Uncertainty Cost Functions for Renewable Generation: A Simplified Approach using a Mixture of Uniform Probability Distribution

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Abstract: - Photovoltaic energy, wind energy, and plug-in electric/hybrid vehicles are being considered as sources and loads, reflecting the increasing importance of renewable energy resources in new microgrids. However, the stochastic behavior of variables such as wind turbine speed, solar irradiation intensity and, plug-in electric vehicle dynamics, introduces uncertainties that could affect the economic dispatch of electric power. This paper employs a mixture of uniform probability distribution (UPDs) techniques to characterize the variability of the available power from renewable energy sources. We propose a new analytical expression derived from the mixture of UPDs to calculate Uncertainty Cost Functions (UCFs), thereby assessing their impact on the economic dispatch of power. Finally, we performed Montecarlo simulations to validate our UCF methodology and its potential applicability in economic dispatch of power. The results demonstrate that our methodology accurately calculates the underestimated and overestimated costs of uncertainty power generation. This methodology holds the potential to optimize economic dispatch, thereby reducing costs and maximizing power generation from the generators.

Key-Words: - Microgrids, Renewable Energy Resources, Stochastic Processes, Economic Dispatch, Renewable Energy Resources: Photovoltaic energy, wind energy, and plug-in electric/hybrid vehicles.

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1 Introduction

Photovoltaic energy generation (PVEG), wind energy generation (WEG), and energy from plug-in electric/hybrid vehicles (PEV/HEV) introduce variable and uncertain scenarios regarding power injection or demand on the grid. In traditional power generation and dispatch, these energy generators are typically treated without specific programming in the optimization of power system operations. However, these resources or loads can be effectively modeled using probability distribution functions (PDFs), [1] and integrated into the economic dispatch of power. The introduction of PEV/HEV into the power network further complicates the uncertainty in modern power systems and smart grids. These vehicles serve as energy storage sources, loads, or power generators, exhibiting probabilistic behaviors. To model this behavior mathematically, Probability Distribution Functions (PDFs) can be employed, specifically, we can use expected values of a mixture of uniform probability distributions (UPDs).

The primary objective of our research is to mathematically formulate Uncertainty Cost Functions (UCFs), derived from a mixture of uniform probability distributions (UPDs). Through this approach, we can define and compute the analytical cost functions associated with photovoltaic power generation (PVEG) and wind power generation (WEG). Furthermore, this cost framework seamlessly extends to the integration of plug-in electric vehicles or hybrid electric vehicles (PEV/HEV). These analytical functions undergo rigorous validation via stochastic simulations of the UPD mixture, ensuring the robustness and reliability of our findings.

The variables of uncertainty associated with economic dispatch in power systems become notably complex with the integration of renewable energy resources like PVEG, WEG, and PEV/HEV.

Evaluating the stochastic characteristics of these resources and loads within power systems can yield uncertainty cost functions as well as marginal expressions. The analytical elaboration of these functions, along with the derivatives of marginal costs, is detailed in [2].

Uncertainty management is also integrated into probability-controlled optimal power flow, distinguishing it from traditional optimal power flow control by incorporating the scheduling of power generation based on state variables with predefined limits. In contingency situations, where a renewable energy source with high uncertainty cannot meet the planned energy demand, the energy flow should remain unaffected, representing a preventive perspective. However, from a corrective standpoint, adjustments in power distribution become necessary to maintain the operating system within acceptable limits post-event. In [3], a strategy programmed and implemented through the Matpower software is employed to provide a preventive solution to optimal energy flow constrained by contingencies. Consequently, these software-based strategies can offer solutions for both preventing and addressing contingencies for ensuring optimal energy flow while accommodating uncertainties.

UCFs are utilized to analyze the variability of solar energy, wind energy, and electric/hybrid vehicle resources, which can be effectively modeled using established probability cost functions, [1]. The stochastic effects of wind turbine speed, solar irradiation intensity, and drive knocks have been analyzed through the application of uniform cost functions (UCFs) in [4]. The novelty of this research lies in the analytical development of uncertainty cost functions and their deterministic verification based on the economic dispatch of power. Uncertainty Cost Functions derived from a mixture of uniform probability distributions (UPDs) are employed to validate the formulated analytical expected cost and penalty cost, [5]. Lastly, the expected value of the penalty cost can be determined based on the mean value of the available power histogram shown in Figure 1.

This research paper is structured into several sections. In Section 2, we introduce fundamental concepts regarding UCFs and UPDs. We explore the derivation of these functions from resulting histograms. Subsections within Section 2 show the mathematical aspects of uncertainty cost functions derived from a mixture of uniform probability distributions (UPDs). Through analytical development, we can determine the UCFs and estimate penalty costs associated with PVEG, WEG, and PEV/HEV. In Section 3, we present the validation and verification process of the analytically developed UCFs based on a mixture of uniform probability distributions. This validation is compared with Monte Carlo simulations to ensure accuracy and reliability. Finally, in Section 4, we summarize our findings and provide insights for future discussions and research directions.

2 Problem Formulation and Analytical Solution: Development of Uncertainty Cost Functions

There are several methods for function optimization including heuristic computational techniques like particle swarm optimization (PSO). Power flow optimization can also be done by injecting the reagents shunt capacitors or transformer taps, [6].

The PVEG, WEG and PEV/HEV resources and loads have uncertainty factors, so the uncertain costs are needed to integrate the injected variable power and its consumption. The variability of factors is based on the probability distribution of sources and loads, [7].

While analyzing microgrids along with renewable energy resources, patent research about scientific and technological developments can be important characteristics to publish scientific and professional technology papers. Especially, international patent classification can impart important and valuable information about the microgrids used in power systems to develop the analytical perspective, [8].

The cost of uncertainty of renewable energy sources and loads can be formulated in the form of uncertainty cost functions in the microgrids operations. Small hydropower plants in this context can be used in the distribution probability of the power plant. The analytical development for uncertainty cost functions of such microgrids can be formulated mathematically to underestimate/overestimate power availability. The validation in this regard is done by using Monte Carlo simulation process, [9].

In an islanded microgrid case, the inverters can surely provide droop control in frequency regulation and required power dispatch even based on reference values. The results of this control show the improvement in frequency regulation due to changes in networked microgrids' inertia, [10].

To optimize the tension profiles and reagents controlling power distribution, we can optimize the capacitors' location in the power system network. The exhaustive search technique is used to optimize it. In this technique, the dimensions are used to evaluate several possibilities to find its solution and algorithm computational iteration visualizations give the solution optimum, [11].

Constrained handling rules in decomposition methodology can be used to provide optimal power flow during iterative computation for a security-constrained problem. The stages for finding the optimal power flow solution are the bases case network and modification of potentially relevant contingencies by updating the constraint limits. The algorithm used is performing computations to find the results in such problems, [12].

Uncertainty of PVEG, WEG, and PEV/HEV generators/sources and loads in terms of economic dispatch is formulated to cost functions analytically. We can integrate these sources and loads to handle the uncertainty mentioned. These uncertainty cost functions for sources and loads can be verified and validated in two ways:

- At any instant, the available power can be verified in the form of a mixture of uniform probability distributions.
- The Uncertainty Cost Functions can be calculated with an analytical development (presented in subsection B), and they can be contrasted by Montecarlo simulations, we can use two and three uniform probability distributions.

2.1 Power Histogram Description with Two Uniform Distributions

To represent the available power of PVEG, WEG, and PEV/HEV, this research considers two uncertain representations: one with two uniform distributions and the other with three distributions. An example of the available power from renewables can be described by using the power histogram shown in Figure 1.

For the representation with two uniform distributions, the scheduled power (P_s) by the operator can be categorized into two regions as follows:

Case: A; Region I: P_s is less than b and bigger than a.

Case: B; Region II: P_s is less than c and bigger than b.

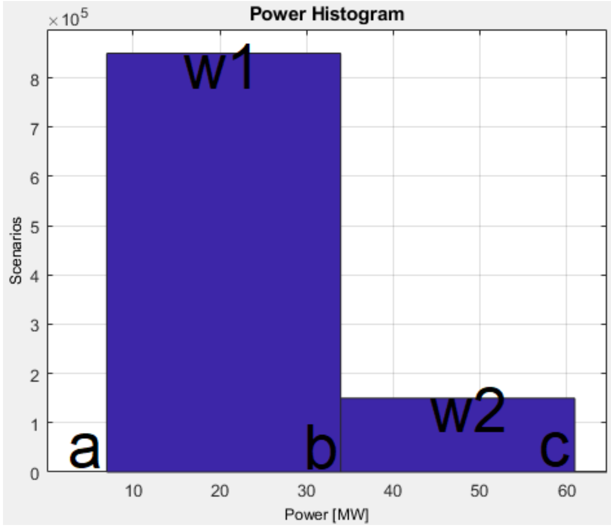

Fig. 1: Available power histogram (two uniform distributions)

2.2 Mathematical formulation of Uncertainty Cost Functions

The uncertainty cost function is a function in terms of the scheduled power, $f(P_s)$, coming from adding two parts.

In this way, there are two components in the uncertainty cost, the overestimation part ($UCF_{overstimation}$), where the scheduled power (P_s) is bigger than the available power (P), and the cost for having a difference between $P_s \wedge P$ represents the use of energy storage systems for storage the difference, valued with an overestimation constant (C_0) and the probability of $P < P_s$.

The second part, the underestimation part ($UCF_{understimation}$), where the scheduled power (P_s) is less than the available power (P), and the cost for having a difference between $P \wedge P_s$ represents the use of energy storage systems for inject to the network the difference, valued with an underestimation constant (C_u) and the probability of $P > P_s$.

The total uncertainty cost functions can have the following development cases based on the analytical development of a mixture of probability distributions (B1 and B2 subsections).

Case A, Region I:

If $P_s < b$

- **Step – 1**

$$UCF_{estimation} = \frac{w_1}{b-a}\int_a^{P_s} C_0(P_s - P)dP \qquad (Ao\text{-}1)$$

$$UCF_{underestimation} = \frac{w_1}{b-a}\int_{P_s}^{b} C_u(P - P_s)dP$$

$$+\frac{w_2}{c-b}\int_b^c C_u(P-P_s)dP \qquad (Au\text{-}1)$$

- **Step – 2**

$$UCF_{estimation} = \frac{w_1C_0}{b-a}\left[P_sP|_a^{P_s} - \frac{P^2}{2}\Big|_a^{P_s}\right] \qquad (Ao\text{-}2)$$

$$UCF_{underestimation} = \frac{w_1C_u}{b-a}\left[\frac{P^2}{2}\Big|_{P_s}^{b} - P_sP_{P_s}^{b}\right] + \frac{w_2C_u}{c-b}\left[\frac{P^2}{2}\Big|_b^c - P_sP_b^c\right] \qquad (Au\text{-}2)$$

- **Step – 3**

$$UCF_{estimation} = \frac{W_1C_0}{b-a}\left[P_s^2 - P_sa - \frac{P_s^2}{2} + \frac{a^2}{2}\right]$$
$$= \frac{W_1C_0}{b-a}\left[\frac{P_s^2}{2} - P_sa + \frac{a^2}{2}\right] \qquad (Ao\text{-}3)$$

$$UCF_{underestimation} == \frac{w_1C_u}{b-a}\left[\frac{b^2}{2} - \frac{P_s^2}{2} - P_sb + P_s^2\right] + \frac{w_2C_u}{c-b}\left[\frac{c^2}{2} - \frac{b^2}{2} - P_sc + P_sb\right]$$

$$= \frac{W_1C_0}{b-a}\left[\frac{P_s^2}{2} - P_sb + \frac{b^2}{2}\right] + w_2c_0\left[\frac{c+b}{2} - P_s\right] \qquad (Au\text{-}3)$$

Case B, Region II:

If $b < P_s < c$

- **Step – 1**

$$UCF_{estimation} = \frac{w_1}{b-a}\int_a^b C_0(P_s-P)dP$$
$$+\frac{w_2}{c-b}\int_b^{P_s} C_0(P_s-P)dP \qquad (Bo\text{-}1)$$

$$UCF_{underestimation} = \frac{w_2}{c-b}\int_{P_s}^c C_u(P-P_s)dP \quad (Bu\text{-}1)$$

- **Step – 2**

$$UCF_{estimation} = \frac{w_1C_0}{b-a}\left[P_sP|_a^b - \frac{P^2}{2}\Big|_a^b\right] +$$
$$\frac{w_2C_0}{c-b}\left[P_sP|_b^{P_s} - \frac{P^2}{2}\Big|_b^{P_s}\right] \qquad (Bo\text{-}2)$$

$$UCF_{underestimation} = \frac{w_2C_u}{c-b}\left[\frac{P^2}{2}\Big|_{P_s}^c - P_sP\Big|_{P_s}^c\right]$$
(Bu-2)

- **Step – 3**

$$UCF_{estimation} = \frac{w_1C_0}{b-a}\left[P_s(b-a) - \frac{b^2}{2} + \frac{a^2}{2}\right] +$$
$$\frac{w_2C_0}{c-b}\left[P_s^2 - P_sb - \frac{P_s^2}{2} + \frac{b^2}{2}\right]$$

$$= \frac{W_1C_0}{b-a}\left[P_s(b-a) + \frac{a^2-b^2}{2}\right] +$$
$$\frac{w_2C_0}{c-b}\left[\frac{P_s^2}{2} - P_sb + \frac{b^2}{2}\right]$$

$$= w_1c_0\left[P_s - \frac{(b+a)}{2}\right] + \frac{w_2C_0}{c-b}\left[\frac{P_s^2}{2} - P_sb + \frac{b^2}{2}\right] \quad (Bo\text{-}3)$$

$$UCF_{underestimation} = \frac{w_2C_u}{c-b}\left[\frac{c^2}{2} - \frac{P_s^2}{2} - P_sc + P_s^2\right] +$$
$$\frac{w_2C_u}{c-b}\left[\frac{P_s^2}{2} - P_sc + \frac{c^2}{2}\right] \quad (Bu\text{-}3)$$

3 Simulation and Validation for the Problem Solution

To evaluate the analytical uncertainty cost functions, we performed Montecarlo simulations to obtain available power values from a mixture of two UDP, representing the variability of a PVEG. For each Ps value, we computed both overestimation and underestimation costs, thereby deriving the uncertainty cost for each specific scenario. Subsequently, separate histograms illustrating the costs attributed to underestimation and overestimation are depicted.

The simulations were conducted to illustrate the two cases outlined in the mathematical formulation section. Figure 2 depicts the simulations for Ps=30MW, corresponding to Case A, Region I: P_s is less than b and bigger than a (P_s). Similarly, Figure 3 presents the results for Ps=70MW, corresponding to Case B, Region II: P_s is less than c and bigger than b.

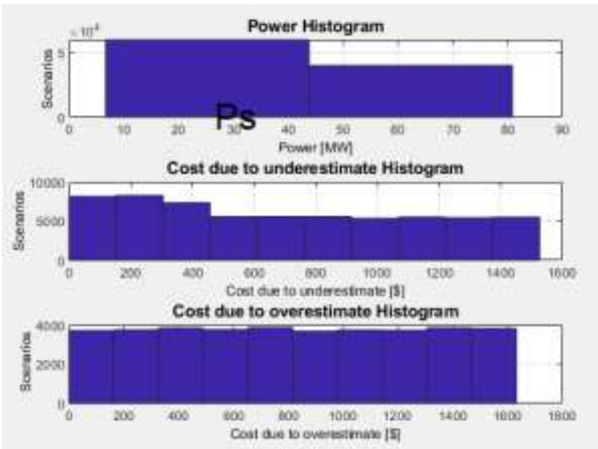

Fig. 2: Power histogram, costs histograms due to

underestimate and overestimate (P_S= 30 MW)

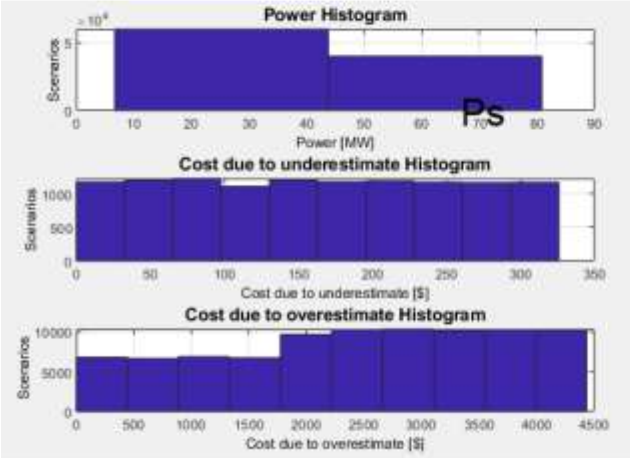

Fig. 3: Power histogram, costs histograms due to underestimate and overestimate (P_S= 70MW)

According to Figure 2, the uncertainty costs for underestimating the available power range between 0 and approximately 1550\$. There is a gradual accumulation of cases between 0 and 400\$. In this scenario, the expected costs for underestimating and overestimating the available power are very similar. In Figure 3, it is observed that the costs of underestimating the available power range between 0 and \$300. However, the costs of overestimating the available power reach values of 4500\$. A significant number of cases are concentrated at the highest cost values. In this instance, the expected costs of overestimating the available power are notably higher than the costs of underestimating.

In Figure 4, the Montecarlo run shows the histogram illustrating the uncertainty cost for all scenarios with P_s= 30MW. The expected cost, derived from these Montecarlo simulations, amounts to 740.9090\$. This value can be compared by employing the expressions outlined in Case A from the previous section:

$$UCF = UCF_{estimation} + UCF_{underestimation}$$

where *a*=6.6780 MW, *b*=43.7734 MW, *c*= 80.8688 MW,
w1= 0.6, *w2*= 0.4 and P_s= 30 MW. In this case, the analytical expressions yield \$741.7588, indicating an error of 0.11%.

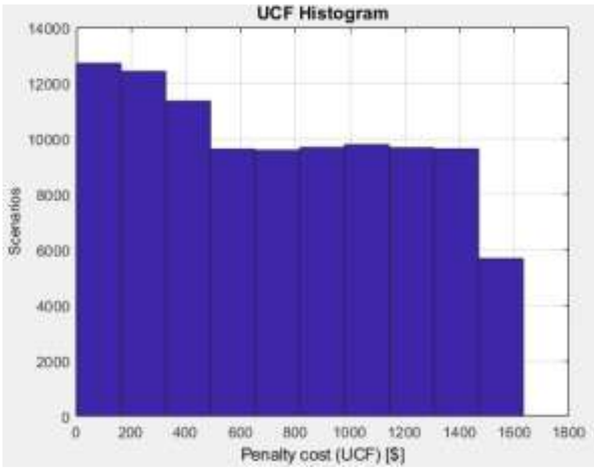

Fig. 4: UCF histogram (P_S= 30 MW).

In Figure 5, Montecarlo run shows the histogram for the uncertainty cost for the whole scenarios in case P_s= 70 MW, the expected cost of these Montecarlo simulations is 2.157 \$. This value can be contrasted by applying the expressions of case A of the previous section:

$$UCF = UCF_{overestimation} + UCF_{underestimation}$$

where *a*=6.6780 MW, *b*=43.7734 MW, *c*= 80.8688 MW, *w1*= 0.6, *w2*= 0.4 and P_s= 30 MW. In this case the analytical expressions give 2.159,2 \$, indicating an error of 0.11%.

Using the UCFs proposed in the previous section, we calculated the uncertainty costs for different values of P_s, generating the total cost function. The results for each value of P_s are detailed in Table 1 and were compared with the Montecarlo simulations. in Figure 6 we analyze the behavior of the cost function in function of the variable P_s.

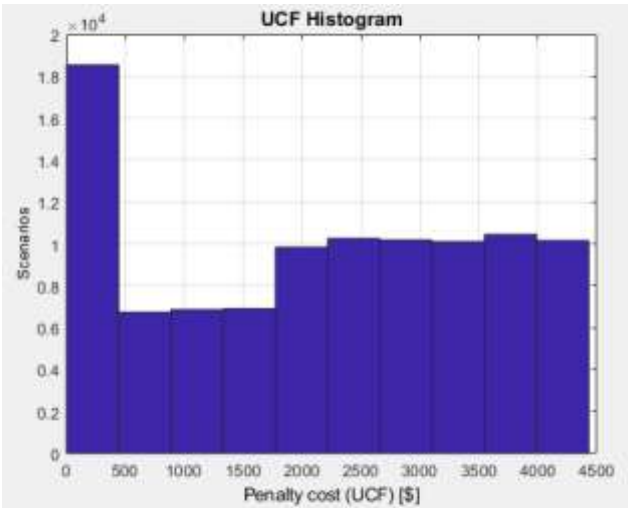

Fig. 5: UCF histogram (P_S= 70 MW)

Table 1. UCF data, Montecarlo, and analytical cases

Scheduled Power (MW)	Montecarlo Uncertainty Cost ($)	Analytical Uncertainty Cost ($)	error (%)
5	10535	1054,2	0.0715
10	910,4	910,9	0.0538
16	792,2	792,2	0.0029
20	745,5	745,4	0.0027
25	723,4	723,4	0.0066
28	728,7	729,6	0.1264
30	742,2	741,8	0.0519
36	817,4	817,2	0.0222
42	950,3	950,9	0.0595
46	1071	1071	0.0015
52	1285,5	1284,8	0.0534
56	1451	1448,9	0.1429
62	1725,6	1727,5	0.1082
66	1934,5	1934,7	0.0071
70	2158,9	2159,2	0.0108
76	2532,3	2528,3	0.1608

According to the results presented in Table 1, the cost values calculated from the proposed UCFs closely align with those generated through Monte Carlo simulations, which validates the efficacy of the proposed approach. An advantage of this approach lies in its ability to calculate costs analytically, which simplifies the calculation process in contrast to the Montecarlo simulation-based determination of uncertainty cost. Moreover, the proposed equations can provide detailed results concerning the costs related to overestimation and underestimation of available power.

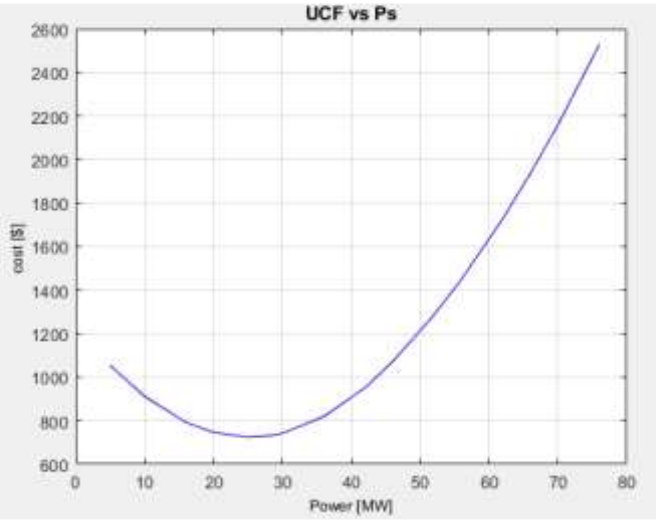

Fig. 6: UCF vs P_S

Figure 6 provides a comprehensive view of the expected value of uncertainty costs across different Ps values. This analysis reveals distinct trends in the behavior of the proposed cost function. For low Ps values, uncertainty costs remain low, as evidenced in Figure 2, where the expected values of underestimation and overestimation costs are comparable. Furthermore, a specific Ps value is identified where uncertainty costs reach their minimum. This value has significant potential for optimizing economic dispatch, enhancing power availability, and mitigating uncertainty costs. Conversely, as Ps increases, a discernible upward trend is observed in uncertainty costs, consistent with the findings in Figure 3, where the cost of overestimation notably surpasses that of underestimation. This analysis provides valuable insights for generating agents and electricity market operators, allowing them to reduce generation costs and optimize economic dispatch more efficiently.

4 Conclusion

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The economic dispatch of the energy scheduling of uncertain sources and loads (PVEG, WEG, and PEV/HEV) must include the tools and techniques to reduce the penalty costs connected with the scheduling. In this paper, an analytical development approach is presented which is better to use as a tool or technique. The mathematical formulation shows it as an optimization technique based on the stochastic economic dispatch technique. To schedule reliable power, the penalty costs can affect the supply of energy generating underestimate or overestimate of power availability by using such sources and loads. The uncertainty cost functions (UCFs) can be used mathematically to calculate the underestimated or overestimated costs of power generation making the power system more stable in electricity market. By using this research concept, simple uncertainty cost functions can be used to optimize the power flow because they have a quadratic shape in nature and consist of optimal solvers of power systems.

The analytically developed equations of uncertainty cost functions are based on parameters applied for several cases of PVEG, WEG, and PEV/HEV sources and loads. We can vary these parameters to optimize the economic dispatch of power based on a mixture of probability distributions and uncertainty cost functions to get the maximum power from the generators. The stochastic behavior of these resources and loads can be used to estimate accurately the uncertainty costs to optimize the power generation.

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Contribution of Individual Authors to the Creation of a Scientific Article

- Romero and Rehman carried out the simulation and the optimization.
- Rivera has implemented the Monte Carlo Algorithm.

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Conflict of Interest

The authors have no conflicts of interest to declare.