance of aligning analytical frameworks with simulation outcomes, [11]. Our study presented in this paper contributes to this discourse by presenting a novel analytical model, grounded in a uniform power distribution (it is an extension of the study presented in [12], with a variation of the ranges of the scheduled power), and validating its performance against Monte Carlo simulations. In this way, section 2 presents the analytical development; section 3 depicts the validation and application calculating the energy to be stored to balance demand and solar generation, and section 4 draws a discussion and conclusion.

2 Controllable Photo-voltaic Cost Function-Analytical Development

The mathematical uncertainty cost functions considering uniform distributions for solar generators are derived considering a scheduled power (P_s) in a determined range, normally from an arbitary maximum and minimum value. In [12], it is considered a range between a minimum power and a maximum power, assuming that the probability density function for the available generated power $f[P]$ (from the technology used to convert primary source (solar irradiation) in electric power) is defined with an uniform distribution:

$$f[P] = \begin{cases} \frac{1}{P_{max}-P_{min}} & \text{for } P_{min} \le P \le P_{max}, \\ 0 & \text{for } P < P_{min} \text{ or } P > P_{max} \end{cases} \tag{1}$$

It is used a linear function in order to handle the penalty cost due to an underestimation $y = C_u[P] = C_u(P - P_s)$ (the same would be in the overestimation case). In this way, it is possible to determine the corresponding expected penalty cost function as follows:

$$E[y] = \int_{-\infty}^{\infty} y f(y) dy \tag{2}$$

$$\rightarrow E[C_u(P)] = \frac{C_u}{P_{max}-P_{min}} \left(\frac{P_s^2}{2} - P_s P_{max} + \frac{P_{max}^2}{2} \right) \tag{3}$$

On the other hand, the expected cost function for the overestimation with $z = C_o[P] = C_o(P_s - P)$ for controllable solar generation can be obtained:

$$E[z] = \int_{-\infty}^{\infty} z f(z) dz \tag{4}$$

$$\rightarrow E[C_o(P)] = \frac{C_o}{P_{max}-P_{min}} \left(\frac{P_s^2}{2} - P_s P_{min} + \frac{P_{min}^2}{2} \right) \tag{5}$$

The previous results (from [12]) make it possible to calculate the expected uncertainty cost function (UCF), which describes a remarkable quadratic pattern, something useful for conventional economic dispatch softwares.

$$\rightarrow E[UCF] = E[C_u(P)] + E[C_o(P)] \tag{6}$$

The need to broaden and continue development of this framework of analysis come from the understanding that scheduled power is complex. Recognizing that scheduled power (P_s) is not a single variable but rather occurs in a variety of operational states, the study presented in this section intends to improve our comprehension by grouping P_s into three different areas (in [12], was used only one area). The study conducted extends beyond the conventional distinction between Pmin and Pmax to encompass areas where P_s is (i) less than P_{min}, (ii) between P_{min} and P_{max}, and (iii) beyond P_{max} but less than P_{min} plus the back Battery Capacity $P_{batterycapacity}$.

In this way, we expanded the work presented in [12], in the following way:

- We consider that the scheduled power (Ps) not only can vary from Pmin to Pmax, but instead the scheduled power could be in three different regions:
  Region 1, $P_s < P_{\min}$
  Region 2, $P_{\min} < P_s < P_{\max}$
  Region 3, $P_{\max} < P_s < (P_{\min} + P_{\text{batterycapacity}})$

In this new development, we update the equations to be presented in three regions:

- One equation (there is not overestimation condition) for Region 1 f_1: For this region in which Ps is less than Pmin, the UCF is denoted by:

$$f_1(P_s) = E[\text{UCF}] \rightarrow where\{ \begin{matrix} P_s & \leq P_{\min} \\ P & \in (P_{\min}, P_{\min}) \end{matrix} \quad (7)$$

$$E[\text{UCF}] = \int_{P_{\min}}^{P_{\max}} (C_u(P - P_s)I(P > P_s))\, dP \quad (8)$$

$$= \frac{C_u}{P_{max} - P_{min}} \int_{P_{\min}}^{P_{\max}} (P - P_s)\, dP \quad (9)$$

$$= \frac{C_u}{P_{max} - P_{min}} (\frac{P_s}{2}\Big|_{P_{min}}^{P_{max}} - P_s P_{max}\Big|_{P_{min}}^{P_{max}}) \quad (10)$$

$$= \frac{C_u}{P_{max} - P_{min}} (\frac{P_{max}^2}{2} - \frac{P_{min}^2}{2} - P_s P_{max} + P_s P_{max}) \quad (11)$$

$$= \frac{C_u}{P_{\max} - P_{\min}} \left(\frac{(P_{\max} + P_{\min})(P_{\max} - P_{\min})}{2} \right. \\ \left. - P_s P_{\max} + P_s P_{\min}\right) \quad (12)$$

$$= C_u(\frac{P_{max} + P_{min}}{2} - P_s) \quad (13)$$

- Two equations (overestimation and understimation conditions) for Region 2 f_2:

$$f_2(P_s) = E[\text{UCF}] \rightarrow where\{ \begin{matrix} P_{min} \leq P_s \leq P_{max} \\ P \in (P_{\min}, P_{\max}) \end{matrix} \quad (14)$$

$$= \frac{C_o}{P_{max} - P_{min}} (\frac{P_s^2}{2} - P_s P_{max} + \frac{P_{max}^2}{2}) \quad (15)$$

$$+ \frac{C_u}{P_{max} - P_{min}} (\frac{P_s^2}{2} - P_s P_{min} + \frac{P_{min}^2}{2}) \quad (16)$$

this is from

Equation A:

$$E[C_o] = \int_{P_{\min}}^{P_{\max}} (C_o(P_s - P)I(P < P_s))\, dP \quad (17)$$

$$= \frac{C_o}{P_{max} - P_{min}} (P_s.P\Big|_{P_{min}}^{P_s} - \frac{P^2}{2}\Big|_{P_{min}}^{P_s}) \quad (18)$$

$$= \frac{C_o}{P_{max} - P_{min}} (P_s^2 - P_{s,min}^2 - \frac{P_s^2}{2} - \frac{P_{min}^2}{2}) \quad (19)$$

$$= \frac{C_o}{P_{max} - P_{min}} (\frac{P_s^2}{2} - P_s P_{min} - \frac{P_{min}^2}{2}) \quad (20)$$

Equation B:

$$E[C_u] = C_u \int_{P_{\min}}^{P_{\max}} (P - P_s)(I(P > P_s))\, dP \quad (21)$$

$$= \frac{C_u}{P_{max} - P_{min}} \int_{P}^{P_{\max}} (P - P_s)\, dP \quad (22)$$

$$= \frac{C_u}{P_{max} - P_{min}} (\frac{P^2}{2}\Big|_{P_{max}}^{P_s} - P_s.P\Big|_{P_s}^{P_{max}})$$

$$= \frac{C_u}{P_{max} - P_{min}} (\frac{P_{max}^2}{2} - \frac{P_s^2}{2} - P_s P_{max} + P_s^2)$$

$$= \frac{C_u}{P_{max} - P_{min}} (\frac{P_s^2}{2} - P_s P_{max} - \frac{P_{max}^2}{2}) \quad (23)$$

- One equation (there is not underestimate condition) for Region 3 f_3:

$$f_3(P_s) = E[\text{UCF}] \rightarrow where \quad \left\{ \begin{array}{l} P_{max} \leq P_s \leq P_{min} + P_{batterycapacity} \\ P \in (P_{\min}, P_{\max}) \end{array} \right. \tag{24}$$

$$E[C_o] = \int_{P_{\min}}^{P_{\max}} (C_o(P_s - P) I(P < P_s))\, dP \tag{25}$$

$$= \int_{P_{\min}}^{P_{\max}} (P_s - P)\, dP \tag{26}$$

$$= \frac{C_o}{P_{max} - P_{min}} (P_s.P\Big|_{P_{min}}^{P_{max}} - \frac{P^2}{2}\Big|_{P_{min}}^{P_{max}}) \tag{27}$$

$$= \frac{C_o}{P_{max} - P_{min}} ((P_s P_{max} - P_s P_{min}) - \frac{P_{max}^2}{2} + \frac{P_{min}^2}{2}) \tag{28}$$

$$= \frac{C_o}{P_{max} - P_{min}} (P_s(P_{max} - P_s P_{min}) - \frac{P_{max}^2}{2} + \frac{P_{min}^2}{2})$$

$$= C_o(P_s - \frac{P_{max} + P_{min}}{2}) \tag{29}$$

3 Validation and Application: Energy to be stored to balance demand and solar generation

If the uncertainty cost functions (developed in the previous section), which has the following units (\$/hour), is multiplied by the system factor (k) that represents the inverse of the energy cost (kWh/\$), we obtain the expected value of the power injected by the solar panels (this expected value considers the probability distribution of the solar power value available in the panels). This study offers a baseline for assessing the financial effects of solar power generation by concentrating on the scheduled scenario and accounting for both the system factor (k) and uncertainty cost functions. In this way, it is possible to calculate the energy to be stored (E_b) to balance power demand (P_{demand}) and solar generation (P_{solar}) in a system:

$$E_b = -\int_0^{24} P_{demand} dt + \int_0^{24} P_{solar} dt$$

where

$$E_b = -\int_0^{24} P_{demand} dt + \int_0^{24} E[\text{UCF}] * k dt$$

It is possible to get the $E[UCF]$ and k from the previous section (analytical method) and with Montecarlo simulations. That is to say, we used a random number generator that follows a uniform distribution between a minimum and maximum value for each time instance. The Monte Carlo simulations descriptions can be described in 5 steps:

i) Solar power (green: schedule power, blue: min, red:max)

The three main accounts situations maximum power (red), scheduled power (green) and minimum power (blue) are taken into consideration. Considering the predictive modeling or operator scheduling data, the scheduled power is displayed by the green curve in the simulation. We present two possible patters (Figure 1 and Figure 2) for scheduling (green curve): A) mean value between P_{min} and P_{max}; and B) an arbitary pattern. The lower limits of solar power generation in our calculations are shown by the blue curve. This scenario takes into consideration times when there is less sunlight or unanticipated changes that cause solar panels to produce the least amount of power. The maximum solar power generation limits in our Monte Carlo simulations are shown by the red curve. In this scenario, ideal circumstances are taken into account, where solar panels provide power to the fullest extent possible.

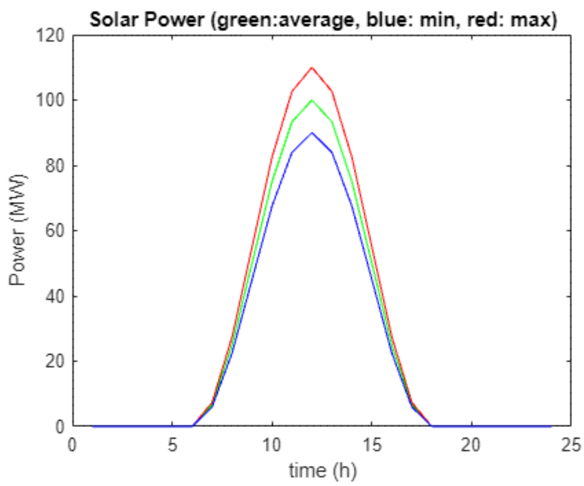

Fig. 1: Solar Power (pattern A).

ii) Monte Carlo scenarios between max and min values The study explores the complex dynamics between the maximum (red) and minimum (blue) values of solar power generation in these

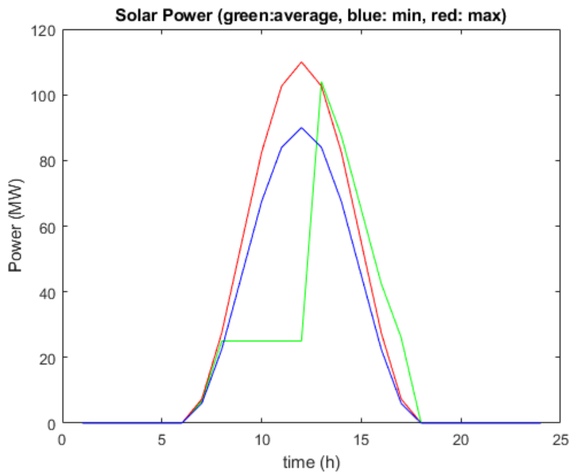

Fig. 2: Solar Power (pattern B).

Monte Carlo simulations. The study captures the subtle differences in solar power generation and their associated economic repercussions by investigating scenarios within this range. The Monte Carlo simulations carried out inside the dynamic range, in contrast to the discrete average, minimum, and maximum scenarios, take into account a spectrum of solar power values between the maximum (red) and minimum (blue) criteria (Figure 3).

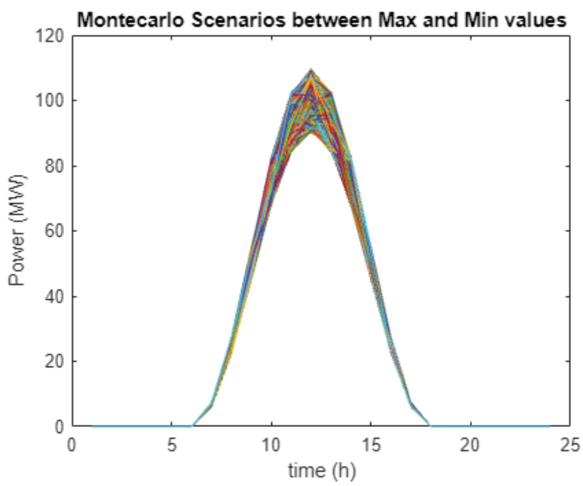

Fig. 3: Montecario scenarios between Max and Min values.

iii) Random Escenario of Solar generation The use of random solar generation scenarios in Monte Carlo simulations has led to a more realistic and flexible method of modeling controlled solar electricity, [3]. Decision-makers can increase the dependability and financial sustainability of renewable energy systems by accepting and valuing the unpredictability of solar power generation, [4]. This methodology generates completely random situations, embracing the stochastic character of solar power generation instead of focusing on specified average, minimum, or maximum values (Figure 4).

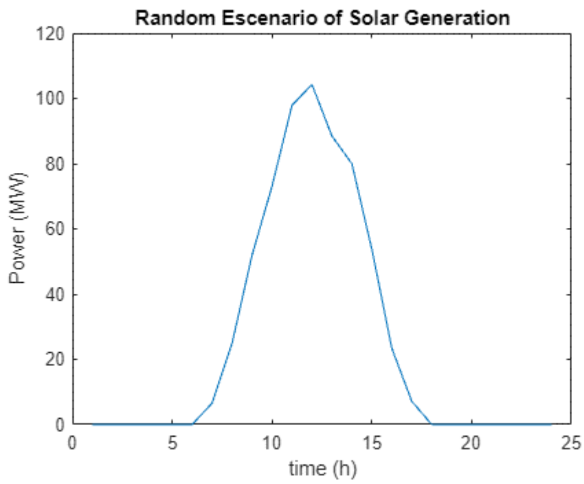

Fig. 4: Random scenario of solar generation.

iv) Expected energy from Montecarlo scenarios When computing predicted energy in Monte Carlo simulations of solar power generation, one must estimate the average or mean power output value over a number of simulation iterations, and it is possible to get the histogram (Figure 5). Because the model is stochastic, every time the Monte Carlo simulation runs, a new solar power scenario is generated, [3]. Next, by adding together all of these iterations' power numbers and figuring out their average, the predicted energy is found. With regard to the predicted average power output, this expected energy value is a useful indicator for decision-makers because it takes into account the inherent unpredictability in solar power generation. Mathematically, the expected energy (Expected) can be calculated as the mean of the power values across all simulation iterations.

v) Constant k to convert UCF in Ps The System Factor, which is commonly expressed as (KW hour / \$), is the inverse of the energy cost. The constant k should be adjusted based on the particular units and scale needed for the application in question, [2]. In order to meaningfully comprehend the financial consequences in a setting of energy generation and consumption, the factor of conversion k basically fills the distinction between the physical features of planned power and the economic considerations in uncertainty cost functions (UCF), [3]. The relation is termed by $P_s = k \times E[UCF]$ in which P_s represents the scheduled power, UCF represents the uncertainty

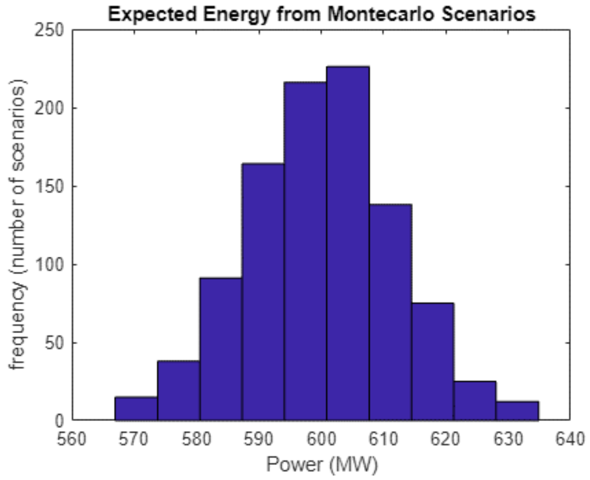

Fig. 5: Expected energy from Monetecarlo scenarios.

cost function, whereas k is the conversion constant. Each scheduling pattern (A and B) will have a different behaviour in the differente time instances. In Figure 6 and Figure 7 are showed the k constant, where it is depicted the results of the analytical calculation (red) and the montecarlos simulation (green).

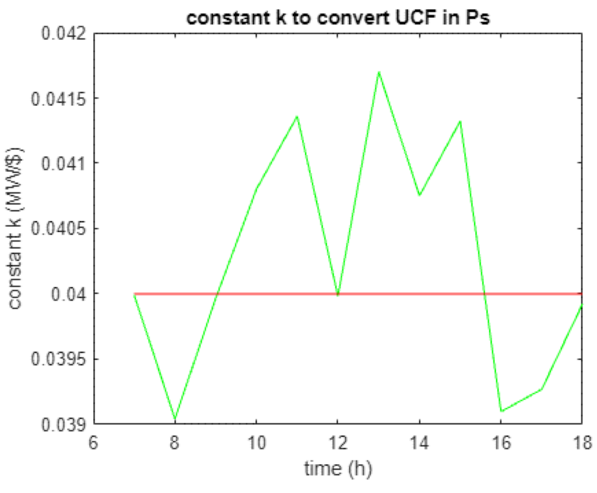

Fig. 6: Constant k to convert UCF in Ps (pattern A).

vi) Uncertainity cost functions (analytical and Montecarlo simulation) An essential tool for evaluating the financial effects of uncertainties related to variables in a system is the Uncertainty Cost Function (UCF). While Monte Carlo simulations offer a more in-depth examination of the stochastic nature of uncertainties, the analytical model offers a rapid insight into the overall functioning of the system, [3]. The specific characteristics of the system itself, the uncertainties (such solar power fluctuations) and related economic factors will determine the details of the uncertainty cost functions. Each scheduling pattern (A and B) will have a different behaviour in the differente time instances. In Figure 8 and Figure 9 are depicted the results of the UCF analytical calculation (red) and the UCF montecarlos simulation (green).

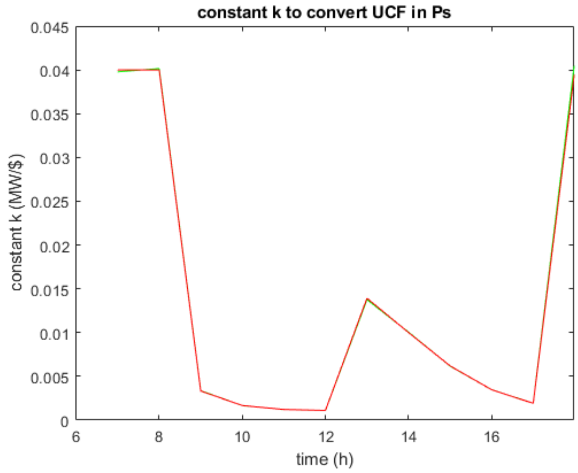

Fig. 7: Constant k to convert UCF in Ps (pattern B).

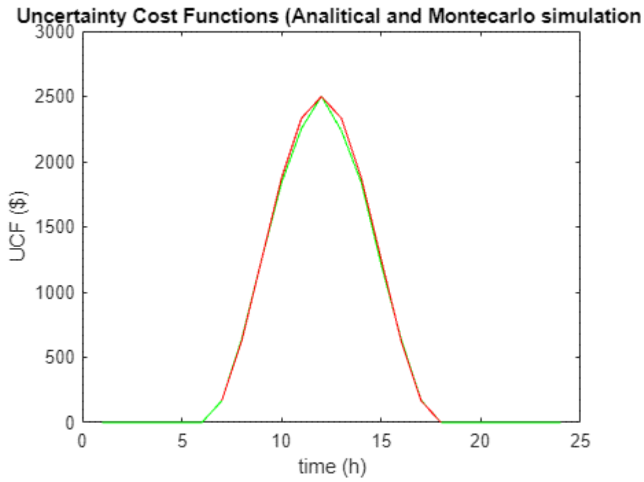

Fig. 8: Uncertianity cost function (Analytical and Monetecario simulation).

4 Discussion and Conclusion

We acknowledge the necessity to update the variance and statistical factors regulating the analytical model in conjunction with our scheduled power classification. The fluctuation, a crucial factor in determining the stability of the system, and related statistical measures need to be in line with

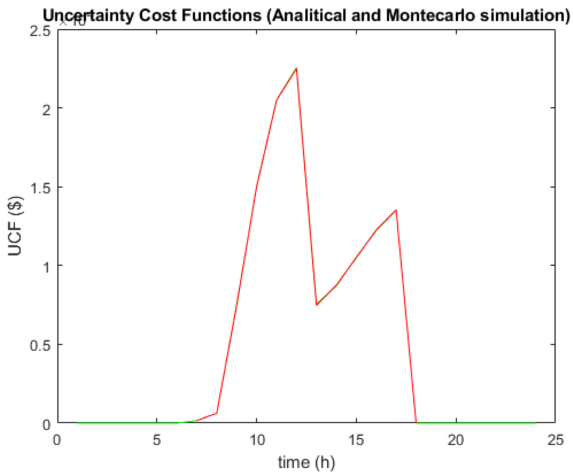

Fig. 9: Uncertianity cost function (Analytical and Monetecario simulation).

the changing face of power generation. These analytucal developments (Equations (7) through (29) are revised in light of this realization, guaranteeing that the mathematical framework will continue to be accurate and flexible enough to accommodate the dynamic conditions of contemporary power systems.

In this way, unlike the traditional model, that somewhat was considering the scheduled power within the range of P_{min} to P_{max}, this expanded approach, presented in this paper, categorizes the P_s into three major regions. In summary, the three regions will conduct the following formulation:

$$f_1(P_s) = C_u(\frac{P_{max} + P_{min}}{2} - P_s)$$

$$f_2(P_s) = \frac{C_o}{P_{\max} - P_{\min}} \left(\frac{P_s^2}{2} - P_s P_{\min} - \frac{P_{\min}^2}{2} \right) + \frac{C_u}{P_{\max} - P_{\min}} \left(\frac{P_s^2}{2} - P_s P_{\max} - \frac{P_{\max}^2}{2} \right)$$

$$f_3(P_s) = C_o(P_s - \frac{P_{max} + P_{min}}{2})$$

Keeping into consideration the likelihood of distribution of the solar power that is accessible; this computation produced the predicted value of the electricity that the solar panels will inject. The next stage was to use Monte Carlo simulations, which are strong computational methods that allow the investigation of various possibilities by introducing unpredictability and randomness.

It is difficult to optimize the distribution of energy in power networks that have a large amount of renewable generation. The unpredictability of renewable sources makes it difficult to achieve the precise target functions that conventional optimization techniques demand. Our research presents a fresh analytical model that has been verified by Monte Carlo simulations.

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