

Simplified Approach for Wind Uncertainty Cost Functions using a Mixture of Uniform Probability Distribution

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Abstract: - Wind energy generation is an individual source of energy considered in renewable energy resources. It is completely dependent on the natural process of air blown in the area or past forecasting of air blown data. It is considered an uncertain source of energy and less reliable in the context of real-time energy provisions. To make this wind energy more reliable and consistent, we are introducing a simplified approach for wind uncertainty cost functions based on uniform probability distributions and its evaluation for usage. We have developed analytical mathematics cost functions based on several assumptions and they are constructed using Weibull and Rayleigh probability distributions. The proposed method produces the required results to make this energy reliable and consistent with the distribution networks. Computing time and complexity are less as compared to the other methods to minimize the uncertainty of wind energy.

Key-Words: - Microgrids, Renewable Energy Resources, Stochastic Processes, Wind Energy, Probability Distribution, Uncertainty Cost Functions, Weibull and Rayleigh Probability Distributions.

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1 Introduction

Wind energy is an integral part of renewable energy resources having environmental impacts as compared to traditional energy resources. Wind energy resources are considered uncertain in behavior producing the factor of a stochastic process. To handle this uncertainty behavior, we need to design and develop appropriate primary methods for variable uncertainty costs, [1]. The distribution system networks have vast pressure to bear such uncertain behavior of wind energies. The exhaustive search approach is most relevant in this situation to see the iterative solutions and visualization based on several possibilities available, [2]. In this regard, the optimal power flow is required to improve the uncertainties in wind power generation and make it more reliable and available to the end customers. A constrained handling approach can be used for this purpose rather than a cost function handling approach, [3]. Being an uncertain energy generation from wind, there is a

vast impact on the charging infrastructure of electric and hybrid electric vehicles.

This uncertain energy generation can affect the power system strategies to operate it properly daily. So, we can use such strategies which are based on time of use energy and demand profile, [4]. The hierarchical coordinated charging system can improve the uncertain power produced by wind. The primary transformer is operating to minimize the energy cost in such situations where induction of energy in the power system is done near the end customer metering system. The distribution system operator is responsible for deciding where to start the charging system or to delay its available feasibility, [5]. Smart grids are playing an important role in developing and delivering to handle uncertain powers to end users, but currently, smart grids are relying on renewable energy resources. Smart grids have the capacity and capability to handle the uncertain power in normal and emergencies. Combinatorial strategy has the potential to cater the several renewable energy resources to optimize the supply and demand of

power, [6]. Energy storage systems can also be the first choice to mitigate the uncertainty of wind energy generation in recent times. It improves the microgrid's scheduling and dispatch of electricity. It also improves the operation of power systems making the grid smart in power outage of several resources. In this regard, the energy storage systems' useful life can have an important impact to decrease the cost of the system and reliability, [7].

The reliability and dispatch of power vary when using renewable energy sources, such as wind generation. We can minimize the variation cost by using the interior point methodology to make renewable energy availability linear, [8]. The real-time control of the operation of microgrids having renewable energy resources is considered an important problem for making the grid; smart. The energy produced from uncertain renewable energy resources can be challenging for the market prices of all stakeholders involved. In this regard, the strategy of parallel computing can provide the maximum power to the market for clearing transactions and prices, [9]. The supply of power needs more reliability and less uncertainty by the distribution companies to create a balance in economic and technical benefits. The fault location and fault tolerance must be precisely catered for in real time to increase the reliability of the power system. To enhance the reliability of the power distribution networks, we need to do the reliability assessment including computing time and improvements, [10].

The quality and reliability of the distribution networks are dependent on the solution of sequential network failure or reduction in dis-connectivity. For this purpose, Markov's decision process in predicting the failure can be identified. Sequential attacks or disconnection from the distribution network or transmission network can be strategically improved by reward functions by using state transition probabilities, [11]. The distribution networks are more complex while talking about the control of their operations. The distribution control strategies are more convenient and robust to handle network operations and variations. Due to the continuous variation behavior of microgrids, dynamic population games can handle variations precisely and maintain the frequency of the group of microgrids, [12].

Currently, the virtual concept of controlling and monitoring the power systems is in discussion. We can develop virtual systems based on several strategies for optimal results rather than real physical systems. Such methods upon evaluation can give optimal results for the power system's

performance, [13]. The energy policies and strategies are formulated globally to test and evaluate the existing energy resources with several existing tools. The Long-range Energy Alternatives Planning (LEAP) system tool can be used to compare and measure the performance of alternative fuels by evaluating their costs and energy generation capacities, [14]. The uncertainty of wind energy in the power systems can affect the electric and hybrid electric vehicles charging conditions. We need to optimize the power provided by the distribution systems operator and to meet the load curves produced by the vehicles. An optimal power profile can be designed to address the uncertainties produced by the wind energy vulnerability, [15].

Based on the previous literature in section I and assumptions made in section II, we have developed the analytical cost functions to deal with the uncertain behavior of wind energy generation.

2 Problem Formulation: Wind Available Power through Three Uniform Distributions

The Weibull and Rayleigh probability distributions are used for modeling the wind speed. Using the power and primary source curve of Eolic technology, it is possible to get the available power from the Eolic generation process to present it as a mixture of three probability distributions. Thus, the available power histogram can be well described by three uniform distributions for different parameters of the Weibull distribution, namely the scale factor (c) and shape factor (k). Additionally, it is well known that the Rayleigh distribution can be viewed as a special case of the Weibull distribution where the shape factor (k) is known to equal 2.

Figure 1(a) data is obtained by setting the parameters shown in Table 1(a) for the power histogram not limited to 3 uniform distributions from the wind speed histogram.

Table 1(a). Weibull- Rayleigh parameters values

Shape factor (k)	Scale factor (c)	Shape factor (k)
15.9577	sqrt (2) *sg	2

The Figure 1(b) data is obtained by setting the parameters shown in Table 1(b) for power histogram limited to 3 uniform distributions from wind speed histogram.

Table 1(b). Weibull- Rayleigh parameters values

Shape factor (k)	Scale factor (c)	Shape factor (k)
15.9577	$\sqrt{2} * sg$	2

The Figure 2(a) data is obtained by setting the parameters shown in Table 2(a) for power histogram not limited to 3 uniform distributions from wind speed histogram.

Table 2(a). Weibull- Rayleigh parameters values

Shape factor (k)	Scale factor (c)	Shape factor (k)
8	$\sqrt{2} * sg$	2

The Figure 2(b) data is obtained by setting the parameters shown in Table 2(b) for power histogram limited to 3 uniform distributions from wind speed histogram.

Table 2(b). Weibull- Rayleigh parameters values

Shape factor (k)	Scale factor (c)	Shape factor (k)
8	$\sqrt{2} * sg$	2

The Figure 3(a) data is obtained by setting the parameters shown in Table 3(a) for power histogram not limited to 3 uniform distributions from wind speed histogram.

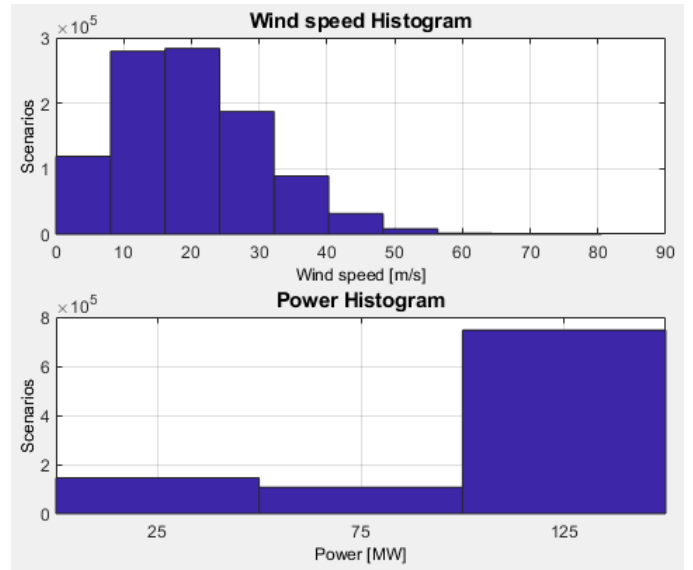


Fig. 1(b): Power histogram limited to 3 uniform distributions.

Table 3(a). Weibull parameters values

Shape factor (k)	Scale factor (c)	Shape factor (k)
16	$\sqrt{2} * sg$	4

The Figure 3(b) data is obtained by setting the parameters shown in Table 3(b) for power histogram limited to 3 uniform distributions from wind speed histogram.

Table 3(b). Weibull parameters values

Shape factor (k)	Scale factor (c)	Shape factor (k)
16	$\sqrt{2} * sg$	4

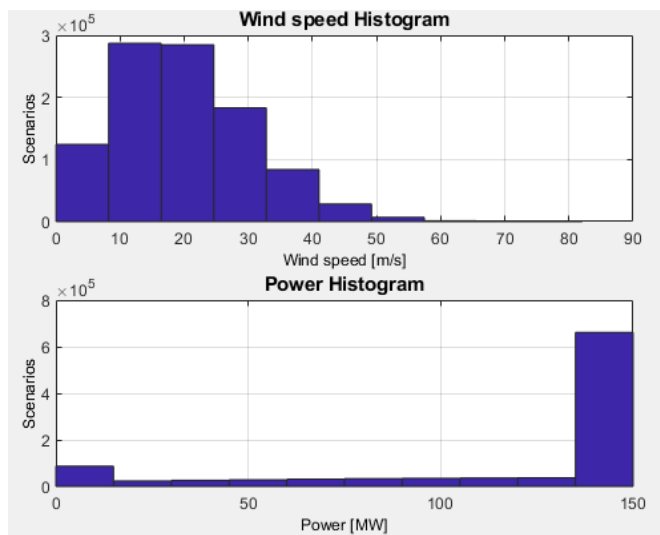


Fig. 1(a): Power histogram not limited to 3 uniform distributions

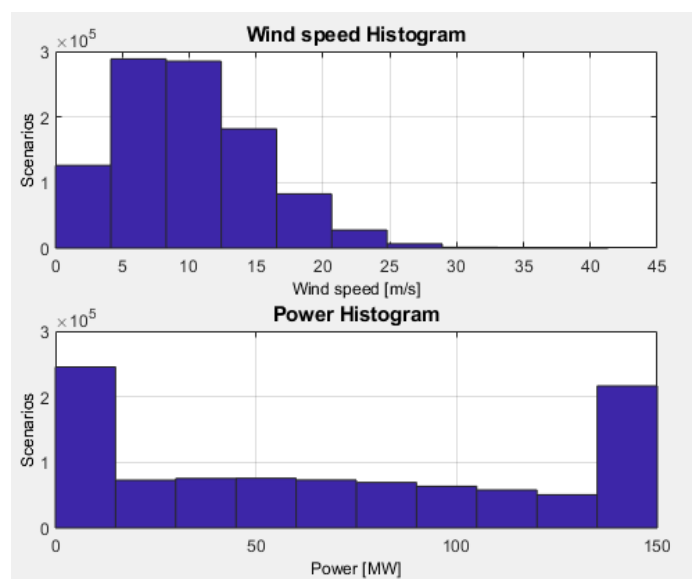


Fig. 2(a): Power histogram not limited to 3 uniform distributions

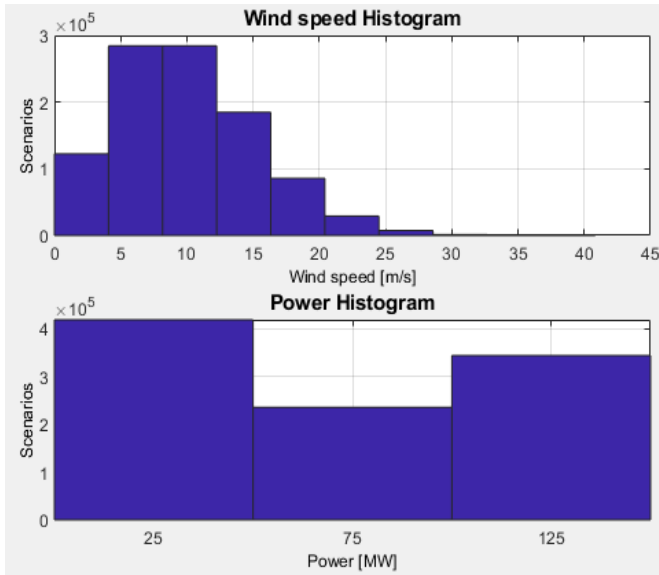


Fig. 2 (b): Power histogram limited to 3 uniform distributions

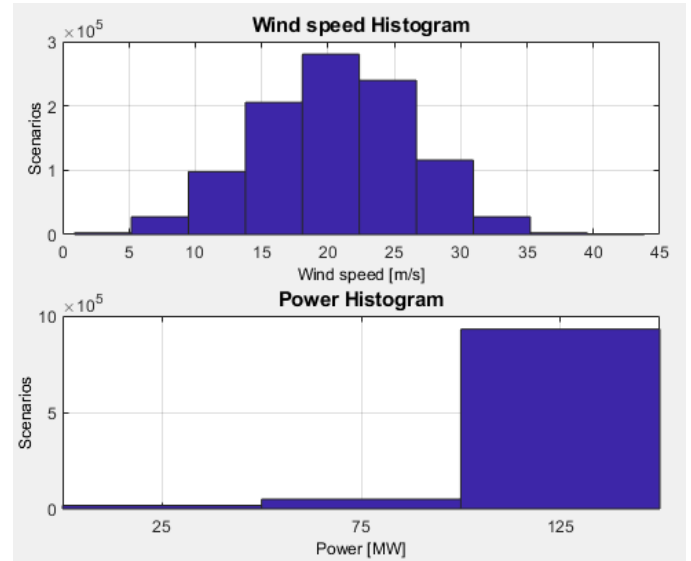


Fig. 3(b): Power histogram limited to 3 uniform distributions

3 Problem Solution: Analytical Development for Wind Uncertainty Cost Functions

3.1 Power Histogram Description

The available power (P) can be described by using the power histogram described in the previous section. The scheduled power to be handled by the operator can be categorized into three regions as follows:

- Case A; *Region I*: P_s is less than b and bigger than a

- Case B; *Region II*: P_s is less than c and bigger than b .
- Case C; *Region III*: P_s is less than d and bigger than c .

Additionally, each uniform distribution will have a ponderation w_1 , w_2 , and w_3 , respectively for each region, as depicted in Figure 4.

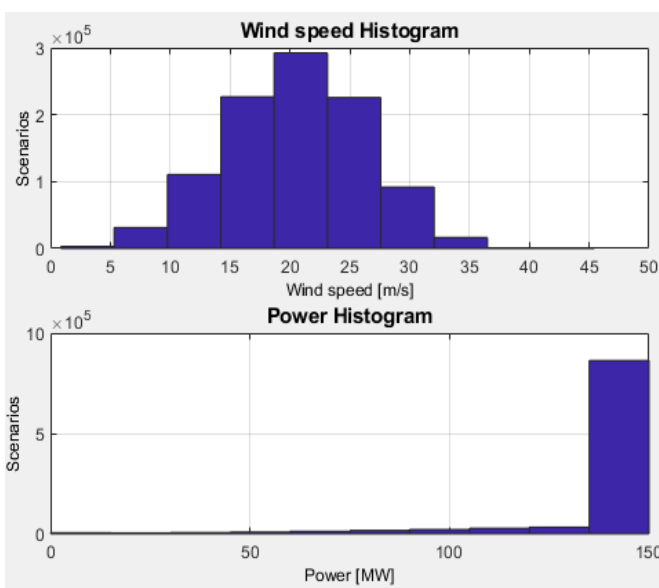


Fig. 3(a): Power histogram not limited to 3 uniform distributions

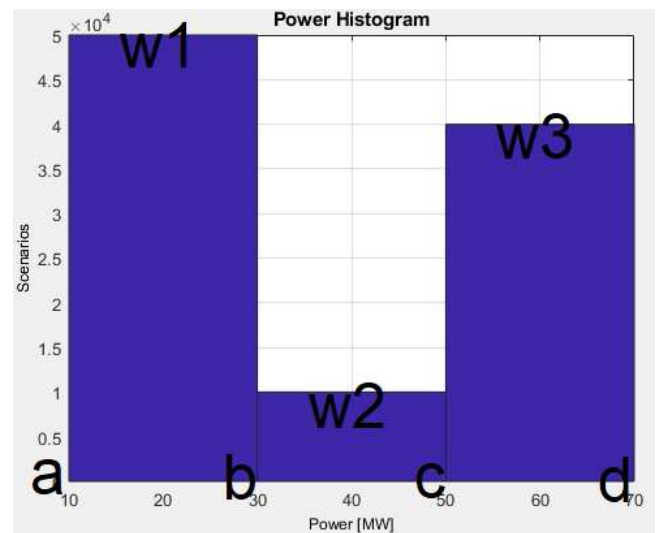


Fig. 4: Available wind power histogram limited to 3 uniform distributions

To develop a mathematical formulation of uncertainty cost functions for wind energy resources, the wind uncertainty cost function must be in terms of P_s , that is to say $f(P_s)$. It can have the following three cases based on the analytical development of a mixture of three probability distributions.

3.2 Analytical Development of UCFs for the Overestimation Part

For wind uncertainty cost function for overestimation part $f(P_s)$, the following cases can be formulated mathematically (c_0 is the cost for using a back storage system in controllable renewable sources, used to inject the power difference from the available power P to the scheduled power (P_s)):

Case A:

If $P_s < b$ then,

$$\frac{W_1}{b-a} \int_a^{P_s} C_0 (P_s - P) dP \quad (Ao-1)$$

Case B:

If $b < P_s < c$ then,

$$\frac{W_1}{b-a} \int_a^b C_0 (P_s - P) dP + \frac{W_2}{c-b} \int_b^{P_s} C_0 (P_s - P) dP \quad (Bo-1)$$

Case C:

If $c < P_s < d$ then,

$$\frac{W_1}{b-a} \int_a^b C_0 (P_s - P) dP + \frac{W_2}{c-b} \int_b^c C_0 (P_s - P) dP + \frac{W_3}{d-c} \int_c^{P_s} C_0 (P_s - P) dP \quad (Co-1)$$

The analytical development for wind uncertainty cost function for overestimation part $f(P_s)$ as follows:

Case A:

$$\begin{aligned} & \frac{W_1 C_0}{b-a} \left[P_s P \Big|_a^{P_s} - \frac{P^2}{2} \Big|_a^{P_s} \right] \\ &= \frac{W_1 C_0}{b-a} \left[P_s^2 - P_s a - \frac{P_s^2}{2} + \frac{a^2}{2} \right] \\ &= \frac{W_1 C_0}{b-a} \left[\frac{P_s^2}{2} - P_s a + \frac{a^2}{2} \right] \quad (Ao-2) \end{aligned}$$

Case B:

$$\begin{aligned} & \frac{W_1 C_0}{b-a} \left[P_s P \Big|_a^b - \frac{P^2}{2} \Big|_a^b \right] + \frac{W_2 C_0}{c-b} \left[P_s P \Big|_a^{P_s} - \frac{P^2}{2} \Big|_a^{P_s} \right] \\ &= \frac{W_1 C_0}{b-a} \left[P_s (b-a) - \frac{b^2}{2} + \frac{a^2}{2} \right] + \frac{W_2 C_0}{c-b} \left[P_s^2 - P_s b - \frac{P_s^2}{2} + \frac{b^2}{2} \right] \\ &= \frac{W_1 C_0}{b-a} \left[P_s (b-a) + \frac{a^2 - b^2}{2} \right] + \frac{W_2 C_0}{c-b} \left[\frac{P_s^2}{2} - P_s b + \frac{b^2}{2} \right] \\ &= W_1 C_0 \left[P_s - \frac{(b+a)}{2} \right] + \frac{W_2 C_0}{c-b} \left[\frac{P_s^2}{2} - P_s b + \frac{b^2}{2} \right] \quad (Bo-2) \end{aligned}$$

Case C:

$$\begin{aligned} & \frac{W_1 C_0}{b-a} \left[P_s P \Big|_a^b - \frac{P^2}{2} \Big|_a^b \right] + \frac{W_2 C_0}{c-b} \left[P_s P \Big|_b^c - \frac{P^2}{2} \Big|_b^c \right] + \frac{W_3 C_0}{d-c} \left[P_s P \Big|_c^{P_s} - \frac{P^2}{2} \Big|_c^{P_s} \right] \\ &= W_1 C_0 \left[P_s - \frac{(b+a)}{2} \right] + W_2 C_0 \left[P_s - \frac{(c+b)}{2} \right] + \end{aligned}$$

$$\frac{W_3 C_0}{d-c} \left[\frac{P_s^2}{2} - P_s c + \frac{c^2}{2} \right] \quad (Co-2)$$

3.3 Analytical Development of UCFs for the Underestimation Part

For wind uncertainty cost function for underestimation part $f(P_s)$, the following cases can be formulated mathematically (c_u is the cost for storage energy in controllable renewable sources, used to storage the power difference from the scheduled power (P_s) to the available power P):

Case A:

If $P_s < b$ then,

$$\frac{W_1 C_u}{b-a} \int_{P_s}^b (P - P_s) dp + \frac{W_2 C_u}{c-b} \int_b^c (P - P_s) dp + \frac{W_3 C_u}{d-c} \int_c^d (P - P_s) dp \quad (Au-3)$$

Case B:

If $b < P_s < c$ then.,

$$\frac{W_2 C_u}{c-b} \int_{P_s}^c (P - P_s) dp + \frac{W_3 C_u}{d-c} \int_c^d (P - P_s) dp \quad (Bu-3)$$

Case C:

If $c < P_s < d$ then,

$$\frac{W_3 C_u}{d-c} \int_{P_s}^d (P - P_s) dp \quad (Cu-3)$$

In this way, the analytical development for wind uncertainty cost function for underestimation part $f(P_s)$ as follows:

Case A:

$$\begin{aligned} & \frac{W_1 C_u}{b-a} \left[\frac{P^2}{2} \Big|_{P_s}^b - P_s P \Big|_a^{P_s} \right] + \frac{W_2 C_u}{c-b} \left[\frac{P^2}{2} \Big|_b^c - P_s P \Big|_b^c \right] + \frac{W_3 C_u}{d-c} \left[\frac{P^2}{2} \Big|_c^d - P_s P \Big|_c^d \right] \\ &= \frac{W_1 C_u}{b-a} \left[\frac{b^2}{2} - \frac{P_s^2}{2} - P_s b + P_s^2 \right] + \frac{W_2 C_u}{c-b} \left[\frac{c^2}{2} - \frac{b^2}{2} - P_s c + P_s b \right] + \frac{W_3 C_u}{d-c} \left[\frac{d^2}{2} - \frac{c^2}{2} - P_s d + P_s c \right] \\ &= \frac{W_1 C_u}{b-a} \left[\frac{P_s^2}{2} - P_s b + \frac{b^2}{2} \right] + W_2 C_u \left[\frac{c+b}{2} - P_s \right] + W_3 C_u \left[\frac{d+c}{2} - P_s \right] \quad (Au-4) \end{aligned}$$

Case B:

$$\begin{aligned} & \frac{W_2 C_u}{c-b} \left[\frac{P^2}{2} \Big|_{P_s}^c - P_s P \Big|_{P_s}^c \right] + \frac{W_3 C_u}{d-c} \left[\frac{P^2}{2} \Big|_c^d - P_s P \Big|_c^d \right] \\ &= \frac{W_2 C_u}{c-b} \left[\frac{c^2}{2} - \frac{P_s^2}{2} - P_s c + P_s^2 \right] + \frac{W_3 C_u}{d-c} \left[\frac{d^2}{2} - \frac{c^2}{2} - P_s d + P_s c \right] \\ &= \frac{W_2 C_u}{c-b} \left[\frac{P_s^2}{2} - P_s c + \frac{c^2}{2} \right] + W_3 C_u \left[\frac{d+c}{2} - P_s \right] \quad (Bu-4) \end{aligned}$$

Case C:

$$\begin{aligned} & \frac{W_3 C_u}{d-c} \left[\frac{P^2}{2} \Big|_{P_s}^d - P_s P \Big|_{P_s}^d \right] \\ &= \frac{W_3 C_u}{d-c} \left[\frac{d^2}{2} - \frac{P_s^2}{2} - P_s d + P_s^2 \right] \\ &= \frac{W_3 C_u}{d-c} \left[\frac{P_s^2}{2} - P_s d + \frac{d^2}{2} \right] \quad (Cu-4) \end{aligned}$$

4 Simulation and Validation

In Figure 5, the Monte Carlo process used original simulation results based on Weibull wind speed and uncertainty cost histograms, and they are shown. A power histogram limited to 3 uniform distributions is used to find out the cost due to overestimating or cost due to underestimating scenarios. Similarly, the other simulation results showed the injected power in the form of a mixture of three uniform distributions as in Figure 1(b), Figure 2(b) and Figure 3(b) also show similar results for cost due to overestimate or cost due to underestimate scenarios. By considering the overestimation and underestimation part (both cases), a total uncertainty cost functions histogram is shown in Figure 6.

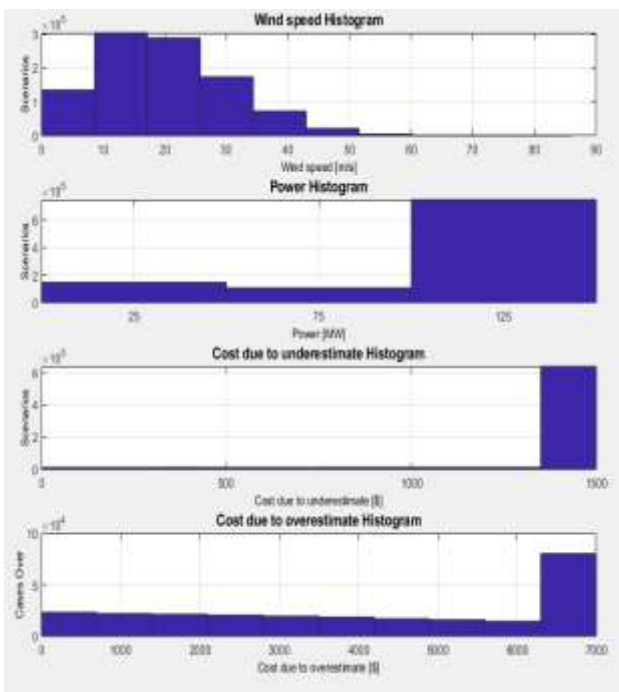


Fig. 5: Weibull wind speed and uncertainty cost

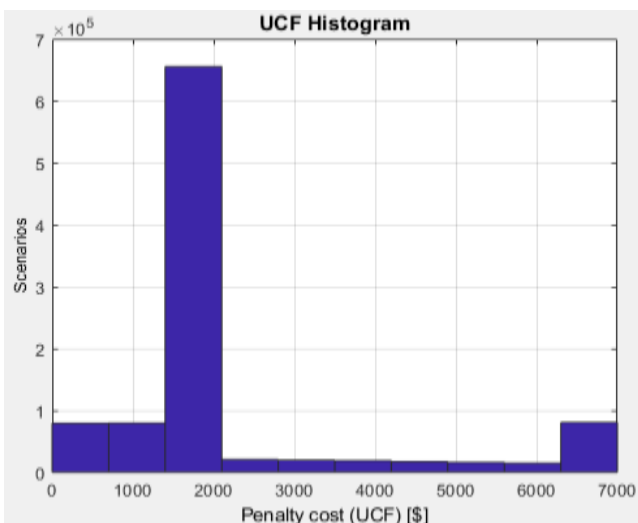


Fig. 6: UCF histogram

Similarly, we can find out the power histograms of other injected power in the form of a mixture of three uniform distributions. The Montecarlo Expected cost of the UCF is $2.0858e+03$ \$, using the Weibull distribution for the simulations. The analytical value using the analytical formulation presented in [5], is in the form of the following equations:

$$E[C_{w,u,i}(W_{w,s,i}, W_{w,i})] = \frac{c_{w,u,i}}{2} \left(\sqrt{2\pi}\rho\sigma \left(\operatorname{erf}\left(\frac{W_{w,s,i}-\kappa}{\sqrt{2}\rho\sigma}\right) - \operatorname{erf}\left(\frac{W_r-\kappa}{\sqrt{2}\rho\sigma}\right) \right) + 2(W_{w,s,i} - W_r) e^{-\left(\frac{W_r-\kappa}{\sqrt{2}\rho\sigma}\right)^2} \right) + c_{w,u,i} \left(e^{-\frac{v_r^2}{2\sigma^2}} - e^{-\frac{v_0^2}{2\sigma^2}} \right) (W_r - W_{w,s,i}) \quad (1)$$

$$E[C_{w,o,i}(W_{w,s,i}, W_{w,i})] = c_{w,o,i} W_{w,s,i} \left(1 - e^{-\frac{v_r^2}{2\sigma^2}} + e^{-\frac{v_0^2}{2\sigma^2}} + e^{-\frac{\kappa^2}{2\sigma^2\rho^2}} \right) - \frac{\sqrt{2\pi}c_{w,o,i}\rho\sigma}{2} \left(\operatorname{erf}\left(\frac{W_{w,s,i}-\kappa}{\sqrt{2}\rho\sigma}\right) - \operatorname{erf}\left(\frac{W_r-\kappa}{\sqrt{2}\rho\sigma}\right) \right) \quad (2)$$

Applying the complex equation (1) and (2), the analytical uncertainty cost functions is $2.0847e+03$ \$. Using the proposed formulation in the previous section, we get the following UCF $2.1536e+03$ \$ which is an error of 3.3 %. Finally, we developed the Montecarlo simulation with the three uniform distributions and we got the following UCF $2.1548e+03$ \$, it is an error of $5.7510e-02$ % between the Montecarlo simulation and the analytical expression of the previous section. That is to say, if we do a variation of the whole P_s possible scheduled power, we can get:

- i. The whole analytical method (equations very complex, (1) and (2))
- ii. The proposed method (simple equations)
- iii. The Montecarlo simulation (In this method the process computing time is so expensive).

5 Conclusion

Wind energy generation produces uncertain power to the distribution networks and has stochastic behavior. We can deal with such uncertain behavior of wind energy generation by designing appropriate methods to handle uncertain costs as well. The produced power histograms are described by three uniform distributions and are used to develop uncertainty cost functions for making the energy

linear. We have used a simplified approach for wind uncertainty cost functions based on a mixture of uniform probability distribution producing required results to make linear and reliable energy availability. The virtual inertia on distribution system networks and the uncertain behavior of wind energies are minimized by using this approach. Optimal power flow is improved in this proposed methodology making the wind energy more reliable and consistent for end users.

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Contribution of Individual Authors to the Creation of a Scientific Article

- L. Acero and M. Rehman carried out the simulation and the optimization.
- S. Rivera has implemented the Montecarlo Algorithm
- L. Acero, M. Rehman and S. Rivera were responsible for the Statistics.

The authors equally contributed in the present research, at all stages from the formulation of the problem to the final findings and solution.

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Conflict of Interest

The authors have no conflicts of interest to declare.

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